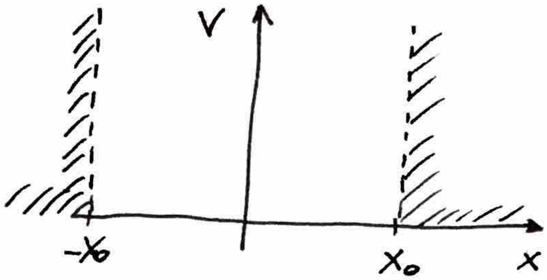
(37) $2x_0 = a = 1\,nm$

$\psi(x) = A(x_0^2 - x^2)$

$\langle E \rangle = ?$



Najprej je potrebno določiti normalizacijo A

$$1 = \int_{-x_0}^{x_0} dx\, A^2 (x_0^2 - x^2)^2 = A^2 \int_{-x_0}^{x_0} (x_0^4 - 2x_0^2 x^2 + x^4)\, dx =$$

$$= A^2 \left(2x_0^5 - \frac{2}{3} 2x_0^5 + \frac{2}{5} x_0^5 \right) = A^2 \frac{2}{15} x_0^5 (15 - 10 + 3) = \frac{16}{15} A^2 x_0^5$$

NORMALIZACIJA : $A = \sqrt{\frac{15}{16}}\, x_0^{-\frac{5}{2}}$

Povprečna energija je podana z : $\int_{-\infty}^{\infty} \psi^* \hat{H} \psi\, dx = \langle E \rangle$,

$$\hat{H} = \hat{T} + \hat{V} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + 0$$

$$\langle E \rangle = -\frac{\hbar^2}{2m} A^2 \int_{-\infty}^{\infty} (x_0^2 - x^2) \underbrace{\frac{d}{dx}(-2x)}_{=-2} dx = \frac{\hbar^2}{m} \frac{15}{8\cancel{16} x_0^{\cancel{5}2}} \underbrace{\left(\cancel{2} \cancel{x_0^3} - \cancel{2} \frac{\cancel{x_0^3}}{3} \right)}_{\frac{2}{3}}$$

ENERGIJA : $\langle E \rangle = \frac{5}{4} \frac{\hbar^2}{m x_0^2}$

Vstavimo številke : $= \frac{5}{\cancel{4}} \frac{(\hbar c)^2}{mc^2\, x_0^2} = \frac{\cancel{5} \cdot \cancel{4} \cdot 10^4\, eV^{\cancel{2}}\, \cancel{nm^2}}{\cancel{0{,}5} \cdot \cancel{10^6}\, \cancel{eV}\, \cancel{nm^2}} = \underline{0{,}4\, eV.}$

$\frac{\cancel{1}}{\cancel{4}}$ nm^2 $\quad 10^5$

Poglejmo si še nedoločenost lege δx ingibalne količine δp

$$\delta x^2 = \langle x^2 \rangle - \langle x \rangle^2 \quad , \quad \delta p^2 = \langle p^2 \rangle - \langle p \rangle^2$$

$$\langle x \rangle = \int_{-x_0}^{x_0} dx\, |\psi|^2 x = 0.$$

liho

sodo

nadaljujemo z 37

$$\langle x^2\rangle = A^2 \int \psi \; x \; \psi \, dx$$

$$= \frac{15}{16x_0^5}\int_{-x_0}^{x_0}(x_0^2-x^2)\,x^2\,(x_0^2-x^2)\,dx$$

$$= \frac{15}{16x_0^5}\int_{-x_0}^{x_0}(x_0^4x^2-2x_0^2x^4+x^6)\,dx$$

$$= \frac{15}{16\cancel{x_0^5}}\left(\frac{1}{3}\,2x_0^{\cancel{7}\,2}-\frac{4}{5}x_0^{\cancel{7}\,2}+\frac{2}{\cancel{7}}x_0^{\cancel{7}\,2}\right)$$

$$= \frac{\cancel{15}\,x_0^2}{8\cancel{16}\cdot\cancel{15}\cdot 7}\left(\cancel{2}\cdot 35-\overset{2}{\cancel{4}}\cdot 21+\cancel{2}\cdot 15\right) = \frac{x_0^2}{\cancel{8}\cdot 7}\cdot\cancel{8}$$

$$\delta x^2 = \langle x^2\rangle - \langle x\rangle^2 = \frac{x_0^2}{7}\,, \qquad \delta x = \frac{x_0}{\sqrt{7}}.$$

$$\langle p^2\rangle = A^2\int_{-x_0}^{x_0}(\cancel{x_0^2-x^2})(+\hbar^2)(+2)\,dx = \frac{15\cdot\cancel{2}\hbar^2}{8\cancel{16}x_0^5}\int_{-x_0}^{x_0}(x_0^2-x^2)\,dx$$

$$= \frac{15\,\hbar^2x_0^{\cancel{3}}}{8\,x_0^{\cancel{5}\,2}}\left(\overset{4/3}{2-\frac{2}{3}}\right) = \frac{5\hbar^2}{2x_0^2}.$$

$$\langle p\rangle = A^2\int_{-x_0}^{x_0}\underbrace{(x_0-x^2)}_{\text{sodo}}(-i\hbar)\overbrace{(-2x)}^{\text{liho}}\,dx = 0$$

$$\delta p^2 = \langle p^2\rangle - \langle p\rangle^2 = \frac{5\hbar^2}{2x_0^5}\,, \qquad \delta p = \sqrt{\frac{5}{2}}\,\frac{\hbar}{x_0}.$$

$$\delta p\,\delta x = \frac{\hbar}{\cancel{x_0}}\cdot\cancel{x_0}\sqrt{\frac{5}{2\cdot 7}} = \frac{\hbar}{2}\sqrt{\frac{10}{7}} \sim \underset{\downarrow}{1{,}2}\cdot\frac{\hbar}{2}$$

20% nad mejo nedoločenosti

(42) $\psi = A(\psi_1 + 3\psi_3)$, $H\psi_n = E_n \psi_n$, $E_n = n^2 E_1$. $E_1 = \frac{(hc)^2}{8mc^2a^2} = 2{,}35\,\mathrm{eV}$

Normalizacija: $\int \psi^2 dx = A^2 \int (\psi_1 + 6\psi_1\psi_3 + 9\psi_3)dx$

$$= A^2(1 + \phi + 9) = 10 \quad , \quad A = \frac{1}{\sqrt{10}}.$$

V splošnem: $\psi = \sum_n c_n \psi_n$, $\int \psi_n \psi_m dx = \delta_{nm}$,

Normalizacija: $\int \psi^* \psi \, dx = \sum_{nm} \int c_n^* c_m \psi_n^* \psi_m = \sum |c_n|^2 = 1$.

V našem primeru: $c_1 = \frac{1}{\sqrt{10}}$, $c_2 = 0$, $c_3 = \frac{3}{\sqrt{10}}$, $c_{i>3} = 0$,

$$\langle E \rangle = \int dx \sum_n c_n^* \psi_n^* \hat{H} \sum_m c_m \psi_m = \sum_{n,m} c_n^* c_m E_m \underbrace{\int \psi_n^* \psi_m dx}_{\delta_{nm}}$$

$$= \sum_n |c_n|^2 E_n.$$

$$\langle E \rangle = \frac{1}{10}E_1 + \phi + \frac{9}{10}E_3 = \underbrace{\frac{1 + 9\cdot 9}{10}}_{=\frac{82}{10}} E_1 = \frac{41}{5}E_1 = 8{,}2 E_1 = 19{,}3\,\mathrm{eV}.$$

Sedaj si poglejmo še ortogonalno stanje $\psi_\perp$

$$\int \psi^* \psi_\perp dx = 0 \quad , \quad \psi_\perp = c_1 \psi_1 + c_3 \psi_3 \qquad c_3 = -\frac{c_1}{3}$$

$$\frac{1}{\sqrt{10}} \int (\psi_1^* + 3\psi_3^*)(c_1\psi_1 + c_3\psi_3)dx = \frac{1}{\sqrt{10}}(c_1 + 3c_3) = 0$$

$$\int \psi_\perp^* \psi_\perp dx = 1 = c_1^2 + c_3^2 \; , = \left(1 + \frac{1}{9}\right)c_1^2 \Rightarrow c_1^2 = \frac{9}{10},$$

$$c_3^2 = -\frac{1}{10}$$

$$\psi_\perp = \frac{1}{\sqrt{10}}(3\psi_1 - \psi_3) \; , \; \langle E_\perp \rangle = \frac{1}{10}(9E_1 + 9E_1)$$

$$= \frac{18}{10}E_1 = \frac{9}{5}E_1 = 4{,}2\,\mathrm{eV} < \langle E \rangle$$

(42) dodatek: časovni razvoj $\psi(t)$, $\sigma(t)$

Schrödingerjeva enačba: $i\hbar \frac{d\psi}{dt} = \hat{H}\psi(x,t)$

Rešimo z razvojem $\psi = \sum_n c_n \psi_n(x,t) = \psi(x,t)$

za vsako lastno stanje: $i\hbar \dot{\psi}_n = E_n \psi_n$, $\psi_n = e^{\frac{E_n}{i\hbar}t}\psi_n(0)$

$$\psi_n(x,t) = e^{-i\frac{E_n}{\hbar}t}\psi_n(x,0)$$

Povprečna energija se s časom ne spreminja:

$$\langle E \rangle = \sum_{n,m} \int c_n^* \psi_n^* e^{i\frac{E_n}{\hbar}t} H \psi_m c_m e^{-i\frac{E_m}{\hbar}t} dx$$

$$= \sum c_n^* c_m \underbrace{e^{i\frac{E_n-E_m}{\hbar}t}}_{e^0 = 1} E_m \underbrace{\int \psi_n^* \psi_m dx}_{\delta_{nm}}$$

$$= \sum |c_n|^2 E_n$$

$\int_0^\pi dt \cos(nt)\sin(mt) = \frac{(-1+(-1)^{m+n})m}{n^2-m^2}$

kar: $\int_0^\pi \sin t \cos 3t = 0$

Poglejmo še ostale operatorje

$$\langle p \rangle = \sum_{n,m} \int c_n^* \psi_n e^{i\frac{E_n}{\hbar}t} \left(-i\hbar \frac{d}{dx}\psi_m\right) e^{-i\frac{E_m}{\hbar}t} dx = 0.$$

to je res tudi v splošnem

$\langle p^2 \rangle = 2m\langle E \rangle$, neodvisno od t

$\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)$ $\quad\sqrt{\frac{2}{a}}\sin\left(\frac{3\pi x}{a}\right)$

$$\langle x \rangle = \frac{1}{10}\int_0^a \left(\psi_1 e^{iE_1t/\hbar} + 3\psi_3 e^{iE_3t/\hbar}\right) x \left(\psi_1 e^{-iE_1t/\hbar} + 3\psi_3 e^{-iE_3t/\hbar}\right)$$

$= \ldots = \frac{a}{2}$ in neodvisen od časa.

$$\langle x^2 \rangle = \frac{1}{10}\int_0^a dx\, x^2 \left(\psi_1^2 + 9\psi_3^2 + 6\psi_1\psi_3 \cos\left(\frac{E_1-E_3}{\hbar}t\right)\right)$$

$$= \frac{a^2}{120\pi^2}\left(10\pi^2 - 12 + 27\cos\left(\frac{E_3-E_1}{\hbar}t\right)\right).$$

46) Določi nedoločenost $\delta p\,\delta x$ za ψ_n potencialne jame

$$\langle x\rangle = \frac{2}{a}\int_0^a \sin^2\Big(\underbrace{\frac{n\pi x}{a}}_{t}\Big)x\,dx = \frac{2}{a}\left(\frac{a}{n\pi}\right)^2\int_0^{n\pi}\sin^2 t\; t\,dt \qquad dx = \frac{a}{n\pi}dt$$

$$= \frac{\cancel{2}a}{(n\pi)^2}\int_0^{n\pi}\frac{1-\cos 2t}{\cancel{2}}\,t\,dt = \frac{a}{(n\pi)^2}\left(\frac{(n\pi)^2}{2} - \int_0^{n\pi}\cos 2t\; t\,dt\right)$$

$$= \frac{a}{2} - \frac{a}{(n\pi)^2}\left(\frac{t}{2}\overset{0}{\cancel{\sin 2t}}\Big|_0^{n\pi} - \frac{1}{2}\int_0^{n\pi}\sin 2t\,dt\right) \qquad \begin{array}{l} t = u \quad dt = du \\ \cos 2t\,dt = dv \\ \frac{1}{2}\sin 2t = v \end{array}$$

$$\cos 2t\Big|_0^{n\pi} = 0$$

$\langle x\rangle = \dfrac{a}{2}$, za vsak n.

$$\langle x^2\rangle = \frac{2}{a}\left(\frac{a}{n\pi}\right)^3\int_0^{n\pi}\sin^2 t\; t^2\,dt = \frac{\cancel{2}}{a}\left(\frac{a}{n\pi}\right)^3\int_0^{n\pi}\frac{1}{\cancel{2}}(1-\cos 2t)\,t^2\,dt$$

$$t^2 = u \quad 2t\,dt = du$$

$$= \frac{a^2}{(n\pi)^3}\left(\frac{(n\pi)^3}{3} - \overset{0}{\cancel{t^2\cdot\frac{1}{2}\sin 2t}}\Big|_0^{n\pi} + \int_0^{n\pi}\frac{1}{\cancel{2}}\sin 2t\,\cancel{2}t\,dt\right)$$

$$= \frac{a^2}{3} + \frac{a^2}{(n\pi)^3}\left(-\frac{1}{2}\cos(2t)\,t\Big|_0^{n\pi} + \frac{1}{2}\int_0^{n\pi}\overset{0}{\cancel{\cos(2t)}}\,dt\right) \qquad \begin{array}{l}\sin 2t\,dt = dv \\ -\cos(2t)\frac{1}{2} = v\end{array}$$

$$= \frac{a^2}{3} - \frac{1}{2}\overset{1}{\cos(\cancel{2n}\pi)}\,\frac{n\pi}{(n\pi)^3} = \frac{a^2}{3}\left(1 - \frac{3}{2(n\pi)^2}\right)$$

$$\langle p\rangle = \frac{2}{a}(-i\hbar)\int_0^{n\pi}\sin t\cos t\,dt = -\frac{i\hbar}{a}\int_0^{n\pi}\overset{0}{\cancel{\sin 2t}}\,dt = 0.$$

$$\langle p^2\rangle = 2m\langle E\rangle = \cancel{2m}\left(\frac{1}{\cancel{2m}}\right)\left(\frac{n\pi\hbar}{a}\right)^2 = \left(\frac{n\pi\hbar}{a}\right)^2.$$

$$\Rightarrow \delta p = \frac{n\pi\hbar}{a}, \quad \delta x^2 = \frac{a^2}{3} - \frac{a^2}{2(n\pi)^2} - \frac{a^2}{4} = a^2\left(\frac{1}{12} - \frac{1}{2(n\pi)^2}\right)$$

$$\delta p\,\delta x = \frac{n\pi\hbar}{\cancel{a}}\,\frac{\cancel{a}}{2}\sqrt{\frac{1}{3} - \frac{2}{(n\pi)^2}} = \frac{\hbar}{2}\sqrt{\frac{(n\pi)^2}{3} - 2} = \frac{\hbar}{2}\begin{cases}1{,}14\\ 3{,}34\\ 5{,}25\ \dots\end{cases} \xrightarrow{n\gg 1} \frac{\hbar n\pi}{2\sqrt{3}}$$