(61) $m = 10^{-30}\,kg$ $\quad \psi(x) = \sqrt{\frac{\alpha}{\sqrt{\pi}}}\, e^{-\frac{\alpha^2(x-a)^2}{2}}$

$k = 50\ \frac{eV}{nm^2}$

premaknjen HO

$a = 0{,}3\ nm$

$\alpha = \sqrt[4]{\frac{mk}{\hbar}}$

Lastne funkcije $\quad \hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} k \hat{x}^2 \,, \quad \alpha = \sqrt[4]{\frac{mk}{\hbar}}$

$$\psi_n = \sqrt{\frac{\alpha}{2^n n! \sqrt{\pi}}}\, H_n(y)\, e^{-y^2/2} \,, \quad y = \alpha x$$

Sedaj uporabimo splošno formulo za koeficiente

$$c_n = \int_{-\infty}^{\infty} \psi_n^* \psi \, dx$$

$$= \sqrt{\frac{\cancel{\alpha^2}}{2^n n! \sqrt{\pi}^2}} \int_{-\infty}^{\infty} H_n(y)\, e^{-y^2/2} \cdot e^{-(y-\overset{\alpha a}{\overset{\|}{y_0}})^2/2} \frac{dy}{\cancel{\alpha}}$$

$$= \frac{1}{\sqrt{2^n n!\, \pi}} \underbrace{\int_{-\infty}^{\infty} H_n(y)\, e^{-y^2 + y y_0 - y_0^2/2}\, dy}_{\sqrt{\pi}\, y_0^n\, e^{-y_0^2/4}}$$

$$c_n = \frac{1}{\sqrt{2^n n!}}\, y_0^n\, e^{-y_0^2/4}$$

$$\int |\psi|^2 dx = 1 \Rightarrow \sum_{n,m} \int \psi_n^* c_n^* \psi_m c_m \, dx = \sum_n |c_n|^2$$

- Preverimo: $\displaystyle\sum_{n=0}^{\infty} c_n^2 = \sum_{n=0}^{\infty} \frac{y_0^{2n}}{2^n n!} e^{-y_0^2/2} = e^{-y_0^2/2} \underbrace{\sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{y_0^2}{2}\right)^n}_{e^{y_0^2/2}} = 1$

· s temi koeficienti lahko dobimo $\langle E \rangle = \sum_{n=0}^{\infty} c_n^2 E_n$

za $n=0$ je 0.

$$\langle E \rangle = e^{-y_0^2/2}\, \hbar\omega \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{y_0^2}{2}\right)^n \left(n+\frac{1}{2}\right)$$

$$= \hbar\omega \left(\frac{1}{2} + e^{-y_0^2/2} \sum_{n=1}^{\infty} \frac{1}{\underbrace{(n-1)!}_{m=n-1}} \left(\frac{y_0^2}{2}\right)^{n-1} \frac{y_0^2}{2} \right)$$

$$= \frac{\hbar\omega}{2}\left(1+y_0^2\right), \quad \omega = \sqrt{\frac{k}{m}}$$

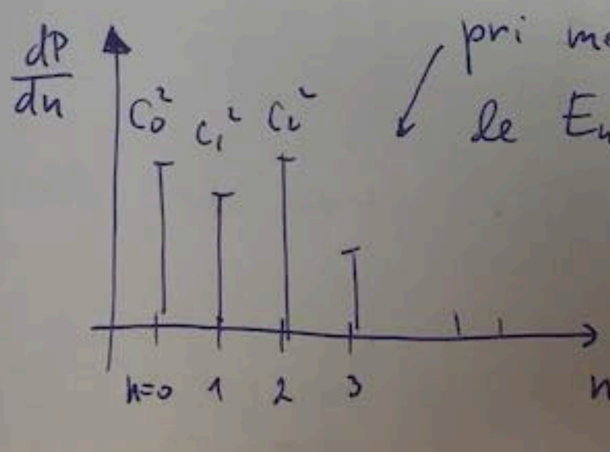
$$y_0 = a\alpha = a \sqrt[4]{\frac{mc^2 k}{(\hbar c)^2}} = a \sqrt[4]{\frac{10^{-30}\; 9\cdot 10^{16} \overset{6\cdot 10^{18}\,\mathrm{eV}}{\left(\frac{\mathrm{kg\,m^2}}{\mathrm{s^2}}\right)}\; 50\,\mathrm{eV}}{200^2\,\mathrm{eV^2\,nm^2}\;\mathrm{nm^2}}}$$

$$= a \sqrt[4]{\frac{5\cdot 10^{-22}\cdot \cancel{6}\cdot 3\cdot 10^{18}}{\cancel{4}\cdot 10^4\,\mathrm{nm^4}}} \sim 5\,\mathrm{nm^{-1}}\cdot a = \underline{1.55}$$

$$\Rightarrow \langle E \rangle = 3.2\,\mathrm{eV}$$

$$P\left(E = \frac{3}{2}\hbar\sqrt{\frac{k}{m}}\right) = |c_1|^2 = \left(\frac{1}{\sqrt{2}}\, y_0 e^{-y_0^2/4}\right)^2 = \frac{y_0^2}{2} e^{-y_0^2/2}$$

$$n + \frac{1}{2} \Rightarrow n = 1 \qquad\qquad = \underline{\underline{0{,}36}}$$



pri meritvi dobimo / izmerimo vedno le E_n = lastne energije z verjetnostjo $|c_n|^2$. V povprečju dobimo seveda $\sum_n |c_n|^2 E_n = \langle E \rangle$.

<u>DODATEK</u>: $\langle E \rangle$ z direktnim računom: ker smo imeli podano $\psi(x)$ lahko vstavimo v H in izračunamo $\langle E \rangle$ z direktno integracijo

$$\langle E \rangle = \int_{-\infty}^{\infty} \left(\frac{-\hbar^2}{2m} \psi \psi'' + \frac{1}{2} \psi x^2 \psi \right) dx$$

$$\psi = \sqrt{\frac{\alpha}{\sqrt{\pi}}}\, e^{-\alpha^2 (x-a)^2/2}, \quad \psi' = \sqrt{\frac{\alpha}{\sqrt{\pi}}} \left(-\alpha^2 (x-a)\right) e^{-\alpha^2 (x-a)^2/2}$$

$$\psi'' = \sqrt{\frac{\alpha}{\sqrt{\pi}}}\, e^{-\alpha^2 (x-a)^2/2} \left(-\alpha^2 - \alpha^2 (x-a) \left(-\alpha^2 (x-a)\right)\right)$$

$$\langle E \rangle = \frac{\alpha}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\alpha^2 (x-a)^2} \left(\frac{\hbar^2 \alpha^2}{2m} \left(1 - \alpha^2 (x-a)^2\right) + \frac{km}{2m} x^2 \right) dx$$

$$\alpha (x-a) = t, \quad \alpha\, dx = dt$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} \frac{(\hbar \alpha)^2}{2m} \left(1 - t^2 + \alpha^2 \left(\frac{t}{\alpha} + a \right)^2 \right)$$

* kvadratični člen se pokrajša, linearni je lih in $\int_{-\infty}^{\infty} \to 0$

Dobimo: $$\langle E \rangle = \frac{(\hbar \alpha)^2}{2m} \frac{1}{\sqrt{\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-t^2} dt}_{\sqrt{\pi}} \left(1 + (\alpha a)^2 \right)$$

$$= \frac{(\hbar \alpha)^2}{2m} \left(1 + y_0^2 \right).$$ ENAKO KOT PREJ.

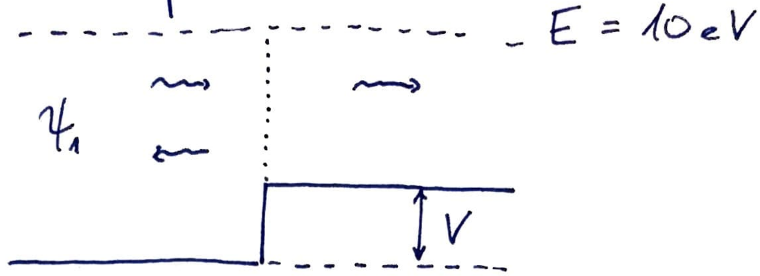
$$\iint e^{-x^2+y^2} dx\,dy = \int_0^{2\pi} \int_0^{\infty} e^{-r^2} r\,dr\,d\varphi = 2\pi \frac{1}{2} \int_0^{\infty} e^{-u} du = \pi \left. -e^{-u} \right|_0^{\infty} = \pi = \left(\int_{-\infty}^{\infty} e^{-t^2} dt \right)^2$$

$$\Rightarrow \int_{-\infty}^{\infty} e^{-t^2} dt = \frac{1}{\sqrt{\pi}}$$

(23) Sipanje na potencialni plasti

$T = 10\,eV < m_e c^2$

$V_0 = 3\,eV$



$\dfrac{j_{odb}}{j_{vp}} = ?$ $\qquad j = \dfrac{\hbar}{2mi}\left(\psi^* \dfrac{d\psi}{dx} - \dfrac{d\psi^*}{dx}\psi\right)$

Ravni val : $\hat{H} = \dfrac{\hat{p}^2}{2m} + V$, $E_k = \dfrac{(\hbar k)^2}{2m}$

$$\psi_1 = A e^{ikx} + B e^{-ikx} \quad , \; k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_2 = C e^{ik'x} \quad , \; k' = \frac{\sqrt{2m(E-V)}}{\hbar}$$

$$\Rightarrow \; j_{vp} = \frac{\hbar k}{m}|A|^2, \quad j_{odb} = \frac{\hbar k}{m}|B|^2, \quad j_{prep} = \frac{\hbar k'}{m}|C|^2$$

$$\Rightarrow \; \frac{j_{odb}}{j_{vp}} = \frac{|B|^2}{|A|^2}$$

Robni pogoj :

$$\psi_1(0) = \psi_2(0) \Rightarrow A + B = C$$

$$\psi_1'(0) = \psi_2'(0) \Rightarrow \cancel{i}k(A-B) = \cancel{i}k'C \quad /:k'$$

Zdelimo in dobimo $\quad A + B = (A-B)\dfrac{k}{k'} \quad /:A$

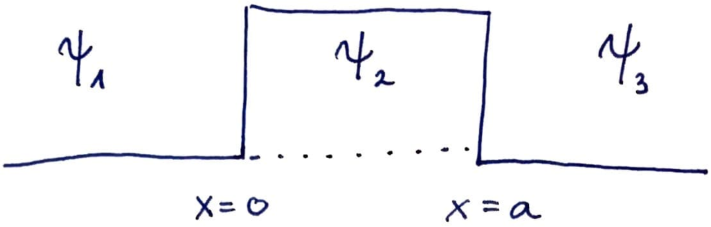
$$1 + \frac{B}{A} = \left(1 - \frac{B}{A}\right)\left(\frac{k}{k'}\right) \Rightarrow \frac{B}{A}\left(1 + \frac{k}{k'}\right) = \frac{k}{k'} - 1$$

$$\Rightarrow \frac{B}{A} = \frac{k-k'}{k+k'} \; ; \quad R = \frac{B^2}{A^2} = \left(\frac{\sqrt{E} - \sqrt{E-V}}{\sqrt{E} + \sqrt{E-V}}\right)^2 = \underline{8 \cdot 10^{-3}}.$$

(27) $a = 0{,}41\,\mathrm{nm}$

$E = 0{,}7\,\mathrm{eV}$

$V = ?$



$$\psi_1 = A e^{ikx} + B e^{-ikx}$$

$$\psi_2 = C e^{ik'x} + D e^{-ik'x}$$

$$T = \frac{|E|^2}{|A|^2} \qquad \psi_3 = E e^{ikx}$$

Robni pogoj:

$$A + B = C + D$$

$$k(A-B) = k'(C-D) \quad /:k \quad)+$$

$$C e^{ik'a} + D e^{-ik'a} = E e^{ika} \quad /:e^{ik'a} \quad)\pm$$

$$k'(C e^{ik'a} - D e^{-ik'a}) = k E e^{ika} \quad /:k'\, e^{-ik'a}$$

$$\cancel{2} \underline{C}\, \cancel{e^{i(k'-k)a}} = E\left(1 + \frac{k}{k'}\right)\frac{1}{2} e^{-i(k'-k)a}$$

$$\cancel{2} \underline{D}\, \cancel{e^{-i(k'+k)a}} = E\left(1 - \frac{k}{k'}\right)\frac{1}{2} e^{i(k'+k)a}$$

$$2A = C\left(1 + \frac{k'}{k}\right) + D\left(1 - \frac{k'}{k}\right) =$$

$$= E(\ldots)$$

$$\Rightarrow \quad \frac{E}{A} = \frac{4kk' e^{i(k'-k)a}}{(k+k')^2 - (k-k')^2 e^{2ik'a}} \Rightarrow T = \frac{1}{1 + \left(\frac{k'^2 - k^2}{2kk'}\right)^2 \sin^2 k'a}$$

• Dobili smo transmisivnost kot $f(E,V)$

$$T = \frac{1}{1+\left(\frac{k'^2 \mp k^2}{2kk'}\right)^2 \sin^2 k'a}$$

max = 1 ko je $\sin^2 k'a = 0$

torej je $k'a = n\pi$

$$k'a = \sqrt{\frac{2m(E+V)}{\hbar^2}}\, a = n\pi \quad /^2$$

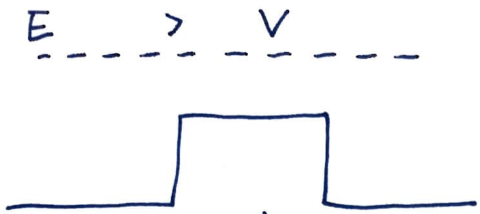
$$E+V = \left(\frac{n\pi\hbar}{a}\right)^2 \frac{1}{2m}$$

$$V = \frac{\pi^2(\hbar c)^2}{2mc^2 a^2} - E$$

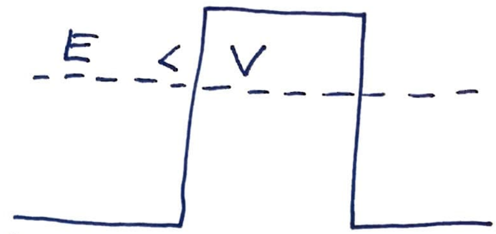
$$\cong \frac{10 \cdot 4 \cdot 10^4\, eV^{\cancel{2}}\, \cancel{nm^2}}{10^6\, \cancel{eV} \cdot (0{,}41\, \cancel{nm})^2} - 0{,}7\,eV$$

$$= (2.23 - 0{,}7)\,eV = \underline{\underline{1{,}53\ eV}}$$

(28) Tuneliranje skozi plast



$\Rightarrow$



$k = \sqrt{2mE}/\hbar$, $k' = \sqrt{2m(E-V)}/\hbar$ $\qquad$ k, $iK = i\sqrt{2m(V-E)}/\hbar = k'$

$$T = \frac{1}{1 + \left(\frac{k^2 - k'^2}{2kk'}\right)^2 \sin^2 k'a} \Rightarrow T = \frac{1}{1 + \left(\frac{k^2 + K^2}{2kK}\right)\frac{\cancel{i^2}}{\cancel{i^2}} \operatorname{sh}^2 Ka}$$

$k = 16{,}2\ \mathrm{nm}^{-1}$, $K = 7{,}3\ \mathrm{nm}^{-1}$, $a = \{0{,}1,\ 1,\ 100\}\ \mathrm{nm}$

$$\Rightarrow \quad T = \{0{,}47,\ 1{,}1 \cdot 10^{-6},\ 9{,}7 \cdot 10^{-631}\}$$

$$Ka \gg 1 \quad \Rightarrow \quad \operatorname{sh} Ka \sim \frac{e^{Ka}}{2} \ \& \ T \propto e^{-2Ka}$$