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We propose a scheme to produce spin entangled states for two interacting electrons. One electron is bound in a well in a semiconductor quantum wire and the second electron is transported along the wire, trapped in a surface acoustic wave (SAW) potential minimum. We investigate the conditions for which the Coulomb interaction between the two electrons induces entanglement. Detailed numerical investigation reveals that the two electrons can be fully spin entangled depending on the confinement characteristics of the well and the SAW potential amplitude.

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I. INTRODUCTION

In recent years it has become appreciated that entanglement, one of the key fundamental features of quantum physics, lies at the heart of numerous interesting research areas. The ability to create entanglement between qubits in a controlled manner is a necessary ingredient for any candidate quantum information processing¹ system. Entanglement between quantum degrees of freedom of interest and those beyond our control—the environment—is responsible for decoherence and the degradation of pure quantum evolution. Entanglement can exist in solids even at thermal equilibrium^{2,3} and it potentially gives a new perspective for critical phenomena.⁴ In solid state systems, whether from the perspective of fundamental quantum phenomena or their assessment as candidate quantum processing devices, a real challenge is to establish and control entanglement between chosen quantum degrees of freedom, while avoiding decoherence due to entanglement with the relevant environment. In this work we study, from a theoretical and modelling perspective, the generation of entanglement between electrons in semiconductor systems that are amenable to current fabrication and experimental techniques.

Single electron transport (SET) in a GaAs/AlGaAs semiconductor heterostructure using a surface acoustic wave (SAW) was demonstrated with a very high accuracy almost a decade ago by Shilton *et al.*⁵ Originally, the SAW-based SET devices were investigated in the context of metrological applications and specifically for defining a quantum standard for the current.⁵⁻⁷ However, many other applications based on this technology have been proposed aiming to manipulate the integer number of electrons in various ways. For example, an extension of a SAW-based SET device is a single photon source⁸ a necessary tool in quantum cryptography.^{9,10}

Barnes *et al.*^{11,12} suggested how quantum computations can be performed and quantum gates can be constructed using the spins of single electrons, trapped in the SAW potential minima, as qubits. The high SAW frequency (~2.7 GHz) allows a high computation rate, which is regarded as an advantage of the SAW-based quantum computer. The electrons are carried by the SAW in a series of

narrow parallel channels separated by tunnel barriers. At the entrance of the channels a strong magnetic field is applied to produce a well-defined initial state for the electrons. As the electrons are driven along the channel they can interact with electrons in adjacent channels. The degree of interaction may be controlled by altering the height and/or the thickness of the barriers between the channels using surface gates. Various readout schemes that use, for example, magnetic Ohmic contacts or the Stern-Gerlach effect were proposed and described.^{11,12}

This proposal of flying qubits has attracted a lot of interest and theoretical work has supported its efficiency, though quantum gates have yet to be demonstrated experimentally. Specifically Gumbs and Abranyos¹³ calculated the entanglement of spins, via the exchange interaction, for two electrons driven by SAWs in two adjacent channels. More recently Furuta *et al.*¹⁴ performed detailed calculations of the qubit dynamics when the qubits pass through magnetic fields.

In recent theoretical work Rodriguez *et al.*¹⁵ and also Bordone *et al.*¹⁶ proposed an experiment to observe quantum interference of a single electron using SAWs. The proposed experiment may provide an estimation of the electron decoherence time which is an important quantity if these devices are to be exploited in the field of quantum information and computation. Finally, the use of single electrons trapped in SAW potential minima for quantum computing was considered briefly in Ref. 17 where a more general scheme to induce entanglement was examined in which ballistic electrons propagate along two parallel quantum wires.

In this paper, motivated by recent work on conductance anomalies¹⁸ and spin entanglement generation in quantum wires,¹⁹ we propose a scheme to produce entangled states for two electrons utilizing SAWs. A schematic illustration of the SAW-based device is shown in Fig. 1. The SAW time-dependent potential is used to carry a single electron through the channel where the second electron is bound in a quantum well. The two electrons interact via the Coulomb interaction and it has been shown in various schemes^{11,13,19-21} that this interaction is capable of inducing entanglement. We investigate the conditions for which the electron in the SAW, after passing through the region of the quantum well, will be entangled with the electron remaining in the well. Considering

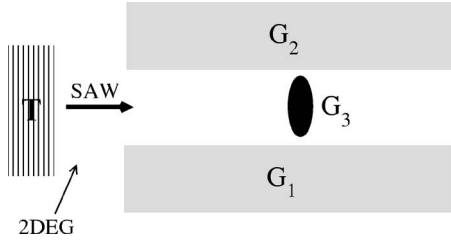


FIG. 1. Schematic illustration of the SAW-based device to generate spin entanglement between a static and a flying qubit. The gates G_1 , G_2 define a pinched-off quasi-one-dimensional channel and the gate G_3 is used to create the open quantum dot which binds the static qubit. A SAW is generated by the transducer (T) above a 2DEG and the (negative) potential on G_1 and G_2 increased until a SAW minimum contains a single electron which interacts with a bound electron under G_3 .

the spins of the electrons as the qubits the proposed scheme belongs to the static-flying qubit category where specifically the qubits interact in the same channel in contrast to Ref. 11 which involves interaction between flying qubits in different channels.

This paper is organized as follows. In Sec. II a single electron study is presented for the bound electron in the well and the propagating electron in the SAW. Section III introduces the two-electron model and considers some typical cases of entanglement generation. In Sec. IV a Hartree approximation is employed to explain the results of the electron dynamics. Section V presents some general important features of the entanglement and the main results are summarized in Sec. VI.

II. SINGLE ELECTRON STUDY

A. Preliminaries

Before examining the dynamics of the two electrons and entanglement generation, we study the two electrons separately. The spin of the bound electron in the well constitutes the static qubit for the proposed scheme and therefore it is necessary to understand how this electron behaves under the SAW propagation. In principle, the static qubit must remain localized in the quantum well during the computation cycle and this means that the SAW-induced time-dependent perturbation must be such that this condition is satisfied. It is also important that the electron in the SAW, whose spin constitutes the flying qubit, remains bound in the same SAW potential minimum at least up to the region where Coulomb repulsion with the bound electron becomes important. Although this could be achieved simply by a large SAW amplitude the degree of screening due to the applied gate bias used to form the quantum wire is uncertain and it may be necessary to form the wire by an etching technique.²² Finally, it is interesting to note that well-defined single SAW pulses can be generated¹¹ which can be employed in order to minimize the interaction between propagating electrons and to allow the read-out process.

The well in the wire could be formed by surface gates, whose geometric design and applied bias would control the

confining characteristics of the well. A single electron turnstile²³ could then be used to launch an electron towards the region of the quantum well. While these aspects of realization are experimentally feasible, details of the formation of the quantum well or the capture process are beyond the scope of this paper.

For all the calculations in the following sections we have employed a one-dimensional model considering only the direction of SAW propagation, that is the positive x direction. The quantum well potential is modelled by the expression

$$V(x) = -V_w \exp\left(\frac{-x^2}{2l_w^2}\right), \quad (1)$$

where the parameters V_w , l_w control the depth and the width of the well, respectively. The SAW time-dependent potential is given by²⁴

$$V_{\text{SAW}}(x,t) = V_o \{\cos[2\pi(x/\lambda - ft)] + 1\}, \quad (2)$$

where the parameter V_o represents the SAW potential amplitude and to be specific we have chosen the typical values $f = 2.7$ GHz for the SAW frequency and $\lambda = 1$ μm for the SAW wavelength.²⁴

B. The bound electron in the well

In order to study the state of the electron in the well we solved the time-dependent Schrödinger equation, using a Crank-Nicholson scheme²⁵ for the Hamiltonian

$$H_o = -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} + V_l(x,t), \quad (3)$$

where $m^* = 0.067m_o$ is the effective mass of the electron in GaAs. The total time-dependent potential is given by the combination of the SAW and the quantum well potential

$$V_l(x,t) = V_{\text{SAW}}(x,t) + V(x). \quad (4)$$

The time evolution of the probability distribution is shown in Fig. 2 for one SAW period $T = 1/f$ and for the parameters $V_o = 2$ meV, $V_w = 6$ meV, and $l_w = 7.5$ nm. The quantum well parameters have been chosen such that there is only a single bound state when $V_o = 0$. This becomes a quasibound at specific times provided $\epsilon_w < 2V_o$, where ϵ_w is the minimum required energy to delocalize the electron from the well when $V_o = 0$. Although this inequality is fulfilled for the chosen parameters, the electron still remains very well-localized in the well for the whole SAW period as we can see from Fig. 2. This is simply because the tunnelling time to escape from the well is much greater than the SAW period.

The instantaneous eigenvalues versus time, obtained by solving the time-independent Schrödinger equation at each instant in time are shown in Fig. 3 and provide insight into the dynamics. Only the first few eigenvalues that are relevant to the time evolution process are shown in order of increasing energy ($E_n, n = 0, 1, \dots$). Eigenvalues corresponding to an odd (even) integer are shown with a dashed (full) line. Note that none of the curves actually cross, though the very small energy difference cannot be resolved in the figure. The characteristic sine feature that develops from the left to the right

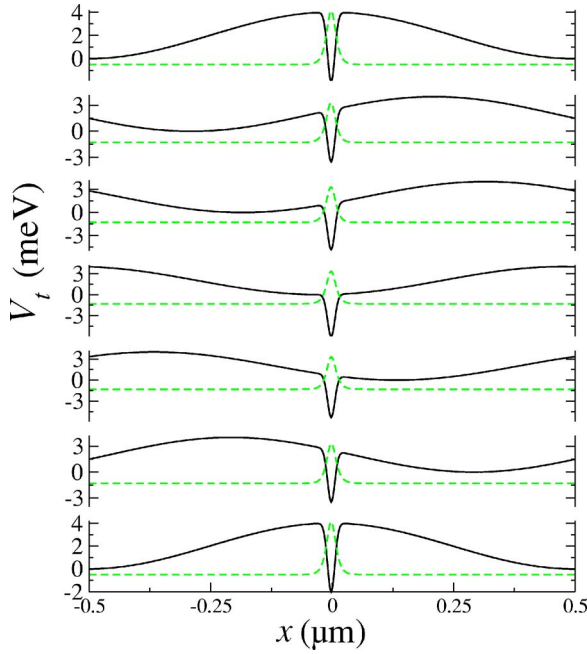


FIG. 2. (Color online) Time evolution of the probability distribution (dashed line in arbitrary units) of the bound state of the quantum well and the total time-dependent potential (full line). The time sequence is from top to bottom and specifically $t/T = 0, 0.2, 0.4, 0.5, 0.6, 0.8, 1$.

of the graph indicates that the state evolves via nonadiabatic Landau-Zener transitions.²⁶ The transition probability at an anticrossing point, that is a point in the graph where the two curves have minimum separation, depends on this characteristic energy gap.^{26,27} Specifically, if the energy gap is large the state cannot undergo the transition and thus it tunnels out of the well losing its initial character, that is bound in the quantum well. On the other hand, for small energy gaps, which is the case here, the electron can successfully undergo Landau-Zener transitions thus retaining the initial character of its state as the time develops and the potential profile changes. Strictly speaking, after very many SAW cycles the electron will be delocalized from its initial quantum well because the Landau-Zener transitions do not occur with probability of exactly one. In our study the sine feature is only shown for one SAW period and it describes how the energy of the bound electron in the quantum well changes with time as the SAW propagates. An important characteristic is that via the Landau-Zener transitions the state retains its initial character by changing the eigenvalue number at each anticrossing point from n to $n \pm 1$. In particular, for $t = 0$ and after one SAW period $t = T$ the SAW potential is maximum at $x = 0$ and therefore the energy level that corresponds to the bound state of the quantum well is maximum. For $t = T/2$ the SAW potential is minimum at $x = 0$ and the energy of the quantum well is the ground state energy of the system. For $t < T/2$ the state lowers its eigenvalue number at each anticrossing point from n to $n - 1$ in order to decrease its energy, whereas for $t > T/2$ the state increases its eigenvalue number from n to $n + 1$ in order to increase its energy, via successfully accomplishing Landau-Zener transitions. For $t > T$ this pattern of transitions is repeated. Increasing the

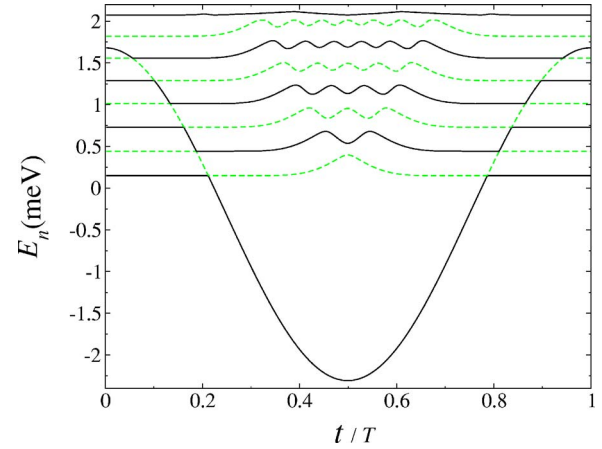


FIG. 3. (Color online) The first few bound instantaneous eigenenergies ($E_n, n=0, 1, \dots$) as a function of time for the total time-dependent potential $V_t(x, t)$ described in the text. Eigenenergies which correspond to an odd (even) integer are shown with a dashed (full) line.

SAW potential amplitude and keeping the characteristics of the well fixed the energy gaps become larger and eventually the sine feature will disappear. In this case the electron escapes from the quantum well tunnelling partly in the SAW potential minimum and in the continuum. Decreasing the SAW amplitude there will be a value such that the state of the well will be a true-bound state at all times. In this case the state evolves adiabatically, its energy changes sinusoidally and the electron remains localized in the well without any effect from the SAW propagation.

To summarize for a particular quantum well there is a regime of a small SAW potential amplitude (that satisfies $\varepsilon_w < V_o$) where the bound electron evolves adiabatically, followed by a regime of stronger SAW amplitude for which the electron evolves via nonadiabatic Landau-Zener transitions. Finally, for an even stronger SAW amplitude the electron escapes from the quantum well. The SAW potential amplitude must be restricted to the first two regimes for a particular well depth. Here we consider the most interesting intermediate case and although we only consider a quantum well with a single bound state, it is straightforward to generalize the results to cases with more bound states in the well.

C. The propagating electron in the SAW

In this section we study how the electron in the SAW potential minimum propagates along the quantum wire far from the quantum well for which we may restrict the potential of the Hamiltonian (3) to the SAW potential only. The set of coefficients $C_m, m=0, 1, \dots$, which satisfy

$$\begin{aligned} \dot{C}_m = & -C_m \langle u_m | \dot{u}_m \rangle \\ & + \sum_{n \neq m} \frac{C_n}{\hbar \omega_{mn}} \left\langle u_m \left| \frac{\partial H_o}{\partial t} \right| u_n \right\rangle \exp \left(i \int_0^t \omega_{mn}(t') dt' \right), \end{aligned} \quad (5)$$

determines the evolution of the wave function $\phi(x, t)$, via the expansion²⁸

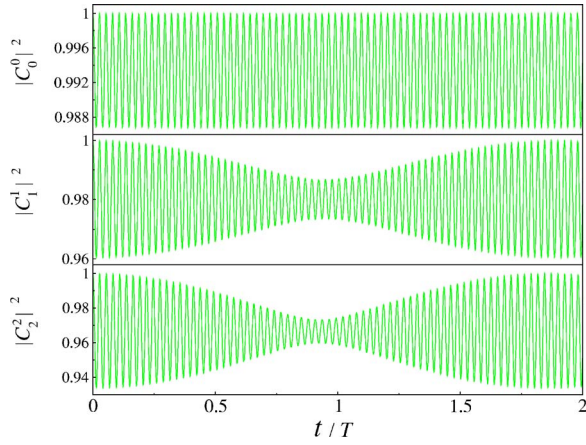


FIG. 4. (Color online) From top to bottom the initial electron state (for $t=0$) is the ground, the first, and the second excited state of a SAW potential minimum, respectively. The plots show how the corresponding probability for each initial state evolves with time for two SAW periods.

$$\phi(x,t) = \sum_n C_n(t) u_n(x,t) \exp\left(-\frac{i}{\hbar} \int_0^t E_n(t') dt'\right), \quad (6)$$

in the basis states of the instantaneous solutions $H_o(x,t)u_n(x,t) = E_n(t)u_n(x,t)$, with $\omega_{mn} = (E_m - E_n)/\hbar$ the Bohr angular frequency. The wave function $\phi(x,t)$ describes the electron in the SAW potential minimum and satisfies the time-dependent Schrödinger equation. The system of Eqs. (5) is solved using a fourth-order Runge-Kutta method,²⁹ although for the calculations we have dropped the first term of Eqs. (5), since it only induces an unimportant phase difference in the final coefficients. Figure 4 shows the variation of the squared modulus of the coefficients, when the initial state is the ground, the first and the second excited state of the SAW potential minimum, ($|C_n^j|^2$, $j=n=0,1,2$ where the superscript j indicates the corresponding initial state) as a function of time for two SAW periods and for a SAW amplitude of $V_o = 4$ meV. As we can see, the electron remains to a very good approximation in the initially populated state of the SAW minimum throughout the time evolution and furthermore the corresponding moduli of the expansion coefficients present an oscillating behavior.

This behavior may be explained within the adiabatic approximation.²⁸ Starting with $C_n^j(t=0) = \delta_{nj}$ (initial state u_j) and assuming that all the coefficients in (5) remain constant with time $C_n^j(t > 0) \approx \delta_{nj}$, we obtain the approximate formula for all $m \neq j$,

$$\dot{C}_m^j \approx \frac{\omega \Lambda_{mj}}{\hbar \omega_{mj}} \exp\left(i \int_0^t \omega_{mj}(t') dt'\right), \quad (7)$$

where we have set $\Lambda_{mj} = \langle u_m | \frac{\partial H_o}{\partial t} | u_j \rangle / \omega$, with $\omega = 2\pi f$ the SAW cyclic frequency. The matrix elements Λ_{mj} and the frequencies ω_{mj} are time-independent and hence the final expression for the squared modulus of the coefficients for $m \neq j$ becomes

$$|C_m^j(t)|^2 \approx \frac{\omega^2 |\Lambda_{mj}|^2}{\hbar^2 \omega_{mj}^4} 4 \sin^2\left(\frac{\omega_{mj} t}{2}\right). \quad (8)$$

For the SAW potential given by (2) the matrix elements are real for bound states and therefore $\Lambda_{mj} = \Lambda_{jm}$. Also, the transitions are allowed when $\Lambda_{mj} \neq 0$ which occurs when $m+j$ is odd. If $\omega |\Lambda_{mj}| \ll \omega_{mj}^2 \hbar$ then $|C_m^j| \sim 0$ and the electron remains at all times in the initial state u_j . This limit corresponds to the adiabatic approximation and is satisfied when the system changes very slowly compared to the transition frequency ω_{mj} associated with the states. In general, the higher the states the smaller the transition frequency between them and as a result the less valid the adiabatic approximation. This can be seen directly from Fig. 4 by observing the minimum magnitude of the oscillations which gives the maximum deviation from the initial state and hence the deviation from the adiabatic approximation. On the other hand, the lower the SAW frequency the better the adiabatic approximation. In the extreme limit of a frozen wave, $\omega=0$, the states become stationary acquiring only a phase.

For the ground state, in the time interval of interest, the adiabatic approximation is excellent. During the time evolution there is a very small contribution from excited states and $|C_0^0|^2 \approx 1 - |C_1^0|^2$ with $|C_1^0|^2$ given by Eq. (8) with $m=1$ and $j=0$. Hence, the frequency of the oscillations is equal to ω_{10} and the amplitude depends on the quantity $\omega^2 |\Lambda_{10}|^2 / (\hbar^2 \omega_{10}^4)$. In the evolution of the first excited state there is some contribution not only from the ground but also from the second excited state. We may approximate the numerical results by $|C_1^1|^2 \approx 1 - |C_0^1|^2 - |C_2^1|^2$ and using Eq. (8),

$$|C_1^1(t)|^2 \approx 1 - D_{01} \sin^2\left(\frac{\omega_{01} t}{2}\right) - D_{21} \sin^2\left(\frac{\omega_{21} t}{2}\right), \quad (9)$$

where the auxiliary constant is $D_{m,j} = 4\omega^2 |\Lambda_{mj}|^2 / (\hbar^2 \omega_{mj}^4)$. Defining the quantities $\delta = (\omega_{10} - \omega_{21})/2$, $\delta_o = (\omega_{10} + \omega_{21})/2$ and $D = (D_{21} - D_{10})/2$, $D_o = (D_{21} + D_{10})/2$ we may further write

$$|C_1^1(t)|^2 \approx 1 - D_o + D_o \cos(\delta_o t) \cos(\delta t) + D \sin(\delta_o t) \sin(\delta t), \quad (10)$$

which explains the sinusoidal variation of the amplitude in the oscillations with a period equal to $2\pi/\delta$. This peculiar form is due to the very small difference between the Bohr frequencies which define the frequency of the oscillations and the small difference between the matrix elements which control the amplitude of the oscillations. A similar situation occurs for the evolution of the second excited state as shown in the bottom frame of Fig. 4. All the relevant quantities can be defined similarly, considering that the small deviation from this state is mainly due to transitions to the first and the third excited state.

To summarize, the electron in the SAW propagates adiabatically along the quantum wire even for a relatively low SAW amplitude. In other words, it remains well localized in the particular SAW potential minimum as it is driven towards the bound electron in the well, which is the main requirement for the entanglement generation described in the next section.

III. ELECTRON DYNAMICS AND ENTANGLEMENT

A. The two-electron model

The dynamics of the two-electron system is governed by the time-dependent Schrödinger equation, with the Hamiltonian

$$H = \sum_{i=1,2} \left(-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x_i^2} + V_i(x_i, t) \right) + V_c(x_1, x_2). \quad (11)$$

The single electron term $V_i(x, t)$ was described in Sec. II B and the Coulomb term $V_c(x_1, x_2)$ is modelled by the quasi-one-dimensional form

$$V_c(x_1, x_2) = \frac{q^2}{4\pi\epsilon_r\epsilon_0\sqrt{(x_1 - x_2)^2 + \gamma_c^2}}, \quad (12)$$

where $\epsilon_r=13$ is the relative permittivity of GaAs. This simplified form of the Coulomb interaction assumes that all excitations take place in the x direction, whereas in the other two directions the electrons occupy at all times the corresponding ground states (transverse modes). This is a good approximation provided that the parameter γ_c that models the confinement lengths in the y and z directions is relatively smaller than the confinement length scales in the x direction. For all the calculations we choose $\gamma_c=20$ nm, for which the restriction to lowest transverse modes is an excellent approximation.

The resulting two-electron time-dependent Schrödinger equation is solved numerically with an explicit scheme based on a finite difference method, which is described in detail for the case of a single electron by Visscher.³⁰ The extension for two electrons is straightforward.

For the initial state at $t=0$ we choose one electron to have spin up in the ground state of the SAW potential minimum, $\psi(x)$, and the other electron to have spin down in the ground state of the quantum well, $\varphi(x)$. It is important to note that $\psi(x)$ and $\varphi(x)$ are exactly orthogonal, with no spatial region of overlap, i.e., ψ peaks around the region where the particular SAW potential minimum is located (far from the quantum well) and φ peaks around $x \sim 0$ where the quantum well is located. To study the dynamics of the electrons, at time $t > 0$ it is necessary to take into account the fact that the electrons are fermions and thus indistinguishable. This is important when the electron carried by the SAW interacts with the electron in the quantum well, giving rise to a spin exchange interaction. The initial state is thus represented by the Slater determinant

$$\Psi_{\uparrow\downarrow}(x_1, x_2, 0) = \frac{1}{\sqrt{2}} \begin{vmatrix} \psi(x_1)\chi_{\uparrow}(1) & \varphi(x_1)\chi_{\downarrow}(1) \\ \psi(x_2)\chi_{\uparrow}(2) & \varphi(x_2)\chi_{\downarrow}(2) \end{vmatrix}. \quad (13)$$

This state is unentangled according to the criteria of Ref. 31. Note that $\Psi_{\uparrow\downarrow}(x_1, x_2, 0)$ can also be expressed as a combination of a singlet and a $S_z=0$ triplet state,

$$\Psi_{\uparrow\downarrow}(x_1, x_2, t) = \frac{1}{\sqrt{2}} [\Psi_{\uparrow\downarrow}^S(x_1, x_2, t) + \Psi_{\uparrow\downarrow}^T(x_1, x_2, t)], \quad (14)$$

which is also the general form of the total two-electron wave function at all times, due to the fact that the Hamiltonian Eq.

(11) contains no spin-dependent terms. Furthermore, for the case of two electrons, the orbital and spin parts factorize, i.e., $\Psi_{\uparrow\downarrow}^S(x_1, x_2, t) = \Phi^S(x_1, x_2, t)\chi_{\uparrow\downarrow}^S(1, 2)$ and similarly $\Psi_{\uparrow\downarrow}^T(x_1, x_2, t) = \Phi^T(x_1, x_2, t)\chi_{\uparrow\downarrow}^T(1, 2)$. With this notation the spin components are given by $\chi_{\uparrow\downarrow}^{S/T}(1, 2) = [\chi_{\uparrow}(1)\chi_{\downarrow}(2) \mp \chi_{\downarrow}(1)\chi_{\uparrow}(2)]/\sqrt{2}$ (with the negative sign for the singlet) and the corresponding orbital components at $t=0$, by $\Phi^{S/T}(x_1, x_2, 0) = [\psi(x_1)\varphi(x_2) \pm \varphi(x_1)\psi(x_2)]/\sqrt{2}$ (with the positive sign for the singlet). For $t > 0$, the spin eigenstates are unchanged whereas the orbital states are given directly by the solution of the time-dependent Schrödinger equation. The form of the orbital components at $t=0$ implies that the two electrons do not interact, i.e., they are well separated with negligible Coulomb interaction, and therefore are written as symmetric and antisymmetric products of noninteracting single-electron states. Finally, in this work we consider only cases where the electron in the quantum well is well localized before ($t=0$) and after the scattering event ($t=t_f$), when the electrons are well separated. The energy parameters (SAW amplitude and quantum well characteristics) are thus chosen such that the final electron probability distribution in the well is to a good approximation the same as before interaction. However, this restriction is not imposed on the propagating electron in the SAW, which can gain energy due to a combination of the effect of the time dependence of the SAW potential and Coulomb repulsion.

B. Entanglement measure

In this paper, concurrence will be used as a measure of spin entanglement. An expression for concurrence may be obtained using the form suggested by Woiters,³² starting with the total density matrix for the pure scattering state and integrating over the orbital degrees of freedom. For the axially symmetric problems considered here, for which total spin projection along the quantization axis is conserved, concurrence is physically related to spin-spin correlation functions for the two domains A and B and takes the form³³ $C = 2|\langle S_A^+ S_B^- \rangle|$, where S^{\pm} are the usual spin-flip operators. Equivalently in terms of the symmetric (singlet) and antisymmetric (triplet) orbital states concurrence is given by the formula³³

$$C(t) = \frac{1}{N(t)} \left| \int_{A,B} dx_1 dx_2 \Phi_-^*(x_1, x_2, t) \Phi_+(x_1, x_2, t) \right|, \quad (15)$$

where $\Phi_{\pm}(x_1, x_2, t) = \Phi^S(x_1, x_2, t) \pm \Phi^T(x_1, x_2, t)$ and the normalization constant N equals

$$N(t) = \int_{A,B} dx_1 dx_2 [|\Phi^S(x_1, x_2, t)|^2 + |\Phi^T(x_1, x_2, t)|^2]. \quad (16)$$

In these expressions the regions of integration A, B are chosen to be regions which the electrons are expected to occupy before and after scattering with A being the domain of one electron and B the domain of the other. Physically, the regions A and B can be viewed as (position) measurement domains. For example, sensing of the presence of an electron charge³⁴⁻³⁶ with sufficient positional information only to

identify it as being located in some region could correspond to such a “fuzzy” position measurement. Since the quantum well always contains at least one electron, we choose A to be this region, i.e., $A=[x_l, x_r]$, where x_l and x_r denote the points where the bound state of the electron in the quantum well has decayed to zero at the left and right, respectively. For the numerical calculations, these points were chosen to correspond to a value of approximately 10^{-4} of the probability density at the peak. The region B is chosen to correspond to the region of occupation of the propagating electron. For the two-electron scattering problem under study, we may choose this to be the total domain excluding the well, i.e., $B=[-L, x_l] \cup [x_r, L]$, where $[-L, L]$ defines the total region of space within which the electron dynamics is studied. Note that the corresponding concurrence is really only meaningful when the electrons are well separated, before and after scattering, i.e., at sufficiently small or large t , though it may be calculated at any time. We refer to this concurrence as the total concurrence, $C(t)$. We may also define (the potentially more useful) reflected or transmitted concurrence, for which the measurement domains are restricted to either the left or the right of the quantum well, respectively, i.e., $B=[-L, x_l]$, or $B=[x_r, L]$, with corresponding concurrences C_r and C_t . A “fuzzy” position measurement (charge sensing) which gives sufficient information to resolve the outgoing electron as reflected or transmitted could project the two electrons into a state with the associated concurrence C_r or C_t . It is also useful to define the quantities $P_t^{S/T}$ and $P_r^{S/T}$ as the transmitted and reflected probabilities for singlet and triplet states. The maximum probability in the whole space of either state equals 0.5 due to the general form (14) of the wave function. The transmitted and reflected concurrence are considered only when the corresponding singlet or triplet probabilities are not negligible. For the numerical calculations the minimum limit was taken to be approximately 10^{-2} . We should also mention that by definition³³ $0 \leq C \leq 1$, where the limit $C=0$ corresponds to an unentangled state and $C=1$ to a fully entangled state. For the time dependent problem under study, the concurrence is also time dependent and it is easily verified that for the initial state, $C(t=0)=0$.

C. Entanglement generation

In this section we present some typical scattering results that take place when the two electrons interact via the Coulomb interaction and demonstrate how the entanglement develops with time due to this interaction.

Figure 5 shows the initial and final electron density $\rho^{S/T}(x, t) = 2 \int dx' |\Phi^{S/T}(x, x', t)|^2$, and Fig. 6 shows how the concurrence and the relative probabilities develop with time for the parameters $V_w=6$ meV, $l_w=7.5$ nm, and $V_o=2$ meV. For these parameters the quantum well can accommodate only a single bound electron, a second electron being delocalized due to the Coulomb interaction. From the figure we see that there is a very high transmission for the singlet state and high reflection for the triplet state after scattering. The concurrence (C, C_t, C_r) varies with time, due to the interactions of the wave packets mediated by the Coulomb interaction, and eventually saturate to a constant value when the

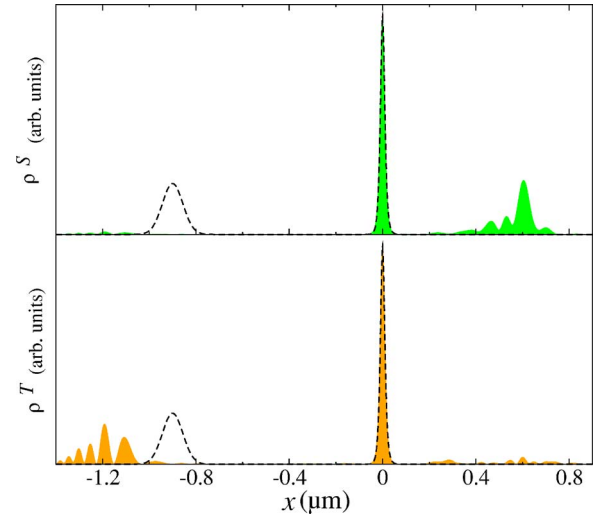


FIG. 5. (Color online) Initial (dashed) and final electron distribution (full) in arbitrary units when the singlet state (top) is mostly transmitted and the triplet state (bottom) is mostly reflected.

overlap is once again negligible. The transient time interval is not of main interest since the degree of entanglement is important after the scattering process when the two electrons are well separated. Note that the reflected concurrence is close to unity since at the left-hand side the reflection probability of the singlet state is very small compared with that of the triplet, and a pure $S_z=0$ triplet is fully spin entangled. In this case, the reflection process may be regarded as a filtering process in which the singlet part of the initial wave function is essentially removed by transmission to the right. On the other hand, if we look in transmission, although the singlet part is almost fully transmitted, the transmission of the triplet is not negligible and interference results in an asymptotic concurrence which is somewhat less than unity. We also see

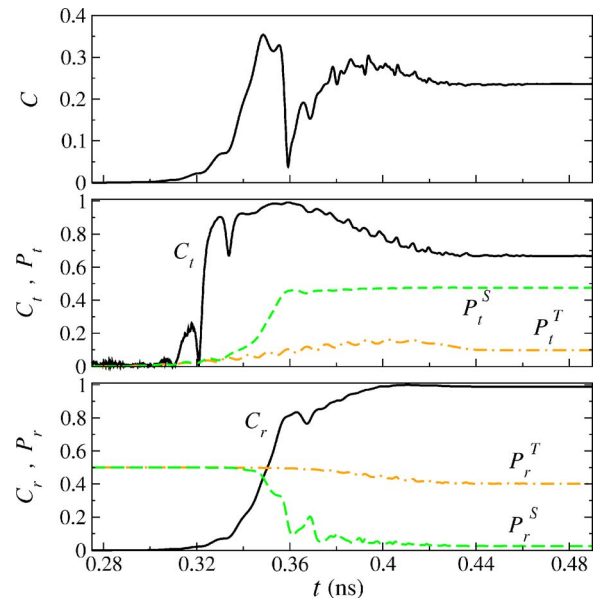


FIG. 6. (Color online) Concurrence and relative probabilities as a function of time when the singlet state is mostly transmitted and the triplet state is mostly reflected.

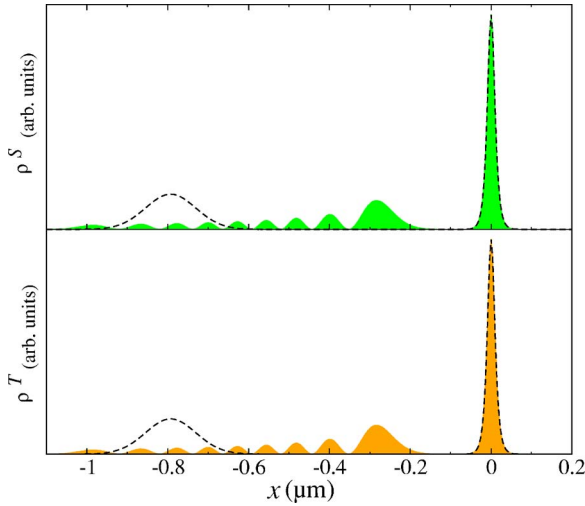


FIG. 7. (Color online) Initial (dashed) and final electron distribution (full) in arbitrary units when both singlet (top) and triplet (bottom) are almost totally reflected.

that the maximum concurrence is also significantly reduced when the measurement domain includes both transmitted and reflected parts after scattering.

In Fig. 7 we present results for a smaller SAW amplitude ($V_o=0.5$ meV) for which the initial and final electron density of both singlet and triplet states are almost totally reflected, the electron carried by the SAW being almost completely reflected by the Coulomb repulsion with the bound electron. Figure 8 illustrates how the concurrence develops in time, the concurrence of the reflected part and that over the whole domain being approximately the same due to the high reflection. We can see again that the concurrence builds up with time due to the Coulomb interaction and saturates to a constant value after reflection. Note however, that this asymptotic value is much smaller, due essentially to the Coulomb repulsion inhibiting significant overlap of the wave packets.

A third regime of interest is when both singlet and triplet are almost fully transmitted. Figure 9 shows the initial and final electron density for such a case with parameters $V_w=70.5$ meV, $l_w=10$ nm, and $V_o=10$ meV. Although for this choice of parameters the quantum well can bind two elec-

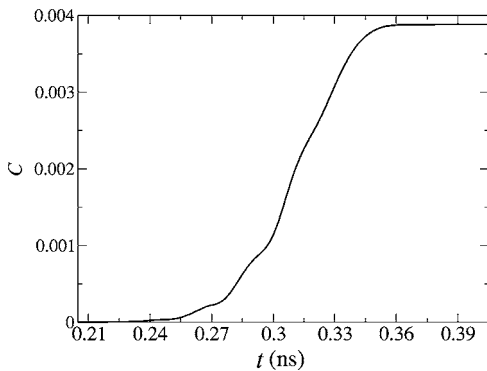


FIG. 8. Concurrence as a function of time when both singlet and triplet are almost totally reflected.

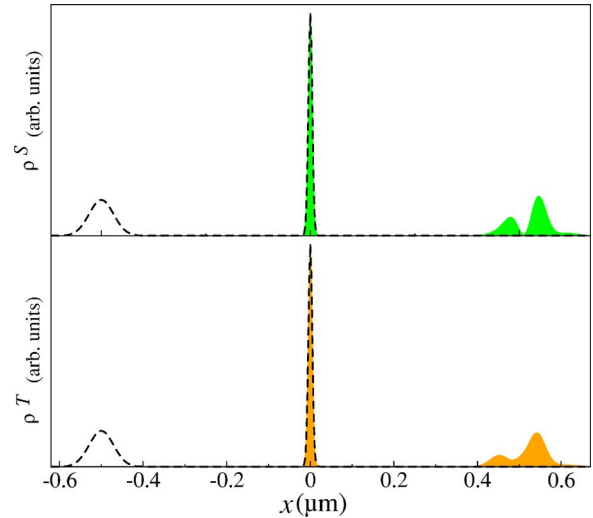


FIG. 9. (Color online) Initial (dashed) and final electron distribution (full) in arbitrary units for a typical case when both singlet (top) and triplet (bottom) are almost fully transmitted.

trons in the absence of the SAW, the second electron does not in fact become bound when it is carried by the SAW and after scattering the probability of finding both electrons in the well is negligible. This is because the SAW period is too short for the second electron to become trapped in the well. We thus see that both singlet and triplet states are almost perfectly transmitted, while the electron in the quantum well remains very well localized. However, the electron in the SAW potential minimum, after passing through the region of the bound electron in the well, gains energy from the interaction, resulting in a superposition state which includes excited states of the SAW. It is easily verified that for this special case the concurrence takes the form³³ $C=|\text{Im}\langle T|S\rangle|$, where $|T\rangle$ and $|S\rangle$ are the single electron states of the transmitted electron in a SAW minimum which result from triplet and singlet states respectively. We see from this formula that $C=1$ only when $|T\rangle$ and $|S\rangle$ differ by a phase factor of $\pi/2$. This is not the case in general, for which not only the phases but also the amplitudes of $|T\rangle$ and $|S\rangle$ are different. Limits which give zero concurrence are when $|S\rangle=|T\rangle$ (such as the trivial case of no interaction between the electrons) and when $|S\rangle$ and $|T\rangle$ are orthogonal. As explained in the next section the latter can occur, or approximately so, when an electron in the well resonantly tunnels out of the well into an excited state of a SAW minima for singlet but not triplet or visa versa. More generally, the overlap (and hence C) may be small but not precisely zero due again to different tunnelling rates for singlet and triplet. Figure 10 shows how the concurrence and the relative probabilities develop in time. Similarly with the previous cases the concurrence increases and saturates to a constant value, while for intermediate times it oscillates. Since the reflected part is very small, we get the expected result that the asymptotic value of the transmitted concurrence approximately equals the total concurrence $C \approx C_t \sim 0.53$.

Finally, we consider the possibility of choosing parameters such that by changing the confining characteristics of the well the singlet and triplet orbital components, after the

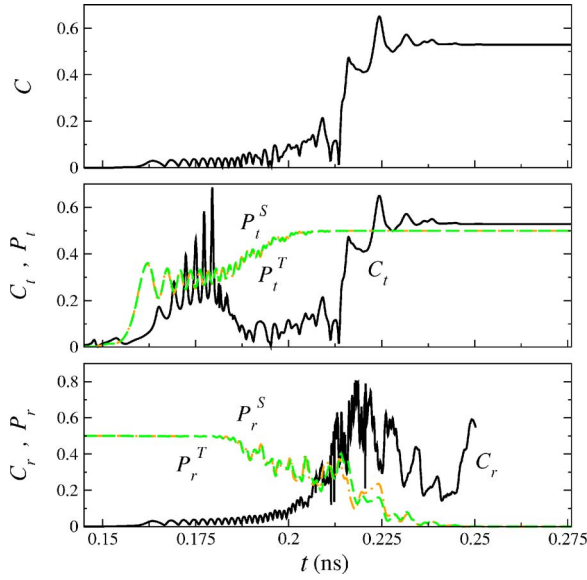


FIG. 10. (Color online) Concurrence and relative probabilities as a function of time for a typical case when both singlet and triplet are almost fully transmitted.

scattering event, may be chosen to differ only by a phase factor $e^{i\delta\varphi}$, with $\delta\varphi = \varphi_S - \varphi_T$. This may be done, at least approximately, for parameters which give almost perfect transmission for both singlet and triplet states. This occurs, for example, when the SAW amplitude is sufficiently large. For this regime the concurrence takes the form³³ $C = |\sin \delta\varphi|$, as can be seen directly from Eq. (15), using the form $\Phi^S(x_1, x_2, t_f) = e^{i\varphi_S} [\psi_f(x_1)\varphi(x_2) + \varphi(x_1)\psi_f(x_2)] / \sqrt{2}$, $\Phi^T(x_1, x_2, t_f) = e^{i\varphi_T} [\psi_f(x_1)\varphi(x_2) - \varphi(x_1)\psi_f(x_2)] / \sqrt{2}$, for the singlet and triplet orbital components, respectively. Note that ψ_f describes the electron in the SAW potential after scattering and φ describes the electron in the well. We see immediately from this form that the concurrence may be controlled by changing the relative phase of singlet and triplet while maintaining approximately full transmission, giving full entanglement when the magnitude of this phase difference is $\pi/2$. However the value of the phase difference cannot be easily controlled and, indeed, the phase difference picture is itself only an approximate since the singlet and triplet probabilities distributions in the SAW minimum after scattering are never precisely identical. However for some special cases this model is an excellent approximation as shown for example in Fig. 11 for the parameters $V_w = 66$ meV, $l_w = 10$ nm for which the well can bind at least two electrons and $V_o = 20$ meV. We see that the electron in the SAW after the scattering event is well bound in the SAW potential minimum occupying the characteristic ground state both for singlet and triplet. In Fig. 12 we show the concurrence as a function of time. In this case the concurrence increases relatively smoothly compared to the previous cases because we are in a regime where the one electron orbital states in the scattering process are the same for singlet and triplet, apart from a phase factor. Note that in order to even have the possibility of achieving high concurrence the electrons must have sufficient time to interact as the SAW propagates. The time scale to give spin entanglement is of order $\hbar/|J|$, with $J = E^T - E^S$ the exchange

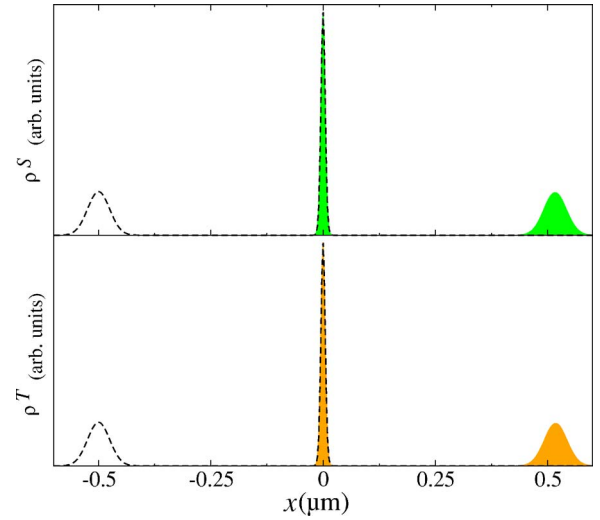


FIG. 11. (Color online) Initial (dashed) and final electron distribution (full) in arbitrary units when both singlet (top) and triplet (bottom) are fully transmitted with a phase difference as it is defined in the text.

energy and E^T , E^S the triplet and singlet energies when we fix the SAW with both electrons in the proximity of the well. Hence the SAW period must be at least as long as this for high concurrence to be possible and this is indeed the case for typical SAWs which used to give high accuracy single electron quantization,⁵⁻⁷ as we demonstrate in next section. Finally, it is worth mentioning that to a good approximation a phase difference may be present even when both states are reflected backwards, as described earlier. However in this case the phase difference is expected very small due to the weak effect of the Coulomb interaction. Furthermore, the regime of near perfect transmission also has the advantage that the electron in the SAW, after the scattering event is very

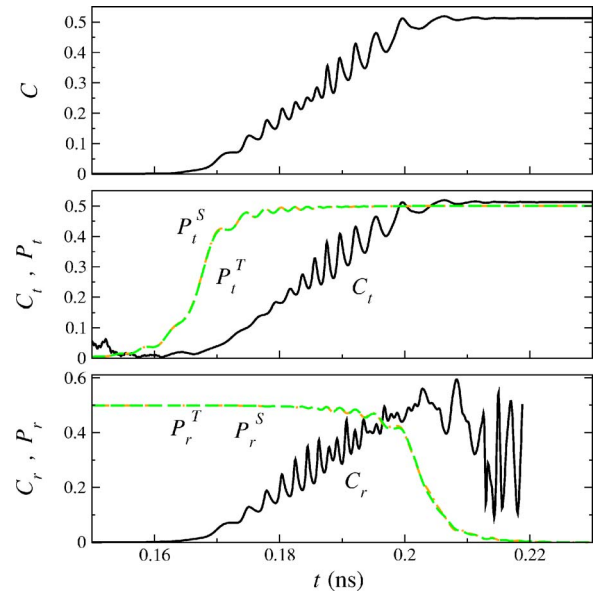


FIG. 12. (Color online) Concurrence and relative probabilities as a function of time when both singlet and triplet are fully transmitted with a phase difference as it is defined in the text.

well localized in a particular SAW minimum driven along the wire.

For all the cases we have described so far the induced entanglement between the two-electron spins is subjected to quantum decoherence which is an undesirable factor present to all solid state systems. The spin lifetime in GaAs, within which the process of generation detection needs to take place, is estimated to be ~ 100 ns (Ref. 37) arising primarily from phonon scattering. Typical times to generate entanglement in the SAW-based system are almost two orders of magnitude shorter while methods to read the final spin states have been described theoretically in Refs. 11 and 14 for the SAW qubit and already demonstrated experimentally for the static qubit.³⁸ Other important sources of decoherence that can affect the entanglement generation are coupling of electron spins to nuclear spins,^{39,40} noise on surface gates and temperature effects. These are discussed in the original proposal for SAW-based quantum computation.^{11,14}

IV. A HARTREE APPROXIMATION

The mechanism that controls the different scattering of singlet and triplet may be understood with an approximate treatment which also gives insight into the origin of the differences in transmission and reflection probabilities and concurrence. In a mean-field approximation, the electron which is carried by the SAW potential feels an effective time-dependent potential of the form $V_{\text{SAW}}^e(x, t) \approx V_t(x, t) + V_H(x, t)$, where the second term represents the Hartree potential due to the Coulomb repulsion of the trapped electron in the well: $V_H(x, t) = \int |\varphi(x', t)|^2 V_c(x, x') dx'$. This assumes that the trapped electron in the quantum well remains well localized and is described at a specific time t by the state $\varphi(x, t)$. Below we explain the different scattering results of Sec. III C by employing the effective potential form V_{SAW}^e for the propagating electron.

A. The single bound energy level regime

First we consider the cases for which the quantum well has a single bound energy level, i.e., the first two cases of Sec. III C. Figure 13(a) illustrates the effective potential $V_{\text{SAW}}^e(x, t)$ when t is such that $V_{\text{SAW}}(x, t)$ is minimum at $x \sim 0$ and for parameters that result in high transmission for the singlet state and high reflection for the triplet (that is the first case that we described in Sec. III C). The effective potential may be described as a triple well structure that changes with time due to the time-dependent nature of the SAW potential. Specifically, due to the SAW propagation, the right well becomes deeper than the left well with increasing time, with the middle well shifting upwards and downwards in energy at $x \sim 0$. Initially, the electron that is carried by the SAW resides in the left well and has a tendency to tunnel through the middle well into the right well, in order to remain bound in the SAW potential. The tunnelling mechanism is more efficient when there are resonance conditions for the electron to first tunnel from the left well into the middle well and then from the middle well to the right-hand well, i.e., in the time interval when resonant bound state energy widths of

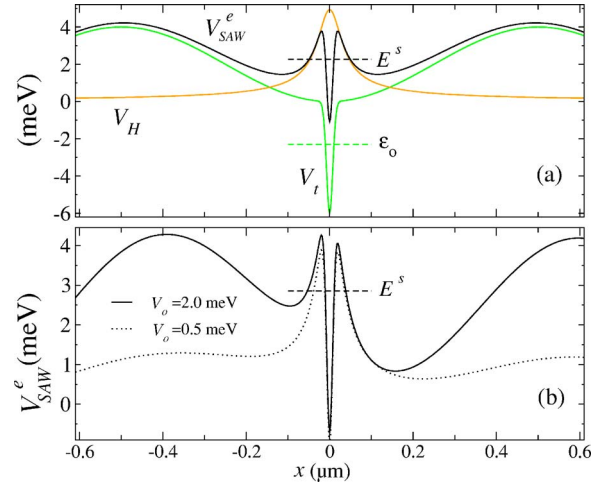


FIG. 13. (Color online) (a) The effective potential, and its constituent parts, that a SAW electron feels at a time for which the SAW potential is minimum at $x=0$ and for a SAW amplitude $V_o = 2$ meV. (b) The effective potential close to the resonant tunnelling regime for the singlet state for a SAW potential amplitude $V_o = 2$ meV (solid line). For $V_o = 0.5$ meV (dotted line) the resonance condition cannot be fulfilled (see text).

the left-hand, middle, and right-hand wells overlap. Of course the SAW potential amplitude, the width of the wire and the characteristic width and depth of the well should be chosen in such a way that the resulting effective potential guarantees at least one resonance energy level for the middle well, that will lie above the bottom of the right-hand and left-hand wells. A sufficiently large SAW amplitude is also necessary to ensure that the barrier between the resonance condition between left-hand and middle wells is satisfied, otherwise the electron in the SAW will be reflected. Finally, a necessary criterion for high transmission is that there must be sufficient time for the whole process to take place, i.e., the tunnelling time into and out of the middle well must be much smaller than the period of the SAW, a condition that is fulfilled in the simulations.

To explain qualitatively the difference in the evolution between the singlet and the triplet states, we also need to consider explicitly the symmetry of the orbital states and take into account the fact that only for the singlet state can both electrons occupy the same one electron orbital state. More specifically, if $\varphi_o(x, t)$ is the lowest resonant bound state of the combined well and SAW potential that peaks in the region of the well $x \sim 0$, then the instantaneous energy of the two-electron singlet state on resonance is $E^S(t) \approx \varepsilon_o(t) + U_o(t)$ where $U_o(t) = \int |\varphi_o(x_1, t)|^2 V_c(x_1, x_2) |\varphi_o(x_2, t)|^2 dx_1 dx_2$ is the Coulomb energy when both electrons occupy the single electron state $\varphi_o(x, t)$. This is analogous to resonant tunnelling in the Anderson impurity model. A similar approach could be applied to the triplet state but in this case the two electrons must occupy different one electron resonance levels, $\varphi_o(x, t)$ and $\varphi_1(x, t)$ due to the Pauli principle. If the quantum well had a second resonant state then the two-electron resonance would occur at the generally higher, triplet resonance energy $E^T(t) \approx \varepsilon_1(t) + U_{1,0}(t) - J_{1,0}(t) > E^S(t)$, where $U_{1,0}(t)$ and $J_{1,0}(t)$ are the Coulomb and the exchange integrals, respectively.

From the above description it is clear that the electron which is carried by the SAW feels an effective potential which is independent of the character of the two-electron orbital state (symmetric or antisymmetric), however this is not the case for the energy levels of the tunnelling process. For the first regime described in Sec. III C the singlet resonance level gives high resonance transmission only for the singlet state, as expected. The transmission of the triplet state is much weaker and is in fact due to nonresonant tunnelling, since with the chosen parameters the energy of the electron in the effective potential V_{SAW}^e is always below the barriers which define the middle well. Note that for this case the SAW potential amplitude is strong enough to drive the propagating electron to the resonance level. On the other hand, in the second regime described in Sec. III C the SAW potential well is so shallow that, for both singlet and triplet, the propagating electron is reflected before it reaches the resonance energy for tunnelling into the middle well. Figure 13(b) shows the effective potential profile close to the resonant tunnelling regime for the singlet state for both SAW potential amplitudes. Note that the resonance level lies above the bottom of the left- and right-hand wells, ensuring that tunnelling may occur. For these two regimes an effective antiferromagnetic exchange interaction controls the scattering process since the singlet scattering involves lower energy levels than the triplet.

B. Beyond the single bound energy level regime

For the last two cases of Sec. III C the quantum well has more than a single bound energy level and can bind at least two electrons. One effect of making the quantum well deeper is to reduce the barriers to the SAW wells to the left and right, as can be seen by comparing Figs. 13(a) and 14(c) for the effective one electron potential. This results in almost perfect transmission for both singlet and triplet states provided the SAW amplitude is much larger than the small residual barriers when the quantum well is at a SAW potential minimum [Fig. 14(c)]. However, the different positions of the singlet and triplet resonances still affect the final orbital states of the transmitted electron in the SAW, depending on the magnitude of the tunnel barrier when the SAW potential minimum energy is close to the resonance level in the quantum well [e.g., Fig. 14(b)].

Specifically, when this barrier is large the propagating electron emerges in the lowest state of the SAW potential minimum. This is illustrated in Fig. 14, where we plot some of the instantaneous eigenenergies of the effective one electron potential V_{SAW}^e for the parameters that result in a phase difference between singlet and triplet (this is the last case that we considered in Sec. III C). Note that the quasibound state levels within the well include the effect of Coulomb repulsion due to the bound electron which shifts the potential well up in energy by V_H and also gives rise to the very small peaks in the effective potential. At $t=0$ [Fig. 14(a)] the propagating electron is in the lowest energy of the V_{SAW}^e potential minimum to the left of the quantum well. This is actually the first excited state of the system since the lowest state is in the well. Between $t=0$ and $t=0.3T$ [Figs. 14(a) and

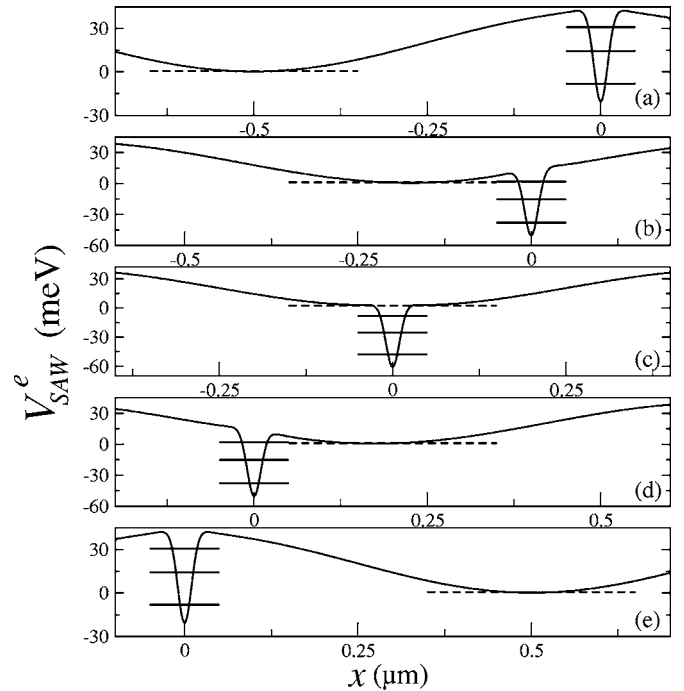


FIG. 14. Propagation of an electron in the V_{SAW}^e minimum passing through the well region. The dashed line indicates the energy of the propagating electron at each particular time, if after the SAW cycle it exits the well region in the lowest state of the SAW minimum. The full lines indicate the energy levels of the quantum well. In the anticrossing regions (b) and (d) the electron makes a nonadiabatic Landau-Zener transition and always remains in the V_{SAW}^e minimum when the tunnel barrier is large and as a result after the SAW cycle the electron emerges in the lowest state of the SAW potential minimum (e). When the tunnelling probability into and out of the quantum well is not negligible, the electron emerges in a superposition state of the low-lying states of the SAW. The time sequence is from (a) to (e) and specifically $t/T=0, 0.3, 0.5, 0.7, 1$.

14(b)] the energy levels corresponding to the electron in the V_{SAW}^e minimum and in the second state of well are almost the same (anticrossing region) but there is insufficient time for the electron to tunnel into the well and therefore it remains in the V_{SAW}^e potential minimum. It therefore makes a (nonadiabatic Landau-Zener) transition from the first to the second excited state of the system. At $t \approx 0.3T$ there is a further anticrossing region and transition to the third excited state of the system with the electron remaining in the SAW. Between $t=0.5T$ and $t=T$ [Figs. 14(d) and 14(e)] there are further transitions back to the initial state. This is the reason that the propagating electron emerges in the lowest state of the V_{SAW}^e potential minimum which actually coincides with the original SAW potential minimum when the electrons are well separated at $t=T$ [Fig. 14(e)] when the SAW cycle is completed. It is clear from Fig. 14(c) that the highest resonance level of the well, in this case the fourth level, gives rise to the largest interaction with the propagating electron as long as tunnelling to lower excited states is blocked due to the large barrier. Of course lower excited resonances are involved for shallower quantum wells.

Although the scattering process does not excite the electron in the SAW, it does affect its wave function by inducing

a phase shift and this phase shift is different for singlet and triplet cases due to Coulomb interaction. In particular, the evolution of singlet and triplet states will be different but, unlike the lowest singlet-triplet pair, the higher-lying levels will generally have the triplet lower in energy than the singlet, due essentially to Hund's rule.^{41,42} This results in a ferromagnetic exchange interaction, rather than the antiferromagnetic exchange of the lowest singlet-triplet pair. It is the small energy difference between the relevant singlet and triplet energy levels which induces the relative phase between singlet and triplet states as a consequence of the interaction time for the two electrons which is set by the SAW period. We show in the next section how the phase difference may be directly related to a ferromagnetic exchange interaction between the spins of the two electrons as they interact, changing their entanglement.

Finally, when the parameters are such that the probability to tunnel from the SAW potential minimum to the well is not negligible in the anticrossing region (e.g., the third case considered in Sec. III C) then the electron will emerge in a superposition state of the low-lying states of the SAW. This can be understood qualitatively again by referring to Fig. 14. Since the tunnelling probability is no longer negligible at the point where the V_{SAW}^e minimum crosses a resonance level, then the electron in the region of the well [e.g., Fig. 14(c)] will emerge in a superposition state of the third and fourth energy levels. Similarly, at later times when the energy level corresponding to the electron in the well sweeps through higher excited levels in the V_{SAW}^e potential minimum the electron will eventually leave the well and emerge in an asymptotic state that is a superposition of the low-lying states of the SAW. In this regime the different orbital states for singlet and triplet are due to different tunnel barriers for the highest-lying singlet-triplet pair due to different positions of the resonances. Note, however, that if the highest-lying resonant level is very close to the top of the well, such that there is strong tunneling into and out of the well, then the propagating electron will emerge in the SAW ground state for both singlet and triplet, though their phases will in general be different.

To conclude, an effective antiferromagnetic exchange interaction controls the scattering events when the quantum well has only one bound state, due to the singlet resonance channel. However, by increasing the depth of the well the ground singlet resonance level becomes inactive simply because it lies much lower than the energy of the propagating electron at all times. In this regime the scattering is controlled by an effective ferromagnetic exchange interaction involving excited states for singlet and triplet in which the triplet is lower. It is interesting to note that in the flying qubit scheme^{11,13} it is always an antiferromagnetic exchange interaction that generates the entanglement, whereas in the scheme that we propose both ferromagnetic and antiferromagnetic type interactions can generate entanglement depending on SAW and well parameters.

V. SOME GENERAL FEATURES OF THE ENTANGLEMENT

In this section we generalize some of the above results and demonstrate quantitatively the sensitivity of the system

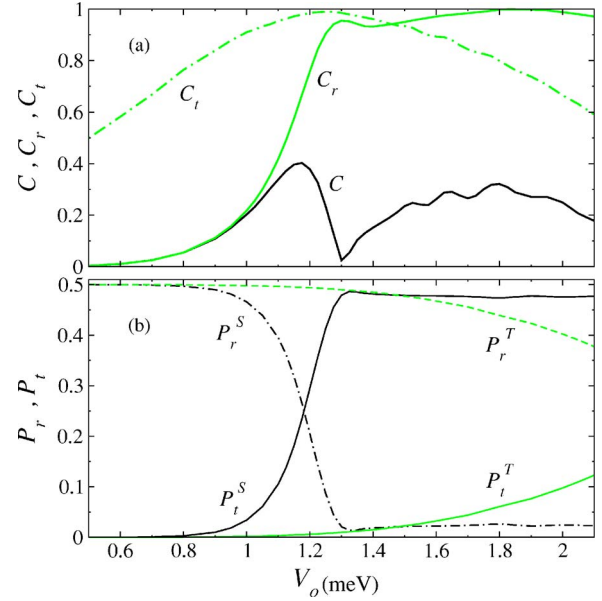


FIG. 15. (Color online) Variation of asymptotic (final time) concurrence (a) and relative probabilities (b), as defined in the text, versus the SAW potential amplitude when the quantum well is such that only a single electron can be bound.

to changes in the well and the SAW characteristics.

Figure 15(a) illustrates the variation of the concurrence versus the SAW potential amplitude for a quantum well with $V_w = 6$ meV and $l_w = 7.5$ nm, which can accommodate only a single bound electron. This plot shows the total, transmitted and reflected concurrence at the final time for which the overlap of propagating and bound electron wave packets is negligible. Figure 15(b) presents the corresponding probabilities. Note that the SAW potential amplitude is restricted to the specific regime for which the electron in the quantum well remains well-localized, as described in Sec. II B. We see that the very small transmission of singlet and triplet (which we include here for completeness), corresponding to the minimum value of the SAW amplitude, gives rise to a concurrence, $C_t \sim 0.5$. With increasing SAW amplitude there is then a regime in which the transmitted concurrence increases to $C_t \sim 1$, for which the singlet state is on resonance and simultaneously there is minimum transmission for the triplet state. We may regard this as a two-electron spin filter for which the initial unentangled state, that is an equal superposition of singlet and $S_z = 0$ triplet states, has its triplet component filtered out (reflected) with resonant transmission of the fully entangled singlet component. For this SAW amplitude, a “fuzzy” position measurement applied to the outgoing electron (say through charge sensing^{34–36}), which merely resolves whether it is transmitted or reflected, could be used to probabilistically prepare a highly entangled state. The form of the state prepared is heralded by the measurement outcome. Further increase of the SAW potential amplitude from this point causes the transmitted concurrence to gradually decrease due simply to the higher transmission of the triplet state. The reflected concurrence is very small for low SAW potential amplitude and approximately equals the total concurrence due to the very high reflection for both states. It

then increases smoothly as the singlet transmitted part increases and remains almost constant and close to unity with further increase of the SAW amplitude, since the reflected component is mainly triplet. Finally, the total concurrence has a relatively more complicated behavior, although it is clear that it has a maximum value $C \sim 0.4$ when $P_r^S = P_t^S \sim 0.25$, namely when the singlet is equally transmitted and reflected. Also it is always lower than the transmitted and reflected concurrence except when $P_t \sim 0$ and then $C \approx C_r$. We may conclude from Fig. 15 that the degree of entanglement for the two electrons can be changed significantly with SAW amplitude in the regime that scans through the singlet resonance and that there exists a point where in principle, through “fuzzy” position measurement, highly entangled states could be prepared.

A more relevant case for simpler experiments, which do not require position measurement to project into the transmitted or reflected outcomes for the outgoing electron, is when the SAW potential amplitude and the quantum well are such that the electron in the SAW is always fully transmitted, or approximately so, leaving the partner electron bound in the quantum well. The backward reflection, which is more likely to occur for a low potential amplitude, may cause undesirable effects, since the reflected electron will occupy multiple wells and involves highly excited components. This case of a reflected nonbound electron is more efficiently studied using kinetic injection without the presence of the SAW potential, as it is described, for example, in Refs. 19 and 21. In addition, a strong SAW potential amplitude has the advantage of preventing the trapped electron from leaking into adjacent minima, thus minimizing possible errors. We have calculated the concurrence as a function of the well depth for two different, though relatively strong, SAW potential amplitudes of $V_o = 20$ meV and $V_o = 10$ meV, and for fixed $l_w = 10$ nm. The SAW potential amplitude that is used in the experiments for SAW-based SET applications can be even stronger than this ($V_o \sim 40$ meV),²⁴ though along the channel there is likely to be some screening due to the gate bias. The chosen parameters guarantee that there is very high transmission both for singlet and triplet states ($P_t^{S/T} \sim 0.5$). In this study the range of the well depth ensures high localization of the trapped electron resulting in a truly bound singlet ground state, as calculated within a Hartree approximation and therefore, a ferromagnetic type exchange interaction generates the entanglement as described in the preceding section.

The results for the total concurrence, which almost equals the transmitted concurrence, are shown in Fig. 16, while Fig. 17 illustrates the singlet and triplet components of the electron in the SAW (after scattering) for various well depths and for the SAW amplitude of $V_o = 10$ meV. Figure 16 presents two distinct maxima for each of the two amplitudes considered with an intermediate region of relatively low concurrence (which is shown in detail in the inset of Fig. 16). Analysis of the data shows that in the rise up to the first maximum, the asymptotic state is approximated well by the simple phase difference picture described in Sec. III C, i.e., with the electron in the SAW potential minimum being in its ground state, to a good approximation, but with a phase difference between singlet and triplet components. This concur-

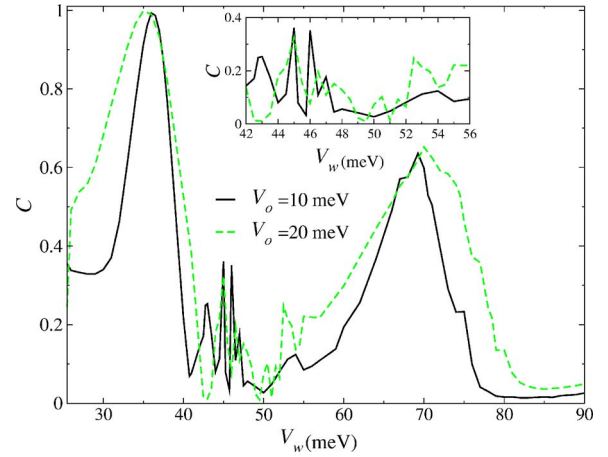


FIG. 16. (Color online) Variation of asymptotic (final time) concurrence for a fixed SAW potential amplitude ($V_o = 10$ meV, $V_o = 20$ meV) as a function of the well depth.

rence of almost unity at the maximum then corresponds to a phase difference of $\delta\varphi \sim \pi/2$. As the well depth is increased from this point the electron in the SAW occupies additional excited states which are different for singlet and triplet (the phase difference picture is no longer valid) and this is why the concurrence decreases. Figure 17 helps us understand how the singlet and triplet components of the electron in the SAW change with well depth and specifically how we pass from a region of different probability distribution to a region where the phase difference picture is valid. This behavior is clear, for example, by considering Figs. 17(a)–17(c). Similar behavior is valid for a SAW amplitude of $V_o = 20$ meV. Within the intermediate region for both SAW amplitudes the concurrence fluctuates due to spin-dependent scattering events which involve excited states of the SAW potential minimum. Figure 17(c) shows an example within this region. Note that zero concurrence corresponds to cases where the orbital states in the SAW for singlet and triplet components

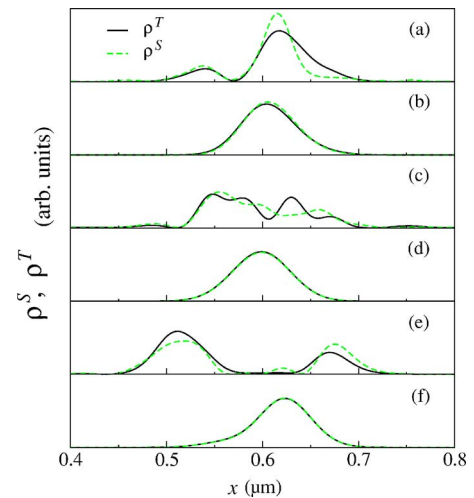


FIG. 17. (Color online) Final electron distribution for singlet and triplet states in the SAW potential minimum, for a SAW amplitude of $V_o = 10$ meV and a well depth from (a) to (f) of $V_w = 26, 36, 47, 62, 75, 85$ meV.

are exactly orthogonal. Further increase of the well depth gives rise to a second concurrence maximum, due to the fact that the final states of the SAW electron for singlet and triplet components are approximated by the ground state of the SAW potential minimum. Similar to the first concurrence maximum, the phase difference picture is again valid as shown, for example, in Fig. 17(d). Excited states with high probability however are involved in the scattering process, different for singlet and triplet as we demonstrate in Fig. 17(e) (reflecting spin-dependent scattering) lowering the concurrence. This occurs up to the regime of the very deep quantum well (at the right-hand side of Fig. 16 and for each SAW amplitude) where the phase difference picture becomes valid as Fig. 17(f) demonstrates. This is because the Coulomb interaction is effectively reduced due to the high confinement of the trapped electron. Singlet and triplet components are scattered mainly due to the presence of the potential well in the same final states with only a small effect from the Coulomb interaction, which will become negligible for extremely deep wells. When this extreme limit is reached the two-electron states will be approximated at all times by single electron states.

As in the preceding sections, the asymptotic value of the concurrence (at the final time) which emerges when a relative phase difference is present between singlet and triplet states, depends on the magnitude of the so-called exchange energy $J(t) = E^T(t) - E^S(t)$ and the SAW period which sets the interaction time. In the phase difference regime an approximate Heisenberg Hamiltonian⁴³ $H(t) = J(t) \mathbf{S}_1 \cdot \mathbf{S}_2$, with \mathbf{S}_i the spin operator of the i th electron can provide insight into the spin entanglement generation. This is because in this regime the two electrons at all times occupy different and well-defined orbital states and as we have described in Sec. IV B these states are the same for singlet and triplet apart from a phase factor. The exchange energy as a function of time can be determined by solving the instantaneous (time-independent) two-electron Schrödinger equation for singlet and triplet states treating time as a parameter, i.e., $H(t) \Phi_n^{S/T}(t) = E_n^{S/T}(t) \Phi_n^{S/T}(t)$, with the Hamiltonian given by Eq. (11). A common diagonalization procedure is described in Refs. 13, 41, and 42. The instantaneous solutions provide the sets $E_n^S(t)$ and $E_n^T(t)$ with n the eigenvalue index. By following the nonadiabatic Landau-Zener transitions which successfully take place in the phase difference regime, we can extract the energy of the two electrons during the scattering event, i.e., $E^T(t)$, $E^S(t)$ and from these the $J(t) = E^T(t) - E^S(t)$ curve.⁴⁴ In Fig. 18 we show the exchange energy as a function of time for two different well depths $V_w = 36$ meV, $V_w = 62$ meV and a SAW potential amplitude $V_o = 10$ meV which result in a phase difference as shown in Figs. 17(b) and 17(d). As we have analyzed in Sec. IV B and we see in Fig. 18, the exchange energy J is negative in the phase difference regime. The lower J for the case of the $V_w = 62$ meV well depth is because a higher excited energy level for the singlet-triplet pair is involved in the scattering process, compared to the $V_w = 36$ meV case and in general higher excited energy levels have a smaller separation.^{41,42} For the phase difference regime and within the Heisenberg model we can calculate the asymptotic concurrence C

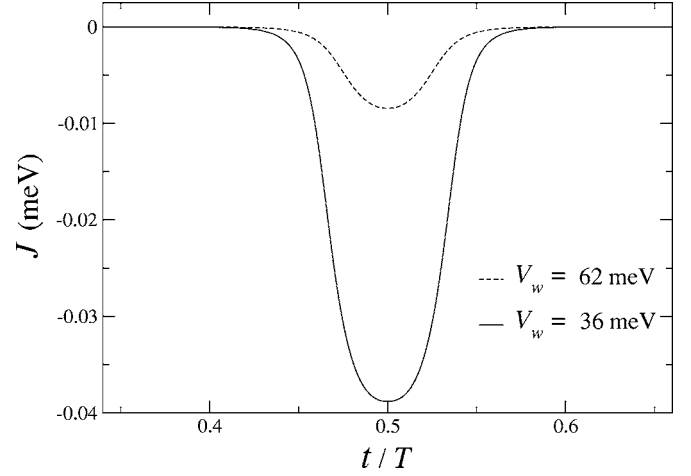


FIG. 18. Exchange energy as a function of time, for two different well depths ($V_w = 36$ meV, $V_w = 62$ meV) and a SAW potential amplitude of $V_o = 10$ meV.

$= |\sin \delta\varphi|$ by extracting the relative phase difference $\delta\varphi$ directly from the $J(t)$ curve as $\delta\varphi = \int_0^T J(t) / \hbar dt$. Note that the time interval of the integration is set by the SAW period T which is fixed in the experiments. The values that we take by this approximate treatment are in excellent agreement with the values that we take by solving the two-electron Schrödinger equation and by calculating the concurrence by the original formula (15).

VI. SUMMARY

In summary, we have presented and investigated a scheme to produce entangled states for two electrons utilizing a SAW. One electron is carried by the time-dependent SAW potential along a semiconductor quantum wire where a second electron is bound in a quantum well. The Coulomb interaction induces entanglement between the two electrons that can cover the full range from zero to full entanglement, depending on SAW potential amplitude and the shape of the confining potential. There are two regimes of interest, depending on the SAW and well parameters.

The first is when there is a significant difference between transmission probabilities for singlet and triplet states. In this regime, entanglement generation may be interpreted as a spin-filtering effect in which the singlet component of an initially unentangled state has a higher transmission probability than the triplet due to spin-dependent scattering. This gives maximal entanglement ($C \sim 1$) for resonant singlet tunnelling with full transmission for the singlet case and almost full reflection for the triplet case. ‘‘Fuzzy’’ position measurement (possibly through charge sensing^{34–36}), just resolving whether the outgoing electron is transmitted or reflected, would be needed to make this useful entanglement. The measurement result would identify and herald the form of entangled state produced in each run of an experiment.

The second regime occurs for the parameters chosen such that there is approximately full transmission for both singlet and triplet cases. Within this regime the transmitted electron in a SAW minimum can be left in an excited state which in

some cases is different for singlet and triplet and in some cases the same (or approximately so). In the latter cases, concurrence is given by a simple expression involving the relative phase difference between the transmitted SAW potential minimum wave functions arising from singlet and triplet. This demonstrates maximal entanglement when the phase difference is $\pi/2$ and we have identified a physically reasonable set of parameters for which this occurs. For other cases, the concurrence cannot reach the unitary limit and can fluctuate significantly due to spin-dependent resonance effects when the electrons interact. In this regime of near full transmission, the concurrence is low when the transmitted SAW minima wave functions are significantly different from each other for singlet and triplet cases, becoming zero in the limiting cases when these wave functions are orthogonal.

The physical system we have considered and the parameter ranges we have investigated suggest that it should be experimentally possible to produce useful entanglement between a travelling and a trapped electron, using a SAW. This could be achieved either by sensing whether the outgoing electron is transmitted or reflected, or by working in a regime where there is essentially complete transmission.

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⁴⁴Note that the phase-difference picture is also valid when the highest resonance level allows very strong tunneling into and out of the well.