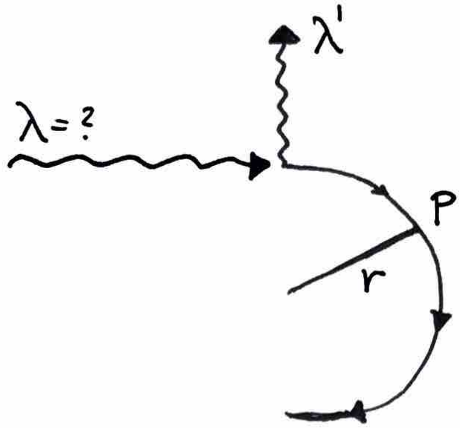
(6) $B = 0{,}002\,\mathrm{T}$

$\theta = 90°$

$r = 2\,\mathrm{cm}$



$p = erB$, $pc = e\, 2\cdot \cancel{10^{-2}}\,\cancel{\mathrm{m}}\; 2\cdot \cancel{10^{-3}}\,\frac{\mathrm{V}\cancel{\mathrm{s}}}{\cancel{\mathrm{m}^2}}\cdot 3\cdot 10^{\cancel{8}\,3}\,\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}}} = \underline{12\ \mathrm{keV}}$,

S tako majhno gibalno količino, lahko uporabimo NR

$$T = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)mc^2 \simeq \left(\cancel{1} + \tfrac{1}{2}\beta^2 - \cancel{1}\right)mc^2$$

in $pc = \gamma\beta mc^2 \sim \beta mc^2$ $\quad T \simeq \frac{(pc)^2}{2mc^2} = \frac{144}{10^3}\,\mathrm{keV} = \underline{0{,}14\,\mathrm{keV}}$

Spremembo energije fotona dobimo iz Comptona:

$$\lambda' = \lambda + \lambda_c(1 - \overset{0}{\cos\underset{90°}{\theta}}) = \lambda + \lambda_c$$

Iz ohranitve 0-te komponente p^μ ali energije, sledi:

$$E_\gamma + \cancel{mc^2} = E_\gamma' + \cancel{mc^2} + T \quad \text{ali:} \quad hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = T.$$

Zanima nas λ, zato iz Comptona vstavimo λ'.

$$\frac{1}{\lambda} - \frac{1}{\lambda+\lambda_c} = \frac{T}{hc} \;/\, \lambda(\lambda+\lambda_c) \Rightarrow \cancel{\lambda} + \lambda_c - \cancel{\lambda} = \frac{T}{hc}\lambda(\lambda+\lambda_c)$$

$$\frac{T}{hc}\lambda^2 + \frac{T}{mc^2}\lambda - \lambda_c = 0, \quad \lambda = \frac{hc}{2T}\left(-\frac{T}{mc^2} \pm \sqrt{\left(\frac{T}{mc^2}\right)^2 + 4\frac{T}{mc^2}}\right)$$

$$= \frac{hc}{2\cancel{T}}\frac{\cancel{T}}{mc^2}\left(-1 + \sqrt{1 + 4\frac{mc^2}{T}}\right)$$

Dobili smo:

$$\lambda = \frac{\lambda_c}{2}\left(-1+\sqrt{1+\underbrace{4\frac{mc^2}{T}}_{\gg 1}}\right) \quad , \quad mc^2 \cong 500\,\text{keV}, \quad T = 0{,}14\,\text{keV},$$

$$\lambda_c = \frac{hc}{mc^2} = \frac{1240}{511}\,\frac{\cancel{\text{eV}}\,\text{nm}}{10^3\,\cancel{\text{eV}}} = \underline{2{,}4\,\text{pm}}$$

$$\lambda \simeq \frac{\lambda_c}{\cancel{2}}\,\cancel{2}\sqrt{\frac{mc^2}{T}} = 2{,}4\,\text{pm}\,\underbrace{\sqrt{\frac{500}{0{,}14}}}_{\sim 60} \simeq \underline{\underline{0{,}14\,\text{nm}}}.$$

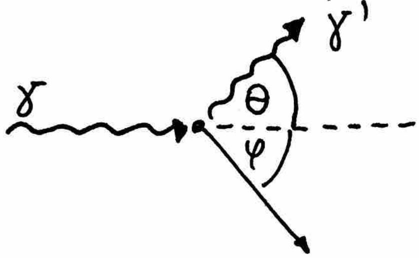
7 Sevanje Čerenkova in Compton

$E_\gamma = \text{MeV}$

$n = 1{,}33 \sim \frac{4}{3}$

$\theta = ?$

$\varphi = ?$



$v > c' = \frac{c}{n}$

Kdaj pride do sevanja Čerenkova?

$$\beta > \frac{1}{n} = \frac{3}{4}, \quad \gamma = \frac{1}{\sqrt{1-\frac{9}{16}}} = \frac{4}{\sqrt{7}}$$

Podobno kot prej bomo uporabili Comptonovo enačbo, da dobimo θ. Energije dobimo iz ohranitve p°

$$\lambda' - \lambda = \lambda_c(1-\cos\theta) \quad \text{in} \quad E_\gamma + \cancel{mc^2} = E_\gamma' + \cancel{mc^2} + T$$

$$\Downarrow$$

$$\frac{1}{E_\gamma'} - \frac{1}{E_\gamma} = \frac{1}{mc^2}(1-\cos\theta)$$

$$T = (\gamma-1)mc^2 = \left(\frac{4}{\sqrt{7}}-1\right)511\,\text{keV} \simeq \frac{1}{4}\,\text{MeV}.$$

$$\Vert$$

$$E_\gamma' = E_\gamma - T \cong \frac{3}{4}\,\text{MeV}$$

$$mc^2\left(\frac{1}{E_\gamma'} - \frac{1}{E_\gamma}\right) \simeq \frac{\cancel{1\,\text{MeV}}}{2\,\cancel{\text{MeV}}}\left(\frac{4}{3}-1\right)$$

$$\Rightarrow \frac{1}{6} = 1-\cos\theta, \quad \cos\theta = \frac{5}{6} \quad \text{in} \quad \underline{\underline{\theta = 35^\circ}}.$$

Drugi kot φ dobimo iz ohranitve v y smeri

$$E_\gamma' \sin\theta = pc \sin\varphi \quad , \quad pc = \sqrt{T(T+2mc^2)}$$

$$\underline{\underline{\varphi}} = \arcsin\left(\frac{E_\gamma' \sin\theta}{\sqrt{T(T+2mc^2)}}\right) = \underline{\underline{47^\circ}}$$

II/8 De Broglie $\overline{\overline{\text{in}}}$ Compton , $\beta = ?$

$$\lambda_{DB} = \frac{hc}{pc} = \lambda_c = \frac{hc}{mc^2} \Rightarrow pc = \gamma\beta mc^2$$
$$= mc^2 ,$$

$$\gamma\beta = 1 \Rightarrow \frac{\beta^2}{1-\beta^2} = 1 \quad , \quad \beta^2 = 1-\beta^2 \quad , \quad \underline{\underline{\beta = \frac{1}{\sqrt{2}}}} .$$

II/9 Zavorno sevanje : $eU = E_\gamma = \frac{hc}{\lambda} = \frac{1240 \, eV \cancel{nm}}{0{,}071 \, \cancel{nm}}$

$\lambda = 0{,}071$ nm

$$= 17 keV \quad , \quad U = \underline{\underline{17{,}5 \, kV}} .$$

$\theta = 45^\circ$

$p = ?$

$U = ?$

$$\text{Compton} : \lambda' = \lambda + \lambda_c (1-\cos\theta)$$

$$= 0{,}071 \, nm + 2{,}4 \cdot 10^{-3} nm \left(1 - \frac{\sqrt{2}}{2}\right)$$

$$\simeq \lambda$$

Zanima nas E_γ', da vstavimo v ohranitev $p^{\mu = 1,2}$

$$\frac{1}{E_\gamma'} - \frac{1}{E_\gamma} = \frac{1-\frac{\sqrt{2}}{2}}{mc^2} \Rightarrow E_\gamma' = \frac{1}{\frac{1}{E_\gamma} + \frac{1-\frac{\sqrt{2}}{2}}{mc^2}} = \frac{E_\gamma}{1 + \frac{E_\gamma}{mc^2}\left(1-\frac{\sqrt{2}}{2}\right)}$$

Compton

$$\frac{17}{511} \ll 1$$

Sedaj imamo $E_\gamma' \simeq E_\gamma$ in nas zanima cp.

$$p_x: \quad E_\gamma = E_\gamma' \cos\theta + cp\cos\varphi \quad /^2$$
$$p_y: \quad 0 = E_\gamma' \sin\theta - cp\sin\varphi \quad /^2 \quad \Big)+$$

$$E_\gamma^2\left(1 - 2\cos\theta + 1\right) = (cp)^2 \Rightarrow cp = E_\gamma\sqrt{2(1-\cos\theta)}$$

$$cp = E_\gamma\sqrt{2\left(1-\frac{\sqrt{2}}{2}\right)} = 17\,\mathrm{keV}\sqrt{2-\sqrt{2}} = \underline{\underline{13{,}4\,\mathrm{keV}}}.$$

Sevanje črnega telesa (IV/14)

$$\frac{dj}{d\nu} = \frac{2\pi h}{c^2}\frac{\nu^3}{e^{h\nu/kT}-1}$$

Bose-Einsteinova distribucija

$T = 1000\,\mathrm{K}$

$\lambda \in (668-670)\,\mathrm{nm}$

$\frac{j}{j_1} = 2{,}3$

$\Delta T = ?$

$$\Delta j = \int_{\lambda_1}^{\lambda_2}\frac{dj}{d\lambda}\,d\lambda \simeq \frac{dj}{d\lambda}\Delta\lambda$$

$$\frac{dj}{d\lambda} = \frac{dj}{d\nu}\frac{d\nu}{d\lambda}$$

$$c = \nu\lambda \quad /\ln,\ d$$

$$\ln c = \ln\nu + \ln\lambda \Rightarrow 0 = \frac{d\nu}{\nu} + \frac{d\lambda}{\lambda}$$

oziroma: $$\frac{d\nu}{d\lambda} = -\frac{\nu}{\lambda} = -\frac{c}{\lambda^2}$$

$$\frac{dj}{d\lambda} = -\frac{c}{\lambda^2}\frac{2\pi h}{\cancel{c^2}}\frac{c\cancel{c^3}}{\lambda^3}\frac{1}{e^{hc/(\lambda kT)}-1} = -\frac{2\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

Poglejmo si količino v eksponentu $\frac{hc}{\lambda kT}$

$$\frac{hc}{\lambda kT} = \frac{1240\,\cancel{eV}\,\cancel{nm}\,\cancel{K}}{66{,}9\,\cancel{nm}\,\cancel{10^{-4}}\,\cancel{eV}\,\cancel{1000K}} = 21{,}5 \gg 1$$

$$\frac{\Delta j}{\Delta j'} = \frac{\cancel{\#}}{\cancel{\#}}\,\frac{e^{hc/\lambda kT'}-1}{e^{hc/\lambda kT}-1} \simeq e^{\frac{hc}{\lambda kT}\left(1-\frac{T}{T'}\right)} = R \overset{2,3}{=} \quad /\ln$$

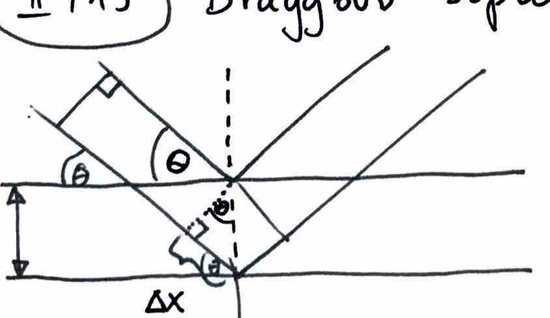
$$\frac{hc}{\lambda kT}\left(1-\frac{T}{T+\Delta T}\right) \simeq \frac{hc}{\lambda kT}\left(\cancel{1}-\cancel{1}+\frac{\Delta T}{T}\right) = \ln R$$

$$\frac{1}{1+\frac{\Delta T}{T}} \sim 1-\frac{\Delta T}{T}$$

$$\Rightarrow \frac{\Delta T}{T} = \frac{\lambda kT}{hc}\ln R \Rightarrow \Delta T = \frac{1000\,K}{21{,}5}\times\begin{cases}\ln 2\\ \ln 3\end{cases}$$

$$= \begin{cases}32\,K\\ 51\,K.\end{cases}$$

II / 13 Braggovo sipanje, $d = 0{,}21$ nm



razlika poti $\Delta l = 2\Delta x = n\lambda$

pogoj za resonančno sipanje na kristalu

$$\sin\theta = \frac{\Delta x}{d}$$

$$\lambda = \lambda_{DB} = \frac{hc}{pc} \Rightarrow n\frac{hc}{pc} = 2d\sin\theta$$

$$pc = \frac{nhc}{2d\sin\theta} \xrightarrow{\min} \frac{hc}{2d} = \frac{1240\,eV\,\cancel{nm}}{0{,}42\,\cancel{nm}} = \underline{\underline{3\,keV}},$$

$n=1$, $\sin\theta = \max = 1$

• Dobili smo $pc = 3\,keV$, $\quad pc \ll \underset{\overset{\approx}{0{,}5\,MeV}}{m_e c^2} < \underset{\overset{\approx}{1\,GeV}}{m_n c^2}$

zato lahko uporabimo NR formulo.

$$T_e = \frac{(pc)^2}{2m_e c^2} = \frac{9\,keV^2}{1\,MeV} = 9\cdot 10^{-3}\,keV = \underline{\underline{9\,eV}},$$

$$T_n = \frac{(pc)^2}{2m_n c^2} = \frac{m_e}{m_n} T_e = \frac{1}{2}\frac{MeV}{GeV} T_e = 5\cdot 10^{-4}\cdot 9\,eV = \underline{\underline{4{,}5\,meV}}.$$