Pseudogap and superconductivity in the $t$–$J$ model

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Abstract

In the framework of the planar $t$–$J$ model for cuprates we analyze the development of a pseudogap in the density of states. For finite doping with short range antiferromagnetic (AFM) order the spectral function retains large incoherent contribution in the hole part and exhibits large Fermi surface. At low-doping we observe the effective truncation of the large Fermi surface and the evolution of hole-pocket-like Fermi surface, reduced electron density of states and at the same time quasiparticle (QP) density of states at the Fermi level. The Green’s functions obtained with this method in the regime of optimum doping are taken as an input and the Eliashberg equations are further solved in the weak coupling limit. Resulting gap function with d-wave symmetry and the critical temperature are in the regime relevant to cuprates.

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The nature of the ground state and of low-energy excitations is one of most intriguing questions in the theory of strongly correlated electrons. The focus is on superconducting (SC) cuprates where there exists now rich experimental evidence for very anomalous low-energy properties [1], besides the most evident open question of the origin of the high-$T_c$ superconductivity. For the theoretical considerations the most straightforward signature of pseudogap physics is the reduction of the specific heat coefficient at low-temperatures in the underdoped cuprates [2] and the observation of the leading-edge shift [3] in the angle-resolved photoemission spectroscopy measurements.

The single-particle spectral function $A(k, \omega)$ and its properties are of crucial importance, since their full knowledge would essentially clarify most open questions concerning the anomalous character of strongly correlated electron systems. In recent years there has been an impressive progress in ARPES experiments [3,1] confirming a well defined large Fermi surface (FS) in the overdoped and optimally doped samples. On the underdoped BSCCO QP are dispersing through the Fermi surface (FS) and are resolved by ARPES only in parts of the large FS, in particular along the nodal $(0,0)\rightarrow(\pi,\pi)$ direction, indicating that the rest of the large FS is truncated.

The prototype single-band model relevant for cuprates which takes explicitly into account strong correlations is the $t$–$J$ model,

$$H = -\sum_{i,j} t_{ij} \tilde{c}_{ij}^\dagger \tilde{c}_{ij} + J \sum_{\langle ij \rangle} \left( S_i \cdot S_j - \frac{1}{4} n_i n_j \right),$$

where fermionic operators are projected ones not allowing for the double occupancy of sites, i.e., $\tilde{c}_{ij}^\dagger = (1 - n_{i,\sigma})c_{ij}^\dagger$. We consider besides $t_{ij} = t$ for the n.n. hopping also $t_{ij} = t'$ for the n.n.n. hopping on a square lattice.

In our analytical approach we analyze the electron Green’s function directly for projected fermionic operators [4,5], which is equivalent to the usual propagator within the allowed basis states of the model. In the equation of motion method one uses relations for general correlation functions and the key quantity is the
commutator $[c_k, H]$ \[4,5\]. In this procedure is derived the effective coupling parameter $m_{kq}$ analogous to the coupling in the spin-fermion Hamiltonian,

$$m_{kq} = 2J\gamma_q + \frac{t}{2}(c_{k-q}^+ c_k^+ + \text{h.c}),$$  
(2)

where $\epsilon_q = -4t\gamma_q - 4t\gamma_q$ is the bare band energy with $\gamma_q = (\cos k_x + \cos k_y)/2$ and $\gamma_q = \cos k_x \cos k_y$.

In the present theory spin susceptibility $\chi(q, \omega)$ is taken as an input. The system is close to the AFM instability, so we assume spin fluctuations of the overdamped form \[6\]. Nevertheless, the appearance of the pseudogap and the form of the FS are not strongly sensitive to the particular form of $\gamma(q, \omega)$ at given characteristic inverse AFM correlation length $\kappa$.

From these starting points the spectral function can self-consistently be determined, as presented in detail in Ref. [5]. Within the present theory the origin of the pseudogap feature is in the coupling to longitudinal spin fluctuations near the AFM wave vector $Q$ which determine the QP properties in the ‘hot’ region, i.e., near the AFM zone boundary. The pseudogap opens predominantly in the same region and its extent is dependent on $J$ and $t'$ but not directly on $t$. Evidently the pseudogap bears a similarity to a $d$-wave-like dependence along the FS (for $t' < 0$) being largest near the $(\pi, 0)$ point. The strength of the pseudogap features depends mainly on $\kappa$. In Fig. 1(a) and (b) is shown the Fermi surface for underdoped and optimally doped regime, respectively. It is important to note that apart from extremely small $\kappa < 0.1$ we are still dealing with a large FS. The QP in the pseudogap have small weight $Z_k \ll 1$.

The problem of superconductivity in cuprates is not completely settled down. However, under assumptions presented above one can apply the present formalism also to the SC phase. The spectral function is approximated only with coherent part, $A(k, \omega) \sim Z_k \delta(\omega - \epsilon_k)$. We study SC order parameter and $T_c$ in the weak coupling Eliashberg approximation \[7\], with partially included retardation effects,

$$A_k = \frac{1}{N} \sum_q \left[2m_{kq} + m_{kq}^2/(k-q)\Theta(\omega_q - E_q)\right]$$
$$\times \frac{\Delta_k Z_k Z_q}{2E_q} \tanh \frac{E_q}{2T},$$
(3)

where $E_q^2 = \epsilon_k^2 + \Delta_k^2$. Static spin susceptibility $\chi(q) = C[(q - Q)^2 + \kappa^2]^{-1}$ is quantitatively determined from exact diagonalization studies \[8\]. Solution of Eq. (4) is strongly momentum dependent (predominantly $d$-wave) and in Fig. 2 is presented temperature dependence of $A_k$ to its maximum for two regimes of doping.

The results for both, normal and SC spectral properties reasonable reproduce typical experimental observations. However, further study within the present formalism is necessary in order to explore possible closer quantitative agreement with experiments.

References