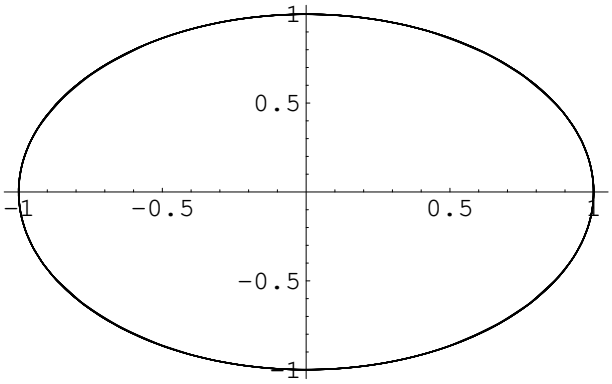
```
sol = NDSolve[{x''[t] == -x[t] - 0.1 (4 x[t]^3 y[t]^2 - 2 x[t] y[t]^4),
    y''[t] == -y[t] + 0.1 (2 x[t]^4 y[t] - 4 x[t]^2 y[t]^3),
    x'[0] == 1, y'[0] == 0, x[0] == 0, y[0] == 1}, {x, y},
  {t, 70}]
```

```
{{x → InterpolatingFunction[{{0., 70.}}, <>],
  y → InterpolatingFunction[{{0., 70.}}, <>]}}
```

```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol],
 {t, 0, 70}, PlotRange -> All]
```




```
- Graphics -
```

```
sol = NDSolve[{x''[t] == -x[t] - 0.1 (4 x[t]^3 y[t]^2 - 2 x[t] y[t]^4),
    y''[t] == -y[t] + 0.1 (2 x[t]^4 y[t] - 4 x[t]^2 y[t]^3),
    x'[0] == 1, y'[0] == 0, x[0] == 0, y[0] == 1}, {x, y},
  {t, 10}]
```

```
x[0] /. Out[33]
```

```
{0.}
```

```
x[0]
```

```
x[0]
```

```
x[0] /. sol
```

```
{0.}
```

```
x'[0]^2 - y'[0]^2 /. sol
```

```
{1.}
```

```
{1 / 2} (x'[0]^2 - y'[0]^2) +
  {1 / 2} (x[0]^2 - y[0]^2) + 0.1 ( - x[0]^2 y[0]^4 + x[0]^4 y[0]^2) /.
 sol
```

```
{{0.}}
```

```
{1 / 2} (x'[1]^2 - y'[1]^2) +
  {1 / 2} (x[1]^2 - y[1]^2) + 0.1 ( - x[1]^2 y[1]^4 + x[1]^4 y[1]^2) /.
 sol
```

```
{{-1.86192 × 10^-6}}
```

```
{1 / 2} (x'[10]^2 - y'[10]^2) +
  {1 / 2} (x[10]^2 - y[10]^2) + 0.1 ( - x[10]^2 y[10]^4 + x[10]^4 y[10]^2) /.
 sol
```

$$\{\{2.39552 \times 10^{-7}\}\}$$

```
t = 6
```

6

```
{1 / 2} (x'[t]^2 - y'[t]^2) +
  {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
 sol
```

$$\{\{-7.31003 \times 10^{-7}\}\}$$

```
H = {1 / 2} (x'[t]^2 - y'[t]^2) +
    {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
  sol
```

$$\{\{-1.2618 \times 10^{-7}\}\}$$

```
t = 60
```

60

```
H =.
```

```
H
```

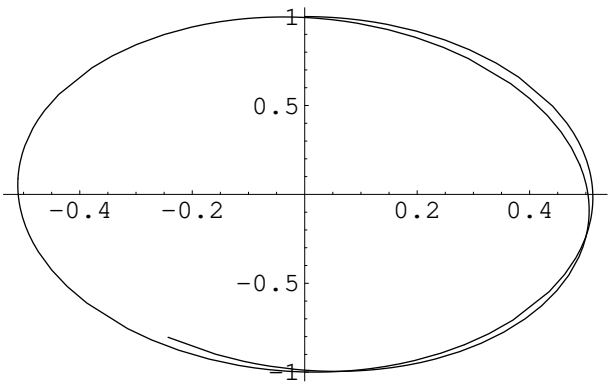
H

```
t =.
```

```
sol = NDSolve[{x''[t] == -x[t] - 0.1 (4 x[t]^3 y[t]^2 - 2 x[t] y[t]^4),
    y''[t] == -y[t] + 0.1 (2 x[t]^4 y[t] - 4 x[t]^2 y[t]^3),
    x'[0] == 0.5, y'[0] == 0, x[0] == 0, y[0] == 1}, {x, y},
  {t, 70}]
```

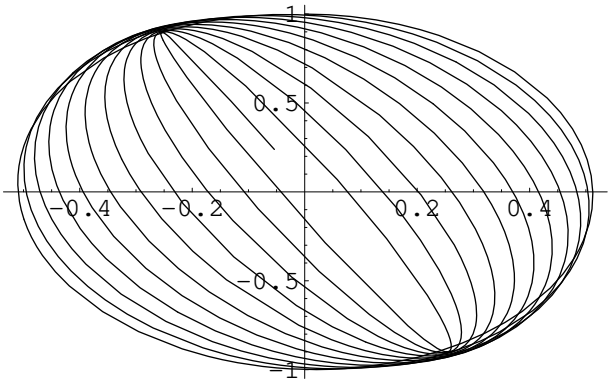
```
{{x → InterpolatingFunction[{{0., 70.}}, <>],
  y → InterpolatingFunction[{{0., 70.}}, <>]}}
```

```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol],
 {t, 0, 10}, PlotRange -> All]
```



- Graphics -

```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol],
 {t, 0, 70}, PlotRange -> All]
```




```
- Graphics -
```

```
t = 0
```

```
0
```

```
H = {1 / 2} (x'[t]^2 - y'[t]^2) +
   {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
  sol
```

```
{{-0.375}}
```

```
{{-0.375`}}
```

```
t = 10
```

```
10
```

```
H = {1 / 2} (x'[t]^2 - y'[t]^2) +
   {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
  sol
```

```
{{-0.374998}}
```

```
{{-0.374997583714709081`}}
```

```
t = 70
```

```
70
```

```
H = {1 / 2} (x'[t]^2 - y'[t]^2) +
   {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
  sol
```

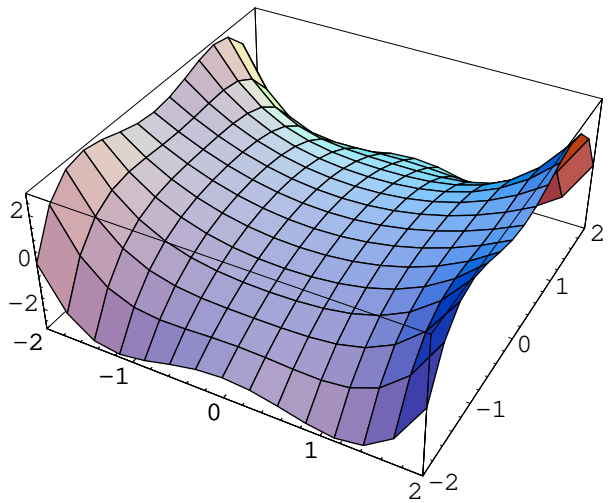
```
{{-0.37498}}
```

```
-0.374980136245943773`
```

```
V = {1 / 2} (x^2 - y^2) + k ( - x^2 y^4 + x^4 y^2)
```

$$\left\{\frac{1}{2}\ (x^2 - y^2) + 0.1\ (x^4\ y^2 - x^2\ y^4)\right\}$$

```
Plot3D[1/2 (x^2 - y^2) + 0.1 (x^4 y^2 - x^2 y^4), {x, -2, 2}, {y, -2, 2}]
```



- SurfaceGraphics -

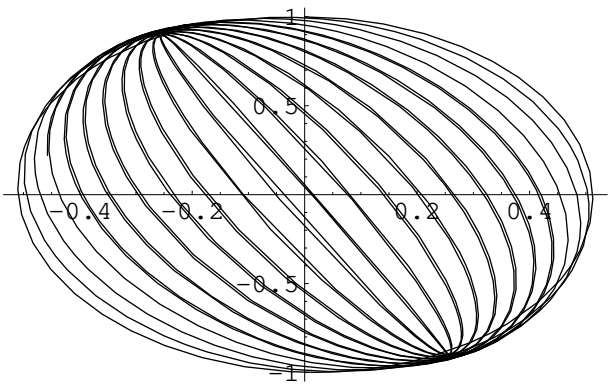
```
sol = NDSolve[{x''[t] == -x[t] - 0.1 (4 x[t]^3 y[t]^2 - 2 x[t] y[t]^4),
    y''[t] == -y[t] + 0.1 (2 x[t]^4 y[t] - 4 x[t]^2 y[t]^3),
    x'[0] == 0.5, y'[0] == 0, x[0] == 0, y[0] == 1}, {x, y},
  {t, 200}]
```

```
NDSolve::mxst : Maximum number of 1000 steps reached at the point t == 120.004899539812903`.
```

```
{{x → InterpolatingFunction[{{0., 120.005}}, <>],
  y → InterpolatingFunction[{{0., 120.005}}, <>]}}
```

```
t =.
```

```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol],
 {t, 0, 120}, PlotRange -> All]
```



- Graphics -

```
t = 120
```

```
120
```

```
H = {1 / 2} (x'[t]^2 - y'[t]^2) +
    {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
  sol
```

```
{{-0.374964}}
```

```
x'[120] /. sol
```

```
{0.0233369}
```

```
y'[120] /. sol
```

```
{-0.954547}
```

```
x[120] /. sol
```

```
{-0.457371}
```

```
y[120] /. sol
```

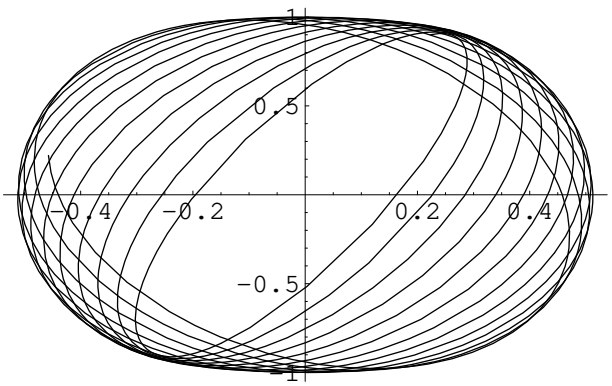
```
{0.220972}
```

```
sol = NDSolve[{x''[t] == -x[t] - 0.1 (4 x[t]^3 y[t]^2 - 2 x[t] y[t]^4),
   y''[t] == -y[t] + 0.1 (2 x[t]^4 y[t] - 4 x[t]^2 y[t]^3),
   x'[120] == 0.0233369307654103286`, y'[120] == -0.954547115960078329`,
   x[120] == -0.457371414236554763`, y[120] == 0.220972056664788096`},
  {x, y}, {t, 120, 200}]
```

```
{{x → InterpolatingFunction[{{120., 200.}}, <>],
  y → InterpolatingFunction[{{120., 200.}}, <>]}}
```

```
t =.
```

```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol],
 {t, 120, 200}, PlotRange -> All]
```




```
- Graphics -
```

```
t = 200
```

```
200
```

```
H = {1 / 2} (x'[t]^2 - y'[t]^2) +
   {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
  sol
```

```
{{-0.374982}}
```

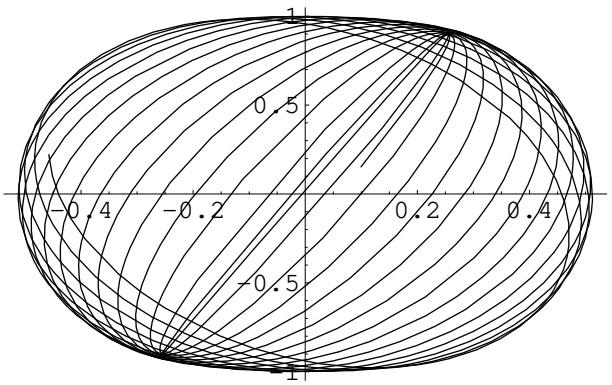
```
t =.
```

```
sol = NDSolve[{x''[t] == -x[t] - 0.1 (4 x[t]^3 y[t]^2 - 2 x[t] y[t]^4),
    y''[t] == -y[t] + 0.1 (2 x[t]^4 y[t] - 4 x[t]^2 y[t]^3),
    x'[120] == 0.0233369307654103286`, y'[120] == -0.954547115960078329`,
    x[120] == -0.457371414236554763`, y[120] == 0.220972056664788096`},
  {x, y}, {t, 120, 300}]
```

```
NDSolve::mxst : Maximum number of 1000 steps reached at the point t == 221.34236502898279`.
```

```
{{x → InterpolatingFunction[{{120., 221.342}}, <>],
  y → InterpolatingFunction[{{120., 221.342}}, <>]}}
```

```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol],
 {t, 120, 220}, PlotRange -> All]
```



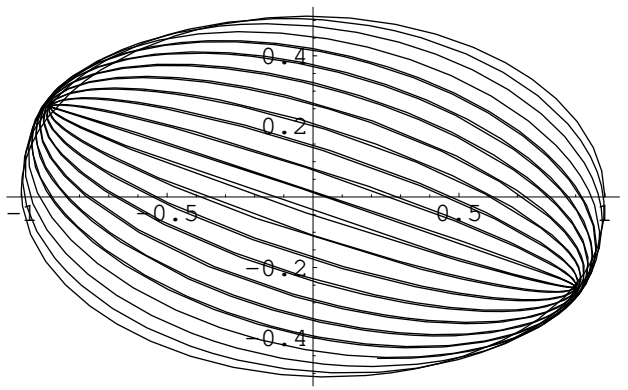
- Graphics -

```
sol = NDSolve[{x''[t] == -x[t] - 0.1 (4 x[t]^3 y[t]^2 - 2 x[t] y[t]^4),
    y''[t] == -y[t] + 0.1 (2 x[t]^4 y[t] - 4 x[t]^2 y[t]^3),
    x'[0] == 0, y'[0] == 0.5, x[0] == 1, y[0] == 0}, {x, y},
  {t, 200}]
```

```
NDSolve::mxst : Maximum number of 1000 steps reached at the point t == 120.004899539812903`.
```

```
{{x → InterpolatingFunction[{{0., 120.005}}, <>],
  y → InterpolatingFunction[{{0., 120.005}}, <>]}}
```

```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol],
 {t, 0, 120}, PlotRange -> All]
```



- Graphics -

```
t = 0
```

```
0
```

```
H = {1 / 2} (x'[t]^2 - y'[t]^2) +
    {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
  sol
```

```
{{0.375}}
```

```
t = 100
```

```
100
```

```
H = {1 / 2} (x'[t]^2 - y'[t]^2) +
    {1 / 2} (x[t]^2 - y[t]^2) + 0.1 ( - x[t]^2 y[t]^4 + x[t]^4 y[t]^2) /.
  sol
```

```
{{0.374964}}
```

```
x'[120] /. sol
```

```
{-0.954547}
```

```
y'[120] /. sol
```

```
{0.0233369}
```

```
x[120] /. sol
```

```
{0.220972}
```

```
y[120] /. sol
```

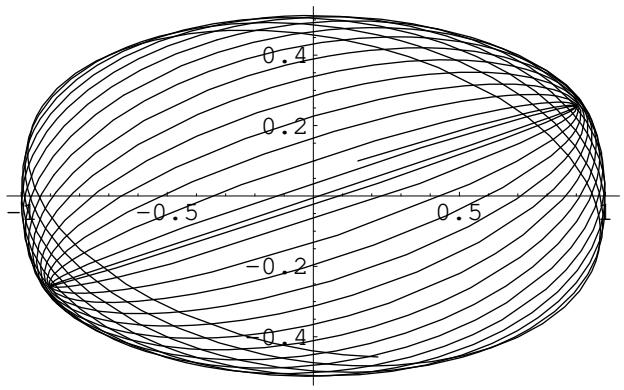
```
{-0.457371}
```

```
sol = NDSolve[{x''[t] == -x[t] - 0.1 (4 x[t]^3 y[t]^2 - 2 x[t] y[t]^4),
   y''[t] == -y[t] + 0.1 (2 x[t]^4 y[t] - 4 x[t]^2 y[t]^3),
   x'[120] == -0.954547115960078329`, y'[120] == 0.0233369307654103286`,
   x[120] == 0.220972056664788096`, y[120] == -0.457371414236554763`},
  {x, y}, {t, 120, 220}]
```

```
{{x → InterpolatingFunction[{{120., 220.}}, <>],
  y → InterpolatingFunction[{{120., 220.}}, <>]}}
```

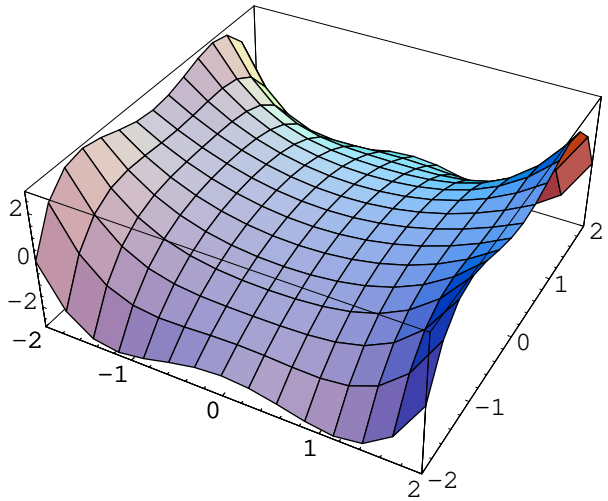
```
t =.
```

```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol],
 {t, 120, 220}, PlotRange -> All]
```



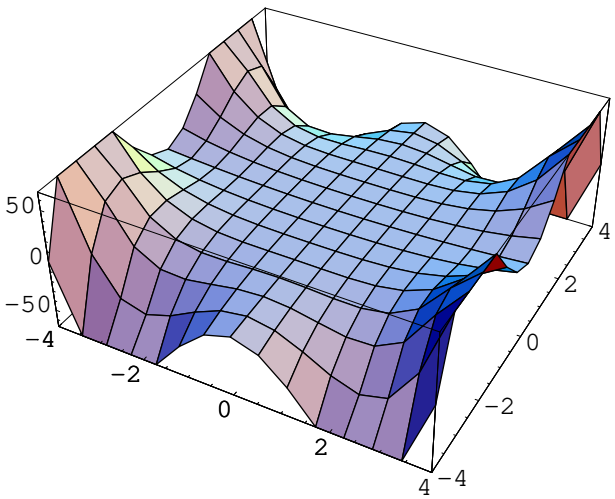

```
- Graphics -
```

```
Plot3D[1/2 (x^2 - y^2) + 0.1 (x^4 y^2 - x^2 y^4), {x, -2, 2}, {y, -2, 2}]
```



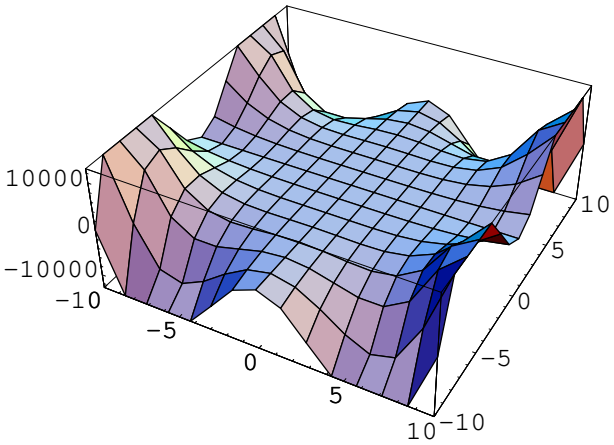

```
- SurfaceGraphics -
```

```
Plot3D[1/2 (x^2 - y^2) + 0.1 (x^4 y^2 - x^2 y^4), {x, -4, 4}, {y, -4, 4}]
```




```
- SurfaceGraphics -
```

```
Plot3D[1/2 (x^2 - y^2) + 0.1 (x^4 y^2 - x^2 y^4), {x, -10, 10}, {y, -10, 10}]
```




```
- SurfaceGraphics -
```