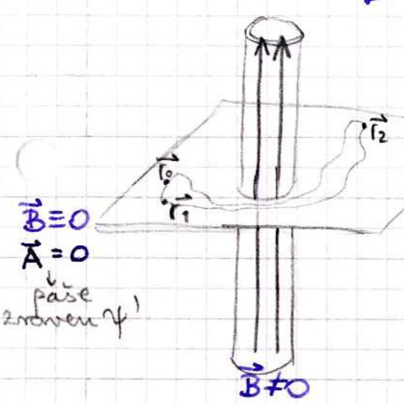
← Le znotraj valja $\vec{B} \neq 0$.



$$\psi(\vec{r}_1, t) = \psi'(\vec{r}_1, t)\, e^{i\frac{e}{\hbar}\int_{\vec{r}_0}^{\vec{r}_1} \vec{A}\, d\vec{s}}$$

ker delec ne more vedet ali je v valju $\vec{B}$ ali ne.

naredimo zanko okrog valja: $$\psi(\vec{r}_1, t) = \psi'(\vec{r}_1, t)\, \underbrace{e^{i\frac{e}{\hbar}\oint \vec{A}\, d\vec{s}}}_{\equiv 1} = \psi'(\vec{r}_1, t)$$

gremo po področju z $\vec{B} = 0$.

ψ' je funkcija potem ko damo v valj $\vec{B} \neq 0$

torej: $$e^{i\frac{e}{\hbar}\phi_m} = 1 = e^{i2\pi n}$$

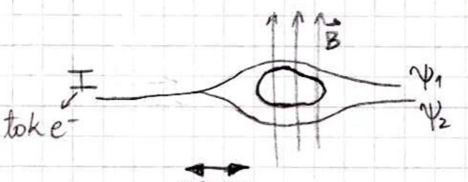
$$\hookrightarrow \phi_m = n\frac{2\pi\hbar}{e} = n\frac{h}{2e}$$

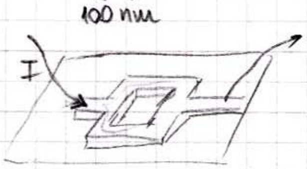
Kvant prevodnosti → to dvojko je treba dodat. Kuiperjevi pari, ko im je $2e_0$.

Z eksperimentom se pokaže, da to je res.
Tako se meri šibka polja.

SQUID → merijo lahko $\vec{B}$ zaradi pretakanja ionov / krvi po žilah

Aharonov - Bohmov pojav

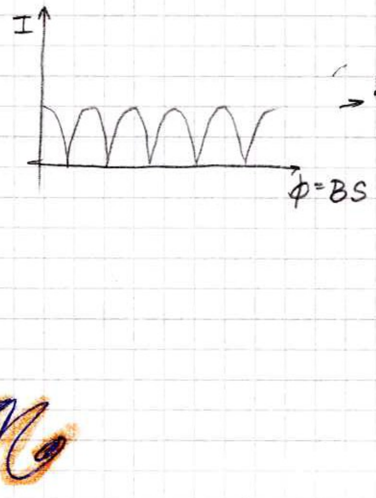




$$|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\,\mathrm{Re}\,\psi_1^* \psi_2$$

$$\downarrow$$

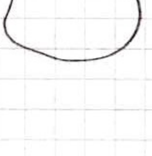
$$\sim e^{i\frac{e}{\hbar}\phi_m}$$



→ ko spreminjaš magnetno polje, tok oscilira.

→ lahko meriš B → zelo male polja, in noro natančno lahko meriš

Spin



zanka, po njej teče tok. klasično: $\vec{p}_m = I\vec{S}$

zadnjič smo pokazali: $\vec{\mu}_L = g_L \frac{e}{2m}\vec{L}$ sklopitev: $-\vec{\mu}_L \cdot \vec{B}$

Landéjev faktor

↳ klasično je giromagn. razmerje 1, delci s spinom 1/2 pa imajo $g_L \neq 1$.

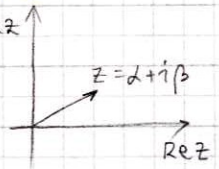
Splošno: $\vec{\mu} = \frac{e}{2m}(\vec{L} + g\vec{S})$

Če vzameš najbolj enostavno teorijo, dobiš za delce s spinom $1/2$ $g = 2$. Zares je nekje $g = 2{,}002319304718$ → to kažejo TUDI eksperimenti

Pauli:

Malo zgodovine:

Hamilton: ~l.1800



$i^2 = -1$

Sorodniki kompleksnih št

kvaternioni → se ni prijelo, nima uporabe

$1,\ \underbrace{i, j, k}$

3 imaginarne komponente

Lastnosti: $i^2 = j^2 = k^2 = -1$

$ij = k,\ jk = i \ldots$

$ji = -k$

Paulijeve matrike:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

↳ elementi matrik so $\mathbb{C}$

* Stern-Gerlach:

↳ poskus, ki je pokazal, da je treba glede vrtilne količine vzeti polovične vrednosti:

$$S_z = \pm \frac{1}{2}$$

ustrezni operator vrtilne količine razpenja 2D prostor:

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma} \qquad \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$$

↳ matrika 2×2

Paulijeve matrike

velja:

- $[\sigma_i, \sigma_j] = 2i\sigma_k \Rightarrow [S_i, S_j] = i\hbar\varepsilon_{ijk}S_k$
- $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = I$
- $\sigma_x\sigma_y = i\sigma_z$; $\sigma_y\sigma_z = i\sigma_x$; $\sigma_z\sigma_x = i\sigma_y$
- $\sigma_x\sigma_y + \sigma_y\sigma_x = 0$ ∀ par
- $\sigma_x\sigma_y\sigma_z = i$
- $\mathrm{Tr}\,\sigma_x = \mathrm{Tr}\,\sigma_y = \mathrm{Tr}\,\sigma_z = 0$
- $\det\sigma_x = \det\sigma_y = \det\sigma_z = -1$

splošno: $\sigma_i\sigma_j = \delta_{ij}I + i\varepsilon_{ijk}\sigma_k$

- $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = (\vec{a}\cdot\vec{b})I + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$

$(\sigma_x a_x + \sigma_y a_y + \sigma_z a_z)$

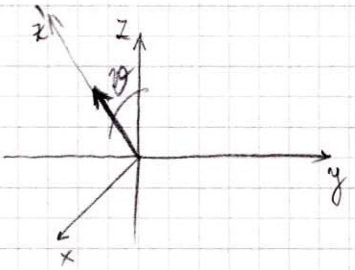
matrika, skalar

- Paulijeve matrike so hermitske: $\vec{S}^{+} = \vec{S}$
- σ so baza:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \lambda_1 I + \lambda_2\sigma_x + \lambda_3\sigma_y + \lambda_4\sigma_z$$

Naredimo Primer:

Imamo vektor v ravnini xz, ki je enotni in označuje z':



delec po SG poskusu preleti ven s spinom v smeri z:

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$ → bazna funkcija, stanje, ki označuje stanje, ki gor kaže.

$$\hat{e}_{z'} = (\sin\vartheta, 0, \cos\vartheta)$$

$$\chi = \sum_{\lambda=1}^{n} c_\lambda \chi_\lambda$$

naj bo: $\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Kakšna je pričakovana vrednost vrtilne količine S vzdolž z' osi?

Torej $\langle \vec{S} \cdot \hat{e}_{z'} \rangle = ?$ za stanje $\chi_\uparrow$.

$$\langle \vec{S} \cdot \hat{e}_{z'} \rangle = \frac{\hbar}{2} \langle \chi_\uparrow | (\sin\vartheta\, \sigma_x + \cos\vartheta\, \sigma_z) | \chi_\uparrow \rangle =$$

$$\hat{e}_{z'} \cdot \langle \vec{S} \rangle$$

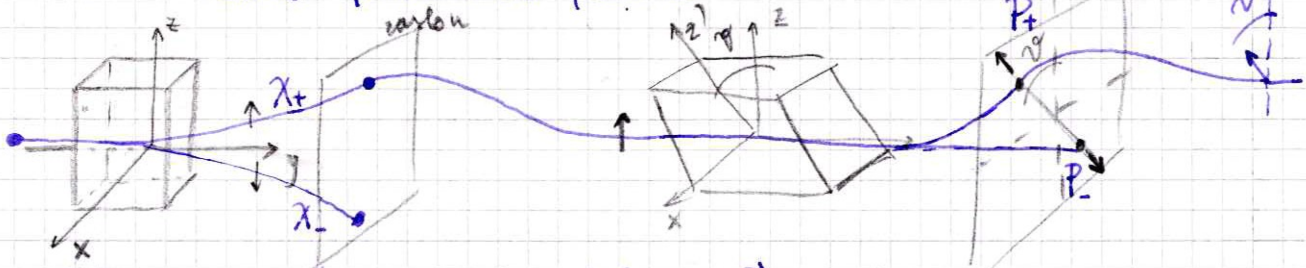
$$= \frac{\hbar}{2}(1, 0) \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ \sin\vartheta & -\cos\vartheta \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}(1, 0) \begin{pmatrix} \cos\vartheta \\ \sin\vartheta \end{pmatrix} = \frac{\hbar}{2}\cos\vartheta$$

Namesto $\uparrow \downarrow$ bomo pisali $\pm$: $\chi_\pm$ → to je naša baza

Recimo, da imamo $|\psi\rangle = c_+ |\chi_+\rangle + c_- |\chi_-\rangle$

→ Kakšna je verjetnost, da pri danem $|\psi\rangle$ izmerimo $|\chi_+\rangle$ oziroma $|\chi_-\rangle$?

Napravimo STERN-GERLACHov poskus



Pošljemo noter srebrov atom: je nevtralen, v $\vec{E}$ ni odklona.

v $\vec{B}$ pa je: $\vec{F} = (\vec{\mu} \cdot \nabla)\vec{B} = \nabla(\mu \cdot \vec{B})$ Polje mora biti nehomogeno.

če $\vec{B} = (0, 0, B_z)$

↳ $\vec{F} = \mu_z \frac{\partial B_z}{\partial z} \vec{e}_z$

Curek atomov se razcepi v 2 curka po prehodu skozi $\vec{B}$.

Recimo, da si izberemo zgornji curek. Po aksiomu, da je atom po meritvi v stanju, ki si ga izmeril, vem, da je curek v χ_+. Ta curek speljemo naprej in poskus naredimo še enkrat. Tokrat spektrometer /škatlo/ zasukamo.

Kakšna je verjetnost, da izmerimo obrnjenega gor ali dol v odvisnosti od ϑ?

Kako izračunati P_+ oz. P_-? Glej sliko ↗

Imamo bazo $\chi_\pm$.

P_+ ... verjetnost, da v stanju $|\psi\rangle$ $= c_+|\chi_+\rangle + c_-|\chi_-\rangle$ najdem stanje, ki je lastno stanje nove smeri z'

$$P_+ = |\langle \chi_+' | \psi \rangle|^2 = |c_+'|^2$$

$$|\psi\rangle = c_+|\chi_+\rangle + c_-|\chi_-\rangle = c_+'|\chi_+'\rangle + c_-'|\chi_-'\rangle$$

→ nova baza

Naloga je poiskat $\chi_\pm'$:

Lastnost je: $\underbrace{(\vec{e}_{z'} \cdot \vec{S})}_{}\chi_\lambda' = \frac{\hbar}{2}\lambda \chi_\lambda'$ $\lambda = +, -$

to je operator $\vec{S}$ v smeri z', v novi smeri

$\vec{e}_{z'} = (\sin\vartheta, 0, \cos\vartheta)$

$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$

$$(\vec{e}_{z'} \cdot \vec{\sigma})\chi_\lambda' = \lambda \chi_\lambda'$$

χ_λ' je novi lastni vektor izražen v stari bazi!

$\chi_\lambda' = a_+\chi_+ + a_-\chi_- = \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$

$$\vec{e}_{z'} \cdot \vec{\sigma} = \left[\underbrace{\begin{pmatrix} 0 & \sin\vartheta \\ \sin\vartheta & 0 \end{pmatrix}}_{\sin\vartheta \cdot \vec{\sigma}_x} + \underbrace{\begin{pmatrix} \cos\vartheta & 0 \\ 0 & -\cos\vartheta \end{pmatrix}}_{\cos\vartheta \cdot \vec{\sigma}_z}\right]$$

$$\left[\begin{pmatrix} 0 & \sin\vartheta \\ \sin\vartheta & 0 \end{pmatrix} + \begin{pmatrix} \cos\vartheta & 0 \\ 0 & -\cos\vartheta \end{pmatrix}\right]\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = \lambda \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$

Poiščemo lastne vrednosti / vektorje:

$$\begin{bmatrix} -\lambda + \cos\vartheta & \sin\vartheta \\ \sin\vartheta & -\lambda - \cos\vartheta \end{bmatrix}\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = 0$$

$$\hookrightarrow \begin{vmatrix} -\lambda + \cos\vartheta & \sin\vartheta \\ \sin\vartheta & -\lambda - \cos\vartheta \end{vmatrix} = 0 =$$

$$= \lambda^2 - \cos^2\vartheta - \sin^2\vartheta = 0 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda_{1,2} = \pm 1$$

za $\lambda = 1$:

$$\begin{bmatrix} -1+\cos\vartheta & \sin\vartheta \\ \sin\vartheta & -1-\cos\vartheta \end{bmatrix}\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = \begin{pmatrix} (-1+\cos\vartheta)a_+ + \sin\vartheta\, a_- \\ a_+\sin\vartheta - (1+\cos\vartheta)a_- \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-2\sin^2\frac{\vartheta}{2}a_+ = -\sin\vartheta\, a_-$$

$$\frac{a_+}{a_-} = \frac{\sin\vartheta}{2\sin^2\frac{\vartheta}{2}} = \frac{2\sin\frac{\vartheta}{2}\cos\frac{\vartheta}{2}}{2\sin^2\frac{\vartheta}{2}} = \frac{\cos\frac{\vartheta}{2}}{\sin\frac{\vartheta}{2}}$$

torej: $\chi_+' = \begin{pmatrix} \cos\frac{\vartheta}{2} \\ \sin\frac{\vartheta}{2} \end{pmatrix} e^{i\delta_+}$ $\delta_+ \in \mathbb{R}$

za $\lambda = -1$ je enak postopek...

$$\chi_-' = \begin{pmatrix} -\sin\frac{\vartheta}{2} \\ \cos\frac{\vartheta}{2} \end{pmatrix} e^{i\delta} \qquad \delta \in \mathbb{R}$$

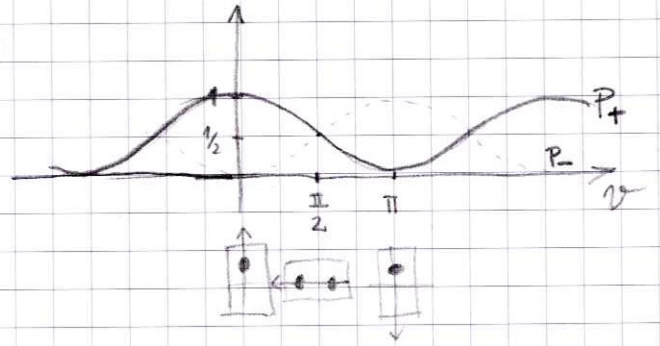
Mi smo začeli z $|\chi_+\rangle$, ki ga po prvem SG peljemo v drug enak poskus:

Torej: $|\psi\rangle = |\chi_+\rangle$

Zanima nas $\langle \chi_+' | \chi_+ \rangle = \left(\cos\frac{\vartheta}{2}, \sin\frac{\vartheta}{2}\right) \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\text{spinor}} = \cos\frac{\vartheta}{2}$

Torej: $P_+ = \cos^2\frac{\vartheta}{2}$

in $P_- = 1 - \cos^2\frac{\vartheta}{2} = \sin^2\frac{\vartheta}{2}$



$\langle \psi | \vec{S} | \psi \rangle$ je pričakovana vrednost.

Recimo: $\langle \chi_+ | \vec{S} | \chi_+ \rangle = \begin{pmatrix} 0 \\ 0 \\ \frac{\hbar}{2} \end{pmatrix}$ ↳ je vektor.

To je pričakovana vrednost spina.

$\hat{e}_z \cdot \langle \vec{S} \rangle$ ↗ pričakovana vrednost operatorja $\vec{S}$ projeciran na $\hat{e}_z$

$\parallel$

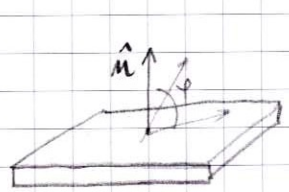
$\langle \vec{e}_z \cdot \vec{S} \rangle = \frac{\hbar}{2} P_+ + \left(-\frac{\hbar}{2}\right) P_- = \frac{\hbar}{2}\left(\cos^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2}\right) = \frac{\hbar}{2}\cos\vartheta$

↳ pričakovana vrednost spina v smeri $\hat{e}_z$

pričakovana vrednost operatorja A → $\langle A \rangle = \sum_n a_n |C_n|^2$ ↳ l. vr.

Rotacija spinorjev

spinor je vektor v prostoru stanj! Ne si ga predstavljat kot vektor v kartez. koord.!!

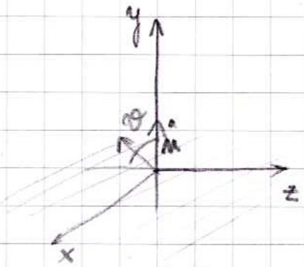


$\psi' = U_\varphi \psi$

$U_\varphi = e^{-i\varphi \frac{\hat{n}\cdot\vec{L}}{\hbar}}$ → smo pokazali, da je to rotacija za φ

Poglejmo kaj naredi U_φ, če je $\vec{L} = \vec{S}_z$

$$U_\varphi = e^{-i\varphi \frac{\hat{n}\cdot\vec{S}}{\hbar}} = e^{-i\varphi \frac{\hat{n}\cdot\vec{\sigma}}{2}}$$



$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \hat{n} = (0,1,0)$$

$$\varphi = \vartheta$$

$$\chi_+' = U_\varphi \chi_+$$
$$= e^{-i\frac{\varphi}{2}\sigma_y} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\rightarrow e^{-i\varphi\frac{\hat{n}\cdot\vec{\sigma}}{2}} = 1 - i\varphi\frac{\hat{n}\cdot\vec{\sigma}}{2} + \left(i\varphi\frac{\hat{n}\cdot\vec{\sigma}}{2}\right)^2\frac{1}{2!} - \dots$$

↳ vrsta, sestavljena iz potenc matrik

poglejmo tole:

$$(\vec{n}\cdot\vec{\sigma})^2 = \left(\sum_{i=1}^{3} n_i \sigma_i\right)^2 = \sum_{ij} n_i n_j \underbrace{\sigma_i \sigma_j}_{\delta_{ij} + i\varepsilon_{ijk}\sigma_k} = \sum_i n_i^2 + 0 = \vec{n}\cdot\vec{n}$$

$$(\vec{n}\cdot\vec{\sigma})^2 = \vec{n}\cdot\vec{n} = 1 \quad !$$

Kaj pa $(\vec{n}\cdot\vec{\sigma})^3 = ?$ Pride $(\vec{n}\cdot\vec{\sigma})^3 = \vec{n}\cdot\vec{\sigma}$

$$(\vec{n}\cdot\vec{\sigma})^4 = 1$$

$$(\vec{n}\cdot\vec{\sigma})^{17} = \vec{n}\cdot\vec{\sigma}$$

torej: $$e^{-i\varphi\frac{\hat{n}\cdot\vec{\sigma}}{2}} = \cos\frac{\varphi}{2} I - i\sin\frac{\varphi}{2}(\overset{(0,1,0)}{\vec{n}}\cdot\vec{\sigma})$$

$$\Rightarrow U_\vartheta = I\cos\frac{\vartheta}{2} - i\sin\frac{\vartheta}{2}\sigma_y =$$

$$= \begin{pmatrix} \cos\frac{\vartheta}{2} & -\sin\frac{\vartheta}{2} \\ \sin\frac{\vartheta}{2} & \cos\frac{\vartheta}{2} \end{pmatrix}$$ ⇒ Matrika, ki suka spinor za kot ϑ

Preverimo: $$\chi_+' = \begin{pmatrix} \cos\frac{\vartheta}{2} & \sin\frac{\vartheta}{2} \\ \sin\frac{\vartheta}{2} & \cos\frac{\vartheta}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\frac{\vartheta}{2} \\ \sin\frac{\vartheta}{2} \end{pmatrix}$$ → je prav!

$$\chi_-' = \begin{pmatrix} -\sin\frac{\vartheta}{2} \\ \cos\vartheta/2 \end{pmatrix}$$

Poglejmo $U_{2\pi}$: $U_{2\pi} = I\cos\pi - i\sin\pi \cdot \sigma_y = -I$

$$\chi_+' = \underset{\begin{pmatrix}1\\0\end{pmatrix}}{U_{2\pi}\chi_+} = -\chi_+ = \begin{pmatrix}-1\\0\end{pmatrix}$$

→ hm... Ramšak pravi, da ta minus ni nič narobe. Če prostor zavrtimo za 2π, dobimo pravotno valovno funkcijo pomnoženo z -1. Če ~~še~~ dvakrat zavrtiš za 2π dobiš pravotno valovno funkcijo.
→ TO NI MERLJIVO.
Valovno funkcijo lahko vedno pomnožiš z -1 če hočeš. Važna je $|\psi|^2$.

Matrika $d^{1/2}_{mn} = \begin{pmatrix}\cos\frac{\vartheta}{2} & -\sin\frac{\vartheta}{2}\\ \sin\frac{\vartheta}{2} & \cos\frac{\vartheta}{2}\end{pmatrix}$ je matrika, ki zavrti spinor za kot ϑ. S tem sukamo bazo.

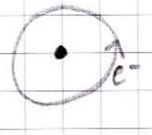
Eulerjevi koti: $D^J_{mn}(\alpha,\beta,\gamma) = e^{-im\alpha} d^J_{mn}(\beta) e^{-im\gamma}$ → poljubna rotacija

SEŠTEVANJE VRTILNIH KOLIČIN

a) Vzemimo 2 delca: $\vec{S}_1, \vec{S}_2$ — delca sta ločena. Ne moreš ju izmenjat / recimo p^+ in e^-. Sta ločljiva. Med njima ni interakcije. Zanima nas spinski del.

$$[\vec{S}_1, \vec{S}_2] = 0$$

b)



Elektron, ki kroži in ima spin: $\vec{L}, \vec{S}$

$$[\vec{L}, \vec{S}] = 0$$

c) Splošen primer: $\vec{j}_1, \vec{j}_2$; $[\vec{j}_1, \vec{j}_2] = 0$

Torej poglejmo primer a):

→ Prostor je produkt dveh prostorov.
$$|\psi\rangle = |\psi_1\rangle|\psi_2\rangle = |m_1\rangle|m_2\rangle = |m_1, m_2\rangle$$

velja: $S^z_{\substack{1\\2}}|m_{\substack{1\\2}}\rangle = \hbar m_{\substack{1\\2}}|m_{\substack{1\\2}}\rangle$ baza: $|\uparrow\rangle_1$, $|\uparrow\rangle_2$, $|\downarrow\rangle_2$, $|\downarrow\rangle_2$

→ sestavimo skupen spin. Definiram $\vec{S} = \vec{S_1} + \vec{S_2}$ $\qquad [\vec{S}, \vec{S_1}] = [\vec{S}, \vec{S_2}] = 0$

$$S^z = S_1^z + S_2^z$$ velja tudi: $[S^z, \vec{S}_{1,2}] = 0$

možna kvantna števila: S_1, S_2, S, S^z

$$S^z|m_1, m_2\rangle = (S_1^z + S_2^z)|m_1\rangle|m_2\rangle = S_1^z|m_1\rangle|m_2\rangle + S_2^z|m_1\rangle|m_2\rangle =$$
$$= m_1\hbar|m_1\rangle|m_2\rangle + m_2\hbar|m_2\rangle|m_1\rangle$$
$$= (m_1 + m_2)\hbar|m_1, m_2\rangle$$

Baza $|m_1, m_2\rangle$ je lastna baza za S^z!

$$\hat{S}_1|m_1, m_2\rangle = \hbar s_1(s_1+1)|m_1, m_2\rangle = \frac{3}{4}\hbar|m_1, m_2\rangle$$

$s_1 = \frac{1}{2}$

↳ $|m_1, m_2\rangle$ je tudi baza za $\hat{S}_1$. In $\hat{S}_2$

Vendar: $S^2\underbrace{|m_1, m_2\rangle}_{\text{ni l. baza za } S^2} \neq C|m_1, m_2\rangle$

Treba je najt lastno bazo

Pišimo takole:

$|\uparrow\uparrow\rangle$
$|\uparrow\downarrow\rangle$
$|\downarrow\uparrow\rangle$
$|\downarrow\downarrow\rangle$

$$\underset{\text{operator}}{S^2} = (\vec{S_1} + \vec{S_2})^2 = S_1^2 + S_2^2 + \underbrace{2\vec{S_1}\cdot\vec{S_2}}$$

↳ tole je treba razumet

Poglejmo: $2\vec{S_1}\cdot\vec{S_2} = 2S_1^z S_2^z + 2(S_1^x S_2^x + S_1^y S_2^y)$

pišimo: $S^{\pm} = S^x \pm iS^y$

torej: $2\vec{S_1}\cdot\vec{S_2} = 2S_1^z S_2^z + S_1^+ S_2^- + S_1^- S_2^+$

$m_1 + m_2 = 1$ ($\frac{1}{2}$, $\frac{1}{2}$)

Vemo: $S^z|\uparrow\uparrow\rangle = \hbar|\uparrow\uparrow\rangle = \hbar|SM\rangle = \hbar|11\rangle$ → proj. je 1

↳ celoten spin je 1

poglejmo: $S^-|\uparrow\uparrow\rangle = S^-|11\rangle$

znižuje projekcijo

v tej notaciji

$$= \hbar\sqrt{S(S+1) - M(M-1)}\,|1\,0\rangle$$
$$= \hbar\sqrt{1(1+1) - 1\cdot 0}\,|1\,0\rangle = \sqrt{2}\hbar|1\,0\rangle$$

torej: → $|1\,0\rangle = \frac{1}{\sqrt{2}\hbar}S^-|11\rangle$

$$|1\,0\rangle = \frac{1}{\sqrt{2}\hbar} S^- |1\,1\rangle = \frac{1}{\sqrt{2}\hbar}(S_1^- + S_2^-)|\uparrow\uparrow\rangle$$

kaj naredi: $S_1^- |m_1\rangle = \hbar |m_1 - 1\rangle$ $\quad\sqrt{s(s+1) - m_1(m_1-1)}$, $\frac{1}{2}$

torej: $S_1^-|\uparrow\rangle = \hbar|\downarrow\rangle$

$S_1^+|\downarrow\rangle = \hbar|\uparrow\rangle$

$S_1^-|\downarrow\rangle = 0$

$S_1^+|\uparrow\rangle = 0$

$$|1\,0\rangle = \frac{1}{\sqrt{2}\hbar}\hbar\left(|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle\right) \rightarrow \begin{matrix} s=1 \\ m=0 \end{matrix}$$

poglejmo:

$$S^-|1\,0\rangle = \frac{1}{\sqrt{2}}(S_1^- + S_2^-)(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) =$$

S_1 na prvega deluje

$\|$ $|1\,-1\rangle$ Poglejmo če je res

$$= \frac{1}{\sqrt{2}}\hbar\left(S_1^-|\uparrow\downarrow\rangle + S_1^-|\downarrow\uparrow\rangle + S_2^-|\uparrow\downarrow\rangle + S_2^-|\downarrow\uparrow\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\hbar\left(|\downarrow\downarrow\rangle + |\downarrow\downarrow\rangle\right) = \sqrt{2}\hbar|\downarrow\downarrow\rangle =$$

$$= \hbar\sqrt{(1(1+1)) - 0(0-1)}\,|1\,-1\rangle = \sqrt{2}\hbar|1\,-1\rangle$$

je vredu.

Zdaj imamo 3 stanja:

$$\left.\begin{aligned} |1\,0\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ |1\,1\rangle &= |\uparrow\uparrow\rangle \\ |1\,-1\rangle &= |\downarrow\downarrow\rangle \end{aligned}\right\}$$ tripletna stanja

Clebsch Gordan koeficienti

kako dobiti to stanje? $\Rightarrow$ $|0\,0\rangle \rightarrow$ singletno

$$|0\,0\rangle = A|\uparrow\uparrow\rangle + B|\downarrow\downarrow\rangle$$

da ~~bo~~ $m=0$ in bo $\langle 0\,0|1\,0\rangle = 0$

v s-ju se razlikujeta!

$$\Rightarrow |0\,0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$$

Dobimo: $S^2|0\,0\rangle = 0$

Akta

Začneš splošno.

$\vec{j}_1, \vec{j}_2, m_1, m_2$ Baza je $|m_1\rangle|m_2\rangle$. Kako iz tega naredit lastno bazo za celotni $\vec{j}$ in celotni m?

(m_1: jih je $2j_1+1$; m_2: jih je $2j_2+1$)

vemo:

$$|j-j_2| \le J \le j_1+j_2$$ (celotni J)

$$M = m_1 + m_2$$

$$|j_1 j_2 JM\rangle = \sum_{m_1, m_2} |j_1 j_2 m_1 m_2\rangle \underbrace{\langle j_1 j_2 m_1 m_2 | JM\rangle}_{\text{koeficienti}}$$

$\Downarrow$ Clebsch-Gordan koeficienti

vzameš stanje: $|J, M=J\rangle$. In nanj deluješ z J^- in zgeneriraš še ostale stanje dokler ne prideš do $|J,-J\rangle$. Stanje $|J-1, M\rangle$ pa zgeneriraš tako, da je ortogonalno na vse ostale.

Oziroma uporabiš tabele.

Primeri: Imamo vrtilno količino $j_1 = 2 = l$ in $j_2 = \frac{1}{2} = s$.
Recimo, da želimo $J = \frac{5}{2}$ in $M = \frac{3}{2}$.
Želimo skonstruirat tako valovno funkcijo.

Stanje: $|2 \; \frac{1}{2} \; \frac{5}{2} \; \frac{3}{2}\rangle$

v tabelah je to $2 \times 1/2$

Dajmo za vajo $|2 \; \frac{1}{2} \; \frac{5}{2} \; \frac{5}{2}\rangle = |2\,2\rangle|\frac{1}{2} \; \frac{1}{2}\rangle = Y_{22}(\vartheta, \varphi)\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}Y_{22}\\0\end{pmatrix}$

$$\Rightarrow |2 \; \tfrac{1}{2} \; \tfrac{5}{2} \; \tfrac{3}{2}\rangle = \sqrt{\tfrac{1}{5}}\, Y_{22}\begin{pmatrix}0\\1\end{pmatrix} + \sqrt{\tfrac{4}{5}}\, Y_{21}\begin{pmatrix}1\\0\end{pmatrix}$$

$$|2 \; \tfrac{1}{2} \; \tfrac{3}{2} \; \tfrac{3}{2}\rangle = \sqrt{\tfrac{4}{5}}\, Y_{21}\begin{pmatrix}0\\1\end{pmatrix} - \sqrt{\tfrac{1}{5}}\, Y_{22}\begin{pmatrix}0\\1\end{pmatrix}$$

Poglejmo še: če $j_1 = 1/2$, $j_2 = 1/2$

$$|\tfrac{1}{2} \; \tfrac{1}{2} \; 1 \; 0\rangle = \tfrac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle\right)$$

tripletno stanje s proj. 0

$$|\tfrac{1}{2} \; \tfrac{1}{2} \; 0 \; 0\rangle = \tfrac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$$