KINETIČNA ENERGIJA IN GIBALNA KOLIČINA

KLASIČNO vs. RELATIVISTIČNO

$\vec{p}_k = m\vec{v}$ $\qquad$ $cp^\mu = (E, c\vec{p}) = c(\gamma mc, \gamma m v)$

$W_k = \frac{1}{2} m \vec{v}^2$

- Limite in kinetična energija T

* samo v x-smeri: $cp^\mu = \gamma mc^2(1, \beta)$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

- Ne-relativistična limita : $\beta \ll 1$ ali $\beta \sim 0$, $\gamma \simeq 1 + \frac{1}{2}\beta^2$,

$\Downarrow$

$$cp^\mu \simeq mc^2\left(1+\frac{1}{2}\beta^2\right)(1, \beta) \simeq (mc^2 + \frac{1}{2}mv^2, cmv)$$

mirovna energija $\qquad$ kinetična energija

$cp^\mu(\beta=0) = (mc^2, 0)$, možno le za masivne delce.

- Ultra-relativistična limita

$cp^\mu \simeq \gamma mc^2(1,1)$ oz za $m=0$ $\quad cp^\mu = cp(1,1) = E(1,1)$

POLNA ENERGIJA : $E = \gamma mc^2$, MIROVNA ENERGIJA : $E_0 = mc^2$.

KINETIČNA : $T = E - mc^2 \simeq \begin{cases} \cancel{mc^2} + \frac{1}{2}mv^2 - \cancel{mc^2} \sim W_k, \\ E \quad , \quad \beta \gg 1. \end{cases}$

• Gibalna količina je četverec in se tako transformira

$$cp^{\mu'} = \Lambda^{\mu}{}_{\nu}\, cp^{\nu} \qquad cp^{\mu} = (E, cp_x, cp_y, cp_z)$$

$$cp^{0'} = E' = \gamma\,(cp^0 - \beta cp^1) = \gamma\,(E - \beta cp_x)$$

$$cp^{1'} = \gamma\,(cp^1 - \beta cp^0) = \gamma\,(cp_x - \beta E)$$

$$cp^{2'} = cp^2 \quad ; \quad p_y' = p_y\,, \quad p_z' = p_z$$

• Iz teh enačb dobimo Dopplerjev zakon za foton $m_\gamma = 0$

$cp^{\mu}_{\gamma} = E_\gamma (1,1)$ v <u>x - smeri</u>, se oddaljuje

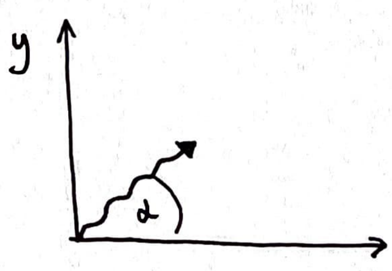
$$\underbrace{E_\gamma'}_{=\,h\nu'} = \gamma\,(E_\gamma - \beta E_\gamma) = E_\gamma \frac{1-\beta}{\sqrt{1-\beta^2}} = \underbrace{E_\gamma}_{=\,h\nu} \sqrt{\frac{1-\beta}{1+\beta}}$$

oziroma : $\nu' = \nu \sqrt{\dfrac{1-\beta}{1+\beta}}$; približuje $_{\beta \to -\beta}$: $\nu' = \nu \sqrt{\dfrac{1+\beta}{1-\beta}}$.

Doppler <u>pod kotom</u>

foton ostaja brezmasen

S|



$$cp^{\mu} = \underbrace{E_\nu}_{=\,h\nu} (1, \cos\alpha, \sin\alpha, 0), \quad p^{\mu}p_{\mu} = 0$$

$$S'|\quad cp'^{\mu} = \underbrace{E_\gamma'}_{=\,h\nu'} (1, \cos\alpha', \sin\alpha', 0), \quad p'^{\mu}p'_{\mu} = 0$$

LT : 0 : $\nu' = \nu\gamma\,(1 - \beta\cos\alpha)$

x : $\nu' \cos\alpha' = \nu\gamma\,(\cos\alpha - \beta)$

y : $\nu' \sin\alpha' = \nu \sin\alpha$

- V RELATIVISTIČNI sliki dobimo Dopplerja tudi v y-smeri, ker se čas, ot $\cancel{0}$-ta komponenta spremeni:

$\boxed{\alpha = 0}$

$$\left.\begin{array}{l} 0 : \\ x : \end{array}\right\} \nu' = \nu\gamma(1-\beta)$$

$$y : \alpha = \alpha' = 0$$

Isto kot prej.

$\boxed{\alpha = 90^\circ}$

$$0 : \nu' = \nu\gamma$$

$$x : \cos\alpha' = -\beta$$

$$y : \sin\alpha' = \frac{1}{\gamma}$$

Transverzalni Doppler #1

$\boxed{\alpha' = 90^\circ}$ Uporabimo obratno LT

$$0 : \nu = \gamma\nu'(1 + \beta\underbrace{\cos\alpha'}_{=0}) = \underline{\underline{\gamma\nu'}}$$

$$x : \nu\cos\alpha = \gamma\nu'(\cos\alpha' + \beta), \quad \cos\alpha = \beta$$

$$y : \nu\sin\alpha = \nu'\sin\alpha', \quad \sin\alpha' = 1, \quad \sin\alpha = \frac{1}{\gamma}$$

Transverzalni Doppler #2

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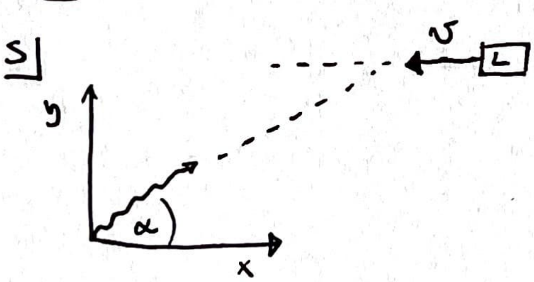
(62) LADJA SE PRIBLIŽUJE POD KOTOM



$$\beta = 0{,}6 \longrightarrow \gamma = \frac{1}{\sqrt{1-0{,}36}} = \frac{1}{0{,}8} = \frac{5}{4} = 1{,}25$$

$$\alpha = 30^\circ$$

$$\nu = 100\ \mathrm{MHz}$$

$$\nu' = \nu\gamma(1 + \beta\cos\alpha) = \nu\,\frac{5}{4}\left(1 + \frac{3}{5}\frac{\sqrt{3}}{2}\right) = \frac{10 + 3\sqrt{3}}{8}\,\nu \sim 1.9\,\nu = 190\ \mathrm{MHz}$$

$$\mathrm{tg}\,\alpha' = \frac{\sin\alpha}{\gamma(\cos\alpha + \beta)} = \frac{1/2}{\frac{5}{4}\left(\frac{\sqrt{3}}{2} + \frac{3}{5}\right)} = 0{,}27 \Rightarrow \alpha' = 15{,}3^\circ$$

(17) $\frac{T}{W_k} = R = (1{,}01\ ,\ 1{,}1\ ,\ 5)$

$$T = E - mc^2 = mc^2(\gamma - 1)\ ,\quad W_k = \frac{1}{2}mc^2\beta^2$$

$$\frac{T}{W_k} = \frac{\gamma - 1}{\frac{1}{2}\beta^2} = R\ ,\quad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{2}\beta^2 R + 1\ ,\quad \beta^2 = x$$

$$\frac{1}{1-x} = \left(\frac{1}{2}xR + 1\right)^2 = \frac{R^2x^2}{4} + Rx + 1$$

$$\cancel{1} = -\frac{R^2x^2}{4} - Rx + \cancel{1} + x + Rx^2 + \frac{R^2x^3}{4} \qquad / : -x$$

$$\frac{R^2}{4}x^2 + \frac{R}{\cancel{4}}\left(\overset{1}{\cancel{4}} - \frac{R}{\cancel{4}}\right)x + 1 - R = 0$$

$$x = \frac{4}{2R^2}\left(R\left(\frac{R}{4}-1\right) + \sqrt{R^2\left(\frac{R}{4}-1\right)^2 - \cancel{4}(1-R)\frac{R^2}{\cancel{4}}}\right)$$

$$= \frac{1}{2R}\left(R - 4 + \sqrt{(R-4)^2 - 16(1-R)}\right) = \frac{R-4+\sqrt{R^2+8R}}{2R} = x$$

$$\Rightarrow \quad \beta = \sqrt{\frac{R-4+\sqrt{R(R+8)}}{2R}} = (0{,}115\ ,\ 0{,}35\ ,\ 0{,}95)$$

LIMITE ZA R in β

$$R = \frac{\gamma - 1}{\frac{1}{2}\beta^2} = \begin{cases} \dfrac{\cancel{1} + \frac{1}{2}\beta^2 - \cancel{1}}{\frac{1}{2}\beta^2} \xrightarrow{\beta \to 0} 1\ , & \beta \xrightarrow{R \to 1} \sqrt{\dfrac{1-4+\sqrt{9}}{2}} = 0\ , \checkmark \\ 2\gamma \xrightarrow{\beta \to 1} 2\gamma \gg 1\ , & \beta \xrightarrow{R \gg 1} \sqrt{\dfrac{2R}{2R}} = 1\ . \checkmark \end{cases}$$

INVARIANTE

$SO(3) \rightarrow$ Minkowski, τ in m

$\vec{r} = (x, y, z)$ $\quad \vec{r}^T \vec{r} = x^2 + y^2 + z^2$ INVARIANTA na rotacije v 3D

$$x^\mu = (ct, x, y, z), \quad x^\mu x_\mu = (ct)^2 - x^2 - y^2 - z^2 \quad \text{je invarianta na LT}$$

$$= (c\tau)^2$$

lastni čas

$$cp^\mu = (E, cp_x, cp_y, cp_z), \quad cp^\mu cp_\mu = E^2 - c^2(p_x^2 + p_y^2 + p_z^2)$$

Vsi opazovalci v različnih S opazijo / izmerijo iste invariante τ in $p^\mu p_\mu$. Poglejmo kaj je $p^2 = p^\mu p_\mu$

$$cp^\mu = \gamma mc^2 (1, \beta) \Rightarrow (cp)^2 = (\gamma mc^2)^2 \underbrace{(1 - \beta^2)}_{1/\gamma^2} = m^2 c^4$$

(MIROVNA MASA $\times c^2)^2$

18 PROTON $\quad m_p \sim \mathrm{GeV}/c^2$, $\quad cp = 0{,}8\ \mathrm{GeV}$, $\quad T = ?$

$$T = E - mc^2 = mc^2(\gamma - 1), \quad cp = \gamma mc^2 \beta$$

VELJA: $(\gamma\beta)^2 = \frac{\beta^2}{1-\beta^2} = \gamma^2 - 1 = \frac{1}{1-\beta^2} - 1 = \frac{\cancel{1-1} + \beta^2}{1-\beta^2}$

$$\Rightarrow \gamma = \sqrt{\left(\frac{cp}{mc^2}\right)^2 + 1} \quad \text{oz.: } T = \left(\sqrt{1 + \left(\frac{cp}{mc^2}\right)^2} - 1\right) mc^2$$

$$= \left(\sqrt{1 + (0{,}8)^2} - 1\right) \mathrm{GeV} = 294\ \mathrm{MeV}.$$

• Zveza MED p in T si najlažje zapomnimo oz. izpeljemo z invarianto in definicijo E

$$E^2 - (cp)^2 = m^2c^4 \quad , \quad T = E - mc^2$$

$$E = m^2c^2\sqrt{1+\left(\frac{cp}{mc^2}\right)^2} \quad = mc^2\left(\sqrt{1+\left(\frac{cp}{mc^2}\right)^2} - 1\right).$$

(22) FOTONSKA RAKETA, pomen ohranitve gibalne količine

$$\beta = 0 \quad \Rightarrow \quad m \to \frac{m}{2} \quad , \quad \beta = ?$$

$$\boxed{m}\ \ v = 0 \quad \Rightarrow \quad \underset{\frac{m}{2}}{\boxed{}} \xrightarrow{v}$$

$$cp^\mu = (mc^2, 0) \quad \Rightarrow \quad cp'^\mu = \gamma\left(\frac{m}{2}c^2\right)(1, \beta) \ , \quad cp_\gamma = (E_\gamma, -E_\gamma)$$

OHRANITEV: $\quad cp^\mu = cp'^\mu + cp_\gamma$

$$0: \quad mc^2 = \gamma\frac{m}{2}c^2 + E_\gamma$$

$$x: \quad 0 = \gamma\frac{m}{2}c^2\beta - E_\gamma \quad \Big| + \ , \ : \frac{mc^2}{2} \ , \quad E_\gamma = \gamma\beta\frac{1}{2}mc^2$$

$$\cancel{mc^2} = \gamma\frac{\cancel{mc^2}}{2}(1+\beta) \quad \Rightarrow \quad 2 = \gamma(1+\beta) = \sqrt{\frac{1+\beta}{1-\beta}}$$

oz.: $\quad 4(1-\beta) = 1+\beta \quad \Rightarrow \quad 3 = 5\beta \ , \quad \underline{\beta = \frac{3}{5}}.$