CONDUCTANCE OF NANOSYSTEMS WITH INTERACTION

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Abstract The zero-temperature linear response conductance through an interacting mesoscopic region attached to noninteracting leads is investigated. We present a set of formulas expressing the conductance in terms of the ground-state energy of an auxiliary system, namely a ring threaded by a magnetic flux and containing the correlated electron region. We prove that the formalism is exact if the ground state of the system is a Fermi liquid. We show that in such systems the ground-state energy is a universal function of the magnetic flux, where the conductance is the relevant parameter. The method is illustrated with results for the transport through an interacting quantum dot and a simple Aharonov-Bohm ring with Kondo-Fano resonance physics.

Keywords: Conductance; Nanosystems; quantum dot

1. Introduction

In the last decade technological advances enabled controlled fabrication of small regions connected to leads and the conductance, relating the current through such a system to the voltage applied between the leads, proved to be the most important property of such systems. There is a number of such examples, e.g. metallic islands prepared by e-beam lithography or small metallic grains,[1] semiconductor quantum dots,[2] or a single large molecule such as a carbon nanotube or DNA. It is possible to break a metallic contact and measure the transport properties of an atomic-size bridge that forms in the break,[3] or even measure the conductance of a single hydrogen molecule. [4] Recent measurements of conductance through single molecules proved that strong electron correlations can play an important role in such systems. [5]

The transport in noninteracting mesoscopic systems is theoretically well described in the framework of the Landauer-Büttiker formalism. The conductance G is at zero temperature determined with the Landauer-Büttiker formula [6]

$$G = G_0 \left| t\left(\epsilon_F\right)\right|^2, \qquad G_0 = \frac{2e^2}{h}. \tag{1}$$

The key quantity here is the single particle transmission amplitude $t(\epsilon_F)$ for electrons at the Fermi energy. The formula proved to be very useful and reliable, as long as electron-electron interaction in a sample is negligible. However, the Landauer-Büttiker formalism cannot be directly applied to systems where the interaction between electrons plays an important role. Several approaches have been developed to allow one to treat such systems. The Kubo formalism provides us with a conductance formula which is applicable in the linear response regime and was intensively studied by Oguri.[7, 8] A more general approach applicable also to non-equilibrium cases was developed by Meir and Wingreen. [9] Recently, ab initio methods to study the transport through small molecular junctions were also applied.[10]

2. Conductance formulas for Fermi liquid systems

The relevant system is schematically presented in Fig. 1(a). A mesoscopic interacting region, which could be a molecule, a quantum dot, a quantum dot array or a similar 'artificial molecule' system, is attached to noninteracting leads. As shown in Ref. [11] (hereafter referred to as RR), the conductance of such a system can be determined solely from the ground-state energy of an auxiliary system, formed by connecting the leads of the original system into a ring and threaded by a magnetic flux, Fig. 1(b). The main advantage of this method is the fact that it is often much easier to calculate the ground-state energy (for example, using variational or quantum Monte Carlo methods) than the Green's function, which is needed in the Kubo and Keldysh approaches. The method is applicable only to a certain class of systems, namely to those exhibiting Fermi liquid properties, at zero temperature and in the linear response regime. However, in this quite restrictive domain of validity, the method promises to be easier to use than the methods mentioned above.

The basic property that characterizes Fermi liquid systems is that the states of a noninteracting system of electrons are continuously transformed into states of the interacting system as the interaction strength increases from zero to its actual value.[12] One can then study the properties of such a system by means of the perturbation theory, regarding the interaction strength as the perturbation parameter. Dynamics of Fermi liquid systems at low temperature and in the linear response regime is governed by quasiparticles. However, the question how quasiparticles propagate in a correlated system is a non-trivial one. The answer can be extracted from the Green's function for a particular problem if it is known. An alternative way, which we advocate in this paper, is to analyze the excitation spectrum of a system directly. If E_M and E_{M+1} are the ground-state energies of an interacting ring system containing M and $M+1$ electrons,

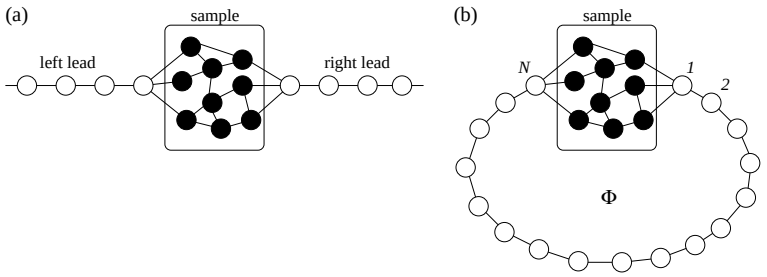

Figure 1. (a) Schematic picture of a sample with interaction connected to noninteracting leads. (b) The sample embedded in a ring formed by joining the left and right leads of the system (a). Auxiliary magnetic flux $\Phi = \frac{\hbar}{e}\phi$ penetrates the ring.

respectively, the energy difference can be attributed to the first quasiparticle energy level $\tilde{\epsilon}$ above the Fermi energy,

$$\tilde{\epsilon} = E_{M+1} - E_M. \tag{2}$$

The variation of the quasiparticle energy with flux threading the ring allows us to determine the conductance of the system. The complete proof of the formalism is given in RR and a brief overview is presented in the next Section. Here we show how the method can be implemented in practice.

The key property of ring systems presented in Fig. 1(b) is the universality expressed in the variation of the ground-state energy with auxiliary magnetic flux through the ring. Here we assume a system obeys the time reversal symmetry. The more general case is presented in the last Section. For an even number of electrons in the system and a large number of sites in the ring $N \to \infty$ the ground-state energy takes a universal form

$$E(\phi) - E\left(\frac{\pi}{2}\right) = \frac{\Delta}{\pi^2}\Big(\arccos^2\left(\mp\sqrt{g}\cos\phi\right) - \frac{\pi^2}{4}\Big), \tag{3}$$

where the average level spacing at the Fermi energy $\Delta = [N\rho(\epsilon_F)]^{-1}$ is determined by the density of states at the Fermi energy in an infinite noninteracting lead $\rho(\epsilon_F)$ and $g = G/G_0$ is the dimensionless conductance. For systems with an odd number of electrons, the ground-state energy is given with

$$E(\phi) - E\left(\frac{\pi}{2}\right) = \frac{\Delta}{\pi^2}\arcsin^2\left(\sqrt{g}\cos\phi\right). \tag{4}$$

It should be mentioned that the ground-state energy of an interacting ring system exactly corresponds to the expression for persistent currents in noninteracting rings, as derived by Gogolin. [14] The only parameter determining the ground-state energy is the conductance g of the original system, Fig. 1(a). In Fig. 2 the

ground-state energy as a function of the flux ϕ for even and odd numbers of electrons is presented. It should be pointed out that as successive quasiparticle levels are being occupied, the points $\phi = 0$ and $\phi = \pi$ interchange their roles (as is also the case for noninteracting systems), and that the periodicity in even and odd cases are π and $\pi/2$, respectively.

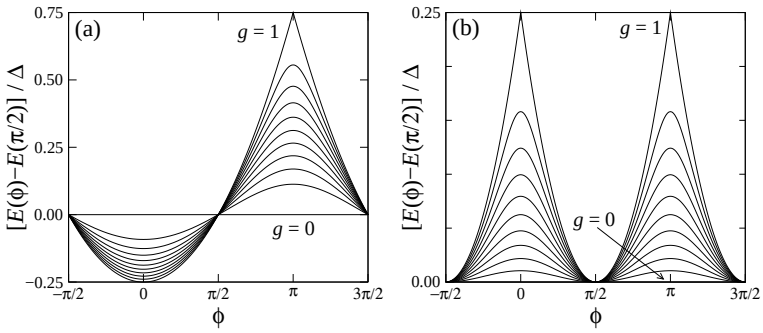

Figure 2. The ground-state energy of an interacting system as a function of flux ϕ for an even (a) and an odd (b) number of electrons for g going from 0 to 1 in steps of 0.1 and $N \to \infty$.

If the ground state of the system in Fig. 1(b) is known, the conductance of the original open system can be extracted from Eq. (3) [or Eq. (4)]. There are several ways how to determine g from Eq. (3). The simplest seems to be the use of the relation

$$g = \sin^2\left(\frac{\pi}{2}\frac{E(\pi) - E(0)}{\Delta}\right), \tag{5}$$

where $E(0)$ and $E(\pi)$ are the ground-state energies of the ring system for $\phi = 0$ and $\phi = \pi$, respectively. The first advantage of this formula is the fact that the energies can be calculated using periodic and antiperiodic boundary conditions, respectively, and thus the wave functions of the system can be taken real. Additional advantage is fast convergence with N, as briefly discussed in the last Section. This formula was derived in the $g \to 0$ limit as $g = \left(\frac{\pi}{2\Delta}[E(\pi) - E(0)]\right)^2$ by Favand and Mila for noninteracting systems and applied to interacting Hubbard chains.[15] More recently, a similar approach was performed in Ref. [16]. In Fig. 3(a) the use of formula Eq. (5) is schematically presented.

The derivative of the ground-state energy with respect to flux gives the persistent current in the ring $j(\phi) = \frac{e}{\hbar}\frac{\partial E}{\partial \phi}$. [17] The second formula relates the conductance to the persistent current at $\phi = \frac{\pi}{2}$,

$$g = \left(\frac{\pi}{\Delta}\frac{\hbar}{e} j\left(\frac{\pi}{2}\right)\right)^2. \tag{6}$$

This relation was recently derived for noninteracting systems [18] and successfully applied to systems with interaction.[18, 19]

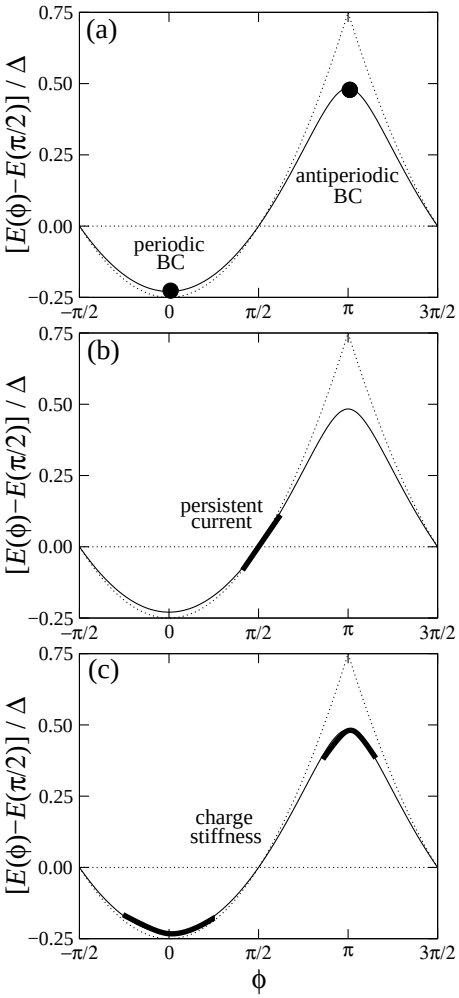

Figure 3 The conductance can be extracted from: (a) the two-point formula, Eq. (5), (b) the persistent current formula, Eq. (6), or (c) the charge stiffness formula, Eq. (7).

At $T = 0$ the charge stiffness is an important quantity describing the charge transport in correlated systems.[20] It is defined as the second derivative of the ground-state energy of the system with respect to the flux in the minimum of the energy vs. flux curve, $D = \frac{N}{2}\,\partial^2 E/\partial\phi^2\big|_{E=\min}$.[21] The sensitivity of the ground state energy to flux has been applied also in the context of electron localisation.[22] One can also define the corresponding quantity for the energy maximum as $\tilde{D} = -\frac{N}{2}\,\partial^2 E/\partial\phi^2\big|_{E=\max}$. From Eq. (3) the conductance can be related to the charge stiffness with an implicit relation,

$$\frac{1}{\Delta}\,\frac{\partial^2 E}{\partial\phi^2}\bigg|_{E=\min,\,\max} = \pm\frac{2}{\pi^2}\sqrt{\frac{g}{1-g}}\,\arccos\left(\pm\sqrt{g}\right). \qquad (7)$$

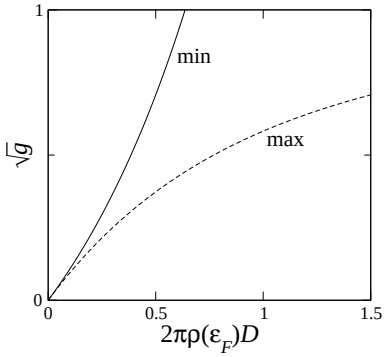

Figure 4 Conductance vs. charge stiffness using D at the energy minimum (full line) and $\tilde{D}$ at the maximum (dashed line). Note the quadratic dependence $g \propto D^2$ for $g \to 0$.

Here the upper and the lower signs correspond to the second derivative at a minimum and at a maximum of the energy vs. flux curve, respectively. In general, this equation has to be solved numerically and the solutions are presented in Fig. 4. In the limit of a very small conductance and in the vicinity of the unitary limit, analytic formulas are available

$$g = \begin{cases} \left[2\pi\rho\left(\varepsilon_F\right)D\right]^2, & g \to 0, \\ \left(\frac{1}{2} + \frac{3\pi}{4}\left[2\pi\rho\left(\varepsilon_F\right)D\right]\right)^2, & g \to 1. \end{cases} \tag{8}$$

Note that there is a quadratic relation between the conductance and the charge stiffness in the low conductance limit. The corresponding formulas for the maximum of the energy vs. flux curve are

$$g = \begin{cases} \left[2\pi\rho\left(\varepsilon_F\right)\tilde{D}\right]^2, & g \to 0, \\ \left(1 - \frac{2}{\left[2\pi\rho(\varepsilon_F)\tilde{D}\right]^2}\right)^2, & g \to 1. \end{cases} \tag{9}$$

It should be stressed that the validity of all formulas presented in this Section is based on an assumption that the number of sites in the ring is sufficiently large according to the condition [11]

$$N \gg \frac{1}{\rho\left(\varepsilon_F\right)} \frac{\partial\sqrt{g\left(\varepsilon_F\right)}}{\partial\varepsilon_F}. \tag{10}$$

This means that if $g\left(\varepsilon_F\right)$ exhibits sharp resonances, as is the case, e.g., in chaotic systems,[23] the calculation has to be performed on a large auxiliary ring system and in such cases the method might be impractical compared to other methods. On the other hand, for systems with strong interaction the method promises to be extremely efficient already for ring systems of a moderate size. [24, 16, 19]

3. Proof of the formalism

The complete proof of the formalism is presented in RR, here we briefly describe the main steps. The proof strongly relies on an assumption that the ground state of the system under investigation is a Fermi liquid.[25]

We start with the linear response conductance of a general interacting system of the type shown in Fig. 1(a). The conductance can be calculated from the Kubo formula [26]

$$g = \lim_{\omega \to 0} \frac{i\pi}{\omega + i\delta} \Pi_{II} \left(\omega + i\delta \right), \tag{11}$$

where $\Pi_{II} \left(\omega + i\delta \right)$ is the retarded current-current correlation function. For Fermi liquid systems, the current-current correlation function can be calculated within the perturbation theory. At $T = 0$, only the bubble diagram gives a non-vanishing contribution [8] and the conductance can be expressed in terms of the Green's function $G_{n'n} \left(z \right)$ of the system,

$$g = \left| \frac{1}{-i\pi\rho \left(\epsilon_F \right)} e^{-ik_F(n'-n)} G_{n'n} \left(\epsilon_F + i\delta \right) \right|^2, \tag{12}$$

where n and n' are sites in the left and the right lead, respectively.

In Fermi liquid systems obeying the time-reversal symmetry,[13] the imaginary part of the retarded self-energy at $T = 0$ vanishes at the Fermi energy and is quadratic for frequencies close to the Fermi energy.[27] Using the Fermi energy as the origin of the energy scale, i.e. $\omega - \epsilon_F \to \omega$, we can express this as

$$\mathrm{Im}\Sigma_{ij} \left(\omega + i\delta \right) \propto \omega^2. \tag{13}$$

Close to the Fermi energy, the self-energy can be expanded in powers of ω resulting in an approximation to the Green's function,

$$\begin{aligned} \mathbf{G}^{-1} \left(\omega + i\delta \right) = \omega \mathbf{1} - \mathbf{H}^{(0)} - \mathbf{\Sigma} \left(0 + i\delta \right) - \\ - \omega \left. \frac{\partial \mathbf{\Sigma} \left(\omega + i\delta \right)}{\partial \omega} \right|_{\omega = 0} + \mathcal{O} \left(\omega^2 \right). \end{aligned} \tag{14}$$

Here $\mathbf{H}^{(0)}$ contains matrix elements of the noninteracting part of the Hamiltonian. The Green's function for ω close to the Fermi energy can then be expressed as

$$\mathbf{G}^{-1} \left(\omega + i\delta \right) = \mathbf{Z}^{-1/2} \tilde{\mathbf{G}}^{-1} \left(\omega + i\delta \right) \mathbf{Z}^{-1/2} + \mathcal{O} \left(\omega^2 \right), \tag{15}$$

where we defined the quasiparticle Green's function

$$\tilde{\mathbf{G}}^{-1} \left(\omega + i\delta \right) = \omega \mathbf{1} - \tilde{\mathbf{H}} \tag{16}$$

as the Green's function of a *noninteracting quasiparticle* Hamiltonian

$$\tilde{\mathbf{H}} = \mathbf{Z}^{1/2}\left[\mathbf{H}^{(0)} + \mathbf{\Sigma}\left(0+i\delta\right)\right]\mathbf{Z}^{1/2}, \tag{17}$$

and introduced the renormalization factor matrix $\mathbf{Z}$. Matrix elements of $\mathbf{Z}$ differ from those of an identity matrix only if they correspond to sites of the central region.

The reason for introducing the quasiparticle Hamiltonian is to obtain an alternative expression for the conductance in terms of the quasiparticle Green's function. Eq. (15) relates the values of the true and the quasiparticle Green's function at the Fermi energy,

$$\mathbf{G}\left(0+i\delta\right) = \mathbf{Z}^{1/2}\tilde{\mathbf{G}}\left(0+i\delta\right)\mathbf{Z}^{1/2}. \tag{18}$$

Specifically, $G_{n'n}\left(0+i\delta\right) = \tilde{G}_{n'n}\left(0+i\delta\right)$ if both n and n' are sites in the leads, as a consequence of the properties of the renormalization factor matrix $\mathbf{Z}$ discussed above. Eq. (12) then tells us that the zero-temperature conductance of a Fermi liquid system is identical to the zero-temperature conductance of a noninteracting system defined with the quasiparticle Hamiltonian for a given value of the Fermi energy.

These conclusions are valid if the central region is coupled to semi-infinite leads. Here we generalize the concept of quasiparticles to a finite ring system with N sites and M electrons, threaded by a magnetic flux ϕ. One can define the quasiparticle Hamiltonian for such a system,

$$\tilde{\mathbf{H}}\left(N,\phi;M\right) = \mathbf{Z}^{1/2}\left[\mathbf{H}^{(0)}\left(N,\phi\right) + \mathbf{\Sigma}\left(0+i\delta\right)\right]\mathbf{Z}^{1/2}. \tag{19}$$

Here the self-energy and the renormalization factor matrix are determined in the thermodynamic limit where, as we prove in RR, they are independent of ϕ and correspond to those of an infinite two-lead system.

Suppose now that we knew the exact values of the renormalized matrix elements in the quasiparticle Hamiltonian (19). As this is a noninteracting Hamiltonian, we could then apply the conductance formulas presented in the previous Section (the proof of validity of energy formulas for noninteracting systems is given in RR and the corresponding result for persistent currents in Ref. [14]). Such a procedure would provide us with the exact conductance of the original interacting system. However, to obtain the values of the renormalized matrix elements, one needs to calculate the self-energy of the system, which is a difficult many-body problem. In RR we study the excitation spectrum of a finite ring system with interaction and threaded with a magnetic flux. We show that

$$\begin{aligned} &E\left(N,\phi;M+1\right) - E\left(N,\phi;M\right) = \\ &\quad = \tilde{\epsilon}\left(N,\phi;M;1\right) + \mathcal{O}\left(N^{-\frac{3}{2}}\right), \end{aligned} \tag{20}$$

where $E\left(N,\phi;M\right)$ and $E\left(N,\phi;M+1\right)$ are the ground-state energies of the interacting Hamiltonian for a ring system with N sites and flux ϕ, containing M and $M+1$ electrons, respectively, and $\tilde{\epsilon}\left(N,\phi;M;1\right)$ is the energy of the first single-electron level above the Fermi energy of the finite ring quasiparticle Hamiltonian (19). The error in Eq. (20) is small enough that the proof of the ground-state energy formulas for noninteracting systems, which involves only the properties of a set of neighboring single-electron energy levels, remains valid also for interacting Fermi liquid systems, provided a system is a Fermi liquid for all values of the Fermi energy below its actual value.

4. Examples

Noninteracting system

In this Section we discuss the convergence properties of the conductance formulas. As the first example we take a double-barrier potential scattering problem presented in Fig. 5. Results of various formulas for different number of sites in the ring are presented in Fig. 6.

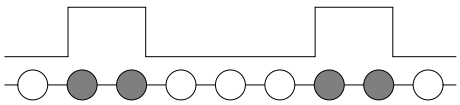

Figure 5. A double barrier noninteracting system. The height of the barriers is $0.5t$, where t is the hopping matrix element between neighboring sites.

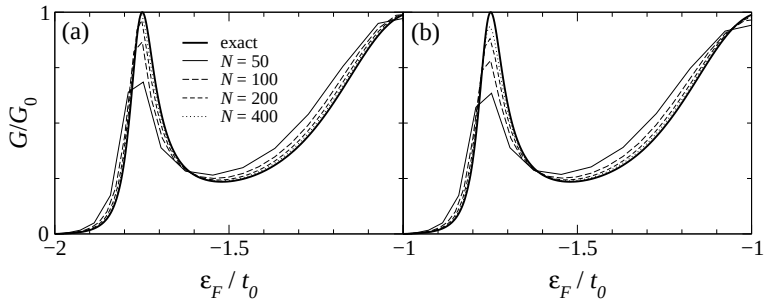

Figure 6. (a) The conductance through the system in Fig. 5 calculated from the two-point formula, Eq. (5), and (b) from the persistent current formula, Eq. (6). Note the different convergence behavior of the two formulas.

The exact zero-temperature conductance for this system exhibits a sharp resonance peak superimposed on a smooth background conductance. We notice immediately that as the number of sites in the ring increases, the convergence

is generally faster in the region where the conductance is smooth than in the resonance region, which is consistent with the condition Eq. (10). Comparing the results obtained employing the two-point formula Eq. (5) and the persistent current formula Eq. (6) we observe that the convergence is better for the two-point formula expressing the conductance in terms of the difference of the energies at $\phi = 0$ and $\phi = \pi$. From the computational point of view there is an additional advantage of the two-point formula. In this case, all the matrix elements can be made real if one chooses such a vector potential that only one hopping matrix element if modified by the flux as then the additional phase factor is $e^{\pm i\pi} = -1$.

Anderson impurity model

As a nontrivial example of the use of the formalism we calculate the zero-temperature conductance of a single impurity Anderson model realized as a quantum dot attached to leads as shown in Fig. 7.

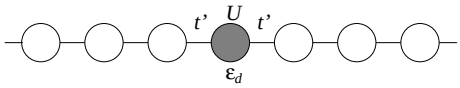

Figure 7. The Anderson impurity model realized as a quantum dot coupled to two leads. The dot is described with the energy level ϵ_d and the Coulomb energy of a doubly occupied level U. t' is the hopping between the dot and leads.

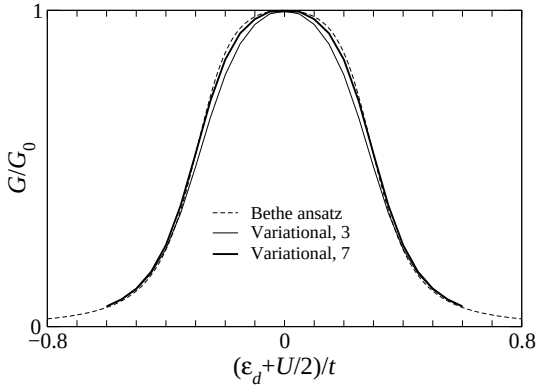

Figure 8 The zero-temperature conductance calculated from ground-state energy vs. magnetic flux in a finite ring system using the variational method described in RR with 3 and 7 basis functions. For comparison, the exact Bethe ansatz result is presented with a dashed line. The system is shown in Fig. 7, with $U = 0.64t$ and $t' = 0.2t$.

In Fig. 8 the results are compared to exact conductance of the Bethe ansatz approach.[29] To calculate the conductance, Eq. (5) was used, with the ground-state energies at $\phi = 0$ and $\phi = \pi$ obtained using a variational method described in RR. For each position of the ϵ_d level relative to the Fermi energy, we increased the number of sites in the ring until the conductance converged. The number of sites needed to achieve the convergence was the lowest in the empty orbital

regime and the highest (about 1000 for the system shown in Fig. 8) in the Kondo regime. This is a consequence of Eq. (10) as a narrow resonance related to the Kondo resonance appears in the transmission probability of the quasiparticle Hamiltonian (17) in the Kondo regime.

Interacting Aharonov Bohm rings

One can generalize the conductance formulas to systems which exhibit time reversal asymmetry, such as is e.g. an Aharonov-Bohm (AB) type of system presented in Fig. 9(a).[13]

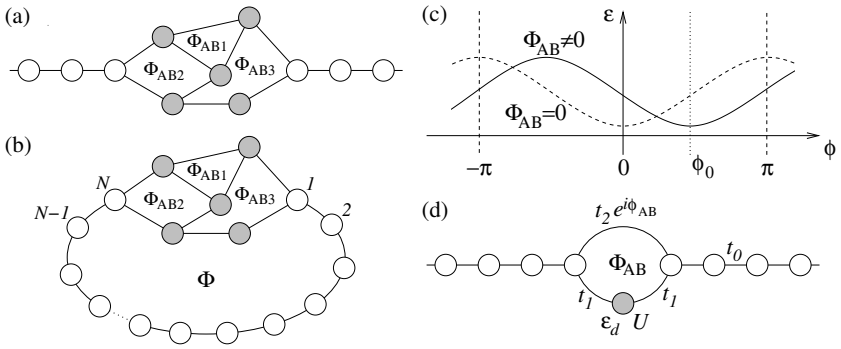

Figure 9. (a) Interacting mesoscopic region (gray-shaded sites), threaded by magnetic flux and coupled to noninteracting leads. (b) Auxiliary ring system. (c) Behavior of energy levels as the flux threading the ring is varied. (d) Example system with a quantum dot embedded in an AB ring.

If there is no AB flux threading the mesoscopic region, the time reversal symmetry is restored and the energy is an even function of ϕ. In the general case, the energy extremum is shifted to a non-trivial point $\phi_0(\varepsilon_F)$, as illustrated in Fig. 9(c). The ground-state energy is then generalized to $E(\phi) = \pi^{-2}\Delta \arccos^2\left(\mp\sqrt{g}\cos\left[\phi - \phi_0(\varepsilon_F)\right]\right) + \text{const.}$ for an even number of electrons in a system and to $E(\phi) = \pi^{-2}\Delta \arcsin^2\left(\sqrt{g}\cos\left[\phi - \phi_0(\varepsilon_F)\right]\right) + \text{const.}$ for an odd number of electrons in a system. From the former expression, the transmission probability can be extracted, and the conductance is given by [13]

$$g = \sin^2\left(\frac{\pi}{2}\frac{E(\phi_0+\pi) - E(\phi_0)}{\Delta}\right), \tag{21}$$

where $\phi_0 \equiv \phi_0(\epsilon_F)$ is determined by the position of the minimum (or maximum) in the energy vs. flux curve. The conductance can also be calculated from the more convenient four-point formula [32]

$$g = \sin^2\left(\frac{\pi}{2}\frac{E(\pi) - E(0)}{\Delta}\right) + \sin^2\left(\frac{\pi}{2}\frac{E(\pi/2) - E(-\pi/2)}{\Delta}\right), \tag{22}$$

and ϕ_0 is determined with the expression

$$\phi_0 = -\arctan \frac{\sin\left(\frac{\pi}{2}\frac{E(\pi/2)-E(-\pi/2)}{\Delta}\right)}{\sin\left(\frac{\pi}{2}\frac{E(\pi)-E(0)}{\Delta}\right)}. \tag{23}$$

If no AB flux is present in the mesoscopic region, we recover the two-point formula Eq. (5) since $E(\pi/2) = E(-\pi/2)$ in this case.

If the time reversal symmetry is broken due to AB flux, Eq. (13) is not valid and the proof of the previous Section has to be reconsidered. Repeating the steps as presented in detail in RR, the proof is restored and basically unchanged if the self energy obeys the relation

$$\frac{1}{2i}\left[\Sigma_{ij}(\omega + i\delta) - \Sigma_{ij}(\omega - i\delta)\right] \propto \omega^2. \tag{24}$$

It follows that the linear response conductance of an interacting AB system at zero temperature is given by the four-point formula Eq. (22). The condition Eq. (24) is fulfilled if the system is a Fermi liquid.

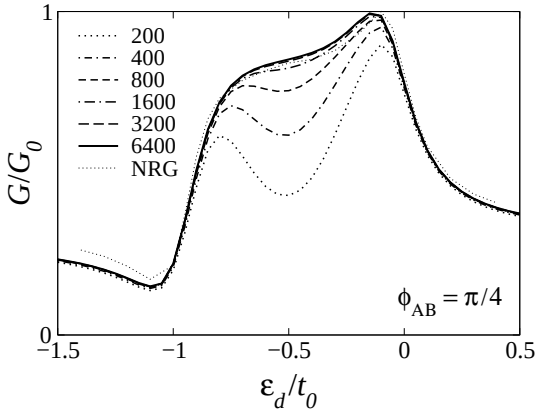

Figure 10 Zero-temperature linear response conductance of the system in Fig. 9(d) as a function of level position ε_d for various number of ring sites N and $\phi_{AB} = \pi/4$. The dotted line is the NRG result from Ref. [31]. Parameters: $t_1 = 0.177t_0$ ($\Gamma = 0.125t_0$), $t_2 = 0.298t_0$, $U = t_0 = 8\Gamma$.

In order to demonstrate the practical value of the method, we quantitatively analyze the conductance through an Aharonov-Bohm ring with a quantum dot placed in one of the arms[30, 31] as presented in Fig. 9(d). In Fig. 10 a convergence test of the method is shown. The convergence with N is fast in the empty orbital regime and becomes progressively slower as ϵ_d shifts toward the Kondo regime, as was also the case for a quantum dot attached to leads. The converged conductance curve is in excellent agreement with the numerical renormalization group result of Ref. [31].

5. Summary

We have demonstrated how the zero-temperature conductance of a sample with electron-electron interaction, attached to noninteracting leads can be determined. The method is extremely simple and is based on several formulas

relating the conductance to the ground-state energy of an auxiliary ring system. The conductance is determined from the ground-state energy of an interacting system, while in more traditional approaches, one needs to know the Green's function of the system. The advantage of the present method comes from the fact that the ground-state energy is often relatively simple to obtain by various numerical approaches, including variational methods. Let us summarize the key points of the method:

(1) The "open" problem of the conductance through a sample coupled to semi-infinite leads is mapped on to a "closed" problem, namely a ring threaded by a magnetic flux and containing the same correlated electron region.

(2) For the case of a Fermi liquid interacting system, even with broken time-reversal symmetry, it is shown that the zero-temperature conductance can be deduced from the variation of the ground state energy with the flux in a large, but finite ring system.

(3) In order to prove this, the concept of Fermi liquid quasiparticles is extended to finite, but large systems. The conductance formulas give the conductance of a system of noninteracting quasiparticles, which is equal to the conductance of the original interacting system.

(4) The results of our method are compared to results of other approaches for problems such as the transport through a quantum dot containing interacting electrons. The comparison shows an excellent quantitative agreement with exact Bethe ansatz results. We have demonstrated the usefulness of the formula also by applying it to a prototype system exhibiting Kondo-Fano behavior. Results based on the four-point formula confirm the results of the numerical renormalization group method.

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