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Shot noise reduction in quantum wires with the 0.7 structure

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Shot noise reduction in quantum wires is interpreted within the model for the 0.7 structure in the conductance of near perfect quantum wires [T. Rejec, A. Ramšak, and J. H. Jefferson, Phys. Rev. B **62**, 12985 (2000)]. It is shown how the Fano factor structure is related to the specific structure of the conductance as a consequence of the singlet-triplet nature of the resonances with the probability ratio 1:3. An additional feature in the Fano factor, related to the 0.25 structure in conductance, is predicted.

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Conductance in various types of quantum wires and quantum point contacts is quantized in units of $G_0=2e^2/h$. Since early experimental evidence for this effect[1,2] many subsequent experiments have supported the idea of ballistic conductance in clean quantum wires. However, certain anomalies remain, some of which are believed to be related to electron-electron interactions and appear to be spin dependent. In the rising edge to the first conductance plateau, a structure appears around $0.7G_0$, merging into the plateau at higher energies.[3] Traces of such an anomaly are present already in the early measurements.[1,4] In quantum point contacts an additional structure appears around $0.25G_0$ with increasing source-drain bias[3] and this structure is also seen at low bias in hard-confined quantum wires.[4–8]

Under increasing magnetic field the 0.7 structure moves down and merges with the $\frac{1}{2}G_0$ plateau at very high fields.[4–8] At elevated temperatures the structures are eventually washed out but surprisingly this also happens at very low temperatures where the disappearance of the 0.7 structure signals the formation of a Kondo-like correlated spin state.[7,9] Related anomalies in thermopower measurements may also be explained though a violation of the Mott law emerges at very low temperatures. This additionally suggests a many-body nature of electron transport in ballistic quantum wires.[10]

Recently the nonequilibrium current noise in a one-dimensional quantum wire was measured[11,12] and this class of measurements opens up a range of possibilities for gaining insight into the problem of the transport anomalies in clean quantum wires. In particular, careful measurements of the Fano factor in the expression for shot noise for this system appears to be correlated with anomalies in conductance, signaling different transmission probabilities for spin subchannels.

In this paper we present a theoretical explanation of the peculiar dependence of the Fano factor on conductance and Fermi energy in ballistic quantum wires. We follow the idea developed in the study of conductance of a clean one-dimensional quantum wire with cylindrical[13] or rectangular cross section[14] and a very weak bulge. Several other scenarios for the 0.7 structure were recently proposed, from a phenomenological model involving enhanced spin correlations,[15] Kondo-like physics due to a localized moment,[16] fluctuation of local electron density[17] to corrections due to the backscattering of electrons due to phonons or Wigner crystal state formation.[18] The relevance of these models to the description of the 0.7 anomaly in shot noise measurements has not yet been investigated.

Here we do not limit the investigation to a particular geometry of the wire but consider the general case of a slightly imperfect quantum wire. Such structures occur naturally, e.g., from a two-dimensional electron gas (2DEG) in which a surface split gate, which depletes the 2DEG below it, gives rise to a quasi-one-dimensional conducting channel at low temperatures. Slight deviation from a perfect one-dimensional confining potential, either accidentally or deliberately, can give rise to a localized potential well with single-electron bound states. This occurs in a wire with a weak symmetric bulge[13] or in the presence of remote gates, impurities, or even electric polarization due to the electron itself. The lowest bound state with energy $\epsilon_b<0$ relative to the bottom of the first conductance channel is shown schematically in Fig. 1(a). Provided the confining potential is sufficiently weak, only a single electron will be bound since the energy of the second electron will be in the continuum due to Coulomb repulsion and the system behaves like an open quantum dot. From the numerically exact solution of two-electron scattering problem, one can extract the transmission probabilities for particular spin configurations.[13] The problem is analogous to treating the collision of an electron with a hydrogen atom, e.g., as studied by Oppenheimer and Mott.[19] This can be visualized as the scattering of the second electron in an effective potential V_{eff} arising from the combined effect of the Coulomb repulsion from the first electron and the initial confining potential [Fig. 1(b)]. In the absence of a magnetic field the appropriate spin subchannels are singlet and triplet and, as illustrated in Fig. 1(b), the two-electron system exhibits singlet and triplet quasi-bound-state resonances in transmission. Summing over all electrons in the leads gives the current,[13,20]

$$I=\frac{2e}{h}\int\left(\frac{1}{4}T_0+\frac{3}{4}T_1\right)(f_L-f_R)d\epsilon, \qquad (1)$$

where $T_S=T_S(\epsilon)$ is an energy-dependent singlet or triplet transmission probability for $S=0$ and $S=1$, respectively.

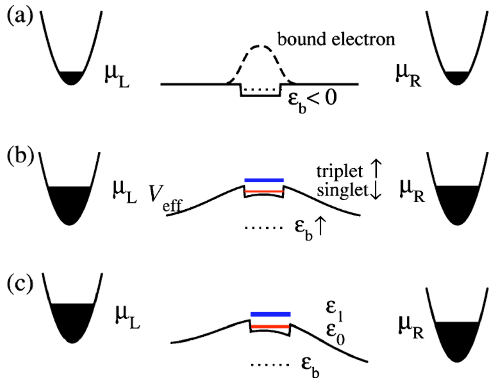

FIG. 1. (Color online) (a) A weak negative potential fluctuation binds one electron (electron density shown with dashed line) with the energy below first channel minimum, $\epsilon_b < 0$ (dotted line) and chemical potential $\mu_L \sim \mu_R$. (b) Finite conduction with electron energy, $\epsilon > 0$, and linear regime, $\mu_L \gtrsim \mu_R$. The two-electron scattering (quasibound) state is singlet or triplet. V_{eff} is an effective double barrier tunneling potential for the scattered electron. (c) A larger source-drain voltage where the triplet (ϵ_1) resonance becomes overdamped with a broader, more robust, singlet (ϵ_0) becoming visible in transport.

$f_{L,R} = \{1 + \exp[(\epsilon - \mu_{L,R})/k_B T]\}^{-1}$ is the usual Fermi distribution function corresponding to left and right lead, respectively, with temperature T and the Boltzmann constant k_B. In the linear regime, $\Delta\mu = \mu_L - \mu_R \to 0$, this reduces to a generalized Landauer-Büttiker formula[13,20] for conductance, $G = eI/\Delta\mu$. The many-electron problem can be mapped onto an extended Anderson model[21,22] for which we have an *open* quantum dot with Coulomb blockade except near the resonances, which are analog to a "mixed-valence" single-electron tunneling regime. At very low temperatures, the effects of Kondo physics are expected and indeed signaled experimentally[9] and studied theoretically.[16] However, at higher temperatures these Kondo effects are suppressed and the extended Anderson model yields a conductance in agreement with Eq. (1).

In accordance with the Lieb-Mattis theorem[23] the singlet resonance is always at lower energies than the triplet, $\epsilon_0 < \epsilon_1$, and consequently the quasibound state has a longer lifetime (the resonance is sharper) than the triplet. This is clearly seen from the results obtained for the case of a cylindrical (or rectangular) quantum wire with a symmetric bulge,[13,14] presented in Fig. 2 (a_1) and (b_1).

Recent high accuracy shot noise measurements enabled the extraction of the Fano factor in ballistic quantum wires.[12] The Fano factor F is a convenient measure of the deviation from Poissonian shot noise. It is the ratio of the actual shot noise and the Poisson noise that would be measured in an independent-electron system.[24] This factor is, in our model,

$$F = \frac{\displaystyle\int [T_0(1-T_0) + 3T_1(1-T_1)](f_L - f_R)^2 d\epsilon}{\displaystyle\int (T_0 + 3T_1)(f_L - f_R)^2 d\epsilon}. \qquad (2)$$

This expression and Eq. (1), are based on the results of a two-electron scattering between a single bound electron and a propagating conduction electron with a summation over all conduction electrons near the Fermi energy. This approximation is only valid at temperatures above the Kondo scale in this system,[16] as discussed in Ref. 22. Eq. (2) directly reflects the fact that singlet and triplet modes do not mix in this pairwise interaction approximation, resulting in contributions to the noise that add incoherently with the probability ratio 1:3 for singlet and triplet scattering.

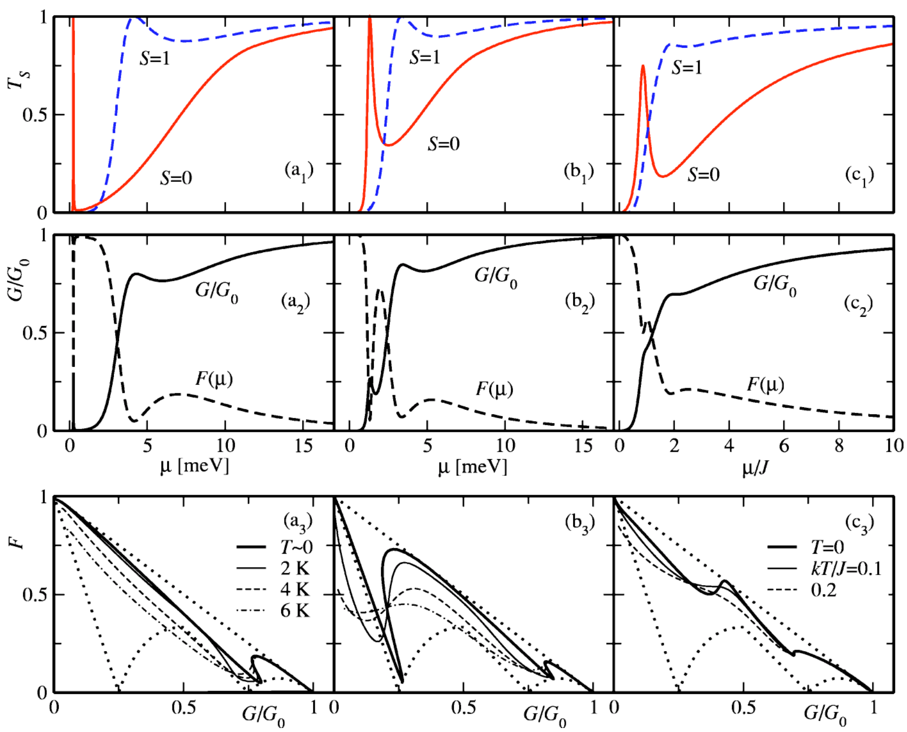

FIG. 2. (Color online) Panels (a_1) and (b_1): Singlet and triplet transmission probabilities as a function of Fermi energy μ measured relative to the bottom of the first channel. Results for cylindrical quantum wires with a symmetric bulge for all parameters as in Figs. 3(b) and 3(c) of Ref. 13. In panels (a_2) and (b_2) the conductance (full lines) and the Fano factor are presented. Panels (a_3) and (b_3): The Fano factor as a function of G (full line) and unitarity limits (dotted line). (c_1) Transmission probabilities as would arise, e.g., for a left-right asymmetric confining potential (in resonance $T_S < 1$). The corresponding energy dependence of the Fano factor and conductance, (c_2), and Fano factor as a function of conductance, (c_3). The energy and temperature scale is here in the units of single-triplet energy difference J.

The conductance $G(\mu)$ and Fano factor $F(\mu)$ are plotted vs μ in Fig. 2 (a_2) and (b_2), with Fig. 2 (a_3) and (b_3) showing the Fano factor F vs G for various temperatures and in the linear response regime, $\mu=\mu_L\sim\mu_R$. The dotted lines show zero temperature boundaries for the allowed values of F, under the assumption of validity of Eq. (2) in the limit $T\to 0$ and the unitarity condition for the transmission probabilities, $0\leqslant T_S\leqslant 1$. Equation (2) is not strictly valid in the limit $T\to 0$ due to many-electron effects that start becoming important at low temperatures. Thus this zero-temperature limit should be regarded as a limiting behavior that would occur in the absence of such many-body effects. The Fano factor exhibits two distinctive features. Firstly, there is a structure for $G/G_0<0.5$ corresponding to the sharp 0.25 singlet conductance anomaly. The second distinctive feature is in the region $0.5<G/G_0<1$ and corresponds to the dip in the singlet channel just above the singlet resonance and also partially to the triplet channel resonance. In our previous work we assumed a symmetric confining potential fluctuation, giving perfect transmission probabilities at resonance energies. However, in real systems left-right symmetry will not be perfect, especially if the fluctuation is of random origin, and also under finite source-drain bias. In these cases $T_S<1$ even on resonance. Such an example is presented in Fig. 2 (c_1) and (c_3). In this case the structure of $F(G)$ is less pronounced, consisting of kinks at $G/G_0\lesssim 0.5$ and $G/G_0\lesssim 0.75$. This behavior is a consequence of the absence of a pronounced triplet resonance and a dip in the singlet channel as mentioned above. Such a situation is typical for very weak confining potential fluctuations, where the triplet resonance is far in the continuum.

The structure at $G/G_0\lesssim 0.5$ has the same origin as the 0.25 structure in conductance, a direct consequence of a sharp singlet resonance. In Fig. 1(c) we show a schematic representation of the nonlinear regime with finite source-drain voltage. In this case the double peak potential barrier is asymmetric and shallower, giving rise to broader singlet and triplet resonances. The triplet resonance can even become overdamped while the singlet becomes more robust to temperature as it broadens. Hence, a pronounced 0.25 structure in the conductance is expected, surviving to a higher source-drain voltage than the triplet (0.7 structure). This is indeed seen in experiments.[4,5,8] If Eq. (2) at least qualitatively holds also in this nonlinear regime, a distinctive feature should appear in the Fano factor, as presented in Fig. 2 (a_3), (b_3), and (c_3).

Thus far we have calculated conductance and the Fano factor from singlet and triplet resonances and found good semiquantitative agreement with experiment. In the regime of linear conductance and low temperature, Eq. (1) and Eq. (2) simplify, with G and F determined by $T_s(\mu)$ taken at the Fermi energy. We can then invert the procedure and use the experimentally determined conductance and Fano factor to determine the transmission probabilities T_0 and T_1 using Eq. (1) and Eq. (2) in this regime. Unitarity for F requires that $F\leqslant 1-G/G_0$ and that F is above some lower limit, $F_{\min}(G)$ [dotted line in inset of Fig. 3(a)]. Unfortunately experimental values of F rise above this limit, possibly due to uncertain temperature corrections, and therefore cannot be used to determine the probabilities unambiguously. However, some estimates can be done. First, we approximate $F(G)=\min(F_I,1-G/G_0)$, where F_I corresponds to the line connecting the experimental points for $B=0$. Such a "fit I" is presented in the inset of Fig. 3(a) (full line). Another choice is $F(G)=F_{II}$, where F_{II} is some "minimal assumption" linear approximation for experimental data at low G and presented in Fig. 3(a) inset with a dashed line, "fit II." The line-shaded areas between the two choices correspond to the experimentally undetermined regime.

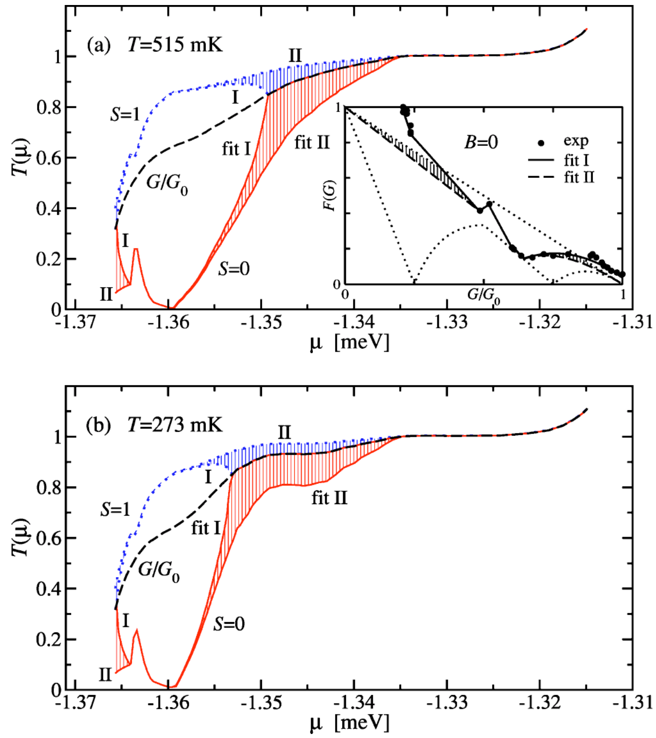

FIG. 3. (Color online) (a) Inset: Experimental values from Ref. 12 for $F(G)$ for $B=0$ (bullets) and two different interpolating (fitting) forms of F. Main figure: Conductance for $T=515$ mK and $B=0$ (dashed line), singlet (full line), and triplet (dotted) transmission probabilities extracted from experimental values of $G(\mu)$ vs μ and using two interpolating forms for F. The shaded area corresponds to the shaded area in the inset. (b) As in (a), for $T=273$ mK.

In Fig. 3(a) are presented the singlet and triplet transmission probabilities extracted from such $F(G)$ and the corresponding experimental values for $G(\mu)$ for $T=515$ mK. In spite of the uncertainty in F, the structure of both, T_0 and T_1 are relatively well determined and remarkably similar to the theoretically predicted cases from Fig. 2, with a much larger triplet transmission probability at lower energies, where the singlet is just above resonance and only slowly approaches unity. A small resonance in the singlet transmission probability corresponds to the kink structure in F at $G\sim 0.5G_0$. However, more accurate measurements of F are necessary in order to reduce the error bars in the estimates of singlet and triplet transmission probabilities. In Fig. 3(b) these probabilities are extracted for the case of the lower temperature $T=273$ mK. The most striking observation is the more rapid increase of the singlet transmission probability, a possible signature of Kondo behavior.[9,16]

The experimentally measured Fano factor in a strong magnetic field clearly suggests the spin-up and spin-down structure of spin subchannels. Here the singlet-triplet concept is not relevant, $T_\uparrow$ and $T_\downarrow$ being the appropriate subchannel division. Equation (2) is therefore not valid in this limit. However, the theoretical results for the conductance of near perfect quantum wires in a magnetic field[14,22] predict that, due to the Zeeman subband splitting at finite magnetic field, only one spin channel is open at lower energies and the conductance reaches $G \sim G_0$ only at higher energies, consistent with experiment. The corresponding Fano factor then follows the unitarity limit for this case, $F \sim 1-2G/G_0$ for $G < 0.5G_0$ forming a bow with a maximum at $G \sim 0.75G_0$, shown in Ref. 12. This is qualitatively consistent with the experimentally determined high field results for F, though a more quantitative description of the transition between the two regimes is still lacking.

To conclude, we have shown that anomalous structures in shot noise Fano factor measurements can be understood within the framework of the theory of 0.7 conductance anomalies in near perfect quantum wires.[13] The analysis of temperature dependence indicates stronger *singlet*-channel temperature dependence, which could be related to the Kondo-like behavior at lower temperature.[9,16] High magnetic field measurements are in qualitative agreement with the results of the theory. Finally, our results for a weak asymmetric confining potential in an otherwise perfect quantum wire predict that in finite source-drain voltage measurements a strong structure in Fano factor should appear for $G \lesssim 0.5G_0$. This structure corresponds to the recently measured 0.25 conductance anomalies.[8] Additional refined measurements of conductance and the Fano factor could more precisely resolve the singlet and triplet transmission probabilities and test the predictions of the theory based on the singlet-triplet resonant scattering.

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