LECTURES

ON

NEUTRINO

MASS

Miha Nemevšek

VIPAVA SUMMER SCHOOL

~ 2016 ~

(Some) References

[1] G. Senjanović ; Neutrino mass : From LHC to grand unification , La rivista del Nuovo Cimento 1, 34 (2011)

[2] A. Strumia , F. Vissani ; Neutrino masses and mixings and... hep-ph / 0606 054

[3] H. Dreiner , H. Haber , S. Martin ; Two component spinor techniques ... 0812 . 1594

Lecture 1

- Neutrino, Pauli to Fermi

- Weak interactions, from Fermi to $SU(2)_L \times U(1)_Y$
  + neutrino oscillations

- Massive QED, symmetries & 4 component spinors

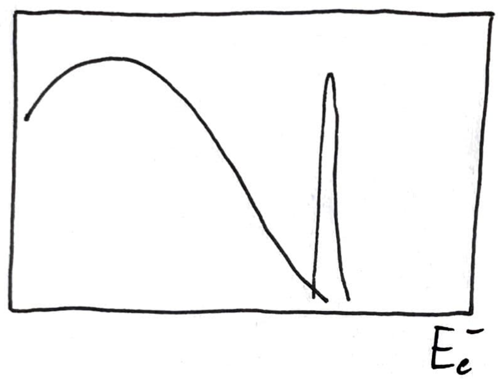
- Weyl & Majorana spinors, 2 & 4 component notations

The birth of neutrino

* 1914 Chadwick measures the β spectrum of

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^{-} \; (\beta^{-} \text{ ray})$$

carbon-6 $\longrightarrow$ nitrogen-7



? Confusion

$\rightarrow \cancel{E}$?

* 1930 Pauli sends a postcard proposing the existence of a neutron

$\downarrow$ 1932 Chadwick (the same) discovers the neutron

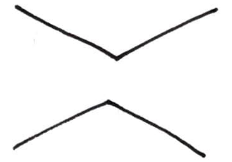
Fermi calls it neutrino ν

* 1933 Fermi sends his theory to Nature only to be rejected

Fermi Theory

* In '33, Dirac equation & EM interactions were known

QED : $j^{\mu}_{em} = \bar{e}\gamma^{\mu}e$ V - interaction

e.g. $j_{\mu} j^{\mu\dagger}$:  $\Rightarrow$ 

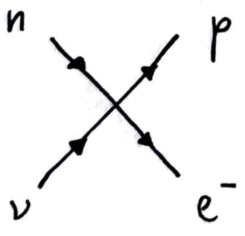
~Dirac QED

Fermi : $n \longrightarrow p + e^- + \bar{\nu}_e$

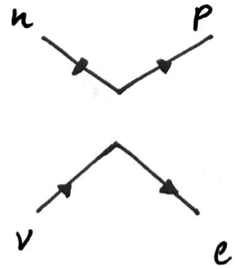
$j^{h} = \bar{p}\gamma^{\mu} n$ also V - interaction

$j^{\ell} = \bar{e}\gamma^{\mu}\nu$ (parity conserving)

today we call it EFT, W heavy



$\Rightarrow$

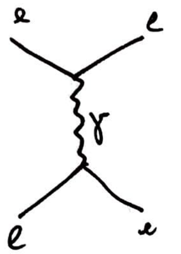


V - V

$[\psi] = \frac{3}{2}$ $\quad \mathcal{O}_F = \frac{G_F}{\sqrt{2}} \underbrace{(\bar{p}\gamma^{\mu} n)(\bar{e}\gamma_{\mu}\nu)}_{d=6}$

~15-20 min

* QED is a RENORMALIZABLE theory



$m_\gamma = 0 \qquad \mathcal{A} \sim \frac{1}{q^2}$

* Fermi theory is NON-RENORMALIZABLE : ∃E at which something must happen

$[G_F] = \frac{1}{\Lambda^2} \qquad G_F^{exp} = 10^{-5}\, GeV^{-2}$

β & μ decay

dimensional analysis : $G_F = \frac{g^2}{\Lambda_F^2}$

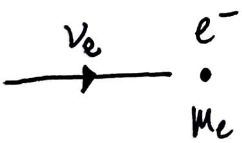
$g = 1 \Rightarrow \Lambda_F \simeq 300\ GeV$

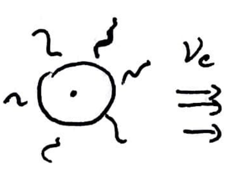
$g = 4\pi \qquad \Lambda_F \simeq 3.6\ TeV$

*now we call $\boxed{\Lambda_F = v}$, the scale of EW breaking

Despite non-renormalizability, it's useful to understand β decays, ν scattering @ $E_\nu \ll \Lambda_F$

* <u>EXAMPLE</u> (S&V)

SK 



$E_\nu \sim 10 MeV$

e^-

$m_e \sim 0.5\ MeV$

$|\mathcal{A}|^2 \sim G_F^2 m_e^2 E_\nu^2$

Do it!

$\sigma \sim \frac{\mathcal{A}^2}{s} \sim \frac{\mathcal{A}^2}{m_e E_\nu} \sim G_F^2 m_e E_\nu \sim 10^{-10}\, GeV^2\ 0.5\, MeV\ 10\, MeV$

$\sigma = 10^{-44} \frac{1}{cm^2} \left(\frac{E_\nu}{MeV}\right)$

$\phi_\odot \sim 10^6 \frac{1}{cm^2 s}, \quad N_e \sim 10^{33}$

$\sim 10^4$ events / year

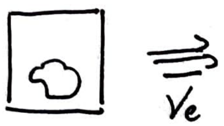
$\ell \sim \frac{1}{\sigma n} \sim 10^8\, km \gg d_{earth} \sim 12 km$

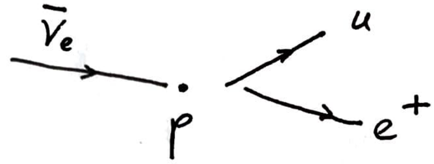
* Pauli vindicated in 1956 by Reines & Cowan

25 min

Detection of reactor neutrinos

$$\phi = 5 \cdot 10^{13} \frac{1}{cm^2 s}$$



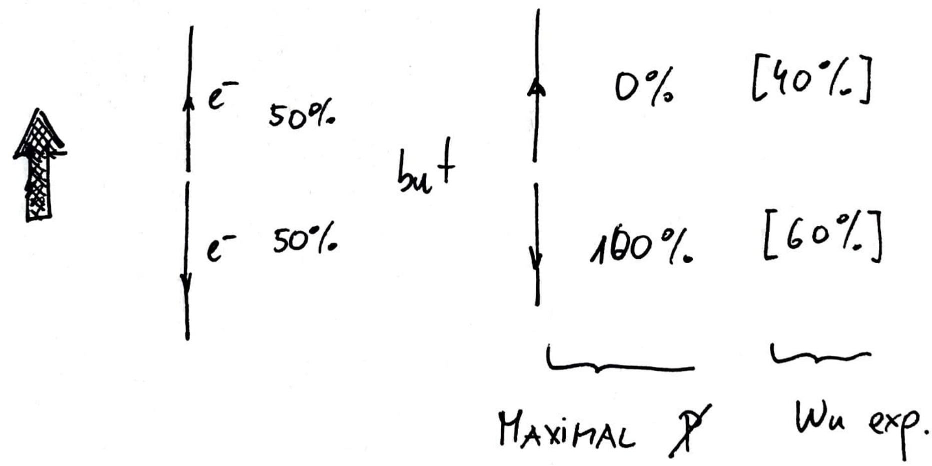


* Another landmark in 1956

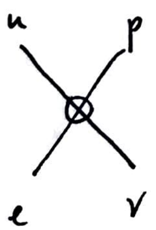
Lee & Yang : Parity is violated in weak interactions

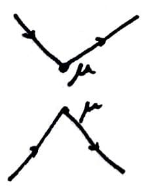
Madame Wu $\quad {}^{60}Co$

β decay



*

If 

$\neq$ 

Fermi's V-V then what is it?

* 1957 Sudarshan, Marshak, Gell-Mann, Feynman : (V-A)(V-A)

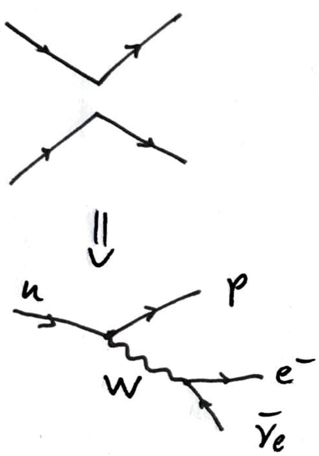
30 m

↷ 1957 SMGF : V-A

$$\mathcal{O}_{(V-A)(V-A)} = \frac{G_F}{\sqrt{2}} (\bar{\nu}_L \gamma^\mu \mu_L)(\bar{e}_L \gamma_\mu \nu_L)$$

- still non-renormalizable but correct low energy theory

1962 : Glashow proposes a (complete) theory based on $SU(2)_L \times U(1)_Y$



$L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $\quad e_R$

W, Z , s_W, ... , $m_\gamma = 0$, ...

$Q = T_{3L} + \frac{Y}{2}$

1964 : Higgs

1967 Weinberg gives a "~~Theory~~ Model of leptons"

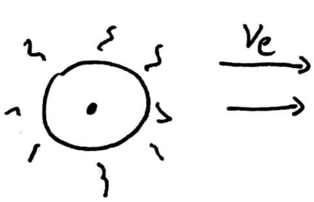
Prediction

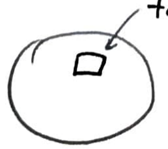
$m_\nu = 0$

W_L $\quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ $\quad e_R$

Z $\quad \varphi = \begin{pmatrix} \varphi^+ \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$

1960 – 2002 : Solar neutrino puzzle





R. Davis measures $\frac{1}{3}$ of the prediction

J. Bahcall predicts the flux

~ 1998 - 2002 SK & SNO confirm both | ~35
 ~11.40

Bahcall & Davis

~ Deficit is due to neutrino oscillations

* Suggested by B. Pontecorvo ('57)

  Gribov & Pontecorvo ('68)

EXAMPLE : 2 neutrino oscillations

$\xrightarrow{\nu_e}$ (E, p) $\qquad\qquad$ $\nu_\mu = ?$

$x = 0$ $\qquad\qquad\qquad$ $x = L$

$$|\nu_1(x)\rangle = e^{ip_1 x}|\nu_1\rangle c_\theta + e^{ip_2 x}|\nu_2\rangle s_\theta$$

$$\Rightarrow P(\langle\nu_\mu|\nu\rangle) = |\langle\nu_\mu|\nu(L)\rangle|^2 \simeq s^2_{2\theta}\sin^2\left(\frac{\Delta m^2_{12} L}{4E}\right)$$

* This is a separate topic but θ & Δm^2 need to be non-zero

$$\Delta m^2_\odot = \Delta m^2_{12} = 7.4 \cdot 10^{-5}\, eV^2$$

$$\Delta m^2_A = \Delta m^2_{23} = 2.5 \cdot 10^{-3}\, eV^2$$

$$s^2_{12} = 0.3,\quad s^2_{23} = 0.44,\quad s^2_{13} = 2 \cdot 10^{-3}$$

2015 Nobel Prize: Neutrinos are massive, contrary to the SM 11.50

HOW CAN WE GENERATE m_ν?

ARE NEUTRINOS FUNDAMENTALLY DIFFERENT FROM OTHER (CHARGED) FERMIONS?

IF NOT THE SM, WHAT IS THE THEORY OF NEUTRINO MASS?

AND HOW WILL WE KNOW (= TEST IT)?

~ A STEP BACK ~

1928 Dirac $\quad (i\not{\partial} - m)\psi_D = 0$

REPEAT LATER

But in 1937 Majorana publishes a paper for a position in Napoli where a theory with less degrees of freedom is advocated, the MAJORANA SPINOR. How does it work?

Dirac & Majorana Spinors

45
11.55

ψ_D ... four component Dirac spinor

$$\mathcal{L}_D = i\,\overline{\psi}_D \not{\partial}\, \psi_D - m_D \overline{\psi}_D \psi_D \quad \text{... Dirac Lagrangian}$$

This is relativistically invariant under LT

$$\not{\partial} \equiv \gamma^\mu \partial_\mu \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \qquad \mu = 0,1,2,3$$

$$\overline{\psi} \equiv \psi^\dagger \gamma^0 \qquad \sigma^\mu = (1, \sigma^i) \qquad i = 1,2,3$$

$$\overline{\sigma}^\mu = (1, -\sigma^i)$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$P_{L,R} = \frac{1}{2}(1 \mp \gamma^5)$$

Lorentz transformation

$$\psi_D \to \Lambda \psi_D \qquad \Lambda = e^{i\,\Sigma^{\mu\nu}\Theta_{\mu\nu}}$$

$$\Sigma^{\mu\nu} = \frac{1}{4i}[\gamma^\mu, \gamma^\nu]$$

&

$$\overline{\psi}_D \to \overline{\psi}_D \Lambda^{-1} \qquad \vec{\theta}^i = i\varepsilon^{ijk}\,\Theta^{jk} \text{ ... rotations}$$

$$\vec{\varphi} = \Theta^{0i} \text{ ... boosts}$$

It is convenient to work with a smaller object than ψ_D, namely with a 2-component Weyl spinor χ. Since $[\gamma_5, \gamma_\mu] = 0$, in the absence of mass terms

$$\psi_L = P_L \psi_D = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}$$

$$\psi_R = P_R \psi_D = \begin{pmatrix} 0 \\ \chi_R \end{pmatrix}$$

$$\text{LT} : \chi_{L,R} \to e^{i \frac{\vec{\sigma}}{2} (\vec{\theta} \pm i \vec{\varphi})} \chi_{L,R}$$

$$\mathcal{L}_D = i \chi_L^+ \bar{\sigma}^\mu \partial_\mu \chi_L + i \chi_R^+ \sigma^\mu \partial_\mu \chi_R$$
$$- m_D (\chi_L^+ \chi_R + \chi_R^+ \chi_L)$$

e.g. $\chi_L^+ \to \chi_L^+ e^{-i \frac{\vec{\sigma}}{2} (\vec{\theta} \mp i \vec{\varphi})}$ $\Rightarrow$ $\underline{\chi_L^+ \chi_R}$ is LI

also $j^\mu = \bar{\psi}_D \gamma^\mu \psi_D$ is a conserved current

$$= \chi_L^+ \gamma^\mu \chi_L + \chi_R^+ \gamma^\mu \chi_R$$

& transforms as a vector under LT.

However, in 1937 Majorana published a theory of fermions with ~~only~~ mass and only two degrees of freedom. Let's take χ_L. Kinetic term is already OK but how to get the mass?

* Hint from $SO(2)$: $\vec{r} \to R r$ $\quad \vec{r} = (x, y)$ $\quad R = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$

$$R^T \mathbb{1} R = \mathbb{1} \Rightarrow \vec{r}^T \vec{r} = x^2 + y^2 = \text{inv.}$$

but also $\quad R^T \underset{\substack{\| \\ \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma_2}}{\varepsilon} R = \varepsilon \Rightarrow \vec{r}^T \varepsilon \vec{r} = \text{inv.}\ (=0)$

$$\vec{r}_1^T \varepsilon \vec{r}_2 \neq 0$$

- for anti-commuting spinors this works well

$$\boxed{\begin{aligned} \chi_L^T i\sigma_2 &\longrightarrow \chi_L^T \left(e^{i \frac{\vec{\sigma}^T}{2} (\vec{\theta} + i \vec{\varphi})} \right) i\sigma_2 \\ &= \chi_L^T (i\sigma_2)\, e^{-i \frac{\vec{\sigma}}{2} (\vec{\theta} + i \vec{\varphi})} \end{aligned}}$$

$\Rightarrow \chi_L^T i\sigma_2 \chi_L$ is LI mass term

$$\Rightarrow \boxed{\mathcal{L}_M = i \chi_L^\dagger \bar{\partial} \chi_L - \frac{1}{2} m_M \left(\chi_L^T i\sigma_2 \chi_L + \chi_L^\dagger i\sigma_2 \chi_L^* \right)}$$

we have $\frac{1}{2}$ wrt ψ_D

In principle this is enough and some people work with 2 component spinors. However to take advantage of the existing 4-comp. machinery one can construct a 4-compound Majorana spinor.

$$\Psi_M = \begin{pmatrix} \chi_L \\ -i\sigma_2 \chi_L^* \end{pmatrix}$$

With $\mathcal{L}_M = \frac{1}{2}\left(i\bar\Psi_M \not\partial \Psi_M - m_M \bar\Psi_M \Psi_M\right)$

kinetic : $\bar\Psi_M \not\partial \Psi_M = \left(\chi_L^\dagger, \chi_L^T (i\sigma_2)\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix}\begin{pmatrix} \partial_\mu \chi_L \\ \partial_\mu \chi_L^* \end{pmatrix}$

$$= \chi_L^\dagger \bar\sigma^\mu \partial_\mu \chi_L + \chi_L^T \underbrace{\sigma_2 \sigma^\mu \sigma_2}_{\bar\sigma^{\mu T}} \partial_\mu \chi_L^*$$

$$= \chi_L^\dagger \bar\sigma^\mu \partial_\mu \chi_L - \left(\partial_\mu \chi_L^T \bar\sigma^\mu \chi_L^\dagger\right)^T$$

$$= 2\chi_L^\dagger \bar\sigma^\mu \partial_\mu \chi_L$$

mass : $\bar\Psi_M \Psi_M = \left(\chi_L^T i\sigma_2, \chi_L^\dagger\right)\begin{pmatrix} \chi_L \\ -i\sigma_2 \chi_L^* \end{pmatrix}$

$$= \chi_L^T i\sigma_2 \chi_L + \chi_L^\dagger i\sigma_2 \chi_L^*$$

$\boxed{\text{OK}}$

ψ_M is a <u>real</u> object in the sense that it does not carry $U(1)$ charges. [R-25]

Remember $\mathcal{L}_D$ is $\mathcal{U}(1)$ invariant

$$\boxed{\mathcal{L}_D = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi}_D \not{D} \psi_D - m_D \bar{\psi}_D \psi_D}$$

is invariant under $\psi_D \to e^{i\alpha(x)} \psi_D$

from this $j^\mu = \bar{\psi}_D \gamma^\mu \psi_D$ is conserved

& $Q = \int \bar{\psi}_D \gamma^0 \psi_D$ charge is obtained

Also if $Q(\psi_D) = q$, we have ψ^c

$$\boxed{\psi^c} = C \bar{\psi}^T = i\gamma^2\gamma^0 (\psi^+\gamma^0)^T = i\gamma^2 \psi^*$$

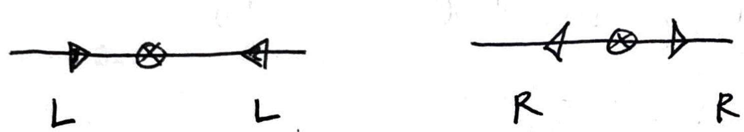
clearly $Q(\psi_D^c) = -Q(\psi_D)$ because of *

it transforms as ψ_D under LT

'31 Dirac : Antiparticles '32 Anderson finds it

$$\mathcal{L}_M = \frac{1}{2}\left(i \bar{\Psi}_M \not{\partial} \Psi_M - m_M \bar{\Psi}_M \Psi_M\right)$$

Since we have this 4-comp notation we can use the same FR (see Haber et al.)



mass insertions break fermion flow & U(1), e.g.

$\boxed{\Delta L = 2}$:

Let's take a closer look at ψ_D^c 12.30

$$\psi_D^c = i\gamma^2\psi^* = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} \chi_L^* \\ \chi_R^* \end{pmatrix}$$

$$= \begin{pmatrix} i\sigma_2 \chi_R^* \\ -i\sigma_2 \chi_L^* \end{pmatrix}$$

Notice that $P_L \psi_D^c = \begin{pmatrix} 0 \\ -i\sigma_2 \chi_L^* \end{pmatrix}$

$$\& \quad \psi_M = P_L \psi_D + P_R \psi_D^c$$

$$= \begin{pmatrix} \chi_L \\ -i\sigma_2 \chi_L^* \end{pmatrix}$$

It is a <u>sum</u> of a LH particle & a RH anti-particle!

Thus $\quad \psi_M^c = i\gamma_2 \psi_M^* = \begin{pmatrix} 0 & +i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} \chi_L^* \\ i\sigma_2^* \chi_L \end{pmatrix}$

$$= \begin{pmatrix} \chi_L \\ -i\sigma_2 \chi_L^* \end{pmatrix} = \psi_M$$

& no charge can be assigned to ψ_M

NB: $\bar{\psi}_M \gamma^\mu \psi_M = (\chi_L^T i\sigma_2 , \chi_L^\dagger)\begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}\begin{pmatrix} \chi_L \\ -i\sigma_2 \chi_L^* \end{pmatrix} = \underline{\underline{0}}$ (check @ home)

- Majorana mass matrix & CP phases

$$(M_M)_{ij}\,\bar{\Psi}_{M_i}\Psi_{M_j} = M_{M\,ij}\left(\chi_{L_i}^{T}\,(i\sigma_2)\,\chi_{L_j} + c.c.\right)$$

$$= M_{M_{ij}}\left(-\chi_{L_j}^{T}\,(i\sigma_2^{T})\,\chi_{L_i} + c.c\right)$$

$$= M_{M_{ij}}\left(\chi_{L_j}^{T}\,(i\sigma_2)\,\chi_{L_i} + c.c.\right)$$

$$= M_{M_{ji}}\left(\chi_{L_j}^{T}\,(i\sigma_2)\,\chi_{L_i} + c.c.\right)$$

$$\Rightarrow \boxed{M_M = M_M^{T}}$$

- Dirac mass arbitrary : $M_D \to U_{uL}^{+}\, m_D\, U_{uR}$ ← diagonal

Bi-unitary diagonalization

- Majorana mass is symmetric $M_M \to U_{\nu L}^{T}\, m_M\, U_{\nu L}$

$\Rightarrow$ extra phases that can't be rotated away.

$\to$ they enter in $\Delta L \neq 0$ processes only (CP even & CP-odd)