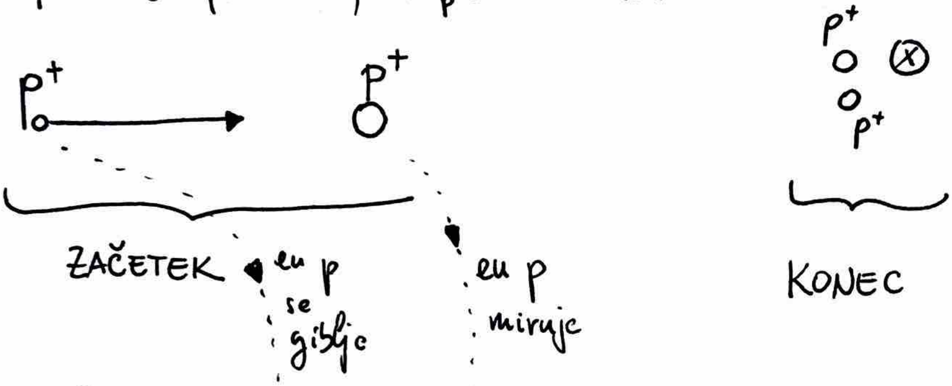
Ohranitev relativistične gibalne količine

$$\sum_{\text{začetek}} p^\mu = \sum_{\text{konec}} p^\mu \quad \text{velja za vsak indeks posebej}$$

(34) $T = 6\ \text{GeV}$, $m_p c^2 \simeq 1\ \text{GeV}$



S] : $c p_z^\mu = (\gamma mc^2 + mc^2,\ \gamma\beta mc^2 + 0)$ in $T = (\gamma - 1) \times mc^2 \Rightarrow \gamma = \frac{T}{mc^2} + 1$

S'] : $c p'^\mu_k = (2mc^2 + E_x,\ 0)$

to je v težiščnem sistemu S', kjer $\sum \vec{p} = 0$ in je največ energije na razpolago E_x za nastanek novih delcev v dogodkih, ko p^+ mirujeta.

Rekli smo, da je $p^\mu p_\mu$ invarianta v S in S'.

$$(c p_z^\mu)(c p_{z\mu}) = m^2c^4\left((\gamma+1)^2 - \underbrace{\gamma^2\beta^2}_{\gamma^2-1}\right) = m^2c^4\left(\cancel{\gamma^2} + 2\gamma + 1 - \cancel{\gamma^2} + 1\right)$$

$$= 2m^2c^4(\gamma + 1) = 2m^2c^4\left(\frac{T}{mc^2} + 2\right)$$

$$c p_k^\mu\, c p_{k\mu} = (2mc^2 + E_x)^2$$

$$\frac{E_x}{T} = \frac{\cancel{2}\,1}{\cancel{6}\,3} \simeq 33\%$$

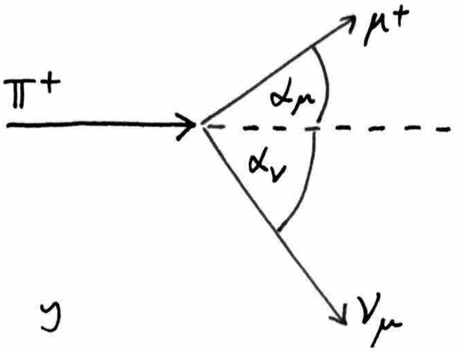
$$\Rightarrow E_x = 2mc^2\left(\sqrt{\frac{T}{2mc^2} + 1} - 1\right) \simeq 2\,\text{GeV}\left(\underbrace{\sqrt{\frac{6}{2} + 1} - 1}_{2-1=1}\right) \simeq 2\,\text{GeV}.$$

V prejšnji nalogi smo delali s skalarno invarianto, sedaj bomo reševali po posameznih komponentah.

(55) $T_{\pi^+} = 100\,\mathrm{MeV}$ Razpad: $\pi^+ \to \mu^+ + \nu_\mu$

$T_{\mu^+} = 80\,\mathrm{MeV}$

$E_\nu = ?$



$m_\mu c^2 \cong 105\,\mathrm{MeV}$,

$m_{\pi^+} c^2 \cong 140\,\mathrm{MeV}$,

$m_\nu c^2 \simeq 0$.

$$\begin{array}{llll} & 0 & x & y \\ \text{ZAČETEK}: \; cp_z^\mu = & (E_\pi, & cp_\pi, & 0), \end{array}$$

$$\text{KONEC}: \; cp_k^\mu = \left(E_\mu + E_\nu,\; cp_\mu \cos\alpha_\mu + \overset{E_\nu}{\overset{\parallel}{\widetilde{cp_\nu}}} \cos\alpha_\nu,\; cp_\mu \sin\alpha_\mu - \overset{E_\nu}{\widetilde{cp_\nu}} \sin\alpha_\nu\right)$$

0-ta komponenta $\mu = 0$: $T_\pi + m_\pi c^2 = T_\mu + m_\mu c^2 + E_\nu$,

Od tod sledi: $E_\nu = T_\pi - T_\mu + (m_\pi - m_\mu)c^2 = (20 + 35)\,\mathrm{MeV}$,

$$E_\nu = 55\,\mathrm{MeV}.$$

Za π^+ in μ^+ vemo kolikšni sta njuni gibalni količini:

$$E_\pi^2 - cp_\pi^2 = m_\pi^2 c^4 \Rightarrow \quad cp_\pi = \sqrt{E_\pi^2 - m_\pi^2 c^4} = 195\,\mathrm{MeV},$$

$$cp_\mu = \sqrt{E_\mu^2 - m_\mu^2 c^4} = 152\,\mathrm{MeV}.$$

Sedaj pogledamo še x in y komponenti ter se poskušamo znebiti α_ν. ↗

$$
\left.\begin{aligned}
x:&\quad cp_\pi - cp_\mu \cos\alpha_\mu = E_\nu \cos\alpha_\nu \,, \quad |^2 \\
y:&\quad cp_\mu \sin\alpha_\mu = E_\nu \sin\alpha_\nu \,. \quad |^2
\end{aligned}\right\} +
$$

$$\Rightarrow \quad c^2\left(p_\pi^2 - 2 p_\pi p_\mu \cos\alpha_\mu + p_\mu^2\right) = E_\nu^2$$

$$\cos\alpha_\mu = \frac{c^2(p_\pi^2 + p_\mu^2) - E_\nu^2}{2\, cp_\pi\, cp_\mu} \quad ; \text{ poznamo } \overset{55\,\mathrm{MeV}}{\overset{\shortparallel}{E_\nu}},\ \overset{195\,\mathrm{MeV}}{\overset{\shortparallel}{cp_\pi}},\ \overset{152\,\mathrm{MeV}}{\overset{\shortparallel}{cp_\mu}}$$

in dobimo: $\alpha_\mu = 11{,}1°$.

Vstavimo nazaj v y: $\sin\alpha_\nu = \sin\alpha_\mu \left(\frac{cp_\mu}{E_\nu}\right)$

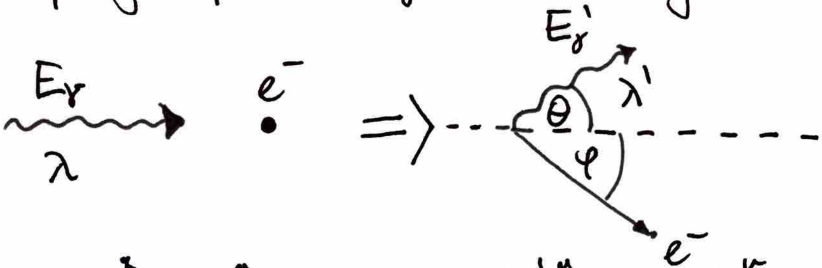
in dobimo: $\alpha_\nu = 33{,}3°$.

To je skladno z intuicijo, da se brezmasni nevtrino odkloni za večji kot $\alpha_\nu > \alpha_\mu$ kot mion μ^+.

Fotone prav tako obravnavamo kot brezmasne delce (kot zgoraj nevtrino), ki imajo svojo gibalno količino p_γ^μ. Za njih velja: $p_\gamma^\mu p_{\gamma\mu} = 0 = m_\gamma^2 c^2$.

Če jim pripišemo: $E_\gamma = cp_\gamma = h\nu = \frac{hc}{\lambda}$, potem lahko iz kinematike izpeljemo Comptonov pojav.

(35) Sipanje fotona γ na mirujočem elektronu e^- (Compton).



vemo: $E_\gamma \gg m_e c^2$

nas zanima: $E_{e\,max} = ?$

$$p_\gamma^\mu + p_e^\mu = p_\gamma'^\mu + p_e'^\mu$$

Ponovno zapišemo četverec p^μ po komponentah:

nas ne zanima

$$0: \quad E_\gamma + mc^2 = E_\gamma' + E_e$$

$$x: \quad E_\gamma = E_\gamma' \cos\theta + c p_e \cos\varphi$$

$$y: \quad 0 = E_\gamma' \sin\theta - c p_e \sin\varphi$$

kvadriramo, da se znebimo φ

$$(E_\gamma - E_\gamma' \cos\theta)^2 + E_\gamma'^2 \sin^2\theta = (cp_e)^2 = E_e^2 - m_e^2 c^4$$

vstavimo iz 0-te komp.

$$E_\gamma^2 - 2E_\gamma E_\gamma' \cos\theta + E_\gamma'^2 = (E_\gamma - E_\gamma' + mc^2)^2 - m^2c^4$$

$$= E_\gamma^2 - 2E_\gamma E_\gamma' + E_\gamma'^2 + 2(E_\gamma - E_\gamma')mc^2 + \cancel{0}$$

Ostane nam: $-\cancel{2}E_\gamma E_\gamma' \cos\theta = -\cancel{2}E_\gamma E_\gamma' + \cancel{2}(E_\gamma - E_\gamma')mc^2$,

kar prepišemo z $E_\gamma = \frac{hc}{\lambda}$, $E_\gamma' = \frac{hc}{\lambda'}$ in $\lambda_c = \frac{hc}{mc^2}$ v:

$$\frac{(hc)^{\cancel{2}}}{\cancel{\lambda\lambda'}}(1-\cos\theta) = \frac{\cancel{hc}\, mc^2}{\cancel{\lambda\lambda'}}(\lambda' - \lambda) \quad \text{oz.:} \quad \underline{\lambda' - \lambda = \lambda_c(1-\cos\theta)}.$$

To je znan izraz za Comptonov pojav. Sedaj nas zanima, kdaj bo E_e ali T_e največji. Iz 0-te komp.: ↗

vidimo $E_e^{max} \Rightarrow E_\gamma^{'min}$ oz. $\lambda^{'max}$.

$$\lambda' = \lambda + \lambda_c \underbrace{(1-\cos\Theta)}$$

max 2, ko $\cos\Theta = -1$, oz. $\Theta = \pi$.

Takrat bo: $\lambda^{'max} = \lambda + 2\lambda_c$, oz. $E_\gamma^{'min} = \frac{hc}{\lambda^{'max}}$

$$E_\gamma^{'min} = \frac{hc}{\lambda + 2\lambda_c} = \frac{1}{\underbrace{\frac{1}{E_\gamma} + \frac{2}{m_0c^2}}} \simeq \frac{mc^2}{2}$$

$E_\gamma \gg mc^2 \Rightarrow \frac{1}{E_\gamma} \ll \frac{2}{mc^2}$

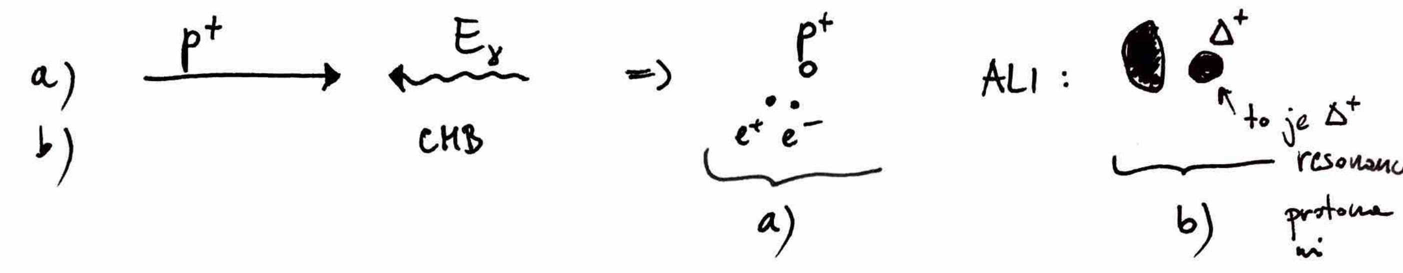
od tod: $E_e^{max} = E_\gamma + mc^2 - E_\gamma^{'min} \simeq E_\gamma + \frac{mc^2}{2} \simeq E_\gamma$.

Podobno kot se lahko fotoni sipajo na mirujočem e^-, se lahko visoko-energijski kozmični žarki (protoni, jedra) sipajo na fotonih. Vemo, da je v celotnem vesolju prisotno kozmično mikrovalovno ozadje fotonov s $T_\gamma = T_{CMB} = 2{,}7\,K$.

$$\langle E_\gamma \rangle \sim 2.7\,kT_{CMB} = 6{,}3\cdot 10^{-4}\,eV \,, \quad k = 8{,}6\cdot 10^{-5}\,\frac{eV}{K}$$

Boltzmannova konstanta

Obstaja več procesov preko katerih p^+ izgubi energijo, poglejmo si: a) Tvorba parov e^+e^- in b) Tvorba pionov preko Δ^+ resonance z $m_{\Delta^+}c^2 = 1{,}2$ GeV.



Podobno kot prej nalogo rešujemo z invarianto.

$$p_p^\mu = (E, pc), \quad E^2-(pc)^2 = m_p^2c^4 \approx 0 \quad \text{oz.: } E \simeq pc.$$

$$p_\gamma^\mu = (E_\gamma, -E_\gamma)$$ takrat je trk centralen, najbolj izgubi energijo.

$$(E+E_\gamma)^2 - (pc-E_\gamma)^2 = \begin{cases} (m_p+2m_e)^2c^4, & (a), \\ (\quad + m_\Delta)^2c^4, & (b). \end{cases}$$

$$\Downarrow$$

$$\cancel{E^2} + 2EE_\gamma + \cancel{E_\gamma^2} - \cancel{(pc)^2} + 2\underbrace{pc}_{\substack{\simeq \\ E}}E_\gamma - \cancel{E_\gamma^2} \simeq (\cancel{m_p^2} + 4m_em_p)c^4$$

a) $$E = \frac{m_pc^2\, m_ec^2}{E_\gamma} = \frac{\text{GeV}\cdot 5\cdot 10^5\,\cancel{\text{eV}}}{6\cdot 10^{-4}\,\cancel{\text{eV}}} \simeq 10^9\,\text{GeV} \sim \underline{10^{18}\,\text{eV}}.$$

b) $$4EE_\gamma + m_p^2c^4 = \quad + m_{\Delta^+}^2)c^4$$

$$E = \frac{(m_\Delta^2 - m_p^2)c^4}{4E_\gamma} = \frac{0.55\,\text{GeV}^2}{2{,}4\cdot 10^{-3}\,\text{eV}} = 2{,}2\cdot 10^{20}\,\text{eV} \ldots$$ GZK cutoff.