Diracova notacija

1) Vektorski prostor (Hilbertov, L_2,...)

$\psi(x,t) \in L_2$

$\varphi_n(x,t)$; $\psi(x,t) = \sum c_n \varphi_n(x,t)$ baza je števna

$= \int c_k \varphi_k(x,t)\, dk$ baza je kontinuumska

→ Banachov prostor

2) $\vec{a}\cdot\vec{b} = \sum_i a_i b_i$

$\varphi(x), \psi(x) \Rightarrow (\varphi,\psi) = (\varphi|\psi) = \langle\varphi,\psi\rangle = \langle\varphi|\psi\rangle$

$= \int_{-\infty}^{\infty} \varphi^*(x)\psi(x)\, dx$

* $\langle\varphi,\psi\rangle = \langle\psi,\varphi\rangle^*$
* $\langle\psi,\psi\rangle \geq 0$; če $= 0 \Rightarrow |\psi\rangle = 0$ (v kvantni mehaniki ni $|\psi\rangle = 0$!!)
* $|\langle\varphi|\psi\rangle|^2 \leq \langle\varphi|\varphi\rangle\langle\psi|\psi\rangle$ ($\vec{a}\cdot\vec{b} = ab\cos\varphi \leq ab$
* $\langle\varphi|(\lambda_1\psi_1 + \lambda_2\psi_2)\rangle = \lambda_1\langle\varphi|\psi_1\rangle + \lambda_2\langle\varphi,\psi_2\rangle$
* $\langle\lambda_1\varphi_1 + \lambda_2\varphi_2|\psi\rangle = \lambda_1^*\langle\varphi_1|\psi\rangle + \lambda_2^*\langle\varphi_2|\psi\rangle$

3) KET

$\psi(x,t) \in L_2 \Rightarrow |\psi\rangle$ (| \Psi \rangle)

$\psi_n(x,t) \rightarrow |n\rangle$ npr: $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$

$\varphi_1(x,t) \rightarrow |1\rangle$

4) Linearni operatorji

$\hat{A} : L_2 \rightarrow L_2 \Rightarrow \hat{A}\psi(x,t) = \psi_1(x,t)$

npr: $-i\hbar\frac{\partial}{\partial x} A e^{-\alpha x^2} = 2i\hbar\alpha A x e^{-\alpha x^2}$

Aleta®

- $\hat{A} = 0 \iff \hat{A}\psi = 0 \ \forall \psi$
- $\hat{A}(\lambda\psi + \eta\varphi) = \lambda\hat{A}\psi + \eta\hat{A}\varphi$
- $(\eta\hat{A} + \lambda\hat{B})\psi = \eta\hat{A}\psi + \lambda\hat{B}\psi$

5) BRA

Linearni funkcional

$$\hat{f}\,\psi(x) = c \in \mathbb{C} \qquad \hat{f} : L_2 \to \mathbb{C}$$

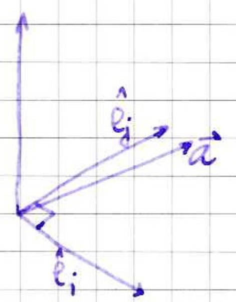
Rieszev izrek:

$$\hat{f}\,\psi(x) = c \quad \exists\ f_c(x) : \underbrace{\int f_c^{*}\,\psi(x)\,dx}_{\langle f_c|\psi\rangle} = c$$

$\langle\varphi| = \hat{f}$ Bra je funkcional : $\hat{f}|\psi\rangle = \langle\varphi|\psi\rangle$

$$\langle\varphi| = \int \varphi^{*}(x) \ldots dx$$

6) Razvoj vektorja po bazi



$$\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$$

$$\vec{a} = (\underset{\text{bra}}{\vec{e}_i \cdot \vec{a}})\underset{\text{ket}}{\hat{e}_i} + (\hat{e}_j \cdot \vec{a})\hat{e}_j$$

V splošnem lahko tudi:

$$\hat{e}_i = \lambda_1 \vec{x} + \lambda_2 \vec{y} \qquad \hat{e}_i = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

$$|\vec{a}\rangle = \langle\hat{e}_1|\vec{a}\rangle|\hat{e}_1\rangle + \langle\hat{e}_2|\vec{a}\rangle|\hat{e}_2\rangle$$

$|\psi\rangle = C_1|1\rangle + C_2|2\rangle \qquad \langle i,j\rangle = \delta_{ij}$

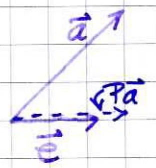
$\langle 1|\psi\rangle = \langle 1|C_1|1\rangle = C_1$

$\langle 2|\psi\rangle = C_2$

$|\psi\rangle = \langle 1|\psi\rangle|1\rangle + \langle 2|\psi\rangle|2\rangle =$

$= \underbrace{|1\rangle\langle 1|}_{P_1}\psi\rangle + \underbrace{|2\rangle\langle 2|}_{P_2}\psi\rangle$

$P_1, P_2 \rightarrow$ projektor



$\vec{e}\cdot\vec{e} = 1 \qquad P\vec{a} = (\vec{e}\cdot\vec{a})\vec{e}$

$P_1 + P_2 + P_3 = 1 \qquad (P_1 + P_2 + P_3)\vec{a} = \vec{a}$

$|\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle$ Tako v splošnem zapišemo funkcijo.

$|n\rangle$ baza $\qquad |n\rangle\langle n|$ sendvič

$\rightarrow \psi(x,t) = \sum_{n=1}^{\infty} A_n \cos(k_n x) \int_{-\infty}^{\infty} A_n^* \cos^*(k_n x')\,\psi(x',t)\,dx'$

$|\psi\rangle = \underset{\uparrow}{I}\,|\psi\rangle$

$\sum_n |n\rangle\langle n| = \sum_n P_n$

7) Hermitski adjungirani operatorji

a) $\hat{p} = -i\hbar\frac{\partial}{\partial x}$

$$\int \varphi^*(x)\,\hat{p}\,\psi(x)\,dx = \int (\hat{p}\varphi)^*\psi\,dx$$

b) $\hat{A} = i$

$$\int \varphi^* \hat{A} \psi \, dx = i \int \varphi^* \psi \, dx$$

$$\int (\hat{A}\varphi)^* \psi \, dx = -i \int \varphi^* \psi \, dx$$

naj bo $A^+ = -i$

$$\int \varphi^* A^+ \psi \, dx = -i \int \varphi^* \psi \, dx$$

$$\int \varphi^* \hat{A} \psi \, dx = \int (A^+ \varphi)^* \psi \, dx$$

A^+ je hermitsko adjungiran k A, če $\langle \varphi | A | \psi \rangle = \langle A^+ \varphi | \psi \rangle$ za vsak φ, ψ.

$$\psi(x,t) \to |\psi\rangle$$

$$\int \varphi^*(x,t) \ldots dx \to \langle \varphi |$$

$$\langle \varphi | A \psi \rangle = \langle \varphi | A | \psi \rangle$$

$$\langle A^+ \varphi | \psi \rangle = \langle \varphi | A | \psi \rangle = \langle \psi | A^+ | \varphi \rangle^*$$

➡ Lastnosti hermitske adjungacije $A \to A^+$

- (cA) $\quad c \in \mathbb{C}$

  $(cA)^+ = c^* A^+ \qquad c^+ = c^*$

  $$\int \varphi^* A \psi \, dx \Rightarrow \int (c^* \varphi)^* A \psi \, dx$$

- $(A+B)^+ = A^+ + B^+$
- $(AB)^+ = B^+ A^+$

$$\langle \varphi | AB | \psi \rangle = \langle (AB)^{+} \varphi | \psi \rangle = \langle A^{+} \varphi | B | \psi \rangle = \langle B^{+} A^{+} \varphi | \psi \rangle \Rightarrow$$

$$\Rightarrow (AB)^{+} = B^{+} A^{+}$$

★ $\hat{A} = |\varphi_1\rangle\langle\varphi_2| = \varphi_1(x) \int \varphi_2(x) \ldots dx$

$\hat{A}^{+} = |\varphi_2\rangle\langle\varphi_1|$

$$\langle \varphi | A | \psi \rangle = \langle \varphi | \varphi_1 \rangle \langle \varphi_2 | \psi \rangle = \langle \psi | \varphi_2 \rangle^{*} \langle \varphi_1 | \varphi \rangle^{*} =$$

$$= \left(\langle \psi | \varphi_2 \rangle \langle \varphi_1 | \varphi \rangle \right)^{*} = \langle \psi | A^{+} | \varphi \rangle^{*}$$

$$\hookrightarrow A^{+} = |\varphi_2\rangle\langle\varphi_1|$$

→ projektor $P_n = |n\rangle\langle n| = P_n^{+}$ $\quad \langle n | n \rangle = 1$

$$\langle \varphi | P | \psi \rangle = \langle P \varphi | \psi \rangle$$

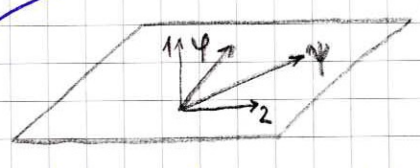
8) Zapis operatorja v bazi, matrični elementi

baza $\{ |n\rangle \}$ $\quad |\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle$

Zgled:

$$|\psi\rangle = c_1|1\rangle + c_2|2\rangle = |1\rangle\langle 1|\psi\rangle + |2\rangle\langle 2|\psi\rangle$$

$$A|\psi\rangle = |\varphi\rangle = c_1 A|1\rangle + c_2 A|2\rangle$$



$\varphi = A\psi$

$$|\varphi\rangle = d_1|1\rangle + d_2|2\rangle = |1\rangle\langle 1|\varphi\rangle + |2\rangle\langle 2|\varphi\rangle =$$

$$= |1\rangle\langle 1|\overbrace{A}^{A_{11}}|1\rangle\langle 1|\psi\rangle + |2\rangle\langle 2|\overbrace{A}^{A_{21}}|1\rangle\langle 1|\psi\rangle +$$

$$+ |1\rangle\langle 1|\overbrace{A}^{A_{12}}|2\rangle\langle 2|\psi\rangle + |2\rangle\langle 2|\overbrace{A}^{A_{22}}|2\rangle\langle 2|\psi\rangle$$

$$|\varphi\rangle = (A_{11}C_1 + A_{12}C_2)|1\rangle + (A_{21}C_1 + A_{22}C_2)|2\rangle = d_1|1\rangle + d_2|2\rangle$$

$$\psi \to \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \vec{\psi}$$

$$\varphi \to \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \vec{\varphi} \qquad \underline{\underline{A}}\vec{\psi} = \vec{\varphi}$$

$$\underbrace{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}}_{\underline{\underline{A}}} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$A_{mn} = \langle m|A|n\rangle = \int \varphi_m^* A \varphi_n \, dx$$ - matrični element

$$|\psi\rangle = \sum_{n=1}^{\infty} |n\rangle \underbrace{\langle n|\psi\rangle}_{C_n}$$

$$A|\psi\rangle = \sum_m \underbrace{\left(\sum_n A_{mn} C_n\right)}_{d_m} |m\rangle$$

$$A|\psi\rangle = IAI|\psi\rangle = \sum_m |m\rangle\langle m| A \sum_n |n\rangle\langle n|\psi\rangle =$$

$$I = \sum_{n=1}^{\infty} |n\rangle\langle n| \qquad = \sum_{m,n} |m\rangle\langle m|A|n\rangle\langle n|\psi\rangle =$$

$$= \left[\sum_{m,n} |m\rangle A_{m,n} \langle n|\right] |\psi\rangle$$

$$\hat{A} = \sum_{m,n} |m\rangle A_{m,n} \langle n|$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x} \qquad |n\rangle \to A_n \sin k_n x$$

$$\hat{p} = \sum_{m,n} A_m \sin k_m x \; p_{mn} \int_a^b A_n^* \sin^* k_n x' \ldots dx'$$

$$p_{m,n} = \int A_m^* \sin^* k_m x' \left(-i\hbar \frac{\partial}{\partial x'}\right) A_n \sin k_n x' \, dx'$$

$$\stackrel{?}{=} \hbar k_n \delta_{m,n}$$ (recimo, da so ortogonalne...)

$\psi \rightarrow$ razvijemo po sin, dobimo c_i

$$\hat{P}\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}$$

Kako poiskati hermitsko adjungiran operator A^+ k A?

$$\hat{A} = \sum_{m,n} |m\rangle A_{m,n} \langle n|$$

$$\hat{A}^+ = \sum_{m,n} |n\rangle A_{m,n}^* \langle m| \underset{\substack{m \to n \\ n \to m}}{=} \sum_{m,n} |m\rangle A_{n,m}^* \langle n| = \sum_{m,n} |m\rangle (A^+)_{m m} \langle n|$$

$$(A^+)_{m,n} = A_{n m}^*$$

$$\hat{A} \rightarrow A_{m,n}$$
$$\hat{A}^+ \rightarrow A_{n,m}^*$$

9. Sebi adjungirani operatorji (Hermitski)

$$\hat{A}, \quad \hat{A}^+ = \hat{A} \qquad A_{m,n} = A_{n,m}^*$$

Lastnosti:

$$A|\psi\rangle = a|\psi\rangle$$
$$\langle\psi|A|\psi\rangle = a\langle\psi|\psi\rangle$$

$$\langle\psi|A|\psi\rangle^* = \langle\psi|A|\psi\rangle = a^*\langle\psi|\psi\rangle^* = a\langle\psi|\psi\rangle$$

$$\hookrightarrow \langle\psi|A|\psi\rangle = \langle A^+\psi|\psi\rangle = \langle\psi|A^+|\psi\rangle^*$$

$$a^* = a$$

Lastne vrednosti so realne

★ $A|\psi\rangle = a|\psi\rangle \quad / \langle\varphi|$

$A|\varphi\rangle = b|\varphi\rangle \quad / \langle\psi|$

$\langle\varphi|A|\psi\rangle = a\langle\varphi|\psi\rangle$

$\langle\psi|A|\varphi\rangle^* = b^*\langle\psi|\varphi\rangle^*$

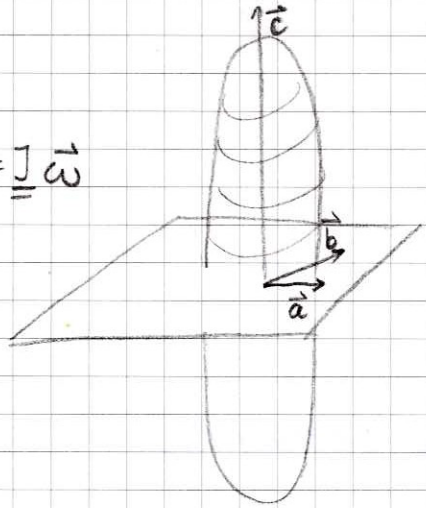
$+$

$0 = (a - b^*)\langle\varphi|\psi\rangle$

a) $\langle\varphi|\psi\rangle = 0 \quad \varphi \perp \psi$

b) $a = b^* = b$

$\vec{\Gamma} = \underline{\underline{J}}\,\vec{\omega}$



$\vec{a}$ in $\vec{b}$ nista nujno pravokotna. Lahko pa tako postavimo / lastni vrednosti sta enaki

Hermitski operatorji imajo realne lastne vrednosti. Lastni vektorji z različnimi lastnimi vrednosti so med sabo pravokotni.

★ $A|\psi\rangle = a|\psi\rangle$

$\langle\psi|A = \langle\psi|a$

10) Opazljivke (observable)

$A = \hat{p}; x, \hat{H}, \hat{l} \dots = A^+$

Operatorji opazljivk so hermitski. Tisto, kar merimo mora biti realno.

Vsaki opazljivki ustreza baza.

$A|n\rangle = a_n|n\rangle$ baza

$B|n'\rangle = b_{n'}|n'\rangle$ baza'

Recimo, da $[A,B]=0$

↳ $A|n\rangle = a_n|n\rangle$

- $AB|n\rangle = BA|n\rangle = a_n B|n\rangle \Rightarrow$ $B|n\rangle$ je tudi lastni vektor operatorja A

- $BA|n'\rangle = AB|n'\rangle = b_{n'} A|n'\rangle$

Ali velja $|n'\rangle = |n\rangle$?

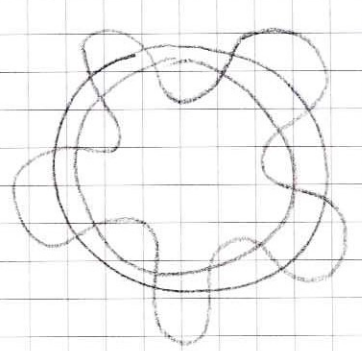
Def: Bazi sta identični, če $|n\rangle = e^{i\varphi}|m\rangle$

Lahko obstaja še $C: [C,A] = [C,B] = 0$. Več v knjigah.

Vedno lahko najdemo sistem opazljivk, ki med seboj komutirajo.

$A|n_1\rangle = a_n|n_1\rangle$
$A|n_2\rangle = a_n|n_2\rangle$ ⇒ degeneracija

Zagotovo obstaje B, da $B|n_1\rangle = b_1|n_1\rangle$ in $B|n_2\rangle = b_2|n_2\rangle$.
B ima lahko spet degenerirana lastna vektorje in poiščemo C. Tako lahko najdemo ~~kompaktni~~ kompletni? sistem operatorjev.



Obroč - noter raven val

Pogoj za val:

$$e^{ikx} = e^{ik(x+l)} \qquad e^{ikl} = 1$$

$$k_n = \pm \frac{2\pi n}{l}$$

$$E_n = \frac{\hbar^2 (2\pi n)^2}{2ml^2}$$ Energije so degenerirane. Vsaki energiji ustrezata 2 vrednosti k ($\pm|k_0|$)

Zaliko najdemo operator, ki bo razklopil stanja. Ugotoviti moramo smer (iz kroženja). Izumimo operator $\hat{p}$.

$$\hat{p}\, e^{\pm ikx} = -i\hbar(\pm ik)\, e^{\pm ikx}$$

lastne vrednosti : $\pm \hbar k$

$$A = \hat{H} \Rightarrow E_n$$
$$B = \hat{p} \Rightarrow \pm k_n$$
$$C = \hat{s} \Rightarrow \pm 1/2$$

sistem operatorjev

$$\psi_{E,k,s}$$

11) Unitarni operatorji

$$U|a\rangle = |a'\rangle \qquad U^{-1}|a'\rangle = a$$

$$U|b\rangle = |b'\rangle$$

imamo : $U, U^{-1} \Rightarrow UU^{-1} = I = U^{-1}U$ (dim jedra mora bit $\emptyset$, da tole velja)

če : $U^{-1} = U^{+}$, potem je U unitaren

$$\langle b'|a'\rangle = \langle Ub|Ua\rangle = \langle b|U^{+}U|a\rangle = \langle b|a\rangle$$

$$\langle b|A|a\rangle = \langle \underset{U^{+}}{U^{-1}} b'|A|\underset{U^{+}}{U^{-1}} a'\rangle = \langle b'|\underbrace{UAU^{+}}_{A'}|a'\rangle = \langle b'|A'|a'\rangle$$

a) Unitarna transf. ohranja sledeče enačbe:

$$U^{-1} = U^{+} \Rightarrow A = \lambda BC + \eta DEF \quad \text{potem: } A' = \lambda B'C' + \eta D'E'F'$$

$\to$ neki operatorji

$\to$ se da dokazat

Glej algebro in matrike...

b) $H = H^+ \to$ je hermitski

recimo, da je: $H|m\rangle \neq C|m\rangle$ $\quad \langle m|H|n\rangle \neq C_{mn}\delta_{mn}$

n je neka baza,
ki je H ne diagonalizira

Izrek: Za vsak hermitski operator obstaja tak U,
da je $H' = U H U^+$ diagonalen v tej bazi.

Torej: $\langle m|H'|n\rangle = E_n \delta_{mn}$

c) H in K

recimo: $H = \frac{p^2}{2m} + V(x)$ in $K = L$

H in K diagonalizira isti $U \Leftrightarrow [H, K] = 0 \to$ komutirata

d) če $H = H^+$, potem je $U = e^{iH}$ unitaren.

kot če bi imel $z \in \mathbb{C}$ in $z = z^* \Rightarrow u = e^{iz}$ je enotni krog. $|u| = 1$

Dodatek k zbirki formul:

IDENTITETA

$A, B \to$ operatorje

$$e^{A+B} = e^{A} e^{B} e^{-\frac{1}{2}[A,B]}$$

$\to$ velja, če $[A, B] = C$, kjer: $[A, C] = [B, C] = 0$

"Dokaz je prav zabaven."

$\hookrightarrow$ zapišeš: $f(\lambda) = e^{\lambda(A+B)} e^{-\lambda A} e^{-\lambda B}$

in $\frac{\partial f}{\partial \lambda}$ ... in dobiš: $\frac{\partial f}{\partial \lambda} \sim \lambda f(\lambda)$ ...

in integriraš. In daš $\lambda = 1$ in dobiš

Potem:

$$\underbrace{e^{iH}}_{U} \underbrace{e^{-iH^+}}_{U^{-1}} = I$$

$H - H^+ = 0$, ker je $H = H^+$.

e) $A + A^+ = (A + A^+)^+$

$i(A - A^+) = (i(A - A^+))^+$

$\frac{\partial}{\partial x}$ je antihermitski.

$-i\hbar\frac{\partial}{\partial x}$ je hermitski.

Vsak operator je vsota hermitskega in antiherm.

12) Primeri unitarnih operatorjev

a) menjava baze

$|n\rangle \qquad |\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle = \sum_n c_n |n\rangle$

$$\vec{\psi} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$$

isti ψ izražen v drugi bazi

$|n'\rangle \qquad |\psi\rangle = \sum_{n'} |n'\rangle\langle n'|\psi\rangle = \sum_{n'} c_{n'} |n'\rangle =$

$$\vec{\psi}' = \begin{pmatrix} c_1' \\ c_2' \\ \vdots \\ c_{n'}' \end{pmatrix}$$

$$= \sum_{n', n} |n\rangle\langle n|n'\rangle \underbrace{\langle n'|\psi\rangle}_{c_{n'}} =$$

$$= \sum_n c_n |n\rangle$$

$$\Downarrow$$

$$\sum_{n'} \langle n|n'\rangle c_{n'}' = c_n$$

$$\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix} = \underline{\underline{U}} \begin{pmatrix} c_1' \\ c_2' \\ \vdots \\ c_n' \end{pmatrix}$$

ista ψ-ja v drugih bazah

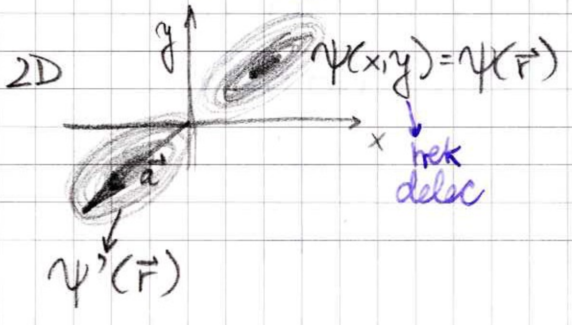
$|\psi'\rangle = \sum_{n'} c_{n'}' |n'\rangle = U^{-1} |\psi\rangle$

$|\psi\rangle = \sum_n c_n |n\rangle = U |\psi'\rangle$

velja : $\langle\psi'|\psi'\rangle = \langle\psi|\psi\rangle$ dokaži!

b) aktivna transformacija

- rotacija
- translacija
- časovni razvoj



translacija:
Definiram nov ψ':

$$\psi' = \psi'(\vec{r}) = \psi(\vec{r} - \vec{a})$$

c) Operacija časovnega razvoja

$\psi_0(x)$

$$|\psi_0\rangle = \sum_n |n\rangle \underbrace{\langle n|\psi_0\rangle}_{C_n} \qquad \vec{\psi}_0 = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$$

baza $|n\rangle$ je taka, da: $H|n\rangle = E_n|n\rangle$

Kako zgleda $\psi(t)$?

$$\vec{\psi}(t) = \begin{pmatrix} c_1 e^{-i\frac{E_1}{\hbar}t} \\ \vdots \\ c_n e^{-i\frac{E_n}{\hbar}t} \end{pmatrix}$$

$$|\psi(t)\rangle = \sum_n C_n \underbrace{e^{-\frac{iE_nt}{\hbar}}}_{\text{"} e^{-\frac{iH}{\hbar}t}} |n\rangle = e^{-i\frac{H}{\hbar}t} \sum_n |n\rangle\langle n|\psi_0\rangle =$$

$$= \underbrace{e^{-\frac{iH}{\hbar}t}}_{U(t)} |\psi_0\rangle$$

↳ unitarni operator časovnega razvoja

velja: $\langle\psi_0|\psi_0\rangle = 1 = \langle\psi(t)|\psi(t)\rangle$

13) RAZNO :

a) operator A : $A|a\rangle = a|a\rangle$

$$A|n\rangle = a_n|n\rangle$$

$$\text{in}: \quad |\psi_1\rangle = \sum_n |n\rangle \overbrace{\langle n|\psi_1\rangle}^{c_n}$$

$$|\psi_2\rangle = \sum_n |n\rangle \overbrace{\langle n|\psi_2\rangle}^{d_n}$$

in nas zanima $\langle\psi_1|\psi_2\rangle$:

$$\langle\psi_1|\psi_2\rangle = \langle\psi_1|\sum_n |n\rangle\langle n|\psi_2\rangle = \sum_n \langle\psi_1|n\rangle\langle n|\psi_2\rangle = \sum_n c_n^* d_n$$

ali pa:

$$\langle\psi_1|\psi_2\rangle = \overbrace{\sum_m \langle\psi_1|m\rangle\langle m|}^{\langle\psi_1|}\sum_n |n\rangle\langle n|\psi_2\rangle = \sum_m c_m^* d_m$$

$$\langle m|n\rangle = \delta_{m,n}$$

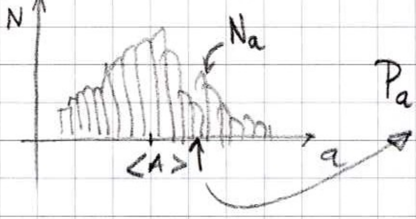
b) $|\psi\rangle = \sum_a |a\rangle\langle a|\psi\rangle$ in $A|a\rangle = a|a\rangle$

$$\langle A\rangle = \langle\psi|A|\psi\rangle = \langle\psi|\sum_a a|a\rangle\langle a|\psi\rangle$$

$$= \sum_a a|\langle\psi|a\rangle|^2 = \sum_a a|c_a|^2$$

vemo: $\sum_a |c_a|^2 = 1$ $\quad P_a = |c_a|^2$ → verjetnost, da bomo izmerili lastno vrednost a, da je delec v stanju $|a\rangle$

Ko ti meriš delec, lahko pri posamezni meritvi le eno lastno vrednost izmeriš. in dobiš:



$P_a = \frac{N_a}{N}$

če bi neskončnokrat meril, bi bilo $P_a = |c_a|^2$

Postulati (aksiomi) kvantne mehanike

1) Stanje sistema je opisano z $|\psi\rangle$. (vektor psi)

2) Opazljivke predstavimo s hermitskimi operatorji. A
↳ za vsako stvar, ki jo lahko meriš $\exists$ operator

3) Pričakovana vrednost $\langle\psi|A|\psi\rangle$ → večkratno merjenje enakega $|\psi\rangle$ ti daje različne meritve (matrika)

4) Dinamika funkcije je podana s Schrödingerjevo enačbo: $i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$

5) $A|a\rangle = a|a\rangle$ Obstaja kompletna baza in $\forall\psi\ \exists$ razvoj:
hermitski operator

$$|\psi\rangle = \sum_a |a\rangle\overbrace{\langle a|\psi\rangle}^{c_a}$$

v zvezi z meritvijo: Merilna aparatura detektira v eni meritvi le eno od možnih lastnih vrednosti $a \in \mathbb{R}$.

→ verjetnost s katero izmeriš a

$$\langle\psi|A|\psi\rangle = \sum_a a P_a \quad ; \quad P_a = |\langle a|\psi\rangle^2| = |C_a|^2$$

$$\sum_a P_a = 1$$

Kopenhagenska šola dodaja : Imamo $|\psi\rangle$ ob $t=0$, izmerim a z verjetnostjo P_a, po meritvi je $|\psi(t)\rangle \equiv |a\rangle$.
↳ za $t>0$

ni 100 % ziher še... se še debatira

↳ Kolaps valovne funkcije. Valovna f. kolapsira. Prej je bila bogata, karkoli, zdaj je $|a\rangle$.

↳ par paradoksov se pojavi, ki pa jih še kar krotijo.
EPR paradoks:
sistem s sferično simetrijo : $s=0$
Sistem razpade (recimo na pozitron in e^-). Kateri ima spin gor, kateri dol? Skupna vrtilna količina je $\emptyset$.
Valovna funkcija obeh skupaj je kombinacije GOR/DOL.
Pozitron naj leti v Stern-Gerlachov poskus. In recimo, da gre gor. V trenutku ko se ukloni gor, ko se pozitron odloči, da je gor, se istočasno e^- odloči, da je dol.
↳ ni tako, da se onadva že ko razpadeta zmenita, jaz bom gor, ti boš dol. Ne, ne. Elektron nima določenega spina dokler ga ne izmeriš.
Problem je ta mehanizem kako se onadva dogovarjata.

vse kar delamo tule je NERELATIVISTIČNO! Delci ubogajo Newtonov zakon.

• "Bohm-ove skrite spremenljivke" → poskuše pokazati da je za KM še nekaj več; spr., ki jih ne vidimo

• Bellove neenačbe → kažejo, da je to kar se mi tle učimo kar prav

Reprezentacija - p

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \quad \dashrightarrow \text{iščemo } \psi_p \text{, da: } \hat{p}\,\psi_p(x) = p\,\psi_p(x)$$
↳ $\in \mathbb{R}$

Taka ψ_p je : $\psi_p(x) = \frac{1}{\sqrt{2\pi\hbar}}\, e^{i\frac{p}{\hbar}x}$

in integriram $\int \psi_{p_1}^*(x)\,\psi_{p_2}(x)\,dx = \delta(p_1 - p_2)$

$$\frac{1}{2\pi}\int e^{i(k_1-k_2)x}\,dx = \delta(k_1-k_2)$$

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$

Ta ψ_p je nepravi vektor. Ne moremo ga normirat. Le takole

→ so posebne. Recimo, lastne vrednosti so zvezne.

→ je pa fajn Ψ_p, ker se da ∀ ψ razvit po njih:

$$\psi(x) = \int \varphi(p)\Psi_p(x)\,dp$$

$$\hat{p}|p\rangle = p|p\rangle \qquad \hat{p}\,\Psi_p = p\Psi_p(x) \qquad |\Psi_p\rangle = |p\rangle$$

Dirac: $|\psi\rangle = \int |p\rangle\langle p|\psi\rangle\,dp$

če je baze števno smo prej imeli: $|\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle$ / ko ni, stvar preide v integral

$\hat{p}|p\rangle = p|p\rangle$ → to so nenormirane funkcije

$\hat{p}\,\Psi_p(x) = p\Psi_p(x)$

$$\boxed{\varphi(p) = \langle p|\psi\rangle}$$ → koeficienti v Fourierjevem razvoju

$\frac{dP_p}{dp} = |\langle p|\psi\rangle|^2$ → verjetnost, da v ψ izmerimo gib. kol. p

$$\langle\psi|\hat{p}|\psi\rangle = \iint \langle\psi|p_1\rangle\langle p_1|\underbrace{\hat{p}|p_2\rangle}_{p_2|p_2\rangle}\langle p_2|\psi\rangle\,dp_1\,dp_2 = \int p\underbrace{|\langle p|\psi\rangle|^2 dp}_{dP_p}$$

$$p_2\delta(p_1-p_2)$$

$|p\rangle$ je vektor.
Ψ_p je funkcija.

Ψ_p je prava funkcija. In ko po teh funkcijah razvijemo, so koeficienti $\varphi(p) = \langle p|\psi\rangle$.

$|\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle$ nezvezno

$|\psi\rangle = \int |p\rangle\langle p|\psi\rangle\,dp$ zvezno

~~Reprezentacija~~ x

$$\hat{p}|\psi\rangle = \hat{p}\int |p\rangle\langle p|\psi\rangle\,dp = \int p|p\rangle\langle p|\psi\rangle\,dp$$

$$\hat{p}\,\psi(x) = -i\hbar\frac{\partial}{\partial x}\psi(x) = \int p\frac{e^{\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}\varphi(p)\,dp$$

$\psi(x) = \int \varphi(p)\Psi_p(x)\,dp$ → $\hat{p}$ na ψ naredi ψ, ki ima pod integralom namesto $\varphi(p)$, $p\cdot\varphi(p)$

→ vidimo: $\left(-i\hbar\frac{\partial}{\partial x}\right)^n\psi(x) \Leftrightarrow p^n\varphi$

Reprezentacija -x

$\hat{X}$ - - → iščemo Ψ_{x_0}, da bo: $\hat{x}\Psi_{x_0}(x) = x_0\Psi_{x_0}(x)$

množenje z x → $\hat{x}$; lastna vrednost → x_0

taka je δ: $x\delta(x-x_0) = x_0\delta(x-x_0)$

ampak δ se ne da normirat. / $\int\delta^2 dx = ?$

Spet = nepravi vektor.

Poglejmo: $\hat{x}\psi_{x_0}(x) = x\psi_{x_0}(x) = \int \varphi_{x_0}(p)\, x\, \psi_p(x)\, dp =$

$\psi_{x_0}(x) = \int \varphi(p)\psi_p(x)\,dp$ → Fourierov integral pač

$$= \int \varphi_{x_0}(p)\, x \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{ipx}{\hbar}}\, dp = \int \varphi_{x_0}(p)\left(-i\hbar\frac{\partial}{\partial p}\overbrace{\frac{1}{\sqrt{2\pi\hbar}} e^{\frac{ipx}{\hbar}}}^{\psi_p(x)}\right) dp$$

spomnimo se: $\int f(x)\frac{\partial}{\partial x}g(x)\,dx = \int\left[-\frac{\partial}{\partial x}f\right] g\, dx$ odvod je antihermitski.

Torej: $\hat{x}\psi_{x_0}(x) = \int \underbrace{\left[\overbrace{i\hbar\frac{\partial}{\partial p}}^{\hat{x} \text{ v p reprezentaciji}}\varphi_{x_0}(p)\right]}_{\tilde{\varphi}_{x_0}(p)}\psi_p(x)\,dp$ → v razvoju se pred $\varphi_{x_0}(p)$ znajde $i\hbar\frac{\partial}{\partial p}$

$= x_0\psi_{x_0}(x)$ ker to velja, mora veljati tole

$$x^n\psi_{x_0}(x) \leftrightarrow \left(i\hbar\frac{\partial}{\partial p}\right)^n \varphi_{x_0}(p)$$

ko funkcijo ψ_{x_0} množiš z x, torej ko narediš $\hat{x}\psi_{x_0}$, spremeniš φ_{x_0} v razvoju v $\tilde{\varphi}_{x_0}$

$\hat{x}\varphi_{x_0}(p) = x_0\varphi_{x_0}(p)$

$$= \int x_0\varphi_{x_0}(p)\psi_p(x)\,dp$$

Torej smo rekli, da mora veljati: $i\hbar\frac{\partial}{\partial p}\varphi_{x_0}(p) = x_0\varphi_{x_0}(p)$

$\Downarrow$

taka je $\varphi_{x_0}(p) = \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{ipx_0}{\hbar}}$

to je pa $\varphi_{x_0}(p) = \psi_p^*(x_0)$

$\hat{x}\psi(x) = x\psi(x)$

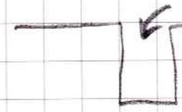
$\psi(x) = \int\varphi(p)\psi_p(x)\,dp$ → $\varphi(p) = \int \frac{e^{-\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}\psi(x)\,dx$

to smo tu pokazali

$\underline{x\psi(x)} = \int\varphi(p)\,x\,\psi_p(x)\,dp = \int\left[i\hbar\frac{\partial}{\partial p}\varphi(p)\right]\psi_p(x)\,dp = x_0\psi(x)$

dobim novo funkcijo

torej mora biti $i\hbar\frac{\partial}{\partial p}\varphi(p) = x_0\varphi(p)$

intermezzo:  ← Kako numerično poiskat rešitev?

numerično reševanje s strelsko metodo. v $-\infty$ začneš z $\psi(-\infty)$ in $\psi'(-\infty)$ in greš naprej, spreminjaš energijo, dokler ne najdeš rešitve, kjer bo $\psi(-\infty)=0$ in $\psi(\infty)=0$.

najdeš na netu luškano igročkanje

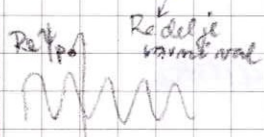
$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \; ; \quad \hat{x} = x$$

$$\hat{p}\psi_{p_0}(x) = p_0\psi_{p_0}(x) \quad , \psi_{p_0}(x) = \frac{1}{\sqrt{2\pi\hbar}}\, e^{\frac{ip_0x}{\hbar}} \qquad \int \psi_{p_1}^*(x)\psi_{p_2}(x)\,dx = \delta(p_1-p_2)$$

$$\psi(x) = \int \varphi(p)\psi_p(x)\,dp$$

$$\psi_{p_0}(x) = \int \varphi_{p_0}(p)\psi_{p}(x)\,dp \Rightarrow \varphi_{p_0}(p) = \delta(p-p_0)$$

razvili smo ravni val po ravnih valovih



Dirac: $\hat{p}|p_0\rangle = p_0|p_0\rangle \qquad \langle p_1|p_2\rangle = \delta(p_1-p_2)$

$$\int |p\rangle\langle p|\,dp$$

$$|\psi\rangle = \int |p\rangle\langle p|\psi\rangle\,dp$$

ψ_p je lastna funkcija operatorja $\hat{p}$. $\qquad |p\rangle$ je lastni vektor operatorja $\hat{p}$.

$$\langle p|\psi\rangle = \varphi(p)$$

$$\hat{p}\psi(x) \longleftrightarrow p\varphi(p)$$

v razvoju se pomnoži z p
v p reprezentaciji se pomnoži z p.

Reprezentacija v p pomeni, da poznamo $\varphi(p)$. Torej p odvisnost, ne x.

V x reprezentaciji je pa $\psi(x)$.

Smo zadnjič pokazali:

$$\hat{x}\psi(x) = x\psi(x) = \int \left[i\hbar\frac{\partial}{\partial p}\varphi(p)\right]\psi_p(x)\,dp$$

$$\hat{x}^n\psi(x) = x^n\psi(x) \longleftrightarrow \left(i\hbar\frac{\partial}{\partial p}\right)^n\varphi(p)$$

v p reprezentaciji

$$\hat{x}\psi_{x_0}(x) = x_0\psi_{x_0}(x) = \int\left[i\hbar\frac{\partial}{\partial p}\varphi_{x_0}(p)\right]\psi_p(x)\,dp = \int x_0\varphi_{x_0}(p)\psi_p(x)\,dp$$

$$\Rightarrow i\hbar\frac{\partial}{\partial p}\varphi_{x_0}(p) = x_0\varphi_{x_0}(p)$$

$$\Rightarrow \varphi_{x_0}(p) = \frac{1}{\sqrt{2\pi\hbar}}e^{-\frac{ipx_0}{\hbar}}$$

$$= \psi_p^*(x_0)$$

$$\psi_{x_0}(x) = \int\varphi_{x_0}(p)\psi_p(x)\,dp = \int\psi_p^*(x_0)\psi_p(x)\,dp = \delta(x-x_0)$$

Lastna funkcija ψ_{x_0} je δ funkcija.

Dirac:

$$\hat{x}|x_0\rangle = x_0|x_0\rangle$$

$$|x_0\rangle = \int|p\rangle\langle p|x_0\rangle\,dp \qquad \text{Torej} \Rightarrow \langle p|x_0\rangle = \varphi_{x_0}(p) = \psi_p^*(x_0)$$

poljuben $|\psi\rangle = \int|p\rangle\langle p|\psi\rangle\,dp \quad /\cdot\langle x_0|$

$$\langle x_0|\psi\rangle = \int\underbrace{\langle x_0|p\rangle}_{\psi_p(x_0)}\underbrace{\langle p|\psi\rangle}_{\varphi(p)}\,dp =$$

$$= \int\varphi(p)\psi_p(x_0)\,dp = \psi(x_0)$$

Pomemben rezultat:

$$\psi(x) = \langle x|\psi\rangle$$

za na platnico zvezka napisat!

vektor: $|\psi\rangle$

: $|x\rangle$

$\langle x|$ → je lastna f. operatorje $\hat{x}$, ki ima l. vr. x

$$\Rightarrow \langle x|\psi\rangle = \int\delta(x'-x)\psi(x')\,dx' = \psi(x)$$

$$\ast\ \langle x|x_0\rangle = \delta(x-x_0)$$

$$\int|x\rangle\langle x|\,dx = I$$

poljuben $|\psi\rangle = \int|p\rangle\langle p|\psi\rangle\,dp =$

$$= \int\underbrace{|x\rangle}_{\delta\text{ funkcije}}\underbrace{\langle x|\psi\rangle}_{\psi(x)}\,dx$$

koeficien v razvoju po δ-ah