LECTURE 2

- Origin of mass - the Higgs mechanism
- The Weinberg operator
- Toy see-saw models
- Dirac vs. Majorana
  * type I vs. Left-Right

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The origin of fermion mass

- $G_{SM} = SU(2)_L \times U(1)_Y$

$$W_L, \quad L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad\qquad e_R$$

$$= (2,-1)_F \qquad\qquad\qquad = (1,-2)_F$$

$$\varphi = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$$

$$= (2,1)_B \qquad\qquad Q = T_{3L} + \frac{Y}{2}$$

- in QED: $\bar{e}_L e_R$ is $U(1)$ invariant $\Rightarrow$ many mass scales, no common origin

- in SM: $\cancel{\bar{e}_L e_R}$ by $SU(2)_L$ & $U(1)_Y$ invariance

introduce the Higgs Yukawa

$$\mathcal{L}_Y = y_e \bar{L}_L \varphi e_R + h.c.$$

$$= \frac{y_e}{\sqrt{2}} \bar{e}_L e_R (v + h)$$

associated field to test this origin

common origin of mass = EWSB scale

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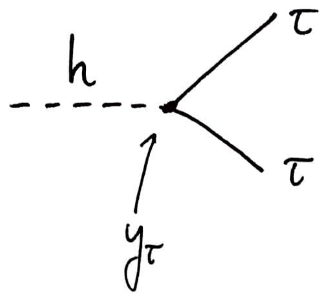
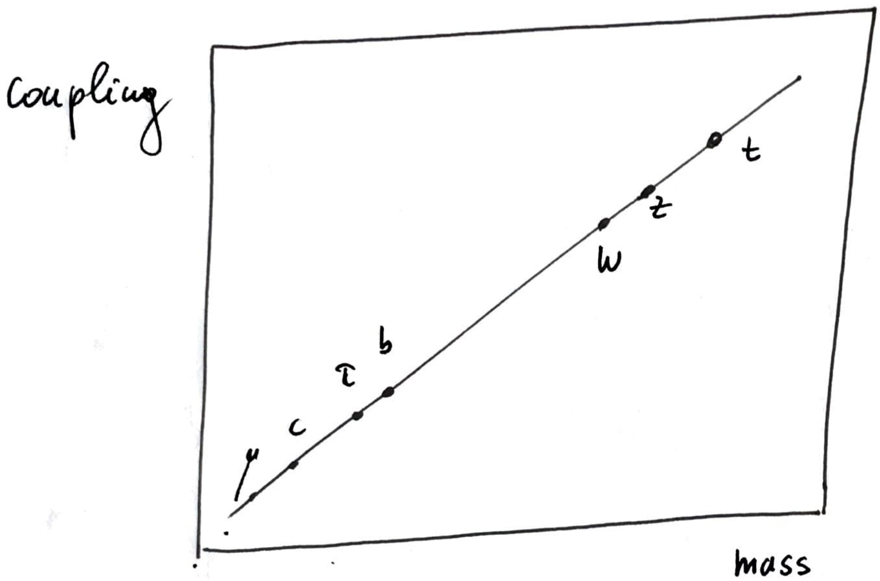
$$m_\tau = \frac{y_\tau v}{\sqrt{2}} \qquad \Gamma_{h\to\tau\tau} = \frac{m_h g^2}{32\pi}\left(\frac{m_\tau}{M_W}\right)^2$$



@ ~ 4σ



However: $\boxed{m_\nu \neq 0}$

due to oscillations

Is there something for neutrinos?

- SM as an EFT, no light particles
- simple UV completions
- complete theories of neutrino mass

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SM as an effective theory

- Weinberg '79 $\quad \mathcal{L} = \mathcal{L}_{SM} + \sum_{n=1}^{\infty} \frac{C_n^{\mathcal{O}}}{\Lambda^{n}} \mathcal{O}^{(n+4)}$

at $n=1$ $\quad d=5$ operators, we have only one.

NB: $\mathcal{L}_{SM}$ conserves B & L accidentally

$$C_1 \mathcal{O}_W^{d=5} = \frac{\tilde{y}}{\Lambda_\nu} (L\varphi)(L\varphi) \qquad \underline{2 \times 3/2 + 2 = 5}$$

What do we mean by this?

$$\boxed{2 \otimes 2 = 3 \oplus 1}$$

Type I

(1) $$\Delta\mathcal{L} = \frac{\tilde{y}}{\Lambda_\nu} \big(\underbrace{L^T i\sigma_2 \varphi}_{(1,0)_F \equiv \text{fermionic singlet}}\big) C \left(\varphi^T i\sigma_2 L\right)$$

$$= \frac{\tilde{y}}{\Lambda_\nu} \left(\nu_L^T \; l_L^T\right) \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} C \left(0 \; \frac{v}{\sqrt{2}}\right) \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$$

$$= \frac{\tilde{y}}{\Lambda_\nu} \left(\nu_L^T \frac{v}{\sqrt{2}}\right) C \left(-\frac{v}{\sqrt{2}} \nu_L\right)$$

$$= -\frac{\tilde{y} v^2}{2\Lambda_\nu} \nu_L^T C \nu_L \quad \Rightarrow \quad \boxed{m_\nu^M = \frac{\tilde{y} v^2}{\Lambda_\nu}}$$

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(II) $\frac{\tilde{y}}{\Lambda_\nu} (L^T C i\sigma_2 \sigma_a \varphi)(\underbrace{\varphi^T i \sigma_2 \sigma_a \varphi}_{(3,2)_B})$ $\Rightarrow$ bosonic triplet

σ_3 gives zero

$\sigma_1 + \sigma_2$: $l_L^T C l_L$ cancels

$\nu_L^T C \nu_L$ sums up

$= -\frac{\tilde{y} v^2}{\Lambda_\nu \cancel{2}} \cdot \cancel{2}\, \nu_L^T C \nu_L$ $\qquad \boxed{m_{\nu H} = \frac{2\tilde{y} v^2}{\Lambda_\nu}}$

(III) $\frac{\tilde{y}}{\Lambda_\nu} (\underbrace{L^T i\sigma_2 \sigma_a \varphi}_{(3,0)_F})(\varphi^T i \sigma_2 \sigma_a L)$ $\Rightarrow$ fermionic triplet

$= + \frac{\tilde{y}}{\Lambda_\nu} \frac{v^2}{2} \nu_L^T C \nu_L$ $\qquad m_{\nu_H} = -\frac{\tilde{y} v^2}{\Lambda_\nu}$

(IV) $\frac{\tilde{y}}{\Lambda_\nu} (L^T C i\sigma_2 L)(\varphi^T i\sigma_2 \varphi)$

$\varphi^T i\sigma_2 \varphi = 0$ $\qquad m_\nu = 0$

$(1,2)_B$

- Physics of σ_w
- UV completions

How to test / probe $\mathcal{O}_W$ & $\tilde{y}$? 15.00

•) $\tilde{y}$ fixed by m_ν & oscillations gap to m_ν^{min}.

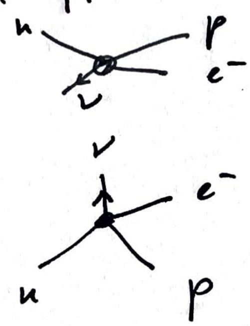
•) LNV

1937 Racah & Furry suggest $0\nu2\beta$

Fermi

β decay

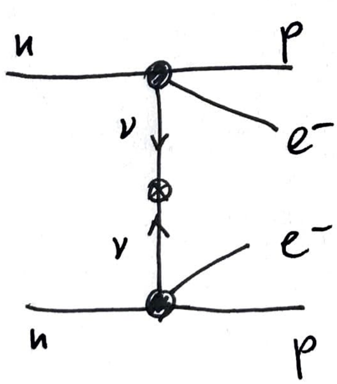
Goeppert-Mayer



$\beta\beta$ decay

$\tau^{Te} = 7.2 \cdot 10^{24}$ yr

Majorana:  $\propto m_\nu^M$



$\Delta L = 0$

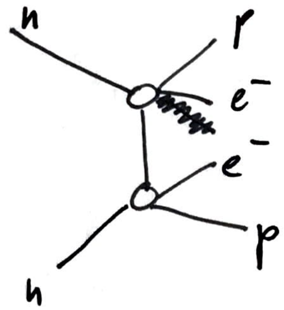
$L = 2 \Rightarrow \Delta L = 2$

$\propto m_\nu^M$

$0\nu2\beta$ decay

• with $\mathcal{O}_W$ we can estimate the rate for

$0\nu 2\beta$



$$6 \cdot \tfrac{3}{2} = 9 \quad d=9 \text{ operator}$$

$$\propto G_F^2 \, m_\nu^{\,1}$$

$$\mathcal{A}_{0\nu\beta\beta} \sim G_F^2 \; \frac{(V_L \, m_\nu \, V_L^T)_{ee}}{p^2}, \qquad (V_L \, m_\nu \, V_L^T)_{ee} = m_\nu^{ee}$$

$$E^{-4} \qquad E^{-1} \quad \Rightarrow \quad \frac{1}{\Lambda^5} \text{ ok for } d=9$$

$$\boxed{\text{Doi, Kotani, Takasugi '85 Prog. Theor. Phys. Suppl. (83) 1}}$$

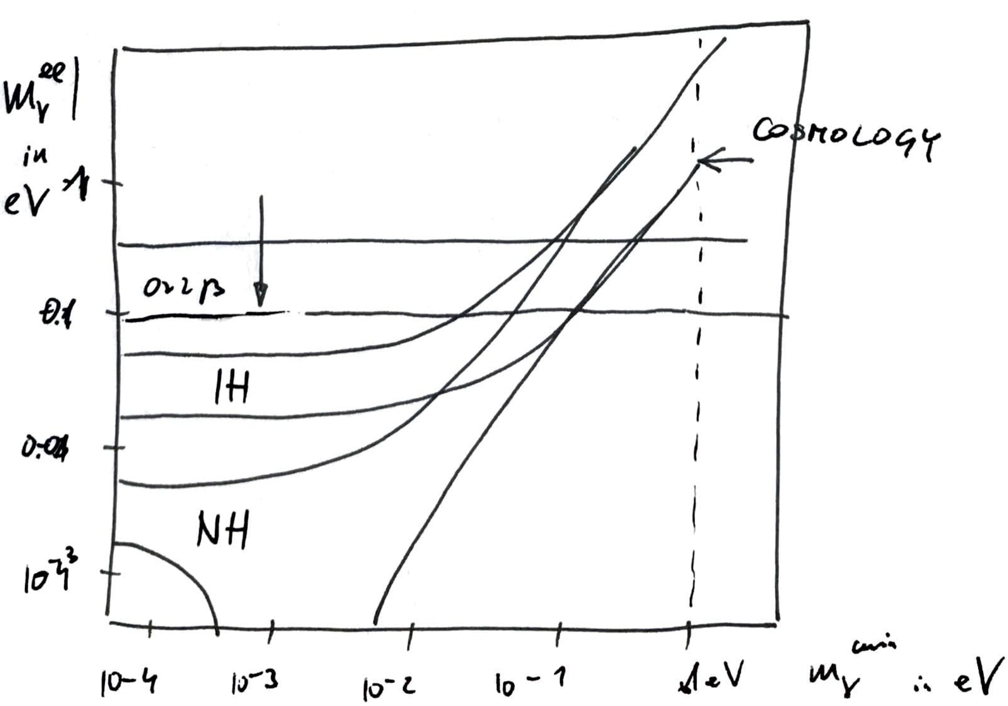
$$\Gamma_{0\nu 2\beta} \sim G \left| \frac{\mathcal{M}}{m_{ec}} \right|^2 \left| m_\nu^{ee} \right|^2$$

G: phase space; $\left|\frac{\mathcal{M}}{m_{ec}}\right|$: NME; $\left|m_\nu^{ee}\right|$: effective Majorana mass

10.5.2016 : KamLAND-Zen in ^{136}Xe : $\tau^{0\nu}_{1/2} > 1.1 \cdot 10^{26}$ yrs

Vissani plot $|m_\nu^{ee}|$ in eV

Many experiments

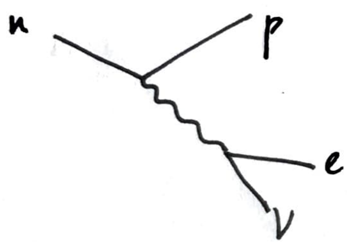


See-saw: Smallness of m_ν related to a new scale $\gg$ EW scale

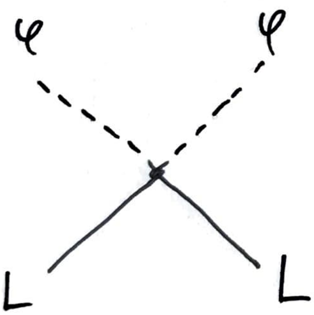
FERMI vs. $\mathcal{O}_W^{d=5}$

$\frac{G_F}{\sqrt{2}} \sim 10^{-5}\,\mathrm{GeV}^{-2} \Rightarrow \frac{1}{\sqrt{G_F}} \sim 300\,\mathrm{GeV}$

$\Downarrow$



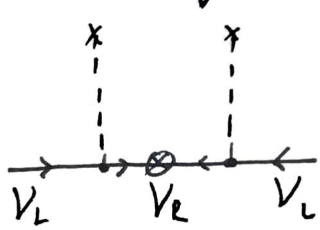
$M_W = 80\,\mathrm{GeV}$



$\frac{\tilde{y} v^2}{\Lambda_\nu} \sim m_\nu \sim 10^{-10}\,\mathrm{GeV}$

$$\Lambda_\nu = \begin{cases} m_D \sim m_t \Rightarrow m_{\nu_R} = 10^{15}\,\mathrm{GeV} \\ m_D \sim m_e \Rightarrow m_{\nu_R} \cong \mathrm{TeV} \end{cases}$$

$\Downarrow$



$m_\nu = -\frac{m_D^2}{m_{\cancel{M}R}}$

COMPLETING the O_W 15.20

(I) $(L^T i\sigma_2 \varphi) \equiv (1,0)_F$ = fermionic singlet

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\nu_R}$$

$$\mathcal{L}_{\nu_R} = -i\frac{1}{2}\bar{\nu}_R \not{\partial} \nu_R - \frac{1}{2}m_M \bar{\nu}_R \nu_R - y_D \bar{\nu}_R \varphi i\sigma_2 L + h.c.$$

$$\mathcal{L}_{mass} = \underset{\substack{\parallel \\ \frac{y_D v}{\sqrt{2}}}}{m_D}(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R) + \frac{m_{\nu_R}}{2}(\bar{\nu}_R \nu_R + h.)$$

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_{\nu_R} \end{pmatrix}$$

One generation: $\operatorname{tr} M_\nu = m_{\nu_R} = m_\nu + m_N \sim m_N$

$\det M_\nu = -m_D^2 = m_\nu \cdot m_N \Rightarrow \boxed{m_\nu = -\frac{m_D^2}{m_{\nu_R}}}$

SEE-SAW

$N = \nu_R + \nu_R^c$ ...heavy Majorana neutrino

$\nu = \nu_L + \nu_L^c$ ... light — " —

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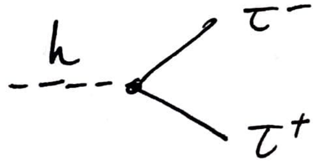
Type I SEE-SAW AMBIGUITY

In the SM : m_τ measured kinematically $\sim 1.7\,\text{GeV}$

$$\mathcal{L}_Y = \underbrace{\frac{y_\tau v}{\sqrt{2}}}_{m_\tau} \bar{\tau}_L \tau_R + \frac{y_\tau}{\sqrt{2}} h \, \bar{\tau}_L \tau_R$$



- only one source of mass, $y_{D\tau}$ predicted unambiguously

a clear prediction for the LHC

$$\Gamma_{h\to\tau\tau} \propto m_\tau^2$$

SEE-SAW

$$M_{\nu N} = \begin{pmatrix} 0 & M_D \\ M_D & M_N \end{pmatrix} \longrightarrow \begin{pmatrix} \overbrace{V_L m_\nu V_L^T}^{M_\nu} & 0 \\ 0 & m_N \end{pmatrix}$$

$$M_\nu \simeq - M_D^T M_N^{-1} M_D$$

$$= - (m_S^{-1/2} M_D)^T (\underbrace{m_S^{-1/2} M_D}_{OS})$$

$S = S^T$

$O O^T = 1$

$$= - S O^T O S = - S^2$$

$\Rightarrow$

$\Rightarrow \boxed{S = i\sqrt{M_\nu}}$ ~ known $\quad O = ?$ free $\in \mathbb{C}$!

We then have

$$m_S^{-1/2} M_D = O \, i \sqrt{M_\nu}$$

$$\boxed{M_D = i\sqrt{m_S} \, O \sqrt{M_\nu}}$$

in 1D ok : $m_D \sim \sqrt{m_N m_\nu}$

i 2gen & more : O ambiguous & possibly large

* This situation is in contrast to the SM where the Yukawa & Higgs interactions are unambiguously predicted. In LR this gets fixed and the LRSM is a complete theory of ν origin.
* Still, one can hope to measure M_D by various means, if ν_R is observed.

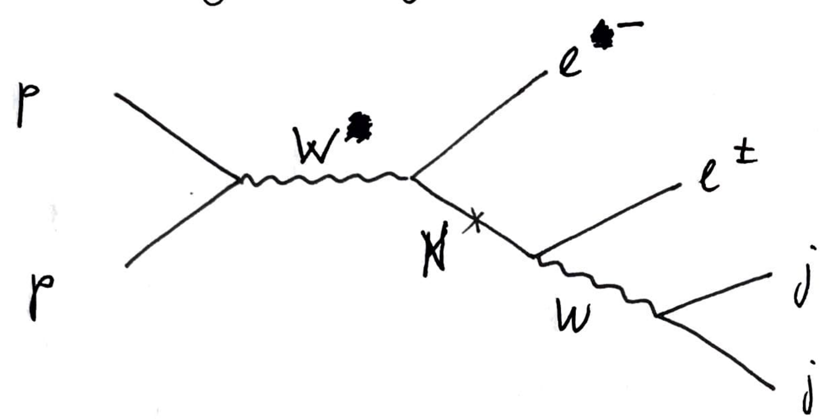
  * meson decays
  * W decays
  * h decays

LNV @ COLLIDERS

'37 Racah & Furry : $0\nu 2\beta$

⋮

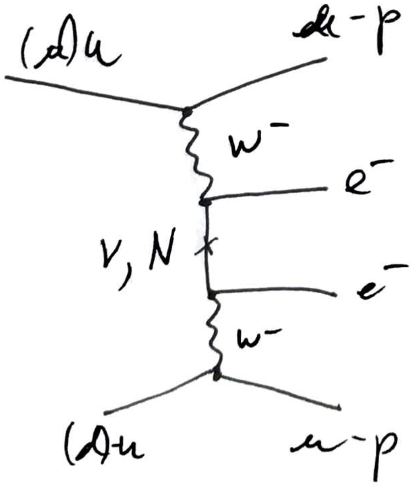
'83 Keung & Senjanović : LNV @ hadronic colliders



$$pp \to W^{(*)} \to e^- e^- jj \Rightarrow \Delta L = 2$$

in type I this goes through the ~~ν~~ $\nu - N$ mixing

$\propto M_D$, analog to $0\nu 2\beta$



* this mixing Θ is typically suppressed

$m_D \sim \sqrt{m_\nu m_N}$

$\Theta \sim \frac{m_D}{m_N} \sim \sqrt{\frac{m_\nu}{m_N}} \sim \sqrt{\frac{10^{-10}}{10}}$

$\underline{\underline{\sim 10^{-6}}}$

$\boxed{\text{Higgs decay?}}$

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Type II SEE-SAW

(11) $(L^T i\sigma_2 \sigma^a L)(\underbrace{\varphi^T i\sigma_2 \sigma^a \varphi}_{\Delta_L^a = (3,2)_B})$

Y=2 triplet "Higgs"

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_\Delta$$

$$\mathcal{L}_\Delta = |D_\mu \Delta|^2 - y_\Delta L^T i\sigma_2 \sigma^a C L \Delta_L^a + V(\;)$$

$$\Delta_L = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \quad \langle \Delta^0 \rangle = v_L$$

$$= \sigma^a \Delta_L^a$$

Then $y_\Delta (\nu_L^T \; \ell_L^T) C \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$

$$= y_\Delta v_\Delta \; \nu_L^T C \nu_L$$

$$\Rightarrow \boxed{M_\nu = Y_\Delta v_\Delta}.$$

the relation $M_\nu = V_L m_\nu V_L^T = Y_\Delta v_\Delta$

is unambiguous and can be tested

directly by producing $\Delta^0, \Delta^+, \Delta^{++}$ at

colliders

$$\text{e.g.} \quad \Delta^{++} \to l_i^+ l_j^+ \propto |(M_\nu)_{ij}|^2$$

In which sense is this see-saw? How

does it relate to $\mathcal{O}_W$ & $\frac{\tilde{y} v^2}{\Lambda_\nu}$?

$$V_{\Delta\varphi} \cong V(h) + \mu\, \varphi^T i\sigma_2 \Delta^* \varphi + m_\Delta \Delta^2$$

for $m_\Delta \gg v$ $\qquad v_\Delta \sim \frac{\mu v^2}{\sqrt{2} m_\Delta^2}$

$$\Rightarrow \mathcal{L}_\nu \sim Y_\Delta \frac{\mu v^2}{\sqrt{2} m_\Delta^2}, \quad \boxed{\Lambda_\nu \sim \frac{m_\Delta^2}{\mu}}$$

* could be tested @ LHC, see later

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TYPE III SEESAW

$$(L^T i\sigma_2 \sigma^a \varphi) \equiv (3,0)_F = T^a_F$$

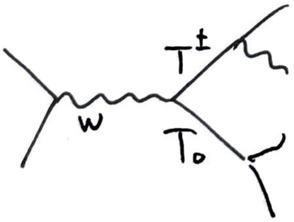
$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{T_F}$$

$$\mathcal{L}_{T_F} = i\bar{T}\not{D}T + y_T L \varphi T + \frac{m_T}{2} T^T C T$$

works like type I, ambiguity is there

$$M_D = i\sqrt{m_T}\, O \sqrt{M_\nu}$$

However $T \in (T^\pm, T^0)$ & can be produced @ LHC



- Since the only coupling that violates N(T) is through M_D (the rest is pair-wise), it is only

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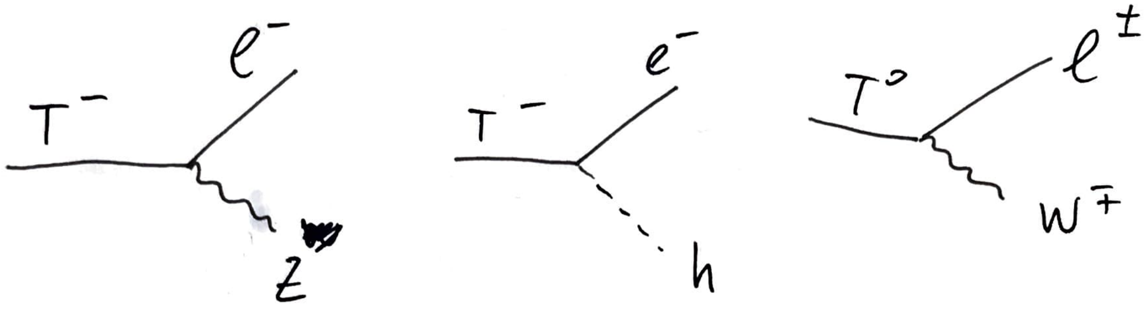
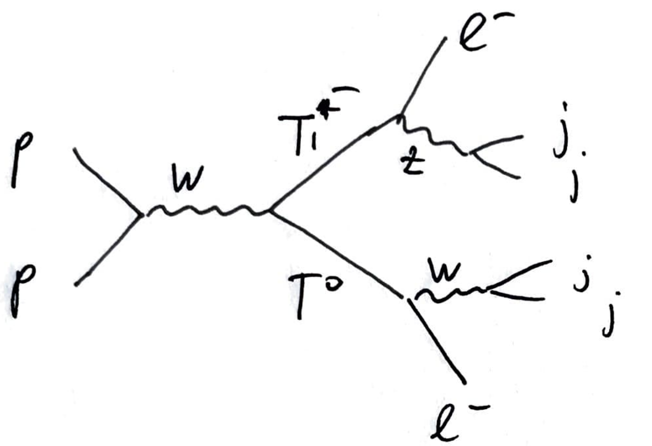
M_J that destabilizes it



e.g.



$\Delta L = 2$

$\boxed{\ell^{\mp}\ell^{\mp} + 4\,\text{jets}}$