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The Aharonov - Bohm effect

Seminar I

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1 Introduction

Maxwell equations form the basis of electrodynamics. The most electric and magnetic fields can be expressed in terms of the vector and scalar potentials $\vec{A}$ and φ in the way

$$\vec{H} = \vec{\nabla} \times \vec{A}, \tag{1}$$

$$E = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi. \tag{2}$$

These equations do not define the potentials uniquely in terms of the fields. If we add an arbitrary vector to the vector potential, the magnetic field will not change. If we add scalar to the scalar potential, the electric field will remain unchanged. Thus the electric and the magnetic fields are invariant under the transformation

$$\begin{aligned} \vec{A'} &= \vec{A} - \vec{\nabla}\psi \\ \varphi' &= \varphi + \frac{1}{c}\frac{\partial \psi}{\partial t}. \end{aligned} \tag{3}$$

This transformation is so called *gauge transformation*. [1] In electrodynamics potentials represent only mathematical aid for expressing fields, and do not have any physical meaning themselves.

2 The original work

The Aharonov-Bohm effect is a quantum-mechanical phenomenon, it claims that charged particles are affected in the form of a phase shift by the existence of electromagnetic fields even though the charged particles travel through field-free regions. In their original work, Aharonov and Bohm suggested two thought experiments which demonstrate the significance of potentials in the quantum theory. [3]

2.1 The electric effect

In the first thought experiment, a single coherent electron beam is split into two parts and each part is then allowed to enter a long cylindrical metal tube. After the beams pass through the tubes, they are combined to interfere coherently. The potential is non-zero only while the electrons are well inside the tube. The purpose of this arrangement is to ensure that the electron is in a time-varying potential without ever being in a field.

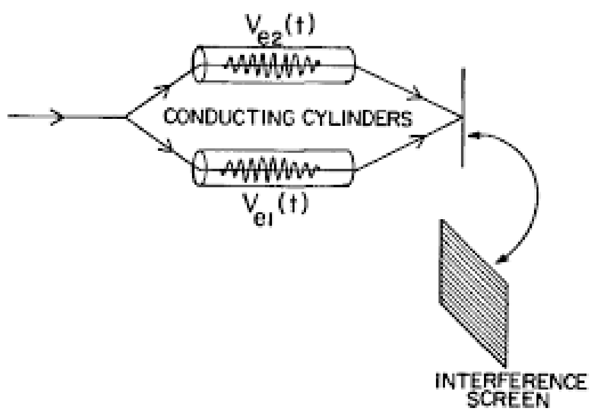


Figure 1: Experimental setup for the electric AB effect.

Hamiltonian for this system is $H = H_0 + V(t)$, and the solution of the Schrödinger equation is

$$\Psi = \Psi_1^0 e^{-iS_1/\hbar} + \Psi_2^0 e^{-iS_2/\hbar}, \tag{4}$$

where $S_1 = e\int \varphi_1 dt$ and $S_2 = e\int \varphi_2 dt$ are phases of chopped waves.
The interference of the two parts will depend on the phase difference [2]

$$(S_1 - S_2)/\hbar,$$

which means there is the potential effect even though there is no applied force. The phase difference can be expressed as the integral

$$\frac{e}{\hbar}\oint \varphi dt.$$

The relativistic generalization of this expression is

$$\frac{\Delta S}{\hbar} = \frac{e}{\hbar}\oint \left(\varphi dt - \frac{\vec{A}}{c}\cdot d\vec{x}\right). \tag{5}$$

2.2 The magnetic effect

The expression (5) motivates the assumption that there might be a similar situation for the vector potential. In this case an electron beam and a solenoid are required. Coherent electron beam must be split into two parts, each of which goes to one side of the solenoid avoiding it. After passing the solenoid, beams are brought together in a point.

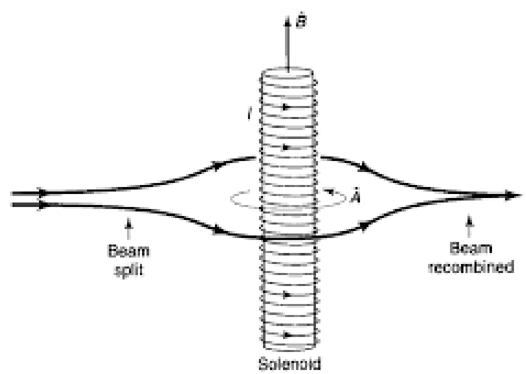


Figure 2: Experimental setup for the magnetic AB effect.

The Hamiltonian is now $H = \frac{\left[\vec{P}-\frac{e}{c}\vec{A}\right]^2}{2m}$. In this experimental set-up we have multiply connected regions. In order to have a single-valued function, we have to split the wave function in two parts, just like in the previous case. The phase difference is now

$$(S_1 - S_2)/\hbar = \frac{e}{\hbar c}\int \vec{A}\cdot d\vec{x} = \frac{e}{\hbar c}\Phi_0.$$

In the previous expression we relied on the fact that the vector potential cannot be zero anywhere outside the solenoid, because the total flux through every circuit containing the origin is equal to a constant

$$\Phi_0 = \int \vec{H} \cdot d\vec{s} = \int \vec{A} \cdot d\vec{x}.$$

This effect will also exist even if there is no magnetic force acting on the electron. In order to protect the electron from any contact with the magnetic field, it is possible to shield the solenoid with the potential barrier and to get the same result. Aharonov and Bohm proposed an experiment with varying magnetic flux, instead of watching the interference pattern with and without magnetic flux.

3 Early experiments

Chambers gave the first experimental confirmation of the fringe shift as predicted by theory. In his experiment, the magnetic flux was supplied by a magnetized iron whisker. The beam was separated by a bi-prism, and two parts of the beam passed on the two sides of the whisker without contact.

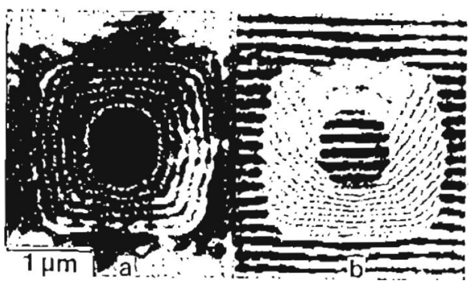
Next confirmation of fringe shift came from Marton, who made an experiment similar to that of Chambers. Finally, Boersch confirmed that vector potentials have a direct effect on the fringes, when he studied the interference pattern of fast electrons passing through thin ferromagnetic layer.

Nevertheless, Aharonov and Bohm, in their second work from 1961. [4], claimed that none of these experiments was in an ideal agreement with theory, because in each case the vector potential was mixed up with the magnetic field, this means that the potential only makes an influence on the total fringe shift effect.

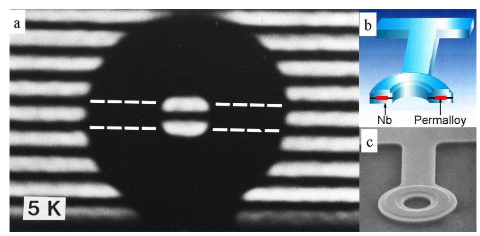
The next great experiment had been carried out by Möllenstedt and Bayh. They used a similar set-up like Chambers, but in contrary, they used a thin solenoid to generate the magnetic flux. They added a frame of iron and nickel to solenoid of wolfram, in order to short-circuit the magnetic field at the ends of the solenoid. They got the visible fringe shifts, due to the vector potential, and they also confirmed the modulus of the flux quantum.

Experiments which definitely showed what Aharonov and Bohm have proposed were made by a Hitachi group of scientists, led by Akira Tonomura.

The first experiment, carried out in 1982., used an optical- and electron-holography. This technique consists of two parts, an electron microscope and an optical system, which transforms the electron waves into light waves. The specimen is realized by a toroidal shaped magnet, which has the advantage of a reduction of leakage fields. The flux is approximately confined to the magnet. This experiment demonstrated that there exists a measurable effect due to the vector potential, because the electron waves that passed on different sides of the magnet, gained a visible phase difference. It also showed that the leakage fields were too small to affect the AB-phase.



(a) First experiment



(b) Second experiment

Figure 3: Hitachi experiments

In their second experiment which was carried out in 1986., scientists led by Tonomura covered toroidal magnet by superconducting material niobium. Due to circulating currents in layers near the surface, the magnetic field cannot fully penetrate the superconducting materials, if they are cooled under their critical temperature. The consequences are that no leakage fields can influence the electron waves and the magnet is completely shielded by the niobium, so the electrons cannot enter the regime of the torus.

This group of scientists have observed the phase shift one more time. Furthermore, they stated that the phase shift is a multiple of π, which is an indication for flux quantization proportional to $\frac{hc}{2e}$.

4 Recent experiments

4.1 Experiment from 2007.

In his work published in 2002 [5], T. H. Boyer stated: "Classical electromagnetic forces can account for the experimentally observed phase shifts seen in an electron interference pattern when a line of electric dipoles or a line of magnetic dipoles (a solenoid) is placed between the electron beams forming the interference pattern". In the case of electric dipoles he relied on Mateucci-Pozzi experiment from 1985, and drew the parallel between them and magnetic dipoles. Following their experiment, Boyer proposed inserting a line of magnetic dipoles between the beams of electrons. Here we place a long, thin solenoid of cross-sectional area A and interior magnetic field B_0 so that its axis of symmetry is along the z-axis of coordinates. The azimuthal surface currents per unit length are given by $\vec{K} = \hat{\phi}\frac{B_0 c}{4\pi}$ and the magnetic dipole moment per unit length is $\hat{k}\mu = \hat{k}\frac{KA}{c} = \hat{k}\frac{B_0 A}{4\pi}$. A charged particle moving perpendicular to the solenoid, in xy-plane, will cause a magnetic field and put a net force on the solenoid. He derived y-component of that force as

$$F_{\mu y} = \frac{e\mu v_0}{c}\frac{4xy}{(x^2+y^2)^2}. \tag{6}$$

The same form was used for the electrostatic case. With the assumption that the net force between charges and the solenoid satisfies Newton's third law, there is a force on the passing electron, $F_e = F_\mu$. There is a relative lag effect for charges passing on opposite sides of the solenoid and hence the relative change in velocity

$$\Delta v_y^{(+)}(t) = \frac{1}{m}\int_{-\infty}^{t'=t} F_{\mu y}(t')dt' = \frac{e\mu}{mc}\frac{2xy}{(x^2+y^2)^2}, \tag{7}$$

the relative displacements

$$\Delta y^{(+)} = \int_{-\infty}^{\infty} \Delta v_y^{(+)}(t)dt = \frac{2\pi e\mu}{m v_0 c}. \tag{8}$$

It follows that the relative displacement between charges passing on opposite sides of the line of dipoles is

$$\Delta Y = \Delta y^{(+)} - \Delta y^{(-)} = \frac{4\pi e\mu}{m v_0 c}. \tag{9}$$

The semi-classical phase shift due to lines of magnetic dipoles is

$$\Delta\phi = \frac{p_y \Delta Y}{\hbar} = \frac{mv_0}{\hbar}\frac{4\pi e\mu}{mv_0 c} = \frac{4\pi e\mu}{\hbar c} = \frac{eB_0 A}{\hbar c}, \tag{10}$$

and that is the exact phase shift proposed by Aharonov and Bohm. This semi-classical explanation has caused considerable controversy, in the first place due to the disagreements with the interpretation which Aharonov and Bohm gave when they suggested the existence of the magnetic phase shift.

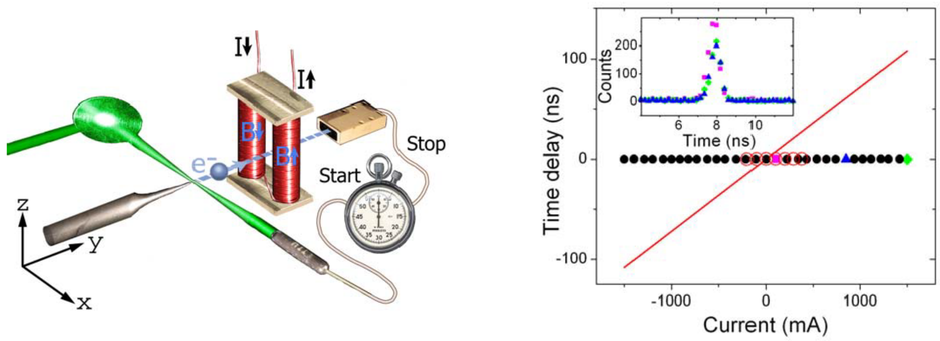
Led by the interpretation that the absence of forces is what makes the AB-effect purely quantum mechanical, and by the fact that the absence of forces has been never shown in type-1 AB-effect (actually the electric effect has escaped detection altogether), a group of scientists in Nebraska carried out an experiment where they searched for time delays associated with the force which represents semi-classical explanation of the AB phase shift for a macroscopic system [6]. Semi-classical theory says that the electron wave packet shifts. Quantum point of view is that the wave packet is only multiplied by the AB-phase factor and not shifted. So they wanted to carry out the experiment which will demonstrate both, the phase shift and the absence of forces simultaneously. A non-zero displacement satisfying the equation (10) introduces a time delay

$$\Delta t = \frac{\Delta y}{v_0} = \frac{eB_0 A}{mv_0^2}. \tag{11}$$

In order to establish the absence or presence of a force it is possible to make a measurement of this time delay. We already saw that Boyer approximated an infinite line of magnetic dipoles by a solenoid. In this experiment, the solenoid has a high permeability iron core. This core can be modelled with magnetic dipoles aligned by the solenoid field. The Nebraska group emphasized that a delay time measurement can rule out all semi-classical force theories. They also pointed out that hidden momentum compensate any force on solenoid so that neither the solenoid nor the passing electron experience any force.

They performed a time-of-flight experiment for a macroscopic solenoid (figure 4). The primary result is that as a function of the current through the solenoids, no time delay is observed, thus signalling the absence of forces. They also observed the effect of the image charge on an electron diffraction pattern for a 100 nm gold coated grating and found time delays associated with the electron - image charges interaction negligible.

Conclusion of this experiment is that no force acts on an electron passing by a macroscopic solenoid of enough magnitude to explain the AB-effect.



(a) Experimental setup

(b) Results

Figure 4: TOF experiment.

4.2 Experiment from 2011.

Subsequently Boyer stated that the observed absence of a classical lag effect for a macroscopic solenoid does not rule out the possibility of semi-classical explanation of the phase shift for a microscopic solenoid [7]. He pointed out that when the resistive energy loss of a solenoid is small, then all solenoids behave in the same way regarding energy conservation for a passing charged particle. When the resistive energy loss is large, then the energy-conserving interaction becomes negligible and the interaction of the particle and solenoid becomes quite different. The magnetic energy of interaction of a charged particle q passing a solenoid with constant currents is given by

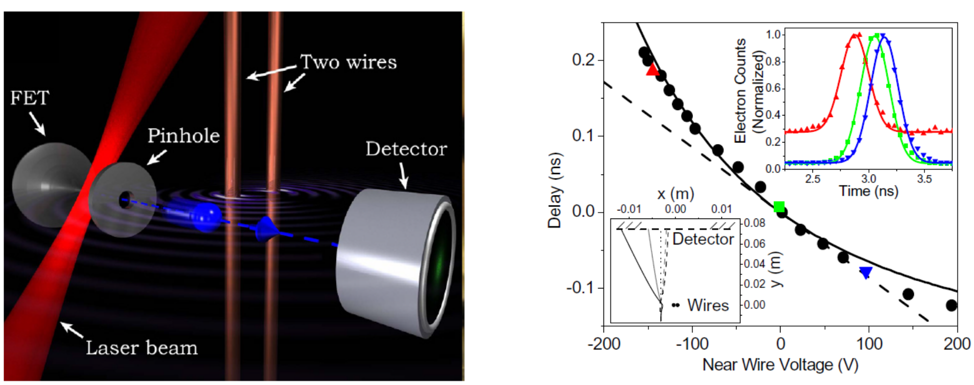
$$U = \frac{q}{c}\vec{v}_q \cdot \vec{A}\left(\vec{r}_q\right). \tag{12}$$

If this magnetic interaction energy is compensated by a change in the kinetic energy of the passing charge $mv_q\Delta v_q = -U$, then there is relative spatial lag and associated semi-classical phase shift given by the equation (10). As already mentioned, crucial test for ruling out the classical lag effect would be to observe the AB phase shift and not the time delay. For in the regimes where the Aharonov-Bohm phase shift has been observed, the time delays would be extraordinarily small.

A group of scientists in 2011. carried out an experiment in which they confirmed a classical time delay for electrons moving in a scalar potential, as predicted by Boyer. Their experiment was very simple and classically motivated [8]. They determined the flight time of an electron passing by two parallel oppositely charged wires, which are mounted in the vacuum chamber and separated for about 1 mm. They held wires at positive and negative voltages of the same magnitude. Electrons, which were emitted via laser induction from the field emission tip, traveled through a grounded pinhole into a region with a scalar potential associated with the two charged wires. They used magnetic shielding to reduce external magnetic fields. When they pass the wires, electrons hit the multi-channel plate electron detector. The laser system provides a start time, while the MCP provides a stop time. This timing mechanism has been used with a drift tube to produce an energy analyser, measure the electron width of the electron pulses emitted from this source and determine the lack of classical forces for electrons passing the solenoid. They observed temporal spectra for different voltages. To estimate the arrival time as predicted by Boyer, the y-coordinate of the force, parallel to the electron velocity, is needed. This component of the force exerted by the electron on the line charges according to Boyer is

$$F_{ey} = -e\mathscr{P}\frac{4xy}{4\pi\epsilon_0\left(x^2+y^2\right)^2}. \tag{13}$$

Time delay that agrees with Boyers prediction had been observed.



(a) Experimental setup

(b) Results

Figure 5: Experiment on time delay.

This shows that the MP-experiment does not demonstrate a true type-2 AB-effect. Type-2 effects are defined as those that arise through local interactions with fields, yet also end up with wave packets accumulating a non-local phase shift, identical in measurable consequences to type-1 AB-effects.

It is interesting to compare this to the magnetic AB-effect. The same experimental and theoretical approaches applied to an electron passing by a solenoid result in a phase shift that is identical to that predicted by Aharonov and Bohm. It has been shown experimentally that no classical time delays are present for electrons passing a solenoid. Furthermore, the magnetic AB-phase shift occurs in experimental configurations where fields are shielded or no evidence of classical forces is shown. Thus, it is generally considered purely quantum mechanical in nature.

5 Conclusion

In 2015, a paper on the interpretation of all mentioned experiments has been published [9]. That paper discussed some loopholes in the interpretation. One is the possibility that magnetized iron cores do not provide a classical back-action reducing the predicted time-delay. The other is the possibility that dispersionless forces exist, because the experiment on the dispersionless nature of the AB effect has never been carried out. The experiments which have been observing time delays were carried out without an iron core. This paper proposed an experiment with a non-tapered magnetized iron whisker, in order to conclude the time-delay experiments.

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