$P_- = |\langle \psi_- | \psi_+ \rangle|^2$ $\quad P_+ = |\langle \psi_+ | \psi_+ \rangle|^2 = |\langle \psi(0) | \psi(t) \rangle|^2$

$\hookrightarrow \psi_+(t)$ $\quad$ ↳ verjetnost, da je v stanju ψ_+

$$|\psi_+(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\omega_1 t}|1\rangle + e^{-i\omega_2 t}|2\rangle\right)$$

$$P_- = \left|\frac{1}{\sqrt{2}}\left(e^{-i\omega_1 t} + e^{-i\omega_2 t}\right)\right|^2 = \cos^2 \overbrace{\frac{\omega_2 - \omega_1}{2}}^{\frac{\Delta\omega}{2}} t = 1 - P_+$$

verjetnost oscilira = UTRIPANJE

→ čeprav je na začetku le $|\psi_+\rangle$, s časom postaje mešanica $|\psi_+\rangle$ in $|\psi_-\rangle$. Ker $|\psi_+\rangle$ NI LASTNO stanje!

Simetrije H

Hamiltonova f. ima lahko simetrije, kar je lušno.

1 • operator parnosti P (naredi refleksijo prostora) $\vec{r} \to -\vec{r}$

$$P f(\vec{r}) = f(-\vec{r})$$

1D : $x \to -x$

* $P f(x) = f(-x) = \lambda f(x)$ $\quad$ če $\lambda = \begin{cases} +1 & \text{soda } f \\ -1 & \text{liha } f \end{cases}$ $\quad$ Lahko pa λ ni ne 1 ne -1...

* $\left(P \frac{\partial^2}{\partial x^2}\right) f(x) = \frac{\partial^2}{\partial (-x)^2} f(x) = \frac{\partial^2}{\partial x^2} f(x)$

P na operatorju nas zanima

$$\frac{\partial}{\partial(-x)}\left[\frac{\partial}{\partial(-x)}\right] = \frac{\partial}{\partial(-x)}\left(-\left[\frac{\partial}{\partial x}\right]\right) = \frac{\partial^2}{\partial x^2}$$

$$P\frac{\partial^2}{\partial x^2} f(x) = P\left(\frac{\partial^2}{\partial x^2} f(x)\right) = \frac{\partial^2}{\partial x^2} P f(x) \Rightarrow \left[\frac{\partial^2}{\partial x^2}, P\right] = 0$$

↳ komutirata

Naj bo: $P V(x) = V(x)$ Potencial je soda funkcija.

$$P[V f(x)] = V P f(x) \to [P, V] = 0$$

$$H = \frac{p^2}{2m} + V \Rightarrow [H, P] = 0$$

$$P H \psi(x) = H P \psi(x)$$

$H\psi(x) = E\psi(x) \Rightarrow PH\psi = H\psi(-x) = EP\psi = E\psi(-x)$

$H\psi(-x) = E\psi(-x)$ → obema ustreza enaka energija

tvorimo: $\psi_\pm(x) = \frac{1}{\sqrt{2}}\left(\psi(x) \pm \psi(-x)\right)$ → ena je soda (+), ena liha (−)

$$P\psi_\pm(x) = \pm\psi_\pm(x)$$

$$\psi_+, \psi_- \to E$$

$\Rightarrow$ če E ni degeneriran $\Rightarrow \psi_E(-x) = \pm\psi_E(x)$

Potem je rešitev soda ali liha. Torej ko vidiš simetrični potencial, in E ni deg, so rešitve sode ali lihe.

če je degenerirano: $\psi_{1,2} = C_+^{1,2}\psi_+ + C_-^{1,2}\psi_-$ → je lin. komb sode in lihe rešitve

→ Prva rešitev je prva lin. komb, druga rešitev je druga lin. komb.

BTW: Soda funkcija ima manjšo energijo.
Ker $\frac{\partial^2}{\partial x^2}$ je manjši, manj je ukrivljeno, ker ni ničel, in je manjša en.

2. Obrat časa T

$$T\psi(x,t) = \psi(x,-t)$$

če: $V \neq V(t)$

$$[T, V] = 0$$

$$i\hbar\frac{\partial}{\partial t}\psi = H\psi(x,t) \quad /\ ^{\text{konjug.}}$$

$$-i\hbar\frac{\partial}{\partial t}\psi^* = (H\psi)^* = H^+\psi^* = H\psi^* \qquad H = H^+$$

nova spr: $\tau = -t$

$$\psi^*(t) = \psi^*(-\tau)$$

$$i\hbar\frac{\partial}{\partial\tau}\tilde{\psi}(\tau) = H\tilde{\psi}(\tau)$$

→ sta enaki enačbi! našli smo: $\tilde{\psi}(t) = \underline{\psi(-t) = \psi^*(t)}$!

$$\boxed{T\psi(t) = \psi(-t) = \psi^*(t)}$$

ravni val: $(e^{ikx-i\omega t})^*$ → gre v drugo smer kot $e^{ikx-i\omega t}$

časovni razvoj: $\psi(x,t) = e^{-\frac{iHt}{\hbar}}\psi(x,0)$

$$\psi(x,-t) = \ \ldots$$

→ Posledica: $H\psi(x) = E\psi(x)$ / Stacionarno stanje, ki nima časa noter.

(če $V \neq V(t)$ lahko dobimo stacion. rešitve)

in: $H\psi^*(x) = E\psi^*(x)$

→ ψ in ψ^* imata enako energijo.

* Če E ni degeneriran: $\psi^* = \psi$ ψ je realna oz: $\psi^* = \psi e^{i\varphi}$ $\psi \in \mathbb{R}$, $\varphi \in \mathbb{R}$

(to je še bolj prav)

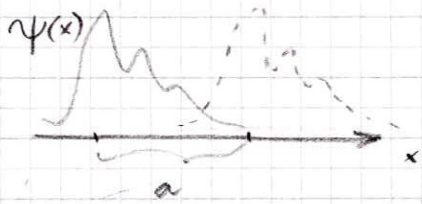
izjema: za $V=0$ → e^{ikx}, e^{-ikx}

→ imata enako energijo

→ drugače to ni prava funkcija...

→ če E je degeneriran, pa so funkcije v splošnem kompleksne.

3. Translacija $U_{\vec{a}}$



$U_a \psi(x) = \psi(x-a)$ = translacija funkcije

$$= \psi(x) - a \frac{\partial \psi}{\partial x}\Big|_x + \frac{1}{2} a^2 \frac{\partial^2}{\partial x^2} \psi(x) \pm \ldots + (-1)^n \frac{a^n}{n!} \frac{\partial^n}{\partial x^n} \psi(x) \pm \ldots =$$

$$= \left[1 - a\frac{\partial}{\partial x} + \ldots + (-1)^n \frac{a^n}{n!} \frac{\partial^n}{\partial x^n} + \ldots \right] \psi(x) =$$

$$U_a \psi(x) = e^{-a\frac{\partial}{\partial x}} \psi(x)$$

$$\rightarrow -\frac{\partial}{\partial x} = \frac{(-i)(-i\hbar\frac{\partial}{\partial x})}{\hbar}$$

$$U_{\vec{a}} = e^{-i\frac{\vec{a}\cdot\vec{p}}{\hbar}}$$

Operator gibalne količine je generator premika.

→ če hočeš računat s tem razviješ v Taylorja:

$$e^{-\vec{a}\cdot\nabla} = 1 - a_x \frac{\partial}{\partial x} - a_y \frac{\partial}{\partial y} + \frac{1}{2!}\left(\frac{\partial^2}{\partial x^2} + 2\frac{\partial}{\partial x}\frac{\partial}{\partial y} + \frac{\partial^2}{\partial y^2}\right) a_x a_y + \ldots$$

$$\rightarrow U_{\vec{a}+\vec{b}} = U_{\vec{a}} U_{\vec{b}} = U_{\vec{b}} U_{\vec{a}}$$

Tvori grupo.

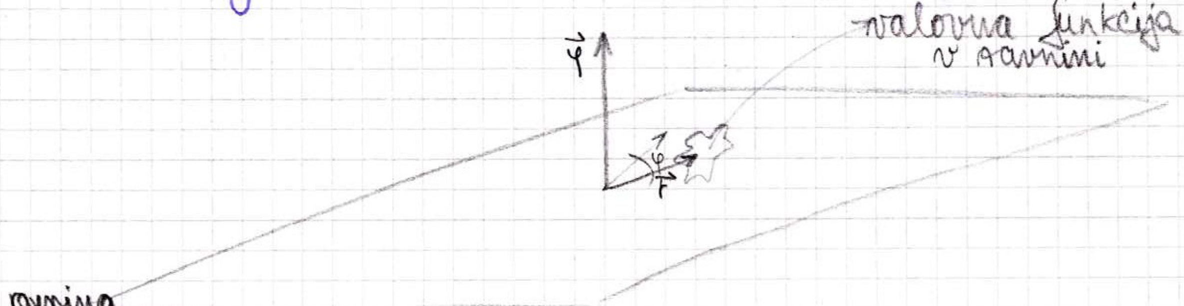
za to pokazat:

vemo: $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}$ in $\vec{a}\cdot\vec{p} = A$, $\vec{b}\cdot\vec{p} = B$

D.N. $U_{\vec{a}}^{\dagger} \vec{r}\, U_{\vec{a}} = \vec{r} + \vec{a}$

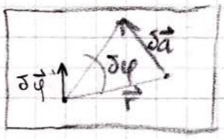
$e^{i\frac{\vec{a}\cdot\vec{p}}{\hbar}}$

4. rotacija $U_{\vec{\varphi}}$



$U_\varphi f(\vec r)$ → kaj bi to bilo

Ravnina od zgoraj:



$\vec r + \delta\vec a = \vec r\,'$ →

$$\delta\vec\varphi \times \vec r = \delta\vec a$$

$$\underline{U_{\delta\vec\varphi} f(\vec r) = f(\vec r - \delta\vec a)}$$

$$= f(\vec r) - \delta\vec a \cdot \nabla f(\vec r)\Big|_{\vec r} + \ldots$$

$$= f(\vec r) - (\delta\vec\varphi \times \vec r)\cdot \nabla f(\vec r) + \mathcal{O}(\delta\varphi^2)$$

$$= f(\vec r) - \delta\vec\varphi \cdot (\vec r \times \nabla) f(\vec r) + \ldots$$

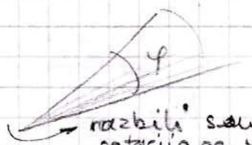
analizija:

$$(\vec\varphi \times \vec r)\cdot \nabla f = \vec\varphi \cdot (\vec r \times \nabla f)$$

$$(\vec a \times \vec b)\cdot \vec c = \vec a (\vec b \times \vec c)$$

$$U_{\delta\vec\varphi} = 1 - \frac{i\delta\vec\varphi\cdot(\vec r \times \vec p)}{\hbar} + \mathcal{O}(\delta\varphi^2)$$

$$U_{\delta\vec b} = 1 - i\frac{\delta\vec b \cdot \vec p}{\hbar}$$ → translacija → mala



razbiti samo rotacijo za φ na $\delta\varphi = \frac{\varphi}{N}$

$$\underbrace{U_{\delta\vec\varphi}\, U_{\delta\vec\varphi} \ldots U_{\delta\vec\varphi}}_{N} f(\vec r) = U_{\vec\varphi}$$

Spomnemo se: $\lim_{N\to\infty}\left(1+\frac{x}{N}\right)^N = e^x$

torej: $U_{\vec\varphi} = e^{-\frac{i\vec\varphi\cdot\vec L}{\hbar}}$ → $\vec L = \vec r \times \vec p$ je generator rotacije

↓ vrtilna količina

$$\hat{\vec L} = \hat{\vec r} \times \hat{\vec p} = -\hat{\vec p} \times \hat{\vec r}$$

vemo: $[r_i, p_j] = i\hbar\delta_{ij}$

$$\vec L = (y p_z - z p_y, \ \ldots \ )$$

komutirata

Recimo: $(\vec r \times \vec p)\psi = \vec r \times (-i\hbar\nabla\psi)$

ali: $-(\vec p \times \vec r)\psi = -(-i\hbar\nabla\times\vec r)\psi$

$$= i\hbar(\nabla\psi \times \vec r)$$

je res enako

mafije:

$$\nabla \times f\vec v = f\nabla\times\vec v + \nabla f \times \vec v$$

$$\nabla\times\vec r = 0$$