IOP Publishing European Journal of Physics
Eur. J. Phys. **37** (2016) 065201 (9pp) doi:10.1088/0143-0807/37/6/065201

Breaking Newton's third law: electromagnetic instances

Fabiana B Kneubil[1,2]

[1] Department of Science Education, University of Copenhagen, Denmark
[2] Department of Physics, Federal University of Santa Catarina, Florianópolis, Brazil

E-mail: fkneubil@gmail.com

Received 8 March 2016, revised 14 July 2016
Accepted for publication 20 July 2016
Published 17 August 2016

Abstract
In this work, three instances are discussed within electromagnetism which highlight failures in the validity of Newton's third law, all of them related to moving charged particles. It is well known that electromagnetic theory paved the way for relativity and that it disclosed new phenomena which were not compatible with the laws of mechanics. However, even if widely known in its generality, this issue is not clearly approached in introductory textbooks and it is difficult for students to perceive by themselves. Three explicit concrete situations involving the breaking of Newton's third law are presented in this paper, together with a didactical procedure to construct graphically the configurations of electric field lines, which allow pictures produced by interactive radiation simulators available in websites to be better understood.

Keywords: radiation, Newton's third law, electric field lines, limit of classical mechanics, accelerated charge

(Some figures may appear in colour only in the online journal)

1. Introduction

Classical mechanics, electromagnetism and relativity are important theories taught in introductory courses of physics and, in general, they appear separately in textbooks. This kind of division of physical knowledge is mainly due to teaching requirements and the textbook culture tends to be rather stable. In basic teaching at university level, selected topics of physics are placed into didactical sectors, which are quite relevant for allowing students to get acquainted with the various particular and unusual modes of thinking associated with each branch of physics.

In particular, the electromagnetic theory, in spite of its rather important intrinsic content, can be also considered as a transition from classical mechanics to relativity, since phenomena involving electromagnetic waves pushed the restructuring of physics and fostered new relationships between mass, energy and momentum. In this and other ways, electromagnetism began to deconstruct classical mechanics and, hence, paved the way for relativity. Nevertheless, important as it is, this issue is not explicitly approached in either lectures or textbooks and tends to reach students just as a kind of vague and distant lore.

In this paper, three instances are discussed, involving charges in both uniform and accelerated motions, which produce clear evidence that Newton's third law is not universal. Although these situations are usually discussed in courses of electromagnetism, their connections with the limitations of Newton's third laws are not. The concrete cases presented here represent just small deviations from the usual tracks followed by courses on electromagnetism and may prove to be important in broadening the perception that students have of physical knowledge and its theories.

2. Newton's third law—uniform motion

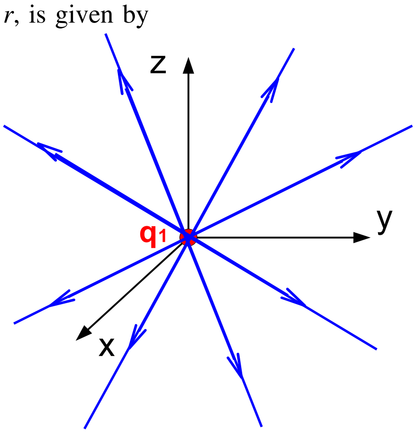
2.1. Charges at rest

Considering a point-like charge q_1 at rest, located at the origin of the coordinate system, the electric field at a point P located at a point $\vec{r}$ is given by

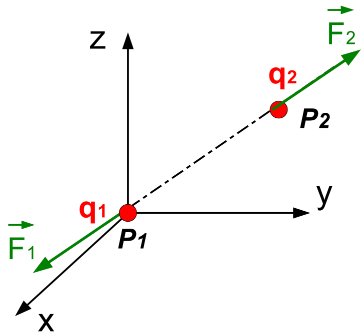
$$\vec{E}_1 = \frac{q_1}{4\,\pi\,\epsilon_o}\,\frac{1}{r^2}\,\hat{r}, \tag{2.1}$$

where $\hat{r} = \vec{r}/r$ and $4\pi\epsilon_o$ is a constant. An important feature of this *Coulomb* field is its dependence on the inverse-distance squared and it is traditionally represented by means of field-lines. In three-dimensional space, these lines are distributed uniformly in all directions and, for a positive charge, they can be represented as in figure 1(a).

If a second charge q_2 is placed close to it, there will be an interaction between the two charges and a force arises. Coulomb's law states that, in empty space, the modulus of the electric force acting between two point-like charges at rest q_1 and q_2, separated by a distance r, is given by



(a) charge q_1 at rest



(b) pair of forces between q_1 and q_2

Figure 1. Field of charge at rest and electric forces.

$$|\vec{F}| = \frac{q_1\, q_2}{4\,\pi\,\epsilon_0}\,\frac{1}{r^2}. \tag{2.2}$$

This result represents, in fact, the moduli of two different forces, as indicated in figure 1(b), where F_1 is *exerted* by charge q_2 and *felt* by q_1 and F_2 is *exerted* by q_1 and *felt* by q_2. Their moduli are equal, $|\vec{F}_1| = |\vec{F}_2|$, and given by equation (2.2). As this interaction is mutual and ascribed to electric fields, one writes

$$\vec{F}_1 \; = q_1\;\vec{E}_2 \qquad \text{and} \qquad \vec{F}_2 \; = q_2\;\vec{E}_1. \tag{2.3}$$

As the forces $\vec{F}_1$ and $\vec{F}_2$ have the same moduli and opposite directions, one has $\vec{F}_1 = -\vec{F}_2$, indicating that the Coulomb forces corroborates the Newtonian *action–reaction principle*, whereby forces always exist in opposite pairs.

2.2. Charges in uniform motion

It is well known that if a charge moves with constant velocity, the field-lines deviate from spherical symmetry. The new configuration can be derived rigorously from Maxwell's equations and, for a given velocity $\vec{v}$, the intensity of field produced by a point-like charge q_1 at a point P is given by

$$\vec{E}_1(r,\,\theta) \;=\; \frac{q_1}{4\,\pi\,\epsilon_o\,r^2}\,\frac{\left[1-\frac{v^2}{c^2}\right]}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2}}\,\hat{r}, \tag{2.4}$$

where $\vec{r}$ describes the positions of P relative to the charge and θ is the angle between $\vec{r}$ and $\vec{v}$ [1]. This result reduces to equation (2.1) for $v \to 0$. This field is no longer spherically symmetric, but still radial and, around the plane perpendicular to the movement, it becomes more intense than the corresponding Coulomb field. This means that the field lines become more concentrated in that region. For example, if the charge moves with constant velocity v along the y-axis, the lines of figure 1(a) tilt, approximating the xz-plane and become more concentrated there [2]. This feature of the field lines can be understood with the help of a pictorial image, in which space would be a kind of uniform net, similar to a graph paper. Considering just a few lines, figure 2(a) shows both this background net and the lines in the yz and xz planes for a charge at rest. For a moving charge, the net contracts along the y-axis and the same lines acquire a higher inclination, as discussed in [3] and shown in figure 2(b).

The field lines of a charge moving with velocity $\vec{v}$ along the y-axis are shown in figure 3(a) and its value at point P is given by equation (2.4). Placing a second charge q_2 at point P, keeping it at rest, one has the situation given in figure 3(b). Again, calling $\vec{F}_1$ the force it exerts on q_1 and $\vec{F}_2$, the force felt by q_2 owing to q_1, one has

$$\begin{aligned} |\vec{F}_1| &= q_1\,\frac{q_2}{4\,\pi\,\epsilon_o\,r^2}, \\ |\vec{F}_2| &= q_2\,\frac{q_1}{4\,\pi\,\epsilon_o\,r^2}\,\frac{\left[1-\frac{v^2}{c^2}\right]}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2}}. \end{aligned} \tag{2.5}$$

These results show that $F_1 \neq F_2$ and hence Newton's third law fails to hold in this instance. In spite of its importance, this result may not look too impressive to students, due to the fact that just the moduli of these forces are different. Cases involving different directions, discussed in the sequence, may be more striking.

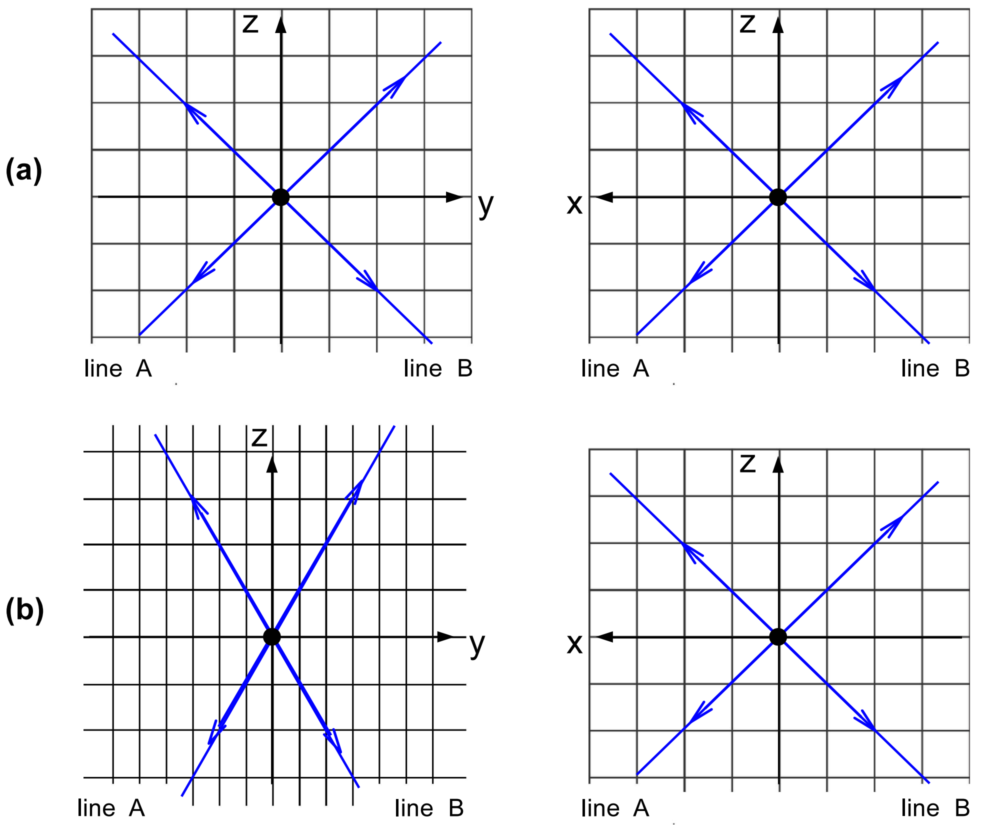


Figure 2. Lines of a moving charge.

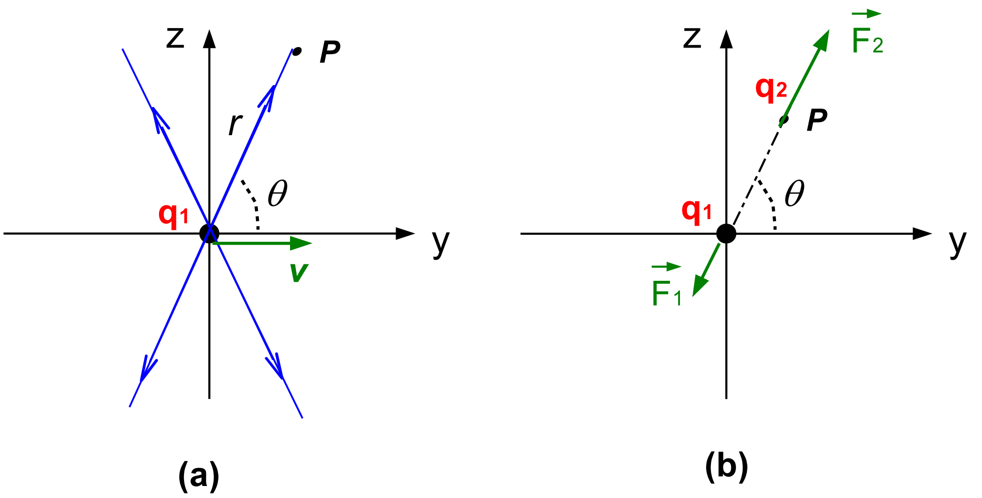


Figure 3. Field of moving charge and electric forces.

3. Accelerated charges

When a charge suffers an acceleration, the electric field lines become distorted and are no longer straight lines. Accelerations correspond to variations of the velocity vector and, in pedagogical situations, it is convenient to discuss separately changes in either its modulus or its direction. In these situations, the breaking of the parallelism in the forces associated with an interacting pair of charges provides clear-cut instances for addressing the limitations of Newton's third law.

3.1. Change in velocity modulus

The features of acceleration relevant for this discussion are mostly qualitative and can be discussed employing field lines only. This allows one to avoid the complications associated with formalism.

The electric field lines of a charge which had the modulus of its velocity changed has been discussed by Purcell [3]. One takes a charge in uniform motion along the y-axis and assumes that it hits an obstacle, which makes it stop [4]. At the instant the charge begins to stop, this information begins to propagate all over space, with the speed of light. This information is carried by a spherical *bubble*, whose radius increases with the speed of light c. Some time later the charge reaches rest and a second bubble begins to propagate, informing all points around the charge that it stopped completely. At any instant later, space becomes divided into three regions, as shown in figure 4. Region R_{I}, internal to the smaller bubble, corresponds to all points informed that the charge is at rest, region R_{II} corresponds to points not reached by the first information bubble, whereas R_{III} is a transition region, associated with the fact that the deceleration process is not instantaneous. This last region lies between two spherical surfaces, with radii $c\ t_F$ and $c\ t_I$, where $t_F - t_I$ is the time interval of the deceleration process.

The existence of these three regions allows one to sketch the field lines. All points belonging to R_{I} are informed that charge is at rest and the field inside the first sphere is electrostatic. In R_{II}, the field is that of a charge with uniform motion, since points there are not informed that an acceleration has occurred. For this reason, field lines in R_{II} point to a spot ahead of charge, which would be the position of the charge in the absence of acceleration. In order to understand what happens with lines in R_{III}, one resorts to Gauss' law, which states that the field lines may only spring from or die in charges. As there are no charges in R_{III}, field-lines must be continuous there. The field lines of a charge which was moving with velocity $\vec{v}$ and was brought to rest are represented in figure 5(a).

The borders of R_{III} are spherical surfaces, with radii increasing with the speed of light. Inside this region, the electric field has important components orthogonal to the radial direction, which are carried in the expansion process. The change of the electric field in time,

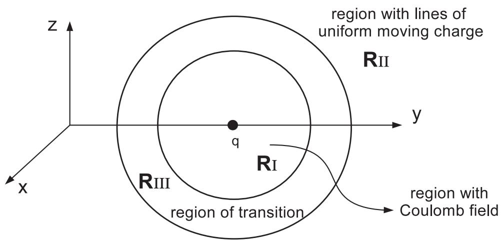


Figure 4. Regions of space after the charge stop.

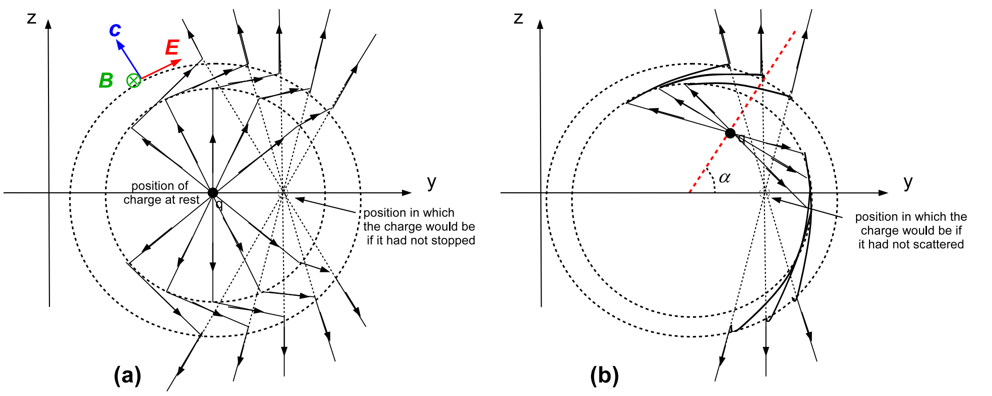


Figure 5. Field lines of an accelerated charge.

caused by the movement of R_{III}, gives rise to a magnetic field, according to the Ampère–Maxwell law, which is perpendicular to the electric field. These are features of an electromagnetic wave existing inside R_{III}.

3.2. Change in velocity direction

Acceleration associated with changes in the direction of the velocity is present in both the classical hydrogen atom and in the scattering of charged particles by heavy targets. A particularly important instance of the latter kind is Rutherford scattering. It is quite common in scattering of charged particles, which the direction of velocity is modified. In figure 5(b), the electric field lines are shown for a charged particle which was initially moving along the y-axis and deflected afterwards by α, moving along the dashed red line.

In general, accelerating charges radiate and the qualitative description presented here rely on deformations of field lines which propagate inside expanding bubbles. These deformations are generated by changes in the velocity of charged particles. In these examples, just single pulse waves were considered. If one wants to create a monochromatic electromagnetic wave, with a well defined frequency, the acceleration of the charge must be periodic, as in the simple harmonic motion.

3.3. Radiation simulator

From a didactical point of view, simulators in physics lessons can both help the visualization of the situations discussed above and allow the inclusion of many other instances. For example, the simulator *Radiation 2D*, which displays the time development of electric field lines in time, was constructed by means of a mathematical method in agreement with electromagnetic theory [5]. Similar versions can be found in specialized sites[3] and can be used in order to clarify this subject. In figure 6, it is shown the field lines of a moving charge brought to rest, as discussed in section 3.1. It is interesting to follow the direction of the lines both inside and outside the circle. As in figure 5(a), field lines inside the bubble are radial and point to the center, where the charge is, whereas the external lines point to the right of the charge.

[3] For example, the website *PhET interactive simulations*: https://phet.colorado.edu/.

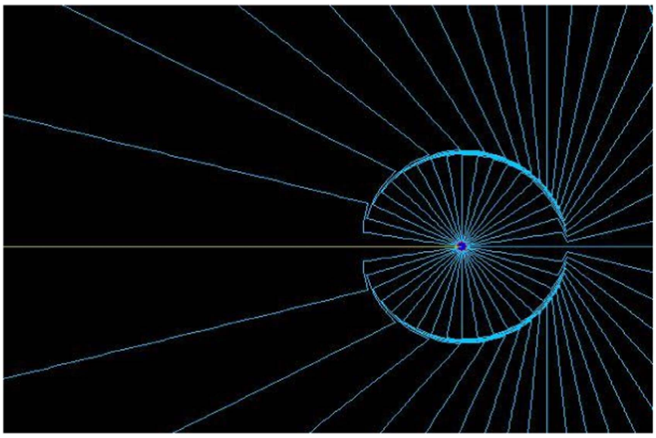


Figure 6. Simulator radiation 2D.

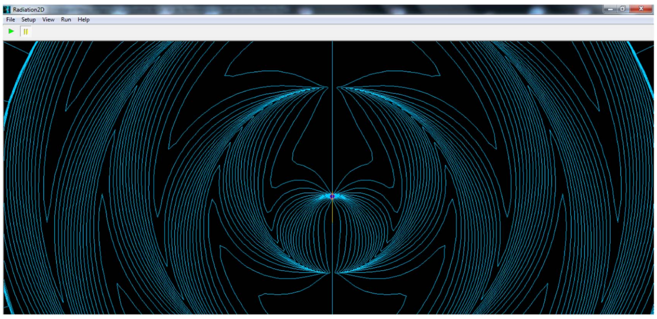


Figure 7. Dipole oscillation.

This simulator is very useful for showing the deformation of electric field lines because it allows the initial velocity of charge to be chosen at will. In general, it is also possible to specify the trajectory of the charge, as *line*, *dipole oscillation*, figure 7, *circle*, figure 8, and others.

4. Newton's third law—accelerated motion

In the case of accelerated charges, it is easy to visualize the breaking of Newton's third law, owing to the fact that one deals with field lines which are not straight. This can be seen, for instance, by going back to the situation described in figure 5(a) and taking a second charge q_2, at rest, placed at point P in region R_{II}, as in figure 9(a). The electric forces on q_1 and q_2 must be parallel to the fields in the points they are located at. Therefore, $\vec{F}_2$, acting on q_2 must be parallel to $\vec{E}_1$, due to q_1. Similarly, $\vec{F}_1$, on q_1 must be parallel to the $\vec{E}_2$ due to q_2. As q_2 is at rest, $\vec{E}_2$ points radially out of q_2. On the other hand, the line associated with $\vec{E}_1$ is not straight, and one has the situation shown in figure 9(b), where $\vec{F}_1 \neq -\vec{F}_2$. This conclusion can be extended to all electric interaction forces between pairs of charges, when one of them is at rest and the other accelerated. These forces *are not* action–reaction pairs.

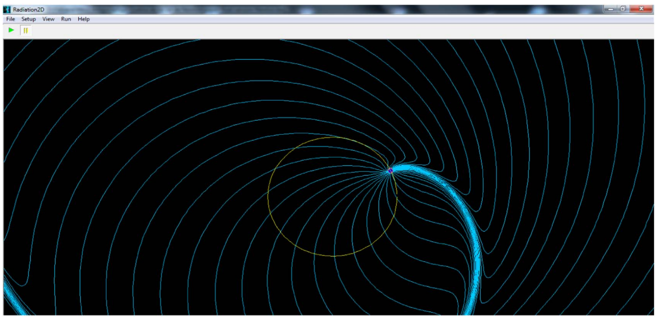


Figure 8. Circular trajectory.

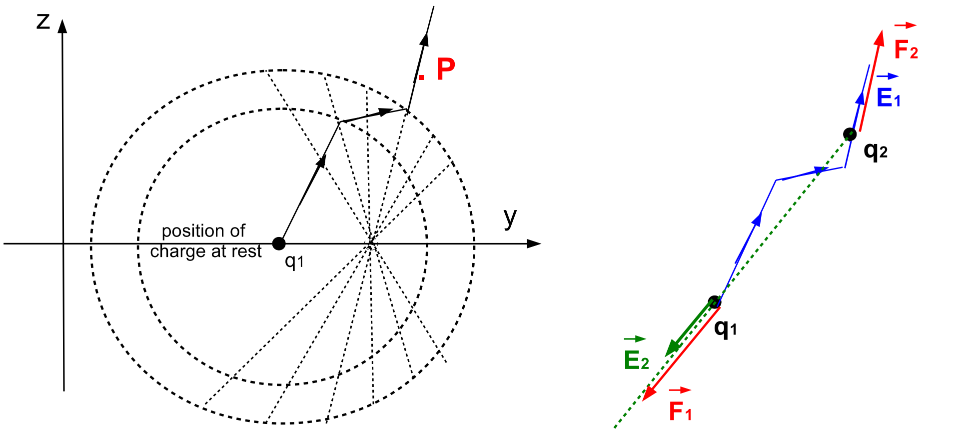


Figure 9. Electric field of accelerated charge and interaction forces.

5. Concluding remarks

Some cases of electromagnetic interactions between two point-like charges were discussed, in order to produce instances in which Newton's third law is not valid. This and other limitations of classical mechanics played an important role in the development of physics and in the transition to relativity. In the teaching context, the discussion presented here may pave the way to more profound discussions to be provided to students. If, on the one hand, the problems of Newton's third law displayed here imply a kind of rupture, on the other, Newton's underlying motivation for that law, namely momentum conservation, has survived. In modern terms, momentum conservation is associated to the features of space known as *homogeneity*, which is rather fundamental. In the restricted context of classical mechanics, Newton's third law and the conservation of momentum are equivalent. In the context of electromagnetism, however, this equivalence breaks and both ideas decouple. Although Newton's third law does not apply to some situations, the conservation of momentum still prevails.

In the examples discussed here, the forces between two charges are not action–reaction pairs and, consistently, the sum of their momenta is not conserved. In order to restore momentum conservation, a third element, the wave, must be considered. As electromagnetic waves also carry momentum, their contributions must also be included in the evaluation of the total momentum of a system.

Acknowledgments

I am very grateful for discussions with M R Robilotta on the content of this paper. I also thank CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior—Brazilian Agency) for financial support (BEX 0816/15-9).

References

[1] Bechara M J, Duarte J L M, Robilotta M R and Salém S 2014 *Física 4, Lecture Notes* (University of São Paulo)
[2] Schroeder D V 1999 Magnetism, radiation, and relativity *Supplementary Notes for a Calculus-Based Introductory Physics Course* available at http://physics.weber.edu/schroeder/mrr/MRRnotes.pdf
[3] Purcell E M *Electricity and Magnetism—Berkeley Physics Course* vol 2 2nd edn (New York: McGraw-Hill) ch 5.6
[4] Ota M I N 1985 Electromagnetism and relativity *MSc thesis* University of São Paulo, Brazil (in Portuguese)
[5] Shintake T 2003 Real-time animation of synchrotron radiation *Nucl. Instrum. Methods Phys. Res.* A **507** 89–92