

EUROPHYSICS LETTERS

OFFPRINT

Vol. 74 • Number 5 • pp. 764–770

**Entanglement between static and flying qubits
in quantum wires**

J. H. JEFFERSON, A. RAMŠAK and T. REJEC



Published under the scientific responsibility of the

EUROPEAN PHYSICAL SOCIETY

Incorporating

JOURNAL DE PHYSIQUE LETTRES • LETTERE AL NUOVO CIMENTO

Entanglement between static and flying qubits in quantum wires

J. H. JEFFERSON¹, A. RAMŠAK² and T. REJEC^{2,3}

¹ *QinetiQ, Sensors and Electronics Division - St. Andrews Road
Great Malvern, UK*

² *Faculty of Mathematics and Physics, University of Ljubljana, J. Stefan Institute
Ljubljana, Slovenia*

³ *Ben-Gurion University - Beer Sheva, Israel*

received 25 November 2005; accepted in final form 3 April 2006

published online 26 April 2006

PACS. 03.67.Mn – Entanglement production, characterization, and manipulation.

PACS. 73.63.-b – Electronic transport in nanoscale materials and structures.

Abstract. – A weakly bound electron in a semiconductor quantum wire is shown to become entangled with an itinerant electron via the Coulomb interaction. The degree of entanglement and its variation with energy of the injected electron, may be tuned by choice of spin and initial momentum. Full entanglement is achieved close to energies where there are spin-dependent resonances. Possible realisations of related device structures are discussed.

A major goal in the rapidly emerging field of quantum information processing is the controlled exchange of quantum information between propagating and static qubits. Purely electron systems have potential as entanglers due to strong Coulomb interactions and although charge-qubit systems suffer from short coherence times, spins in semiconductor quantum wires and dots are sufficiently long-lived for spin-qubits to be promising candidate for realizing quantum gates involving both static and propagating spins [1–5]. Electron entanglers have been proposed using a double dot, exploiting the singlet ground state [6], the exchange interaction between conduction electrons in a single dot [7, 8], and the exchange interaction between electron spins in parallel surface acoustic wave channels [9]. The measurement of entanglement between propagating electron pairs has also been proposed using an electron beamsplitter [10]. In this letter we propose a scheme whereby a single propagating electron interacts strongly with a bound electron in a quantum wire. This differs from the quantum-dot systems referred to above in several respects. Firstly, entanglement is induced between the spins of one propagating and one bound electron, rather than two propagating electrons, and this entanglement is detected by measuring a single-electron spin directly (via a spin-filter), rather than indirectly through current-current correlations which average over many spins. Secondly, unlike in a quantum dot, there are no imposed barriers, only a shallow potential well. Spin-dependent resonances are a consequence of the Coulomb interaction and electron asymmetry and give rise to broad resonances that enable entanglement to be controlled by changing the kinetic

energy of the propagating electron with external gates. This allows considerable flexibility in controlling the entangling interactions via the kinetic energy of the incident electron.

Consider a semiconductor quantum wire in which there is a weak confining potential which is capable of binding one, and only one, electron. Slight deviation from a perfect 1D confining potential, either accidental or deliberate, can give rise to fully bound states for electrons. When the confining potential is very weak, such as occurs with a weak symmetric bulge in an otherwise perfect wire, there is one and only one bound state [11]. Furthermore, only a single electron can be bound in this confining potential since the energy of a second electron will be in the continuum due to Coulomb repulsion. We have shown that the spin-dependent interaction between a single propagating electron and the weakly bound electron can induce entanglement between them, giving rise to a two-electron quantum gate. In this scenario, the flying qubit is realised as the spin of the propagating electron and the static qubit is the spin of the bound electron. A possible realisation of such a system is a clean semiconductor quantum wire in which the propagating electron is injected through a single-electron turnstile [6] and the bound electron is trapped in a shallow potential well along the wire, controlled by a gate electrode.

The two-electron system may be modeled by the effective Hamiltonian

$$H = - \sum_{i=1}^2 \left[\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_i^2} + v(x_i) \right] + V(x_1, x_2), \quad (1)$$

where m is the effective mass of an electron in the lowest conduction miniband, $v(x)$ is an effective one-electron potential and $V(x_1, x_2)$ is an effective two-electron potential. This effective Hamiltonian accurately describes the system provided: i) confinement in the transverse dimensions is sufficiently large and the kinetic energy sufficiently low that only the lowest miniband is occupied, ii) the lowest transverse mode is non-degenerate, iii) the energy scale is sufficiently low that non-parabolicity is negligible and iv) the change in effective potential $v(x)$ is sufficiently slow that coupling to higher minibands is negligible. The effective potential in eq. (1) is generic in that it may be explicitly induced, using surface gates, or implicitly by an expansion in the transverse dimensions of the quantum wire, or a combination of both. It may also have contributions from remote charge centres or defects or other remote gates. The sources of this confining potential are unimportant, and even the condition that potential is slowly varying may be relaxed, *e.g.*, by use of a very narrow nanoscale gate. However, the effective potential well must be sufficiently weak to bind only a single electron, though it may have more single-electron bound states. The problem is analogous to treating the collision of an electron with a hydrogen atom, *e.g.*, as studied by Oppenheimer and Mott [12]. The present paper shows how such spin-dependent scattering may be realised and controlled in a semiconductor quantum wire containing an open quantum dot binding a single electron, giving rise to resonance scattering and associated spin entanglement in this one-dimensional system. Such a system can show exotic behaviour similar to the Kondo effect observed in more conventional quantum-dot systems with high confining barriers [13] and this behaviour has also been related to the conductance anomalies referred to earlier and considered previously by the present authors [14, 15] and others [16].

For the two-electron case we will show that the scattering of the flying qubit from the static qubit can induce entanglement in a controlled fashion and may thus be regarded as a candidate for realising a general two-qubit gate and explicitly demonstrating exchange of quantum information between a static qubit and a flying qubit. Consider an unentangled state in which the quantisation axis is chosen to be in the direction of the propagating electron spin and the bound-electron spin is in some general state on the Bloch sphere, *i.e.* $\cos(\vartheta/2)|\downarrow\rangle + e^{i\phi}\sin(\vartheta/2)|\uparrow\rangle$. We may write the antisymmetrised incoming scattering states for the two

electrons as $\Psi_{in} = c\Psi_{\uparrow\downarrow}^+ + e^{i\phi}s\Psi_{\uparrow\uparrow}^+$, where $c = \cos(\vartheta/2)$, $s = \sin(\vartheta/2)$ and

$$\Psi_{\sigma\sigma'}^{\pm} = \begin{vmatrix} e^{\pm ikx_1}\chi_{1\sigma} & e^{\pm ikx_2}\chi_{2\sigma} \\ \psi_b(x_1)\chi_{1\sigma'} & \psi_b(x_2)\chi_{2\sigma'} \end{vmatrix}.$$

Here $\psi_b(x)$ is the ground-state wave function of the bound electron and the injected electron has quasimomentum k and spinor χ_{\uparrow} . After scattering, the propagating electron will be reflected or transmitted and, asymptotically, will have the same magnitude of momentum, k , leaving the bound electron again in its ground state, ψ_b , provided that the initial energy of the incoming electron is smaller than the energy separation, ΔE , from the the bound state to the next allowable state. In cases where there is only one bound state, ΔE is the ionisation energy, otherwise it is the threshold energy for inelastic scattering via intra-dot transitions. For elastic scattering, the reflected and transmitted part of the asymptotic states are

$$\begin{aligned} \Psi_{out}^- &= c(r_{nsf}\Psi_{\uparrow\downarrow}^- + r_{sf}\Psi_{\downarrow\uparrow}^-) + e^{i\phi}sr_{\uparrow\uparrow}\Psi_{\uparrow\uparrow}^-, \\ \Psi_{out}^+ &= c(t_{nsf}\Psi_{\uparrow\downarrow}^+ + t_{sf}\Psi_{\downarrow\uparrow}^+) + e^{i\phi}st_{\uparrow\uparrow}\Psi_{\uparrow\uparrow}^+. \end{aligned} \quad (2)$$

We see that both the reflected and transmitted waves show spin entanglement after scattering provided $\cos(\vartheta/2)$ and the amplitudes for spin-flip and non-spin-flip scattering, r_{sf} , r_{nsf} , t_{sf} and t_{nsf} are non-zero. Furthermore, fully entangled states occur when $\vartheta = 0$ and $|r_{sf}| = |r_{nsf}|$ or $|t_{sf}| = |t_{nsf}|$. Although it is clear that the interaction between electrons will induce entanglement, it is not obvious that this can be controlled or indeed that maximum entanglement can be achieved. Full entanglement seems plausible for the following reasons. Writing the asymptotic states in the basis of spin eigenstates and comparing with eqs. (2) we see directly that

$$t_{nsf} = \frac{t_T + t_S}{2}, \quad t_{sf} = \frac{t_T - t_S}{2} \quad \text{and} \quad t_{\uparrow\uparrow} = t_T. \quad (3)$$

This two-electron system has at least a singlet resonance ($|t_S| = 1$) at some energy for which the triplet state is off resonance ($|t_T| \ll 1$), as may be seen from explicit solutions of the scattering problem. We have obtained results for GaAs quantum wires with an effective mass $m = 0.067m_0$, a wire width of 10 nm giving an energy separation of 125 meV between the lowest and first excited transverse modes, and an effective Coulomb interaction $V(x_1, x_2)$ given by integrating the bare 3D Coulomb interaction over the lowest transverse mode. The shallow effective potential, $v(x)$, is first chosen such that there is only a single one-electron bound state at energy $\epsilon_b = -12$ meV with a well depth of 26 meV and a width of ~ 20 nm (fig. 1). As described in previous work [14, 15], the bound electron has a long-range Coulomb interaction with the propagating electron and, when combined with the well potential, gives rise to a double-barrier structure which has a singlet resonance energy at approximately $\epsilon_S \sim \epsilon_b + U$, where ϵ_b is the energy of the lowest bound state and $U = 15$ meV is the Coulomb matrix element for two electrons of opposite spin occupying this state. This is also shown in fig. 1 where we have plotted the Hartree-Fock potential due to the bound electron, $v_{HF}(x)$, *i.e.* the self-consistent potential seen by the propagating electron in the ‘‘frozen’’ potential due to the bound electron of opposite spin. For this case, the shallow well is sufficiently narrow that only a single one-electron bound state is possible. For wider wells a second bound state is allowed and this will give rise to a triplet resonance (and a further singlet resonance) at higher energy provided. The singlet-triplet separation may be controlled by changing the width and depth of the well whilst maintaining the condition that only one electron be bound. With increasing well width (and decreasing well depth) the singlet-triplet separation reduces and eventually the resonances overlap.

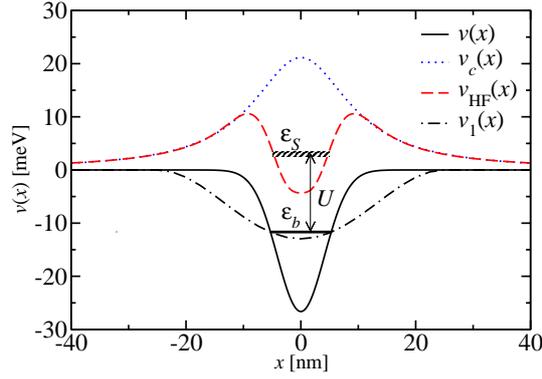


Fig. 1 – Shallow potential well $v(x)$ (full line) and corresponding Hartree-Fock potential seen by a second electron of opposite spin (dashed line). The single resonant bound state, ϵ_b , and quasi-bound singlet state, ϵ_s , within the double-barrier structure are also indicated. The dotted line is the Coulomb repulsion energy due to the bound electron and the dash-dotted line represents the shallow potential well $v_1(x)$ corresponding for the results from fig. 3.

At the lowest (singlet) resonance, $|t_{sf}| \approx |t_{nsf}| \approx \frac{1}{2}$ and the state is close to being fully entangled when spins are initially antiparallel ($\cos(\vartheta/2) = 1$). Similarly, in reflection, $|r_S| \approx 0$, $|r_T| \approx 1$ and $|r_{sf}| \approx |r_{nsf}| \approx \frac{1}{2}$, which is also close to being fully entangled. Note that at such a resonance, the propagating electron has approximately equal probability ($\frac{1}{2}$) of being either transmitted or reflected. The precise condition for full entanglement in transmission is $\vartheta = 0$, $|t_T + t_S| = |t_T - t_S|$ and this is satisfied when the complex numbers t_S and t_T are at right angles in the Argand diagram, *i.e.* $\delta_S - \delta_T = (2n + 1)\frac{\pi}{2}$, where δ is the phase shift due to scattering and n is an integer.

When the states are not fully entangled, a measure of their degree of entanglement is given by the concurrence [17] which, for the pure states considered here, is defined as

$$C = \frac{2}{\langle \Psi | \Psi \rangle} \left| \begin{array}{cc} \langle \alpha \alpha | \Psi \rangle & \langle \alpha \beta | \Psi \rangle \\ \langle \beta \alpha | \Psi \rangle & \langle \beta \beta | \Psi \rangle \end{array} \right|, \quad (4)$$

where $|\alpha\rangle$ and $|\beta\rangle$ are orthogonal qubit base states in any representation. For transmitted electrons the asymptotic spin concurrence after scattering is thus [18] from eqs. (2), (3), and (4)

$$C_t = \frac{2c(T_{sf} T_{nsf})^{1/2}}{c^2 [T_{sf} + T_{nsf}] + s^2 T_{\uparrow\uparrow}} = \frac{2c[(T_T + T_S)^2 - 4T_T T_S \cos^2(\delta_T - \delta_S)]^{1/2}}{T_T + T_S + s^2(T_T - T_S)}, \quad (5)$$

where $T_\lambda = |t_\lambda|^2$ for corresponding labels λ . Similarly, $|r_T + r_S| = |r_T - r_S|$ for full entanglement in reflection with antiparallel spins initially. We again see that full entanglement is plausible at $\vartheta = 0$ for energies for which either pure singlet or pure triplet states are near a resonance since we know, for example, that the phase shift for resonant singlet scattering changes rapidly as we sweep through the resonance energy, whereas the phase shift for the triplet varies only slowly provided the overlap of the resonance widths is small. Thus, provided that singlet and triplet energies are not too close in energy, there will be some energy for which $\cos(\delta_T - \delta_S) = 0$ in eq. (5). We also note from eq. (5) that the concurrence approaches unity when either $T_S \gg T_T$ or $T_T \gg T_S$. This is simply due to the fact that one of the spin channels (singlet or triplet) is “filtered out” leaving the other channel which is fully entangled.

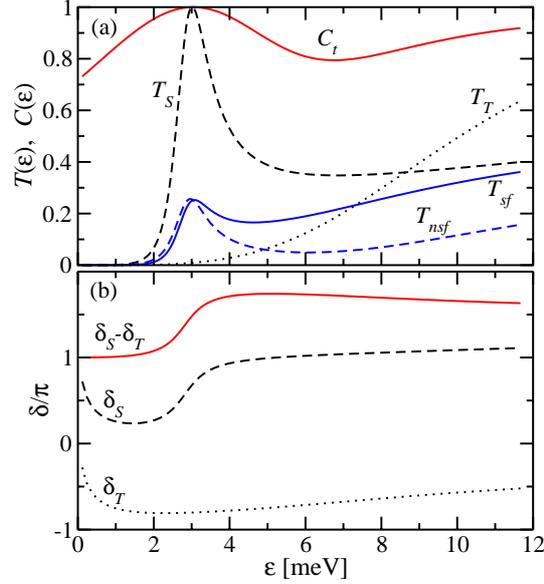


Fig. 2 – (a) Singlet and triplet transmission probability (T_S , T_T), spin-flip and non-spin-flip transmission probability (T_{sf} , T_{nsf}) and corresponding concurrence C_t for the confining potential $v(x)$ from fig. 1. (b) Phase shifts corresponding to transmission probabilities in (a).

Further illustration of this behaviour is seen by solving the scattering problem explicitly for specific cases. Numerical solutions for symmetrised (singlet) or antisymmetrised (triplet) orbital states yield directly the complex amplitudes t_S , r_S , t_T , r_T from which the amplitudes for spin-flip and non-spin-flip scattering may be calculated using eq. (3).

In fig. 2 we plot the singlet and triplet transmission and reflection probabilities, showing a single maximum of unity for the transmission. We also plot concurrence, which is very close to the corresponding singlet resonance, occurring when the spin-flip probabilities are equal and approximately $\frac{1}{4}$ in both transmission and reflection. The phase angle of the singlet changes rapidly with energy through the resonance, whereas the triplet resonance is fairly flat. Since the total change in singlet phase angle is somewhat in excess of $\pi/2$, there is a point where the singlet-triplet phase difference is precisely $\pi/2$ and the transmitted state is fully entangled. The behaviour is similar in reflection. Note, however, that at low energy, the difference in phase angle for singlet and triplet tends to π in transmission and the limiting concurrence is non-zero, whereas in reflection the limiting behaviour is zero phase shift and concurrence. This can be understood when we consider that at low-energy the total transmission probability is very small and hence this somewhat unexpected behaviour results from a very improbable transmission event. When this does occur, the spin-flip process dominates since the incoming up-spin electron simply displaces the down-spin electron due to Coulomb repulsion. Neglecting the non-spin-flip process we see, from eq. (5) that $t_S \approx -t_T$, *i.e.* a phase difference of π . Actually, the limiting non-spin-flip process, though small, is not negligible, as can be seen from the finite concurrence. This shows a limiting value of around 0.7 giving $T_{nsf}/T_{sf} \sim 0.1$. In reflection, the non-spin-flip process is dominant at low energy by the same argument (as seen explicitly in the plot) and hence the phase difference tends to zero as does the concurrence.

In fig. 3 we show results for a shallow potential well of depth 12 meV and width ~ 40 nm. With these parameters there are two single-electron bound states at energies -8 meV and

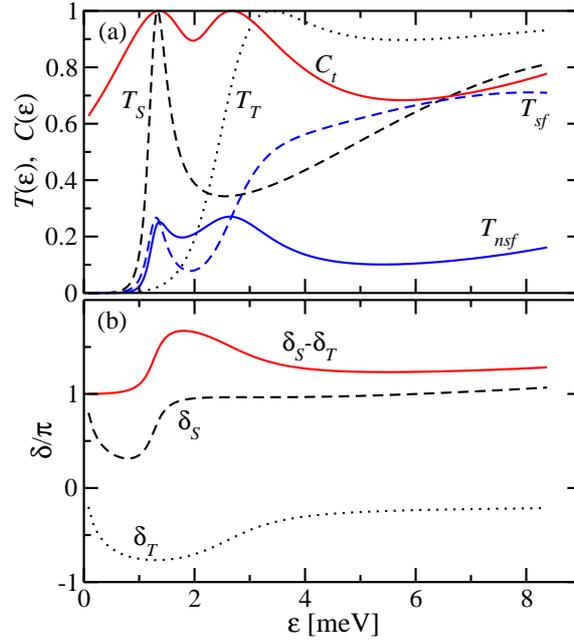


Fig. 3 – (a) Singlet and triplet transmission probability (T_S , T_T), spin-flip and non-spin-flip transmission probability (T_{sf} , T_{nsf}) and corresponding concurrence C_t for the confining potential $v_1(x)$ from fig. 1. (b) Phase shifts corresponding to transmission probabilities in (a).

–10 meV. This gives both a singlet and a triplet resonance, the latter corresponding to one electron in the lowest bound state and the other in the higher bound state which becomes a resonance obeying Hund’s rule under Coulomb repulsion, with a further singlet resonance outside the energy window for elastic scattering. We see that there are two unitary peaks of concurrence in transmission with the second close to, but clearly discernable from, the peak of the rather broad triplet resonance. We have shown in other examples, where singlet and triplet resonances are very close, that the concurrence does not always reach the unitary limit, since both singlet and triplet phase shifts vary rapidly with energy with their difference not reaching $\pi/2$.

In conclusion, we have shown that spin entanglement and exchange of quantum information occurs via the Coulomb interaction when a propagating electron interacts Coulombically with a single bound electron in a shallow potential well in a one-dimensional semiconducting quantum wire. The degree of entanglement may be controlled by kinetic energy of the incoming electron and the shape of the effective potential well and unitary concurrence occurs near a singlet or triplet resonance. Potential realisations of such a system are semiconductor quantum wires and carbon nanotubes.

A possible sequence of operations to demonstrate that entanglement has been achieved would be as follows. Initialisation would consist of first loading the open quantum dot with a single electron using a turnstile injector with surface and back gates to control the shape and depth of the potential well. A second electron is then injected through the turnstile, incorporating a Zeeman quantum dot spin filter [19] in a global magnetic field. The spin filter is tuned such that only minority spins are resonant, resulting in a propagating electron with opposite spin to the bound electron. Alternatively, both static and propagating spins may have the same polarisation with the spin of the bound electron flipped by a microwave

π -pulse prior to interaction. Any further spin rotation due to the global magnetic field may then be accounted for explicitly. This would, of course, depend on the group velocity of the injected electron which may be controlled by the source drain bias, enabling the kinetic energy of the incident electron to sweep the resonances. The measurement of spin for the propagating electron after interaction could also be done using a quantum-dot spin filter in which a transmitted electron would be detected by a single-electron transistor. The static spin would then be inferred indirectly by injecting a second propagating spin of known polarisation and correlating its measured spin after interaction with that of the first propagating electron.

* * *

We thank A. BRIGGS, C. BARNES, S. BENJAMIN, M. FEARN, G. GIAVARAS, D. GUNLYCKE, D. PETTIFOR, S. POPESCU, and I. SEGA for helpful discussions. Authors acknowledge support from the EU, the UK MoD, the UK IRC on QIP (GR/S82176/01), and the Ministry of Higher Education, Science and Technology of Slovenia under grant PI-0044.

REFERENCES

- [1] LOSS D. and DiVINCENZO D. P., *Phys. Rev. A*, **57** (1998) 120.
- [2] ELZERMAN J. M., HANSON R., WILLEMS VAN BEVEREN L. H., WITKAMP B., VANDERSYPEN L. M. K. and KOUWENHOVEN L. P., *Nature*, **430** (2004) 431.
- [3] HANSON R., WITKAMP B., VANDERSYPEN L. M. K., WILLEMS VAN BEVEREN L. H., ELZERMAN J. M. and KOUWENHOVEN L. P., *Phys. Rev. Lett.*, **91** (2003) 196802.
- [4] HANSON R., WILLEMS VAN BEVEREN L. H., VINK I. T., ELZERMAN J. M., NABER W. J. M., KOPPENS F. H. L., KOUWENHOVEN L. P. and VANDERSYPEN L. M. K., *Phys. Rev. Lett.*, **94** (2005) 196802.
- [5] PETTA J. R., JOHNSON A. C., YACOBY A., MARCUS C. M., HANSON M. P. and GOSSARD A. C., cond-mat/0412048.
- [6] XUEDONG HU and DAS SARMA S., *Phys. Rev. B*, **69** (2004) 115312.
- [7] COSTA A. T. jr. and BOSE S., *Phys. Rev. Lett.*, **87** (2001) 277901.
- [8] OLIVER W. D., YAMAGUCHI F. and YAMAMOTO Y., *Phys. Rev. Lett.*, **88** (2002) 037901.
- [9] BARNES C. H. W., SHILTON J. M. and ROBINSON A. M., *Phys. Rev. B*, **62** (2000) 8410.
- [10] BURKARD G. and LOSS D., *Phys. Rev. Lett.*, **91** (2004) 087903.
- [11] LANDAU L. D. and LIFSHITZ E. M., *Quantum Mechanics: Non-Relativistic Theory*, Vol. **3** (Pergamon) 1977.
- [12] OPPENHEIMER J. R., *Phys. Rev.*, **32** (1928) 361; MOTT N. F., *Proc. R. Soc. London, Ser. A*, **126** (1930) 259.
- [13] GOLDHABER-GORDON D., SHTRIKMAN H., MAHALU D., ABUSCH-MAGDER D., MEIRAV U. and KASTNER M. A., *Nature*, **391** (1998) 156.
- [14] REJEC T., RAMŠAK A. and JEFFERSON J. H., *Phys. Rev. B*, **62** (2000) 12985; *J. Phys.: Condens. Matter*, **12** (2000) L233.
- [15] REJEC T., RAMŠAK A. and JEFFERSON J. H., *Phys. Rev. B*, **67** (2003) 075311.
- [16] MEIR Y., HIROSE K. and WINGREEN N. S., *Phys. Rev. Lett.*, **89** (2002) 196802; SUSHKOV O. P., *Phys. Rev. B*, **67** (2003) 195318; STARIKOV A. A., YAKIMENKO I. I. and BERGGREN K.-F., *Phys. Rev. B*, **67** (2003) 235319; CORNAGLIA P. S., BALSEIRO C. A. and AVIGNON M., *Phys. Rev. B*, **71** (2005) 024432.
- [17] WOOTTERS W. K., *Phys. Rev. Lett.*, **80** (1998) 2245.
- [18] The same results are obtained in time-dependent approach and wave packets with sharp momentum resolution, RAMŠAK A., SEGA I. and JEFFERSON J. H., unpublished.
- [19] RECHER P., SUKHORUKOV E. V. and LOSS D., *Phys. Rev. Lett.*, **85** (2000) 1962.