Information on photonuclear transition matrix elements from measuring the emitted nucleon polarization in the reaction plane

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A compact general expression for the differential cross section of the photonuclear reaction (γ, N), in which the incident photon is circularly polarized and any polarization of the emitted nucleon observed, is presented. Angular distribution coefficients relevant to the nucleon polarization in the scattering plane are expressed as bilinear combinations of matrix elements for a special case of the ^{16}O(γ,N$_0$) reaction. The polarization P_x, calculated within the direct-semidirect model at $E_\gamma = 60$ MeV, is shown as an example of the theoretical result.

It is well known that by measuring the angular distribution of the analyzing power in addition to the differential cross section for a radiative nucleon capture reaction, (N,γ), additional information on the reaction transition matrix elements or their combinations is obtained.[1] For example, from the data for the reaction ^{15}N($\vec{\rm p}$,γ_0)^{16}O in the giant resonance region, it is possible, under the reasonable assumption that only electric dipole and quadrupole photons are emitted, to determine in a model independent way the four transition amplitudes (two dipole, corresponding to the s and d partial wave of the incident proton and two quadrupole, corresponding to p and f partial waves) and the three relative phases. Indeed, in such an experiment, the cross section for the quadrupole capture was obtained.[2]

The same reasoning is of course valid for the reactions inverse to the radiative nucleon capture, i.e., for the photonuclear reactions (γ,N), where N is the emitted nucleon. Labeling by θ the angle between the direction of the incident photon momentum and the direction of the emitted nucleon momentum, and by ↑ and ↓ the orientation of the spin of the emitted nucleon normal, up and down respectively, to the reaction plane, it is possible to expand the measured differential cross section $(d\sigma/d\Omega)(E;\theta)$ as

$$\frac{d\sigma}{d\Omega}(E;\theta)=\frac{d\sigma^{\uparrow}}{d\Omega}(E;\theta)+\frac{d\sigma^{\downarrow}}{d\Omega}(E;\theta)$$
$$=A_0(E)\left[1+\sum_{k=1}^{k_{\rm max}}a_k(E)P_k(\cos\theta)\right], \qquad (1)$$

and the measured analyzing power $A(E;\theta)$ as

$$\frac{d\sigma}{d\Omega}(E;\theta)A(E;\theta)=\frac{d\sigma^{\uparrow}}{d\Omega}(E;\theta)-\frac{d\sigma^{\downarrow}}{d\Omega}(E;\theta)$$
$$=A_0(E)\sum_{k=1}^{k_{\rm max}}b_k(E)P_k^1(\cos\theta)\,.$$

Here A_0 is given by the total cross section

$$A_0(E)=\frac{1}{4\pi}\int\frac{d\sigma}{d\Omega}(E;\theta)d\Omega\,,$$

and a_k and b_k are the angular distribution coefficients associated with Legendre polynomials, P_k, and with associated Legendre polynomials, P_k^1, respectively, and $k_{\rm max}=2J_{\rm max}$, where $J_{\rm max}$ is the maximum multipolarity involved. The coefficients a_k and b_k are expressed by bilinear combinations of transition matrix elements of particular electromagnetic operators and wave functions of the nuclear initial and final states. They have been given elsewhere, for example, a coefficients in Ref. 3 for the case of ^{16}O(γ,n$_0$)^{15}N.

For the most general case of a polarized incident photon, and the nucleon emitted and polarized in arbitrary directions, the expansion of the cross section in terms of Legendre polynomials has to be generalized. Here we present a compact expression, which seems to be most appropriate for the experimental analysis and which can be readily reduced to the special cases given by Eqs. (1) and

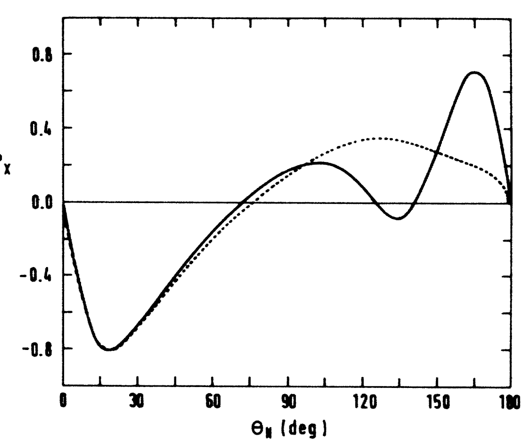

FIG. 1. Polarization P_x of protons from the reaction ^{16}O($\vec{\gamma}$, $\vec{\rm p}_0$)^{15}N calculated within the direct-semidirect model using the optical potential of Ref. 6 (full curve) and of Ref. 7 (dashed curve). The strength of particle vibrations coupling was adjusted to the experimental cross section in the giant dipole resonance region.

(2). Let us assume that the photon is incoming along the direction of the z axis, and that the reaction plane is defined by the Euler angles $(\theta_N,\varphi_N,0)\equiv\hat{k}$ with respect to the x-z plane of the right-handed coordinate system.[4] Then for an incident photon with circular polarization of helicity λ and an emitted nucleon of spin polarization characterized by Euler angles $(\theta_s,\varphi_s,0)\equiv\hat{s}$, the expanded differential cross section can be expressed by:

$$\frac{d\sigma}{d\Omega}(E,\lambda;\hat{k},\hat{s})=A_0(E)\sum_{k,r,\varkappa,n} {}_\lambda a_k^{r\varkappa n}(E)\left(\tfrac{1}{2}\delta_{r0}+rm_s\right)P_r^{\varkappa}(\cos\theta_s)P_k^{\varkappa}(\cos\theta_N)P_\varkappa^{n}(\cos(\varphi_s-\varphi_N)) .$$

Here the index k is the order of the (associated) Legendre polynomial. The indexes r, $\varkappa$, and n can take only the values 0 or 1. From the definition of the associated Legendre polynomials it is evident that $\varkappa\leq k$, $\varkappa\leq r$, and $n\leq\varkappa$. The coefficients with $r=0$ measure the part of the cross section corresponding to the emission of "unpolarized" nucleons. The coefficients with $r=1$, $\varkappa=0,1$, and $n=0$, i.e., the coefficients a^{100} and a^{110} account for the emitted nucleons with spin orientation in the scattering plane, the first one along the z axis and the second one along the x axis. The coefficients with $n=1$, a^{111}, correspond to the nucleons with spins normal to the reaction plane. We introduce m_s to reverse the spin orientation simply by converting $m_s=\frac{1}{2}$ into $m_s=-\frac{1}{2}$.

The meaning of the coefficients is more easily comprehended if the expression is written slightly more explicitly:

$$\begin{aligned}\frac{d\sigma}{d\Omega}(E,\lambda;\hat{k},\hat{s})=&A_0\sum_{k=0}^{k_{\max}} {}_\lambda a_k^{000}\tfrac{1}{2}P_k(\cos\theta_N)+A_0\sum_{k=0}^{k_{\max}} {}_\lambda a_k^{100}m_s\cos\theta_s P_k(\cos\theta_N)\\ &+A_0\sum_{k=1}^{k_{\max}} {}_\lambda a_k^{110}m_s\sin\theta_s P_k^1(\cos\theta_N)\cos(\varphi_s-\varphi_N)\\ &+A_0\sum_{k=1}^{k_{\max}} {}_\lambda a_k^{111}m_s\sin\theta_s P_k^1(\cos\theta_N)\sin(\varphi_s-\varphi_N) .\end{aligned}$$

Taking into account the symmetry properties of the coefficients $a^{r\varkappa n}$ with respect to λ (${}_{-\lambda}a^{000}={}_\lambda a^{000}$, ${}_{-\lambda}a^{111}={}_\lambda a^{111}$, ${}_{-\lambda}a^{110}=-{}_\lambda a^{110}$, and ${}_{-\lambda}a^{100}=a^{000}-{}_\lambda a^{100}$) it is easy to write the cross section for the unpolarized photons. The special cases, presented by Eqs. (1) and (2) can easily be reproduced by taking the reaction plane identical to x-z plane ($\varphi_N=0$) and aligning the spin with the y axis ($\theta_s=\varphi_s=\pi/2$). Then it is evident that

$$a_k^{000}\equiv a_k \text{ and } a_k^{111}\equiv b_k .$$

Another special example is easily obtained, namely the polarization $P_{\parallel}$, measuring the helicity of the outgoing nucleons. In this case, $\theta_N=\theta_s$ and $\varphi_N=\varphi_s$ are used in the general expression for the polarization of $\hat{s}=(\theta_s,\varphi_s,0)$, ${}_\lambda P_{\hat{s}}$:

$${}_\lambda P_{\hat{s}}=\frac{\sum_{kr\varkappa n} r\,{}_\lambda a_k^{r\varkappa n}P_r^{\varkappa}(\cos\theta_s)P_k^{\varkappa}(\cos\theta_N)P_\varkappa^{n}(\cos(\varphi_s-\varphi_N))}{\sum_k a_k^{000}P_k(\cos\theta_N)} .$$

In Fig. 1 we present the polarization in the direction of the x axis of protons emitted from the reaction $^{16}O(\overset{\circlearrowleft}{\gamma},\vec{p}_0)$ when photons have the positive helicity ($\lambda=1$) and an energy of 60 MeV, calculated within the direct-semidirect model.[5] This polarization is given by

$${}_1P_{\hat{x}}=\frac{\sum_k {}_1a_k^{110}P_k^1(\cos\theta_N)}{\sum_k a_k^{000}P_k(\cos\theta_N)} .$$

The coefficients a_k^{110} are given, like a_k and b_k, in terms of bilinear combinations of transition matrix elements. For example, using the same definition for the relative phase as in Ref. 3 and considering only electric dipole and electric quadrupole transitions, we have the following:

$${}_1a_1^{110}=1.1859|s||p|\cos(p,s)+0.3354|d||p|\cos(d,p)-0.4108|d||f|\cos(d,f) ,$$

$${}_1a_2^{110}=-0.5303|s||d|\cos(d,s)-0.3750|d|^2+0.6250|p|^2-0.0729|p||f|\cos(p,f)-0.4762|f|^2 ,$$

$${}_1a_3^{110}=+0.6455|s||f|\cos(f,s)-0.3354|d||p|\cos(d,p)-0.7303|d||f|\cos(f,d) ,$$

$${}_1a_4^{110}=-0.4374|p||f|\cos(f,p)-0.3571|f|^2 .$$

The full and the dashed curve in the figure present P_x obtained by using the optical model potentials of Refs. 6 and 7, respectively. The deviation from the antisymmetry around $\theta_N=90$ is due to the contribution of electric multipoles higher than dipole.

In the near future it might be possible to compare calculated polarizations P_x as well as polarizations P_z (related to the coefficients a_k^{100}) with data extracted from measured polarization in the scattering plane along the momentum of the emitted particle and the perpendicular to it, $P_{\parallel}$ and $P_{\perp}$, respectively.[8]

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