$$\frac{\partial \psi}{\partial x^\mu} = i[\psi, H_\mu], \tag{1.199}$$

where

$$H = \int \mathrm{d}^D x \, \Theta = -\frac{\Lambda}{2} \int \mathrm{d}^D x (\partial_\mu \psi^\dagger \partial^\mu \psi - \kappa^2 \psi^\dagger \psi), \tag{1.200}$$

$$H_\mu = \int \mathrm{d}^D x \, \Theta_\mu = -i \int \mathrm{d}^D x \, \psi^\dagger \partial_\mu \psi. \tag{1.201}$$

Using the commutation relations (1.179)–(1.181) we find that eq. (1.198) is equivalent to the field equation (1.67) (the Schrödinger equation). Eq. (1.198) is thus *the Heisenberg equation* for the field operator ψ. We also find that eq. (1.199) gives just the identity $\partial_\mu \psi = \partial_\mu \psi$.

In momentum representation the field operators are expressed in terms of the operators $c(p)$, $c^\dagger(p)$ according to eq. (1.175) and we have

$$H = -\frac{\Lambda}{2} \int \mathrm{d}^D p \, (p^2 - \kappa^2) c^\dagger(p) c(p), \tag{1.202}$$

$$H_\mu = \int \mathrm{d}^D p \, p_\mu c^\dagger(p) c(p). \tag{1.203}$$

The operator H is the Hamiltonian and it generates the τ-evolution, whereas H_μ is the generator of spacetime translations. In particular, H_0 generates translations along the axis x^0 and can be either positive or negative definite.

ENERGY–MOMENTUM OPERATOR

Let us now consider the generator $G(\Sigma)$ defined in eq. (1.170) with $T^\mu{}_\nu$ given in eq. (1.160) in which the classical fields ψ, ψ^* are now replaced by the operators ψ, $\psi^\dagger$. The total energy–momentum P_ν of the field is given by the integration of $T^\mu{}_\nu$ over a space-like hypersurface:

$$P_\nu = \int \mathrm{d}\Sigma_\mu \, T^\mu{}_\nu. \tag{1.204}$$

Instead of P_ν defined in (1.204) it is convenient to introduce

$$\widetilde{P}_\nu = \int \mathrm{d}s \, P_\nu, \tag{1.205}$$

where $\mathrm{d}s$ is a distance element along the direction n^μ which is orthogonal to the hypersurface element $\mathrm{d}\Sigma_\mu$. The latter can be written as $\mathrm{d}\Sigma_\mu = n_\mu \mathrm{d}\Sigma$. Using $\mathrm{d}s\mathrm{d}\Sigma = \mathrm{d}^D x$ and integrating out x^μ in (1.205) we find that τ-dependence disappears and we obtain (see Box 1.1)

$$\widetilde{P}_\nu = \int \mathrm{d}^D p \frac{\Lambda}{2} (n_\mu p^\mu) \, p_\nu (c^\dagger(p) c(p) + c(p) c^\dagger(p)). \tag{1.206}$$