• Dobili smo $pc = 3\,\text{keV}$, $pc \ll m_e c^2 < m_n c^2$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \overset{\wr}{0{,}5\,\text{MeV}} \qquad \overset{\wr}{1\,\text{GeV}}$

zato lahko uporabimo NR formulo.

$$T_e = \frac{(pc)^2}{2m_e c^2} = \frac{9\,\text{keV}^2}{1\,\text{MeV}} = 9\cdot 10^{-3}\,\text{keV} = \underline{\underline{9\,\text{eV}}},$$

$$T_n = \frac{(pc)^2}{2m_n c^2} = \frac{m_e}{m_n} T_e = \frac{1}{2}\frac{\text{MeV}}{\text{GeV}} T_e = 5\cdot 10^{-4}\cdot 9\,\text{eV} = \underline{\underline{4{,}5\,\text{meV}}}.$$

II./14.

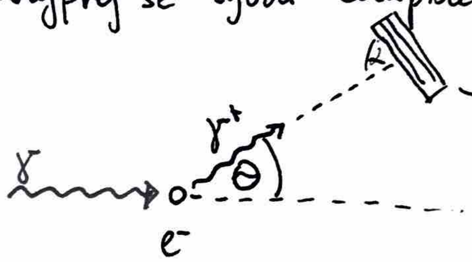
$U = 17.4\,\text{kV}$

$d = 0.3\,\text{nm}$

$\theta = 75^\circ$

$\delta\alpha = ?$

Najprej se zgodi Compton



→ potem pa Bragg

a) Nastavimo Bragg kristal z $\lambda = \dfrac{hc}{eU}$ in

$$\lambda = \frac{1240\,\cancel{\text{eV}}\,\text{nm}}{17{,}4\,\cancel{\text{keV}}} = 0.071\,\text{nm}.$$

Braggov pogoj

$$2d\sin\alpha = \lambda \Rightarrow \sin\alpha = \frac{\lambda}{2d} = \frac{0.071}{0.6}\frac{\cancel{\text{nm}}}{\cancel{\text{nm}}} = 0{,}12, \quad \alpha = 6{,}9^\circ$$

Potem dodamo Comptonsko sipanje pod kotom $\theta = 75^\circ$.

$$\lambda' = \lambda + \lambda_c(1-\cos\theta) = 0{,}071\,\text{nm} + \frac{1{,}24\,\cancel{\text{keV}}\,\text{nm}}{511\,\cancel{\text{keV}}}(1-0{,}26)$$

$$= 0{,}071 + 0{,}0018\,\text{nm}, \quad \lambda' = 0.073\,\text{nm}, \quad \delta\lambda = 0{,}002\,\text{nm}$$

$$\frac{\delta\lambda}{\lambda} \simeq \frac{2}{73} \ll 1; \qquad \sin\alpha' = \frac{\lambda'}{2d} = \frac{0.073}{0.6} = 0{,}122, \quad \delta\alpha \sim 10^{-3}$$

$$\delta\alpha \cong \frac{\delta\lambda}{2d} = \frac{\lambda_c(1-\cos\theta)}{2d} \sim 0{,}17^\circ.$$

PRINCIP NEDOLOČENOSTI

1) e^- pospešimo z $U = 0{,}1\ \mathrm{MV}$, $dU = 50\ \mathrm{V}$. Kolikšna je nedoločenost lege elektrona?

$\Delta x\, \Delta p \gtrsim \frac{\hbar}{2}$ ... princip nedoločenosti

Iz napetosti dobimo kinetično energijo $T = eU$

$T = 0{,}1\ \mathrm{MeV} \lesssim m_e c^2 = 0{,}5\ \mathrm{MeV}$, $dT = e\,dU = 50\ \mathrm{eV}$

Rabimo zvezo med T in p. $E = T + mc^2$, $E^2 - (pc)^2 = m^2c^4$

$$p^2c^2 = T^2 + 2Tmc^2 \quad / d$$

$$\cancel{2}p\,dp\,c^2 = \cancel{2}T\,dT + \cancel{2}mc^2\,dT$$

$$dp = \frac{T + mc^2}{\sqrt{T^2 + 2Tmc^2}}\,dT = \frac{1 + \frac{mc^2}{T}}{\sqrt{1 + 2\frac{mc^2}{T}}}\,dT = \underline{91{,}2\ \mathrm{eV}}.$$

$\hbar = \frac{h}{2\pi}$, $\hbar c = 197\ \mathrm{eV\,nm} \simeq 200\ \mathrm{eV\,nm}$.

$$\Delta x = \frac{\hbar c}{2\Delta pc} = \frac{1\cancel{2}00\ \cancel{\mathrm{eV}}\ \mathrm{nm}}{\cancel{2}\cdot 91{,}2\ \cancel{\mathrm{eV}}} = \frac{1}{1 - \frac{9}{100}}\ \mathrm{nm} = 1{,}09\ \mathrm{nm} \doteq \underline{\underline{1{,}1\ \mathrm{nm}}}$$

2) $m_\eta c^2 = 549\,\mathrm{MeV}$ $\Delta E \Delta t \geq \frac{\hbar}{2}$

$\tau = 7 \cdot 10^{-19}\,\mathrm{s}$ $\Delta t < \tau$

$\Delta m = ?$

↳ toliko časa imamo na voljo za opazovanje /meritev mase

$$\Delta E = \frac{\hbar c}{2c\tau} = \frac{1200\,\mathrm{eV}\,\cancel{\mathrm{nm}} \cdot 10^{-9}}{\cancel{2} \cdot 3 \cdot 10^{8}\,\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}}}\, 7 \cdot 10^{-19}\,\cancel{\mathrm{s}}} = \frac{5 \cdot 10^{-7}\,\mathrm{eV}}{\cancel{5.21} \cdot 10^{-\cancel{11}\,9}} = 5 \cdot 10^{+2}\,\mathrm{eV}$$
$$= 470\,\mathrm{eV}.$$

$$\delta m_\eta c^2 = \frac{\Delta E}{m_\eta c^2} = \frac{470\,\mathrm{eV}}{549\,\mathrm{MeV}} \simeq 10^{-6} = 8{,}5 \cdot 10^{-7}.$$

3) Z načelom nedoločenosti bomo ocenili energije osnovnih stanj za kvantnomehanske sisteme:

i) Harmonski oscilator (HO), $V \propto x^2$,

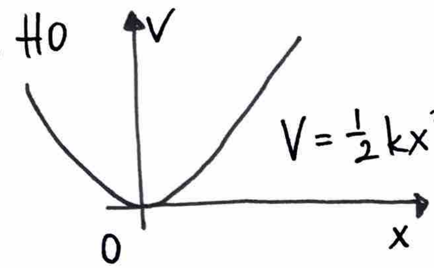
ii) Delec v linearnem potencialu (QCD), $V \propto x$,

iii) Atom vodika, $V \propto \frac{1}{x}$.

Ideja je, da ne rešimo eksaktno Schrödingerjevo enačbo $H\Psi = E\Psi$, ampak da aproksimiramo Ψ. S tem dobimo $\langle E \rangle$, ki bo funkcija $\langle p^2 \rangle$ iz kinetičnega člena in $\langle V \rangle \sim \langle x^p \rangle$ iz potenciala. Ker je sistem na miru (oz. ker smo v težiščnem sistemu) je $\langle p \rangle = 0$.

$$\delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} \quad , \quad \delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

i) HO



$$V = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

$\langle x\rangle = 0$ (izbira koordinatnega sistema)

$\langle p\rangle = 0$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \, , \quad \langle E\rangle = \langle H\rangle = \frac{\langle p^2\rangle}{2m} + \frac{1}{2}m\omega^2\langle x^2\rangle$$

$$\Rightarrow \langle E\rangle = \frac{\delta p^2}{2m} + \frac{1}{2}m\omega^2\delta x^2 \quad , \quad \delta p\,\delta x \gtrsim \frac{\hbar}{2} .$$

Ne moremo postaviti $\delta p = 0$ ali $\delta x = 0$, obstaja pa nek minimum, ki ga še dovoljuje princip nedoločenosti.

$$\langle E(\delta p)\rangle = \frac{\delta p^2}{2m} + \frac{1}{2}m\omega^2\frac{\hbar^2}{4\,\delta p^2} \qquad \Big| \cdot \frac{d}{d(\delta p^2)}$$

$$\frac{dE}{d\delta p^2} = \frac{1}{2m} - \frac{(\hbar)^2 m\omega^2}{\cancel{8}4(\delta p^2)^2} = 0 \quad \Rightarrow \quad \delta p^2 = \frac{\hbar m\omega}{2}$$

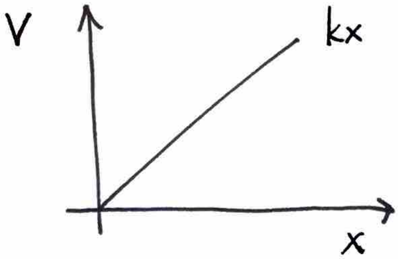
$$\delta x^2 = \frac{\hbar^2}{4\delta p^2} = \frac{\hbar\cdot\cancel{2}}{\cancel{4}m\omega 2}$$

$$\delta p^2 = \frac{1}{2}\hbar m\omega \, , \quad \delta x^2 = \frac{1}{2}\frac{\hbar}{m\omega} .$$

$$\langle E\rangle = \frac{1\,\hbar\cancel{m}\omega}{4\cancel{m}} + \frac{1}{2}\cancel{m}\omega^{\cancel{2}}\frac{\hbar}{2\cancel{m\omega}} = \hbar\omega\left(\frac{1}{4}+\frac{1}{4}\right) = \frac{1}{2}\hbar\omega$$

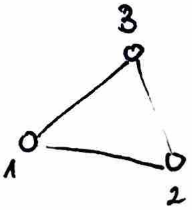
Eksakten rezultat: $E_n = \hbar\omega(n + 1/2)$, $n=0$... osnovno stanje

ii) Linearni potencial $\quad V = kx \quad , k = 0{,}09 \frac{GeV^2}{\hbar c}$



$$m_u = m_d = 340 \frac{MeV}{c^2} = m$$

$$H = \sum \frac{p_i^2}{2m} + k\,(x_3 - x_1 + x_3 - \cancel{x_2} + \cancel{x_2} - x_1)$$



$$\langle E \rangle = \frac{1}{2m}\left(\langle p_1^2 \rangle + \langle p_2^2 \rangle + \langle p_3^2 \rangle \right) + 2k\,(\langle x_3 \rangle - \langle x_1 \rangle)$$

$$\langle p \rangle = 0 \,, \quad \delta x_i \simeq \langle x_i \rangle, \quad \delta p_i \delta x_i = \frac{\hbar}{2}$$

$$\langle E \rangle = \frac{1}{2m}\left(\delta p_1^2 + \delta p_2^2 + \delta p_3^2 \right) + \frac{\cancel{2}k\hbar}{\cancel{2}}\left(\frac{1}{\delta p_3} - \frac{1}{\delta p_1} \right)$$

Sedaj iščemo minimum, odvod po $\delta p_i = 0$, $\forall i$.

$$\frac{d\langle E \rangle}{d\delta p_1} = \frac{1}{m}\delta p_1 + \hbar k \frac{1}{2\delta p_1} = 0 \qquad \delta p_1 = -\sqrt[3]{\hbar k m},$$

$$\frac{d\langle E \rangle}{d\delta p_2} = \frac{1}{m}\delta p_2 = 0 \implies \delta p_2 = 0,$$

$$\frac{d\langle E \rangle}{d\delta p_3} = \frac{1}{m}\delta p_3 - \hbar k \frac{1}{\delta p_3^2} = 0 \qquad \delta p_3 = \sqrt[3]{\hbar k m}.$$

$$\langle E \rangle = \frac{1}{\cancel{2}m}\cancel{2}\cdot(\hbar k m)^{2/3} + \hbar k\left(2(\hbar k m)^{-1/3} \right)$$

$$= \left(\frac{\hbar^2 k^2 \cancel{m^2}}{\cancel{m^3}} \right)^{1/3} + 2\left(\frac{\hbar^3 k^3}{\cancel{\hbar k} m} \right)^{1/3} = 3\left(\frac{\hbar^2 k^2}{m} \right)^{1/3}$$

Dobimo oceno energije vezanega stanja treh kvarkov

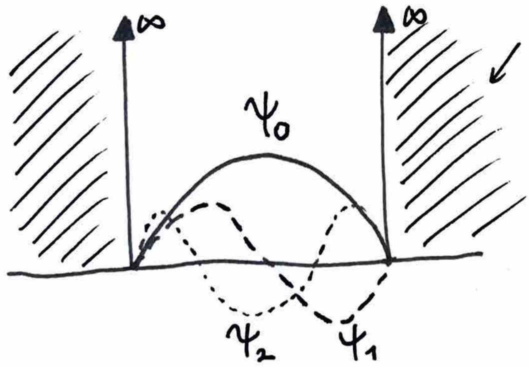
$$\langle E\rangle \cong 3\left(\frac{(\hbar k)^2}{m}\right)^{1/3}, \quad k = 0.09\,\frac{GeV^2}{\hbar c}, \quad mc^2 = 0{,}340\,GeV$$

$$= 3\left(\frac{(\hbar c)^2 k^2}{mc^2}\right)^{1/3} = 3\,\cancel{\left((200\cdot 0{,}09)^2\, eV^2\, GeV^2\right.}$$

$$= 3\left(\frac{0{,}09^2\, GeV^{\cancel{4}}}{0{,}34\,\cancel{GeV}}\right)^{1/3} = 3\cdot 0{,}3\,GeV = \underline{\underline{0{,}9\,GeV}}.$$

Osnovno stanje treh vezanih kvarkov je proton/nevtron z maso $m_p \sim m_n \sim 940$ MeV.

(34) NESKONČNA POTENCIALNA JAMA



prepovedano območje, tu je $\psi = 0$

Robni pogoj: $\psi(x=0) = \psi(x=a) = 0$

zanimajo nas lastna stanja operatorja $\hat{H} = \hat{T} + \hat{V}$, $\hat{H}\psi = E\psi$

$$\hat{T} = \frac{\hat{p}^2}{2m}, \quad \hat{V} = 0, \quad \hat{p} = -i\hbar\frac{d}{dx}$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{d^2\psi_n}{dx^2} + 0 = E\psi_n, \quad \psi_n'' = -\underbrace{\frac{2mE_n}{\hbar^2}}_{=k_n^2}\psi_n$$

$$\psi_n = A_n \sin(k_n x) + B_n \cos(k_n x) \qquad n\pi \quad \Rightarrow k_n a = n\pi,$$

$$\psi_n(0) = B_n = 0 \qquad \psi_n(a) = A_n \sin(\underset{\shortparallel}{k_n a}), \quad k_n^2 = \left(\frac{n\pi}{a}\right)^2$$

$$k_n^2 = \frac{2mE_n}{\hbar^2} = \left(\frac{n\pi}{a}\right)^2, \quad E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2.$$

$\downarrow$

kvantiziran spekter lastnih Energij

Še zadnji korak je, da dobimo normalizacijo A_n:

$$\int_0^a |\psi_n|^2 dx = A_n^2 \int_0^a \sin^2\Big(\underbrace{\frac{n\pi x}{a}}_{t}\Big) \underbrace{dx}_{\frac{a}{n\pi}dt} = A_n^2 \underbrace{\int_0^{n\pi} \sin^2 t \, dt}_{\frac{1}{2}\cdot n\pi} \left(\frac{a}{n\pi}\right) = 1$$

$$\frac{1}{2} = \underset{\frac{1}{2}}{\text{povprečje}} \times \underset{n\pi}{\text{dolžina}}$$

$$\Rightarrow A_n^2 \cdot \frac{a}{\cancel{n\pi}} \cdot \frac{1}{2}\cancel{n\pi} = 1, \quad A_n = \sqrt{\frac{2}{a}}$$

Neskončna potencialna jama:

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad E_n = \frac{1}{2m}\left(\frac{\hbar n\pi}{a}\right)^2.$$

* Pri valovnih funkcijah ψ_n bi lahko dodali poljubne faze $e^{i\alpha_n}$.

(34) "Makroskopska" potencialna jama, $h \simeq 7\cdot 10^{-34}$ Js

$m = \mu g$ $\quad E_n = n^2 E_1, \quad E_1 = \left(\frac{\pi\hbar c}{a}\right)^2 \frac{1}{2mc^2} = \frac{h^2}{8ma^2}$

$a = 1\,cm$ $\quad \Rightarrow n = \sqrt{\frac{E_n}{E_1}} = \sqrt{\frac{10^{-7}}{10^{-55}}} = 10^{24}$ $\quad = \frac{49\cdot 10^{-68}\, J^2 \cancel{s^2}}{8\cdot 10^{-19}\, kg \cdot 10^{-4}\, \cancel{m^2}} = 6\cdot 10^{-55} J$

$E_n = 10^{-7} J$

$$\Delta E_n = \left((n+1)^2 - n^2\right)E_1 = 2nE_1 \sim 10^{24}\cdot 10^{-55} J \sim 10^{-31} J.$$