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Precession of planets and generation of their magnetic fields (a review)

Sh. Sh. Dolginov

St. Petersburg Institute of Terrestrial Magnetism, Ionosphere, and Radio Wave Propagation, Russian Academy of Sciences

Abstract. Comparison of intensities of the dipole magnetic fields of the Earth and all the surveyed planets and satellites of the solar system leads one to the conclusion that the planetary dynamo motive force is the external gravitational forces of the Sun and the nearest satellites which causes the precession of planets and tidal phenomena in their magnetically active regions. The rate of variation of the Earth's external magnetic energy calculated from the dynamic parameters of a model of the precession dynamo and the dissipative parameters of the Earth's core (the kinematic ν and magnetic η_m viscosities) amounts to $dE_m/dt = 6 \times 10^{17}$ erg s^{-1}. A comparison of this value with that observed in the current epoch shows that the precession dynamo mechanism is adequate with respect to the energy required for the geodynamo generation, if its efficiency is only 6–8%. The allowance for the Earth's core viscosity $\nu \approx 10^5$ cm^2 s^{-1} [*Hide*, 1971] rules out the inference that precession has no implication to the geomagnetic field generation [*Loper,* 1975] and is in line with the conclusions of the present paper and that of *Stacey* [1973]. In the precession dynamo mechanism, the conditions are realized under which the action of external forces is minimized (Le Chatelier principle). Synchronously rotating planetary satellites cannot have dynamo fields.

1. Introduction

Among the debatable problems of dynamo theory (which deals with the origin of planetary magnetic fields) is the identification of sources of energy of the fields and the nature of the "engines" of the planetary "dynamo machines." Three possible Geophysical mechanisms of generating convective currents in the Earth's core have been presented . They are discussed below.

The first mechanism involves the continued chemical differentiation of the outer liquid core, in which the heavy melt component (iron) replenishes the solid core, and light components (silicon, sulfur) remain in the liquid phase and "float up" under the action of the buoyancy force [*Braginskiy,* 1964; *Gubbins*, 1977; *Gubbins and Masters*, 1979; *Loper*, 1978; *Loper and Roberts*, 1983].

The second possible mechanism invokes the continued crystallization of the liquid core, in which the internal heat of melting is released creating in the core a temperature gradient exceeding the adiabatic value. No radioactive heat sources in the core are presumed [*Verhoogen*, 1973]. For the Earth, the mechanism of thermal convection has been ruled out on the grounds that thermal machines have a low efficiency. Besides, it is not possible to accurately distinguish the thermal flux from the core to the mantle from the background heat flow caused by the radioactive elements of the Earth's crust.

The third mechanism involves the precession of the Earth under the action of the external gravitational fields of the Sun and the Moon. The precession-related secondary acceleration $\dot{\omega} = [\omega \times \Omega] = 2.25 \times 10^{-16}$ s^{-2}. The force experienced by a unit volume mass ρ at a distance r from the axis of rotation $F_{\mathrm{P}} = \rho[\omega \times \Omega] \times \mathbf{r}$ is called the Poincare force (ω and Ω are Earth's speeds of rotation and precession, respectively) [*Malkus*, 1968]. The speed of precession Ω is proportional to the dynamic compression $f = (C - A)/C$, where C and A are the centroidal and equatorial moments of inertia. For the Earth, $f = 3.27 \times 10^{-3}$. For the greater density of material and smaller dimensions of the Earth's core, the dynamic compression of the core $f_c = 3/4f$. The core's precession thus should proceed at a lower speed. Because of the difference in acceleration between the core and mantle $\Delta\Omega = 0.6 \times 10^{-16}$ s^{-2}, the speed of the core relative to the mantle could reach 156 m s^{-1} over the precession period $T = 25,800$ years. No such high speeds are found by geophysical methods on the

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Earth's surface. Therefore mechanisms of "coupling" between the mantle and the core must exist which reduce the magnitude of the difference between the angular momenta of rotation of the core $L_c = I_c\omega_c$ and the mantle $L_m = I_m\omega_m$, both in magnitude ($\omega_c \neq \omega_m$) and direction (an angle ε exists between L_c and L_m).

Different estimations of the coupling mechanisms in the rotating and precessing mantle-core system have resulted in different notions of the character of flows in the liquid core and in opposite estimates of the effectiveness of the precession in generating the geomagnetic field. Disagreement has developed in answering the two principle questions, can the precession mechanism provide the energy needed to generate and maintain the geomagnetic field, and can the external gravitational forces stimulate the motions of liquid required by kinematic models of the geodynamo?

Progress in solving these problems has been made in the field of comparative planetology. In this paper we describe the results of investigations that have been carried out. Before proceeding to the body of the paper, we give a short overview of the main stages of studying these geophysical problems since the results of these studies bring us closer to an understanding of the precession dynamo mechanism.

2. Can the Earth's Precession Be the Engine of Its Magnetic Dynamo?

Attempts to answer this question date back to 1910 when Poincare studied the flow of a nonviscous incompressible liquid confined in a rotating spheroidal container undergoing low-amplitude precession. The resulting analysis concluded that the character of the flow was determined by the pressure on the rigid spheroidal smooth container shell which brings about a steady laminar flow indistinguishable from "solid state rotation" [see *Mel'khior*, 1976].

This type of coupling in the mantle-core system was later called by *Toomre* [1966] "inertial" in which no energy dissipation takes place in the core. In this case, precession cannot be a means for deriving the energy required for the geodynamo from the kinetic energy stored by a rotating Earth. These features of currents in the core were used by *Bullard* [1949] as a main argument to rule out the participation of precession in the geodynamo, who at the same time believed that among all irregularities of the Earth's rotation it is precession which may have influence.

Elsasser [1950] investigated the possible implication of external forces in the dynamo process and pointed out the importance of nonlinear effects leading to a constant component in the diurnal periodicity of tidal phenomena induced by low-latitude horizontal currents under the action of radial shifts. He also indicated the possible influence of the unevenness of the lower boundary of the mantle and resonances in the mantle-core system. Presuming all these effects insufficient not only for field generation but also for the explanation of the secular variation, *Elsasser* [1950] dismissed precession as an engine for the dynamo. Nevertheless, his analysis has provided a list of effects which are important for understanding the precession mechanism of the dynamo process.

The possibility of motion of material in the meridional planes due to radial motions at high latitudes was suggested by *Pariyskiy* [1963]. *Braginskiy* [1964] considered the diurnal variability of the Poincare force to be the main limitation of precession in the dynamo process and pointed out the importance of allowing for nonlinear effects. The possible influence of resonance effects on the regime of currents in the core mentioned by *Elsasser* [1950] was discussed in more detail by *Hinderer et al.* [1987] who, however, refrained from making quantitative estimates.

The Poincare problem was studied with allowance for considerable precession amplitudes ($\alpha \sim 23.5°$), by *Bondi and Littleton* [1953]. In the second phase of their investigation, they considered the influence of a thin boundary layer with a viscosity θ. The first phase had disclosed centers at latitudes of 30°S and 30°N, with which the investigators associated the onset of turbulence propagating in the liquid. The investigators related these features to observed irregularities in the Earth's rotation speed and to peculiarities of the secular variations of the geomagnetic field.

Stewartson and Roberts [1963] came to different results in analyzing the Poincare problem. They considered the motion of a viscous liquid confined in a spheroidal container with semiaxes a and b ($a > b$) rotating at a speed ω and undergoing precession at speed Ω and amplitude α ($\omega \gg \Omega$). The motion is controlled by the three dimensionless parameters: $R_1 = 2\xi\omega/\Omega$, $R_2 = \Omega a^2/\theta$, and $R_3 = \omega a^2/\theta$; where $\xi = (a^2 - b^2)a^2$ is the compression of the container. The analysis showed that the liquid within time $\tau = a^2/\theta$ begins to precess with the container shell if $\Omega a^2/\theta \gg 1$ and $\Omega a^2 \ll 1$. These conditions seem to be met in the Earth's core at the presumed limiting values of viscosity $\theta = 10^{-3}$ cm^2 s^{-1} and 10^{11} cm^2 s^{-1}. This is "solid state" rotation.

Superimposed on this primary solid state motion is a secondary motion with speed $v = \Omega \sin\alpha\, r/2\xi$. For the Earth's core parameters, these speeds provide for Reynolds numbers great enough for the dynamo process. However, the liquid streamlines are stable, ellipsoidal, and strictly symmetrical and therefore do not fit the properties of the generating speeds of kinematic models of the dynamo process.

The solution of the problem indicates the existence of a boundary layer with a laminar liquid flow and a substantial viscosity (the Eckman layer). The Eckman number $E = \theta/a^2\omega$ and at $\theta = 10^{-2}$ cm^2 s^{-1} is $E = 10^{-15}$.

Toomre [1966] was led to the exclusion of any implication of precession of the Earth in the generation of its magnetic field on the basis of an analysis of different mechanism of coupling in the mantle-core system. He compared the total mechanical torque experienced by the core under the action of external forces, $G_c = 3/4[L_c \times \Omega] = 3/4 I_c\omega\Omega \sin 23.5° = 2.7\times 10^{27}$ dyn cm, with the coupling torque of the mantle-core system through a viscous friction mechanism, $G_\nu = 8\times 10^{24}$ dyn cm (assuming a laminar liquid flow and $\nu = 10^{-2}$ cm^2 s^{-1}), with the electromagnetic coupling torque $G_m = 10^{23}$ dyn cm (a *MacDonald and Ness* [1961] induction model) to infer that given all the assumptions and estimates made, none of these dissipative mechanisms is capable of providing the required coupling torque in the mantle-core system. Yet, this is attained by the inertial mechanism first suggested by Poincare. As is mentioned above, with an inertial coupling mechanism, the energy necessary for magnetic field generation cannot be derived.

Advances in the study of the nature of liquid currents likely to arise in the Earth's core because of precession have been made based on laboratory experiments with liquids confined in rotating and precessing spheroidal and cylindrical containers (*Malkus* [1968] and his colleagues *Suess* [1970] and *Gans* [1970]). An original technique made it possible to visualize the motion of liquid and detect changes in the character of the liquid flow. These investigations showed that the flow of liquid in a precessing container differs markedly from the laminar flows considered by Poincare. The Eckman boundary layer differed from that considered by *Stewartson and Roberts* [1963] in nonlinear effects. The following main features of turbulent liquid flow in a precessing container have been discovered.

1. The flow is opposite to the container rotation (westward drift) with streams of forward and backward flows on both sides at a distance of $0.86r$, where r is the container radius. These features are explained by *Busse* [1968] with an allowance for the nonlinear effects.

2. The liquid westward drift velocity is proportional to the cosine of the angle of inclination of the container rotation axis.

3. The westward drift is accompanied by wave motion to the east which is at a higher speed than that of the drift.

4. The occurrence of turbulent cylindrical tension zones is localized in the latitude range $\pm 30°$ relative to the liquid equator.

5. The existence of an axial tension zone (a linear vortex) is oriented along the liquid vorticity vector.

These experiments have shown that in a rotating and precessing container, liquid rotates, on the whole, as a solid at an angular momentum somewhat deviating from the momentum of a spheroidal container. Thus in a rotating and precessing spheroidal container filled with a liquid, turbulence and energy dissipation develop.

Malkus [1968] estimated the liquid velocity, the field intensity in the core, the poloidal field component, and the precession dynamo power proceeding from the assumption of equality of the Coriolis, Lorentz, and Poincare forces in the core. The values of these quantities are in reasonable agreement (given the stated uncertainties) with those assumed in the dynamo models. The limitations of the "numerical law" of planetary magnetism represented by the formulae reported by *Malkus* [1968] became evident as soon as the first attempt was made of using these for determining the fields and other parameters of other planets [*Stevenson*, 1974]. The assumption of the balance of the Coriolis and Poincare forces in the core leaves room for doubt, since the Poincare force varies with a diurnal periodicity [*Rochester et al.,* 1975].

We note that all the numerical laws of the planetary mechanism based on the assumption of the balance of the Coriolis and Lorentz forces in the core fail to describe the magnetic fields of other planets [*Dolginov*, 1989].

The experiments of *Malkus* [1968] and the results of *Toomre* [1966] on the significant role of the inertial force in the mantle-core coupling mechanism led *Stacey* [1973] to consider the effectiveness of precession in geomagnetic field generation under the combined action of inertial and dissipative forces. *Stacey* [1973] ignored the viscous friction mechanism (at $\nu = 10^{-2}$ cm^2 s^{-1}) and adopted electromagnetic coupling as a dissipative mechanism in the mantle-core system; he managed without deciding upon a specific mechanism of induction in the mantle and a law of variation of the conductivity therein. All he did was assume that the coupling coefficient K_R determined from observations of fluctuations and changes in the Earth's rotational speed (spinup) coincides in magnitude with the coupling coefficient determining the mismatch of the angular momenta of the core L_c and the mantle L_m by an angle ε_d (spinover).

The relaxation time of fluctuations in day length is determined from experimental data $\tau_R = 1.6$ years [*Stacey*, 1972]. Based on these data, *Stacey* [1972] reports a solution of the problem of energy dissipation in the mantle-core system that is attractive in its simplicity. With the external gravitational forces switched off, the continued interaction in the mantle-core system would have led to the coincidence in direction of the angular momenta L_c and L_m over the period of precession $T = 25,800$ years at a speed

$$\frac{d\varepsilon_d}{dt} = \frac{0.25\,2\pi\sin\alpha}{T}$$

with $\alpha = 23.5^\circ$ and $d\varepsilon_d/dt = 7.7\times10^{-13}$ rad s^{-1}. The kinetic energy of the mantle-core system at an angle ε_d between L_c and L_m is

$$E_k = \frac{1}{2}\frac{I_c I_m}{I_c + I_m}\omega_m^2\varepsilon_d^2$$

The momenta of inertia $I_m = 7.92\times10^{44}$ g cm^2 and $I_c = 1.75\times10^{43}$ g cm^2. Then,

$$\frac{dE_k}{dt} = \frac{I_c I_m}{I_c + I_m}\omega_m^2\varepsilon_d^2\frac{d\varepsilon_d}{dt} = 6.96\times10^{22}\varepsilon_d \text{ erg s}^{-1}$$

$$\varepsilon_d\frac{\varepsilon_d}{dt}\tau_R; \quad \tau_R = 1.6 \text{ year} \sim 5.04\times10^7\text{s}$$

$$\varepsilon_d = 3.0\times10^{-5} \text{ rad}; \quad \frac{dE_k}{dt} = 2.4\times10^{18} \text{ erg s}^{-1}$$

The estimate of ε_d from $\tau_R = 1.6$ years has been challenged by *Rochester et al.* [1975]. According to a model of electromagnetic coupling reported by *Rochester et al.* [1975], induction of the coupling currents in the mantle occurs owing to the diurnal rotation of the mantle relative to the "frozen in" field of the core. The coupling coefficient and therefore the rate of energy dissipation will be about 580 times lower. According to this model, with a laminar liquid flow in the mantle-core boundary region, the energy dissipation rate for the precession mechanism can not exceed $\simeq 10^{15}$ erg s^{-1}, which is insufficient for geomagnetic field generation.

In a model of the combined action of the conservative inertial force and the dissipative electromagnetic force of coupling reported by *Stacey* [1973], the latter force tends to bring the angular momenta L_c and L_m into coincidence, whereas the former tends to prevent this by causing the core to precess about the mantle. These forces are mutually orthogonal. A lag appears between L_m and L_c determined by the angle Φ ($\Phi = 0.158$ rad). The action of the inertial force leads to a decrease of the dissipation angle ε_d to $\varepsilon = 6.06\times10^{-6}$ rad and the occurrence of a dependence of the dissipation rate on $\sin\Phi$. At $\Phi = 0.158$ rad, $dE_k/dt \simeq 5.8\times10^{16}$ erg s^{-1}. Such a rate of energy dissipation in the core is adequate to that required for geomagnetic field generation.

Loper [1975] came to the opposite conclusion. In a model of the core with a laminar liquid flow the following three boundary regions turned out to be active: (1) the Eckman layer characterized by the number $E = \nu/\omega R_c^2$; (2) a diffusion layer F in the liquid core, characterized by a dimensionless number $F = 1/(\omega\sigma_c R_c^2)$, where σ_c is the core conductivity. (3) A diffusion layer in the mantle $f_m = 1/(\omega\sigma_m R_c^2)$, where σ_m is the mantle conductivity. The square roots $E^{1/2}$, $F^{1/2}$, and $f_m^{1/2}$ determine the ratio of the thickness of these layers to the characteristic dimensions of the core R_c. At $\nu = 4\times10^{-2}$ cm^2 s^{-1}, $R_c = 3.48\times10^8$ cm, $\omega = 7.27\times10^{-5}$ s^{-1}, $E = 5\times10^{-15}$, and $\delta_E = 3$ m. Likewise, at $\sigma_m = 2\times10^3\,\Omega^{-1}$ m^{-1}, $\sigma_c/\sigma_m = 10^2$, $F = 3\times10^{-9}$, $\delta_F = 2$ km, $f_m = 5\times10^{-7}$, and $\delta_{f_m} = 24$ km.

The coupling in the mantle-core system, the angles between L_c and L_m, and the energy dissipation are all functions of the parameters E, F, and f_m. *Loper's* [1975] model differs from *Stacey's* [1973] in that while in the former all the dissipative processes occur in relatively small boundary belts, in the latter, however, the entire core can be assumed to be a magnetoactive medium.

We restrict ourselves to the final formulae of this detailed analysis.

1. The momentum of inertial force (liquid pressure) is

$$G_p = \frac{8\pi}{15}i\rho\omega^2 R_c^5\varepsilon f$$

where f is the Earth's flattening ($f = 3.27\times10^{-3}$), and ε is the angle between L_c and L_m.

2. $\varepsilon = (1 - f_c/f)\,\Omega\sin\alpha/\omega f$, where the core's flattening $f_c = 3f/4$. At $\varepsilon = 4.3\times10^{-6}$ rad, $G_p = i\,1.36\times 10^{27}$ dyn cm.

3. γ is the angle between the plane $\Omega - \omega_m$ and the plane $\omega_m - \omega_c$ (the "lag" angle)

$$\gamma = \frac{2^{3/2}F f_m^{-1/2}a^2 + 2.62E^{1/2}}{f} \qquad (1)$$

where a is the field intensity in the mantle in a dimensionless form. As can be seen from the latter formula, the value of angle γ is determined by the action of two dissipative forces; the angle itself, however, arises under the combined action of the dissipative and inertial forces. In this sense, the angle γ is an analog of the angle φ in *Stacey's* [1973] model and amounts to 5×10^{-4} rad. In *Stacey's* [1973] model, $\varphi = 0.158$ rad. The ratio $\gamma/\varphi \sim 10^{-3}$; that is why *Loper* [1975], as well as *Rochester et al.* [1975], think that the energy dissipation rate in *Stacey's* [1973] model is overestimated about 300 times.

4. The viscous friction torque is

$$G_\nu = -\frac{8\pi}{15}(2.62 + i\,0.259)\rho\omega^2 R_c^5 E^{1/2}\varepsilon$$

For $\nu = 4\times10^{-2}$ cm s^{-1}, $E = 5\times10^{-15}$, and $G_\nu = 3.8\times10^{23}$ dyn cm.

5. The electromagnetic coupling torque G_m is

$$G_m = -\frac{8\pi}{15}(2^{3/2} + i2^{3/2})\rho\omega^2 R_c^5 E f_m^{-1/2}\varepsilon$$

For $a = 3.5$, and $G_m = 2.3\times10^{24}$ dyn cm.

6. The energy dissipation rate is

$$\frac{dE_k}{dt} = \frac{8\pi}{15}(2^{3/2}Ff_m^{-1/2}a^2 + 2.62E^{1/2})\rho\omega^3 R_c^5\varepsilon^2 \quad (2)$$

For the parameter values adopted above, it amounts to 6.27×10^{14} erg s^{-1}. *Loper* [1975] concluded that this energy, too, dissipates in a thin boundary layer and does not serve as a sources of energy for the geomagnetic field, and thus precession has nothing to do with the dynamo mechanism. *Loper's* [1975] conclusion contradicts the experimental data available. We will show in section 7 that the precession dynamo model satisfies the set of data of dipole fields of all the planets and the satellites surveyed.

One might ask the question; How could such a thorough analysis of the problem of effectiveness of precession in geomagnetic field generation conducted by *Loper* [1975] yield a negative result? We are inclined to attribute this to the neglect of the well-known at that time important determinations of viscosity of the core performed by *Hide* [1969, 1970, 1971]. An analysis of geomagnetic field disturbances caused by the unevenness of the boundary of the lower mantle in a rotating magnetized liquid led *Hide* [1971] to the conclusion that the combined kinematic and turbulent viscosity of the core $\nu \leq 10^6$ cm^2 s^{-1}.

Loper [1975] adopted the value $\nu = 4 \times 10^{-2}$ cm^2 s^{-1}. Mechanical torques of the magnetic and viscous coupling in the mantle-core system: $G_m/G_\nu \sim 7$. The ratio of the magnetic and viscous components determining the lag angle γ is 4 : 1. Roughly the same ratio of these components is in formula (2) determining the energy dissipation rate.

No dynamo process would be possible without a considerable role for the viscous coupling mechanism in the mantle-core system for, in the initial dynamo process, the magnetic fields can be so small in intensity that only a low-efficiency magnetic coupling would result. If one takes $\nu = 4 \times 10^5$ cm^2 s^{-1}, then formula (1) leads to the value $\gamma = 0.173$ rad, and the energy dissipation rate, according to (2), $dE_k/dt = 3.2 \times 10^{17}$ erg s^{-1}. In this case, these parameters are in agreement with those reported by *Stacey* [1973].

Note that at $\nu = 10^5$ cm^2 s^{-1} and $v = 0.08$ cm s^{-1}, the kinetic Reynolds number $R_k = Lv/\nu = 280$. In a nonmagnetized liquid, at a similar value of R_k, the laminar flow becomes unsteady and predisposed to turbulence [*Leorat et al.,* 1983]. Thus the assumption of a significant, and more likely dominating, role of the viscous friction mechanism in a precessing mantle-core system reconciles the solutions by *Stacey* [1973] and *Loper* [1975] and the experimental results of *Malkus* [1968].

It follows from the foregoing that the lengthy discussion of the effectiveness of precession in geomagnetic field generation, within the scope of geophysics, while yielding no final solution, still provided a better understanding of mechanisms of its possible effectiveness. For the criteria of the effectiveness, parameters were used that could not be tested directly. Therefore both positive and negative answers to the question of the possible role of precession in geomagnetic field generation could not be considered plausible enough.

The situation changed after the discovery of the low-intensity intrinsic magnetic field of Mars, the somewhat more intense field of the smaller and slowly rotating Mercury, and practically no field at an Earth-like Venus. These experimental facts could not be reconciled with the model of convection under the action of the buoyancy force without special assumptions [*Braginskiy,* 1964] and, moreover, without a radical change in calculations of the thermal evolution of the terrestrial planets [*Stevenson,* 1983]. By contrast, these experimental data made it possible to begin the study of a model of a precession dynamo within the scope of comparative planetology. Each planet has added important evidence to this problem. In the beginning of tests of the possible role of precession in planetary magnetic field generation was a comparison of the magnetic fields of the Earth and Mars, two planets having nearly similar rotational angular velocities ω and precession amplitudes α (Sh. Sh. Dolginov, unpublished manuscript, 1973).

The magnetic field generation at a small Mercury was associated with thermal convection under the action of heat which was assumed to be released in the continual cooling down of the planet and the crystallization of its liquid core [*Gubbins,* 1977]. This publication, however, contained a warning: a "stumbling block" for the hypothesis of the thermal origin of the magnetic field of Mercury could result from the measurement of the level of heat flux from the planet's surface. Scientific reviewer *Eberhart* [1990] communicated the report by J. A. Burns and M. J. Ledlow at a meeting of the American Astronomical Union stating the absence of evidence of an outflow of intrinsic heat from the surface of Mercury as established from an analysis of the planet's radio emission charts. The absence of heat outflow from the interior of Mercury and the presence of a magnetic field are congruous in a dynamo process caused by external gravitational forces, under the action of which convection is possible at a subadiabatic temperature gradient in the core.

Unlike the Moon which is in synchronous rotation with the Earth and which experiences no precession under the action of the Earth's gravitational field, Mercury rotates in the regime of resonance 3/2 and precesses at a low amplitude α but at a considerable speed. Hence the precession dynamo can be the mechanism of its magnetic field. The radial tensions arising in the high and low latitudes when Mercury is orbiting the Sun with a

Table 1. Precession dynamo model parameters for terrestrial planets and the moon

Parameter	Earth	Mercury, 1	Mercury, 2	Venus	Mars	Moon
R_P	6371	2440	2440	6051	3390	1738
R_c	3480	1600	1800	2840	1290	360
Q	1831	1.525	1.353	2.130	2.620	4.828
$\omega \times 10^{-5}$	7.27	0.123	0.123	0.0299	7.30	0.269
f	3.27×10^{-3}	2.13×10^{-4}	1.34×10^{-4}	$(6+2) \times 10^{-6}$	5.24×10^{-3}	3.1×10^{-4}
Ω angular, s per yr	50.2	1149	722.7	20	7.4	14.8
α, degree	23.5	1	1	2.6	25.2	1.5
ρ, g cm^{-3}	11	8.5	8.0	9.6	(85)	6
σ, Ohm^{-1} m^{-1}	σ_E	$2\sigma_E$	$2\sigma_E$	σ_E	$0.54\sigma_E$	σ_E
$H_{\rm var}$, nT	30.900	165	165	1	35	0.2
$H_{\rm deduc}$, nT		165	165	0.2	35	0.2
F_p, dyn cm^{-3}	8.61×10^{-7}	5.2×10^{-9}	3.46×10^{-9}	6.63×10^{-11}	3.89×10^{-8}	3.45×10^{-11}

great eccentricity [*Burns*, 1976] must stimulate a current of turbulent nature in the liquid core along longitudinal and latitudinal circles, thereby promoting the dynamo process. The possible implication of Mercury's precession in the generation of the planet's magnetic field was tested in 1976 by predicting the amount of dynamic flattening f of Mercury from the dipole field magnitude $H = 170$ nT, at a precession amplitude $\alpha = 3°$ [*Dolginov,* 1976]. The value $f \geq 10^{-4}$ turned out to be in reasonable agreement with the value $f = (1.2 \pm 0.9) \times 10^{-4}$ determined from the value of harmonic J_2 of the gravitational field of Mercury reported by *Esposito et al.* [1977] to be $J_2 = (8 \pm 6) \times 10^{-5}$ (at a period of rotation $T = 58$ days and $f = 3/2J_2$).

In the present paper, we show that with allowance for the known parameters of the solar planets and their satellite systems, the magnetic conditions of planets can be described by a model of the dynamo wherein the motive force in the mechanism of field generation is the external gravitational forces causing precession of the planets and tidal phenomena in their interiors.

3. A Model of the Precession Dynamo

The Earth, rotating at a frequency ω, precesses with an amplitude $\alpha = 23.5°$ with an angular velocity $\Omega = 8\times 10^{12}$ rad s^{-1}. This results in a secondary acceleration $\dot\omega = [\omega \times \Omega = 2.25 \times 10^{-16}$ s^{-2}. The force experienced by a unit mass ρ at a distance r from the axis of rotation is $F_P = \rho[\omega \times \Omega] \times r$.

From the main equation of magnetohydrodynamics

$$\frac{\partial B}{\partial t} = \nabla^2 B \eta_m + \ \mathrm{rot}[\mathbf{v} \times \mathbf{B}]$$

where η_m is the magnetic viscosity, and $\mathbf{v}$ is the liquid velocity, it follows that at $v = 0$, the field takes a time $\tau = L^2/\eta_m$ to decay, where $L = R_c$ is the scale size of magnetoactive region, and τ characterizes the "magnetic capacity" of the electroconducting liquid cores of the planets. Assuming the predominating role of the Poincare force (F_P) in the generation mechanism, it is natural to presume that $B \propto \tau F_P$, i.e.,

$$B \propto R_c^3 \rho \omega \Omega \sin \alpha / \eta_m \tag{3}$$

where $\eta_m = c^2/4\pi\sigma$, c is the velocity of light, and σ is the electric conductivity.

Magnetic fields of a number of planets are strikingly similar in the inclinations of the dipole axes to the axes of rotation of the planets, in the relations of the dipole, quadrupole, and octupole harmonics of their magnetic fields, and the dipole polarities (see Tables 1 through 3).

The principle of the theory of physical similarity is that if two physical phenomena are similar, then, from the given characteristics of the one, it is possible to obtain the characteristics of the other via a simple recalculation analogous to a change from one system of units to another [*Sedov*, 1965]. Proceeding from the principle of similarity and the assumption of a common motive force of the dynamo mechanism for all of the planets, it is correct to allow the possibility of determining the dipole field of any planet (H_P) from the Earth's dipole field (H_E) using the formula

$$H_P = H_E \frac{Q_E^3}{Q_P^3} \frac{\tilde R_c^3 \tilde\Omega \tilde\omega \sin \tilde\alpha \tilde\rho}{\tilde{\eta_m}} \tag{4}$$

where $Q = R_p/R_c$; R_p, R_c are the radii of the planet and the core, and the tilde indicates that the value is normalized by the appropriate value for the Earth.

$$\Omega = 1.5 GMf \cos \alpha / \omega r^3 \tag{5}$$

G is the gravitational constant, M and r are the mass and distance, respectively, to the disturbing body causing the precession, and f is the dynamic flattening of

Table 2. Parameters of magnetic fields of the Earth and giant planets

Planet	R_c, km	H_1, nT	H_2/H_1	H_3/H_1	$\Delta R/R$	i	$W \times 10^{-4}$, W cm^{-2}	Field
Earth	3480	30500	0.13	0.09	0.067	11.5	0.046	S
Jupiter	64340	419200	0.20	0.15	0.11	10	5.44±0.43	N
Saturn	40480	21000	0.07	–	0.04	1	2.01±0.14	N
Uranus	14700	22840	0.7	?	0.3	60	0.042±0.047	N
Neptune	19150	14240	1.16	0.78	0.55	46.8	0.433	N

the planet. The action of the forces arising in the magnetoactive regions in convection of material and those of the magnetic fields brought about (the Coriolis and Lorentz forces) is assumed to be similar to the model both in the "base" planet (Earth) and any other planet.

4. Terrestrial Planets in the Model of Precession Dynamo

Data on the magnetic and gravitational fields of the terrestrial planets and the Moon come from space mission flights conducted by the former Soviet Union and the United States over 1959–1989.

Table 1 lists the parameters for formulae (4) and (5) for the Earth, Mercury, Venus, Mars, and the Moon. The initial data are based on the results of investigations briefly described below.

4.1. Mercury

The magnetic field of Mercury was studied during the first (1974) and third (1975) flights of the Mariner 10 spacecraft. The magnetosphere of Mercury was crossed at an angle $\sim 110°$ from the subsolar point, under the limitations of determining the solar wind parameters. Measurements were made along the flight orbits with a small coverage of the planet's surface. All this presented difficulties in determining the dipole moment of Mercury. A review of the magnetic data and models of the internal structure is reported by *Ness* [1978].

Different models show the intensity of the dipole field of Mercury on the surface at the equator in the range of 165–330 nT. An intensity of the dipole of $\sim 165-190$ nT is more real. Such values are obtained in independent analyses representing the observed field by a sum of a dipole and quadrupole or by an eccentric dipole of a similar intensity [*Wang*, 1979; Ng and Beart, 1979].

The Mariner 10 data were used to refine the rotation period (T) and precession amplitude (α): $T = 58,646 \pm 0.005$ days, and $\alpha = 1°$ with uncertainty in an ellipse with axes of $\sim 2.5°$ and $6°$ [*Klassen*, 1976]. In most models of internal structure of Mercury, the outer radius of the liquid care $R_c = 1800$ km.

Representation of the magnetic field of Mercury by a sum of a dipole and quadrupole (which is supported by reasonable arguments of *Ng and Beart* [1979]) makes it possible to estimate the upper boundary of the magnetoactive region of Mercury, and to determine the distance r at which the first g_1^0 and the second g_2^0 harmonics of the field are equal from the formula of the spectrum of a Gaussian series

Table 3. Determination of boundaries of magnetoactive regions of Uranus and Neptune from the radii of convergence of harmonics of their magnetic fields

Neptune		Uranus 1		Uranus 2		Uranus 3	
r, km	$\Delta H_{1,2}$	r, km	$\Delta H_{1,2}$	r, km	$\Delta H_{1,2}$	r, km	$\Delta H_{1,2}$
20000	664	14000	3569	14000	−1095	13000	3330
19500	297	13900	2497	14200	−619	12800	2070
19200	57	13800	2136	14300	−92	12600	596
19150	0	13700	1638	14350	141	12550	209
19100	−15	13600	710	14400	350	12500	−200
19000	−123	13500	548			12450	−619
18500	−694	13400	−70				
18000	−1317	13300	−713				

$$H_P = (2n+1)^{-1/2} \left(\frac{R_P}{r}\right)^{n+2} \times$$
$$\times \sum [(g_n^m)^2 + (h_n^m)^2]^{1/2} \qquad (6)$$

According to *Wang* [1979], $g_1^0 = 165$ nT and $g_2^0 = 117$ nT. The results of calculations are as follows:

r, km	1500	1550	1600	1610
$\Delta g_{1,2}$	-183	-96	$+3$	$+50$

The distance $r = 1600$ km at $R_c/R_P = 0.66$ is close to the boundary of the core in the model of Mercury's interior [*Ness*, 1978], wherein $R_c/R_P = 0.69$.

It is a widespread notion that the conductivity σ in the terrestrial planets decreases with distance from the Sun. In particular, for Mercury $\sigma_{Me} = 2\sigma_E$ [*Gubbins*, 1977], and for Venus, $\sigma_V = \sigma_E$.

The values of the dipole field of Mercury are determined with a smaller error ($\sim 30\%$) than the values of the dynamic flattening of the planet ($\sim 70\%$). Hence given in Table 1 are the values of f for which the calculated values of H coincide with the measured ones.

4.2. Venus

The data listed in Table 1 are based on the work of *Zharkov* [1985] containing an overview of models of the planet's interior, observational data, and parameters of the planet's form. The value of dynamic flattening of Venus is given according to *Akim et al.* [1978]. The magnetic condition of Venus is surveyed by the spacecraft Mariner 5 [*Smith et al.,* 1965], Venera 4 [*Dolginov et al.,* 1968], Venera 9 and Venera 10 [*Dolginov et al.,* 1978], and Pioneer-Venus [*Knudsen et al.,* 1982; *Luhmann and Russell,* 1983]. The upper limit of the possible field of Venus $H_V \leq 10^{-4} H_E$ [*Phillips and Russell,* 1978].

4.3. Mars

An overview of models of the interior of Mars and the results of investigations of the planet's magnetic field are given in *Dolginov* [1991].

1. A determination of the dipole magnetic moment of Mars from the Phobos 2 satellite data is reported by *Dolginov and Zhuzgov* [1991] and from a set of data from the spacecraft Mars 2, 3, and 5 and Phobos 2 by *Dolginov* [1992].

2. It is customary to assume that the conductivity of the core of Venus $\sigma_V = \sigma_E$, and that of the core of Mars, $\sigma_{Ma} \leq \sigma_E$.

The radius of the liquid core of Mars is assumed to be $R_c = 1290$ km, according to a model of the planet's interior [*Binder and Davis*, 1973]. With such a value of R_c, agreement is attained with the observed value of the dipole field at $\sigma_{Ma} = 0.56\sigma_E$. The current value of the parameters H_{Ma}, R_c, and σ_{Ma} of Mars allows for their variation in a range 1–1.3 with the model similarity of the dipole fields of the Earth and Mars remaining true.

4.4. The Moon

The problems of the magnetic fields of the Moon are discussed in section 6 of the present paper.

Listed in the bottom line of Table 1 are the values of the Poincare force per unit of volume of the liquid cores of the terrestrial planets. In particular, the value F_P for Venus explains (without any speculative assumptions on the possible differences of its interior from that of the Earth) why Venus features no noticeable magnetic field: $F_{PV} : F_{PE} = 10^{-4}$ because of the smallness of the dynamic flattening of the planet and the amplitude of precession under the action of the Sun's gravitational field ($\alpha \leq 1°$).

The foregoing, in conjunction with the results of Table 1, allow one to infer that the magnetic conditions of the Earth, Mercury, Mars, and the contemporary Moon can be described numerically through a model of the precession dynamo, wherein the engine of the planetary dynamos is the external gravitational forces of the Sun and the nearest satellites.

5. Giant Planets in the Model of Precession Dynamo

The role of external gravitational forces in the dynamo mechanism manifests itself clearly when one analyzes the fields of giant planets having complex satellite system.

Current models of the interior structure of the giant planets, data on the magnetic, gravitational, and thermal fields of these planets are based on experimental results obtained over 1974–1989 due to the outstanding missions of the United States spacecraft Pioneer 10 and 11 and Voyager 1 and 2. The results of investigations of the magnetic fields near Jupiter and Saturn have been reported by *Smith et al.* [1974, 1975, 1980].

The results of investigation from the spacecraft Voyager 1 and 2 near Uranus and Neptune have been reported by *Ness et al.* [1986], *Connerney et al.* [1987], *Ness et al.* [1989], *Connerney et al.* [1991], *Anderson et al.* [1987], *Tyler et al.* [1989], and *Pearl and Conrath* [1991]. Models of the internal structure of the giant planets can be found in the works by *Podolak et al.* [1990], *Hubbard et al.* [1991], and *Zharkov* [1983].

Table 2 lists the boundaries of the magnetoactive regions of the Earth and giant planets (R_c), the values of dipole fields (H_1), the ratios of the quadrupole H_2 and octupole H_3 harmonics to H_1, the shift ΔR from the planet center when representing the field by an equivalent eccentric dipole, the inclinations of the dipole axes

Table 4. Parameters of the precession dynamo model for giant planets and the Earth

Parameter	Earth	Jupiter	Saturn	Uranus 1	Uranus 2	Neptune
R_p, km	6371	71492	60268	25000	25000	24765
R_c, km	3480	64340	40980	13800	14700	19150
Q	1841	1.111	1.471	1.855	1.741	1.293
ρ, g cm^{-3}	10	0.82	0.97	3.4	3.1	1.9
$\omega \times 10^{-5}$ s^{-1}	727	17.58	16.38	10.12	10.12	10.83
α, degree	235	Table 5	Table 6	satellite	satellite	21
f	3.27×10^{-3}	0.0651	0.094	0.0195	0.0195	0.0181
Ω angular, s per yr	50.2	Table 5	Table 6	satellite	satellite	48.39
σ, s^{-1}	2.7×10^{15}	5.93×10^{14}	2×10^{14}	1.81×10^{15}	1.81×10^{15}	1.08×10^{13}
$H_{\rm var}$, nT	30500	419200	21000	22840	22840	14240
$H_{\rm deduc}$, nT	–	417940	21000	16885	22915	14240

to the axes of rotation of the planets (i), the heat flux densities (W), and the hemispheres where the northern magnetic pole (N) is found. A striking feature is the similarity of parameters of the magnetic fields of the Earth and Jupiter, planets differing greatly in size, chemical composition, and aggregation state.

The practical coincidence of the axes of rotation and dipole at Saturn and the great angles i at Uranus and Neptune pose problems for the dynamo theory. For one, it is nearness and not coincidence of the axes of rotation and dipoles which is considered a prerequisite of the mechanism of generation of dynamo fields (one of the corollaries of the Cowling theorem).

Comparison of the values of H_1 and W allows one to conclude that there is no correlation between the intensities of the dipole fields and the densities of the heat fluxes from the interiors of the planets.

The huge sizes of the gas-liquid planets with their complex satellite systems have created the conditions in which supercritical Reynolds numbers required for the field generation are attained at moderate values of the conductivity (σ) of materials.

According to *Stevenson* [1983], in the layers of molecular hydrogen at pressures $P = 1$ Mbar and $T = 5000$ K, the magnetic viscosity is found to be in the range $10^5 - 10^6$ cm^2 s^{-1} (σ correspondingly being within $7.2 \times (10^{14} - 10^{13})$ s^{-1}).

Laboratory studies of the constituent materials of Uranus and Neptune at pressures $P \leq 0.5$ Mbar have shown the value of σ to be in the range (1–3) $\times 10^{13}$ s^{-1} [*Nellis et al.*, 1988].

Tables 5 and 6 list the giant planet parameters reported by *Ruskol* [1986]. The values of R_c of the magnetoactive regions of Jupiter and Saturn are determined from models of their interior structures. For Uranus and Neptune, whose magnetic fields contain comparable dipole (H_1) and quadrupole (H_2) harmonics, the values of R_c are also determined by (6), the Gaussian series spectrum. As is mentioned above, R_c is taken to be the distance r at which $\Delta H_{1,2} = 0$. The indicated errors possible in determining H_2 of Uranus have resulted in two possible values of R_c: models of Uranus 1, 2 listed in Table 4. The results of determinations of R_c are listed in Table 3.

Since the values H_1 of dipole fields of all the giant planets are known to a high enough precision, formulae (4) and (5) make it possible to refine the values of conductivity σ. Table 4 shows the values of σ for Jupiter, Saturn, and Uranus at which the measured and calculated magnitudes of the fields coincide. These values agree with the results reported by *Stevenson* [1983] while refining them. The values of σ measured directly under laboratory conditions [*Nellis et al.*, 1988] are adopted in calculating the field of Neptune. Because the dipole magnetic field of Jupiter, Saturn, and Uranus are determined by the action of the gravitational fields of their complex satellite system, Table 4 is supplemented by Tables 5–7 containing the results of calculations by formulae (4) and (5) of the contribution of each satellite to the total field of the planet.

Table 5 lists the magnitudes of the fields, calculated by formulae (4) and (5), generated at Jupiter under the action of the gravitational forces of the Sun and the nearest satellites.

The calculation results lead to the following important conclusions:

1. The observed field is the total effect of generation of the fields of each of the satellites and the Sun. It will be recalled that in the dynamo theory, the observed field of planet is a sum of the fields of individual cyclonic cells.

2. Precession, even at low amplitudes α, is efficient in the generation of a field because it has been proceeding in the same rhythm for millennia.

The field of Jupiter is generated in the inner molecular shell, and not in the region of metallic hydrogen.

Table 5. Magnetic field of Jupiter in the precession dynamo model: fields generated by the gravitational forces of the Sun and satellites

Source of precession	M, 10^{25} g	r, 10^6 km	α, degree	Ω angular, s per yr	H_1, nT
Io	8.94	421.6	0.04	2.86	46593
Europe	4.80	610.9	0.47	0.381	2709
Ganymede	14.823	1070	0.195	28.97	235575
Callisto	10.766	1883	0.281	3.82	13565
Sun	1.989×10^{33}	778.9×10^6	3.04	1.015	119397
				$\sum H_6 = 417840$	

Table 6 lists the corresponding parameters of the satellite system of Saturn and the results of calculation of the field by formulae (4) and (5). In the case of Saturn, one must allow for the great difference in the inclination α of the axis of rotation of Saturn to the normal of the planet's orbit around the Sun ($\alpha = 26.5°$) and also in the normals to the orbits of the satellites of Saturn ($\alpha_{\rm sat}$). Vector addition of the fields provides the explanation of the unique feature of the field of Saturn, i.e., the practical coincidence of the axis of the dipole and that of rotation. If one ignores the small differences in $\alpha_{\rm sat}$, then the total effect on the field generation due to the action of satellites will be $H_{\rm sat} = 28,075$ nT. Consider a triangle ABC, wherein $B = 21,000$ nT (the measured field oriented along the axis of rotation of the planet) and $C = 28,075$ nT (the field $H_{\rm sat}$). It is plotted at an angle $\alpha = 10°$ to the axis of rotation (an angle equal to the angles of inclination of the dipoles of the Earth and Jupiter to their axes of rotation). Side A, opposite the angle α, completes the triangle. Define side A and angle α opposite side B:

$$A = B\sin\alpha/\sin\beta$$

$$\beta = \arctan(B\sin\alpha/C - B\cos\alpha)$$

$A = 8189$ nT, which, accurate to 6%, coincides with the field H_S generated by the gravitational field of the Sun (Table 6); $\beta = 26.5°$ which is in excellent agreement with the astronomical value of Saturn's axis of inclination to the normal of its orbit. The magnetic field of Saturn is generated in the inner molecular shell.

It can be seen from Table 3 that at Uranus, a twin of Neptune, the field is generated at a greater depth than at Neptune, where the conductivity $\sigma_U \simeq 10^2\sigma_N$. The low-mass satellites of Uranus are capable of generating speeds necessary for attaining supercritical Reynolds numbers, allowing for field generation at only $\sigma_U \simeq 0.67\sigma_E$. The following questions then follow:

1. Why does a massive Triton, causing a precession of Neptune at a speed and amplitude exceeding that at Uranus under the action of its satellite system, generate a magnetic field at Neptune of a lower intensity than that of Uranus?

2. Why does the field at Neptune fail to be generated at deeper layers where the conductivity is higher?

It may be that answers to these questions lie in the general principle of thermodynamics: a system takes such states at which the action of external forces is minimized (Le Chatelier principle) [*Landau and Lifshitz*, 1976].

According to *Rochester et al.* [1975], the losses due to Joule heating in the core are in inverse proportion to the electric conductivity squared. The generation of the field of Uranus at great depths at $\sigma_U \simeq 10^2\sigma_N$ may be one reason of the hardly noticeable heat flux of Uranus and the quite significant heat flux of Neptune.

The magnetic fields of Uranus and Neptune, represented by equivalent dipoles with a large eccentricity and inclination of the axis, or by comparable (in magnitude) harmonics of Gaussian series, are reminiscent of geomagnetic field structure during periods of polarity reversal. In dynamo theory, reversals are associated

Table 6. Magnetic field of Saturn in the precession dynamo model: fields generated under the action of the gravitational forces of the Sun and satellites

Source of precession	$M, 10^{23}$ g	r, km	α, degree	Ω angular, s per yr	H_1, nT
Mimas	0.38	185.52×10^3	1.53	0.22	495
Tethys	7.6	294.60×10^3	1.09	11.04	15282
Titan	1345.7	1221.8×10^3	0.33	27.42	12298
Sun	1.989×10^{33}	14267×10^6	26.44	0.22	7695

with a change in the structure of the cyclonic cells in the core under the action of internal and external forces, and the dipole inclination may be determined by peculiarities of the dynamo process. Because the material has finite conductivity, the dimensions of the cyclonic cells must increase (to maintain the conditions of generation of the field) and hence their number n must decrease. A simple dependence of the inclination is assumed $J \sim n^{-1/2}$ [*Parker*, 1969].

At Reynolds numbers approaching critical valves (which is possible at the moderate conductivity of material in the core), the conditions are more favorable for the generation of a quadrupole field. During excursions, the geomagnetic dipole inclination can reach 90°.

In the current epoch, the terrestrial planets have their north magnetic poles in the southern hemisphere. However, while all planets have the same sign of angular momenta, the opposite polarity is found at all the giant planets. This law remains an enigma of planetary magnetism, which is why the assumption that the field of Uranus is in a state of excursion seemed so attractive. That assumption became unlikely after the similar magnetic field structure of Neptune was discovered, since it is unlikely that the state of excursion, so rare an occasion in the history of the geomagnetic field, should occur simultaneously upon two giant planets.

The intensity and unique topology of the magnetic fields of Uranus and Neptune can be explained by the action of the two effects of the external gravitational forces: precession and the generation of tidal currents reaching considerable depths.

These issues have been discussed by *Dolginov* [1993], who offered the following scenario of the dynamo process at Neptune:

1. The lower boundary of the tidal currents where current velocities are maintained which are comparable to those generated by precession ($v = r\Omega \sin\alpha/2f$) is found in Neptune at depths of 2000–3000 km from the surface. Still deeper, 2500–3000 km from the lower boundary of the region, the upper boundary of the magnetoactive region is found, where the field of Neptune is generated (19,150 km, Table 3). The conductivity of material in the region of tidal currents is not sufficient to provide for supercritical Reynolds numbers but is adequate for the magnetohydrodynamic action upon the magnetic field permeating this region. The processes proceeding in the region of zonal currents capable of altering the orientation of the field beyond the region of generation have been discussed (for Jupiter and Saturn) by *Kirk and Stevenson* [1987]. The zonal currents at Uranus and Neptune differ, in their latitudinal dependencies, from those of Jupiter and Saturn.

2. However, the deformation of the magnetic fields by the deep zonal currents cannot immediately lead to the generation of a field with comparable (in the order of magnitude) dipole, quadrupole, and octupole harmonics, such as found at the fields of Uranus and Neptune. But one can assume that the deformations favor the processes which lead to directional changes in the planetary field, such as excursions and reversals. According to *Levy* [1972], the process of deformation and reversal of a field begins with the origination in the equatorial region of a great number of cyclonic cells having opposite polarity. Then the region of opposite polarity starts to propagate to higher latitudes. In this period, the field turns multidipolar. Further expansion of the reversed field region into high latitudes results in a complete field reversal. The model discussed by *Levy* [1972] becomes unstable after 20–30% of the cyclonic cells have reversed their sign. *Hoffman* [1977] inferred that such a sequence in the scenario of field reversal complies with the discovered distribution of the virtual magnetic poles of the paleomagnetic field of the Earth.

Table 7. Magnetic field of Uranus in the precession dynamo model: generation of fields under the action of the gravitational forces of satellites

Satellite	$G \times M$, 10^{15} cm^3 s^{-2}	r, 10^3 km	α, degree	Ω angular, s per yr
Miranda	4.6 ± 1.2	129.4	4.22	3.96
Ariel	84 ± 12	191.01	0.31	22.60
Umbriel	89 ± 13	266.3	0.36	7.38
Titania	232 ± 9	435.9	0.14	51.58
Oberon	202.7	583.4	0.10	1.90

3. The phenomenon of synchronization [*Goldreich*, 1965] widespread in the solar system, and the property of rotating bodies to assume the same rhythm at, however, weak links between the objects [*Blekhman*, 1981], thoroughly studied under laboratory conditions, allow one to explain the coincidence in time of sojourn in the state of excursion of the magnetic fields of two planets, a rare event in the history of the geomagnetic field.

The results of calculations of the values of the planetary magnetic moments $M = H_1 R_P^3$ by formula (4) are shown in Figure 1. Plotted along the vertical axis are the values of log M_P/M_E and along the horizontal axis, the values log D_P/D_E of all the planets surveyed and the Moon. $D(R_c^6 \omega \Omega \rho \sin\alpha/\eta_m)$ is the dynamo number of the precession dynamo model.

The assumption of a dominant role of external gravitational forces in the mechanism of planetary magnetic field generation reveals a rigorous law in the seemingly chaotic picture of distribution of intensities of the planetary dipole fields. According to *Parker* [1992], a correct determination of the energy sources of the dynamo process should result in a simple dependence of the field intensities on a number of parameters. Within the scope

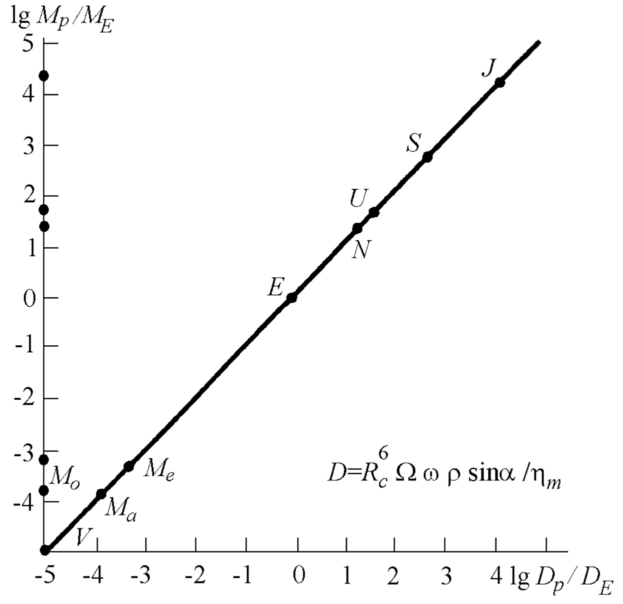

Figure 1. Dependences of the ratios of observed magnetic momenta of planets and the Earth M_P/M_E on the ratios of their dynamo numbers D_P/D_E in the precession dynamo model.

of comparative planetology, this problem is solved by the "numerical law" of the precession dynamo model discussed above.

6. Magnetic States of Planetary Satellites

Can satellites have melted cores? Dissipation of tidal energy in a satellite-planet system is inversely proportional to the orbit eccentricity squared. The heating by tidal friction is an important component of the heat balance of a planet [*Kaula*, 1966].

Energy dissipation in the Earth-Moon system, caused by tidal friction, reaches 2.7×10^{19} erg s^{-1} in the solid parts of the Earth. In the Moon, this dissipation of energy is currently 100 times less.

At Mercury, the tidal friction mechanism results in energy dissipation at a rate of $\sim 10^{16}$ erg s^{-1} [*Burns*, 1976]. A great event was the discovery of volcanic activity at Io, one of the Galilean satellites of Jupiter. The possible melting of Io by tidal energy dissipation was discussed by *Peale et al.* [1979].

Burns [1968] suggested that the observed perturbations in radio emissions from Jupiter which are dependent on the angular positions of the internal Galilean satellites can be attributed to perturbations of the magnetic field of Jupiter at its radiation belt, caused by the satellites if these have either intrinsic magnetic moments or a noticeable permeability of material.

These theoretical estimates and experimental facts might be grounds for the assumptions that some of the giant planet's satellites can have melted cores. Then, by assuming particular mechanisms of convection, one could estimate the possible dynamo fields of these satellites in anticipation of direct measurements of these fields by magnetometers of spacecraft. Such a prediction was made by *Neubauer* [1978] in the assumption that the radii of the liquid conducting cores of the biggest satellites of Jupiter and Saturn are $R_c = 0.4R_{\rm sat}$, where $R_{\rm sat}$ is the satellite radius. Then, on the assumption that the dynamo fields are generated under the action of thermal convection and also that the numerical law of the planetary magnetism suggested by *Buse* [1976] is met, it was estimated that the satellite Io can have a field of an intensity of 2200 nT; Europe, 810 nT; Ganymede, 220 nT; Callisto, 60 nT; and Titan, 100 nT. *Kivelson et al.* [1979] considered the possible processes of reconnection of the lines of force of the field of Jupiter and of hypothetical fields of satellites for the case where the magnetic dipoles of the latter and Jupiter are antiparallel.

Within the precession dynamo model, none of these satellites can have a modern dynamo field, since all of these satellites are in a synchronous rotation at their own orbital periods and hence experience no precession under the action of their gravitational fields.

Precession of these satellites under the action of the gravitational field of Sun, in the assumption that their dynamic flattenings are of the same order of magnitude as the Moon's, can lead to fields of less than 1 nT. Voyager 1 has flown at a distance of 4400 km from the surface of Titan. So far, these is reliable evidence of a tail of the "induced" magnetosphere of Titan generated by the lines of force of the Saturn's field deformed by a rotating Titan featuring an ionosphere. No explicit features of an intrinsic field of Titan have been discovered. Of course, this does not prove the inefficiency of thermal convection until the existence of conducting cores in the giant planet's satellites is unambiguously established.

More definitive results in favor of the precession dynamo mechanism have been provided by studies of the current magnetic state of the Moon (Table 1) and of the astonishing paleomagnetic properties of the rocks of the Earth's satellite. Successive explorations in the vicinity of the Moon by the Luna 2, Luna 10, and Explorer 35 spacecraft have shown that currently the Moon has no general dipole field. A possible upper limit of this magnitude $H_{Mo} \leq 0.05$ nT according to data from the Apollo spacecrafts. However, direct evidence of magnetization of the rocks of the Moon at the sites of landing of the manned Apollo spacecraft has been obtained from their lunar excursion modules and from the Lunokhod 2 automatic station.

Striking results were obtained from investigations of the magnetic properties of the rocks brought to Earth by the Apollo's crews. Magnetic properties of these samples were studied by various methods in laboratories of United Kingdom, Unite States, and Canada. Morphology of the magnetic fields of the lunar surface was studied in the near-equator belt in the range of latitudes $\pm 30°$ at the latitudes of the Apollo flybys with the aid of magnetometers, and on the surface, by the reflected electron method. These original investigations have been cited in a number of reviews dealing with the hypotheses of the origin of the enigmatic paleofields of the Moon [*Daily and Dyal,* 1979; *Collinson,* 1993]. A history of investigation of lunar magnetism addressed to the general public has been published by *Runcorn* [1988].

The numerical model of the precession dynamo of the Moon [*Dolginov,* 1985, 1986] is based on experimental facts and substantiated hypotheses. So far, only the model of the precession dynamo has been capable of describing the principle features of lunar magnetism.

1. Calculations of the dimensionless moment of inertia of the Moon yielded the value $I = 0.3900 \pm 0.00023$, which is in agreement with the idea of the existence of a core in the Moon with a radius $R_c \simeq 350 - 400$ km [*Ferrart et al.,* 1988]. It is assumed that the core consists of Fe, N, and S and is melted [*Brett*, 1973; *Levin*, 1979]. The existence of a conducting core with a radius $R_c = 400$ km is suggested by the results of an electromagnetic probing of the Moon by solar wind fields [*Wiskerchen and Sonnet*, 1977] and by the Earth's geomagnetic tail field [*Russell et al.,* 1981].

2. Based on a large number of moonquakes, it is established that these are associated with the action of external gravitational tidal forces upon irregularities in the Moon's interior at depths of about a half a lunar radius [*Nakamura,* 1992]. This indicates a role of the external forces in the energetic processes of the Moon even in the current epoch.

3. A most important feature of the Moon's paleofield is established, notably, its variation with time. Samples of lunar rocks, aged $\sim 4.2 - 3.5$ billion years, had been magnetized in fields of $\sim 10^{-4}$ T. Younger samples had been magnetized in fields of lower intensities. The field decreased to 10^{-6} T over 2.5 billion years [*Cisowski et al.,* 1983; *Stephenson et al.,* 1977].

4. The observed magnetic anomalous regions can be modeled by layers several hundred kilometers long, uniformly magnetized [*Hood et al.,* 1979]. This is evidence of the rocks having been magnetized in the field of the Moon's intrinsic dipole.

5. In the seemingly chaotic complex morphology of the field, complicated by the mechanical and possibly magnetic action of the meteoric bombardment of the Moon [*Hood,* 1981], *Runcorn* [1983, 1984, 1988] distinguished three different poles of magnetization found on three different axes. The poles have their antipodes about 180° apart.

6. *Runcorn* [1983, 1984, 1988] suggested that these three directions relate to different epochs of formation of the lunar surface. Under the action of major bodies falling on the Moon, the position of the surface relative to the stationary axis of rotation of the Moon varied [*Runcorn,* 1983, 1984, 1988].

7. The major bodies falling on the Moon could be debris of satellites that orbited the Moon and disrupted upon reaching the Roche limit. The fall of the debris accounts for the formation of the annular seas [*Reid*, 1973].

8. The tidal current in the Earth-Moon system resulted in recession of the Moon from the Earth. In the past, the Moon could be at a distance of no closer than $20 R_E$ from the Earth [*Zharkov and Trubitsyn*, 1980].

All these experimental facts and substantiated scenarios of the Moon's history suggest that the rocks of the Moon had acquired magnetism in the Moon's intrinsic dynamo field that for some reasons, has gradually been decayed.

Runcorn and his colleagues did not fail to consider possible sources of energy for the lunar dynamo mechanism. In so doing, they were guided by the plausible condition that this source of energy must vanish. So, *Collinson* [1993] names the three possible sources: radioactive elements in the core, a partial solidification of the liquid core, and the slowing down of the Moon's rotation. The first two sources presume the action of heat convection, and the third source, the dominating role of the Coriolis force. *Runcorn* [1983] dismisses the earlier proposed thermal source in the Moon's core in the form of short-lived radioactive transuranic elements and infers that the nature of the sources of energy of the dynamo of a young Moon remains an unsolved, intriguing problem.

One can hardly agree with the suggestion of *Collinson* [1993] that the cause of the disappearance of the Moon's field is the slowing down of its angular speed. Compared to the time of diffusion $\tau = R_c^2/\eta_m \sim 200$ years, the Moon has been rotating rather "fast" in the current epoch. Mercury rotates about 2 times slower than the Moon but enjoys a contemporary dynamo field, which is explained by the precession dynamo model.

The assumption that the rocks of the Moon had been magnetized in the intrinsic dynamo field of a young Moon has been made by *Dolginov* [1975]. According to *Runcorn* [1983, 1984, 1988] the migration of the Moon's magnetic paleopoles through the migration of its axis of rotation relative to its surface is caused by the fall of major bodies. The arguments of *Reid* [1973] of these bodies possibly being ancient satellites of the Moon have made it possible to revise the model of precession dynamo of the Moon to include the entire set of the observed data discussed above. *Dolginov* [1985, 1986] has determined the parameters of a hypothetical satellite of the Moon which, while orbiting the Moon, would have caused additional precession of the Moon that was responsible for the generation of the $\sim 10^{-4}$ T dynamo field on the Moon's surface. Excluding the unknown rotational speed ω of the young Moon from formulae (4) and (5) yields

$$H_{Mo} = \frac{3}{4}\frac{Q_E^3 R_{cMo}^3 \rho_{Mo} G f M r^{-3} \sigma_{Mo}}{Q_{Mo}^3 R_{cE}^3 \rho_E \omega_E \Omega_E \sigma_E \sin\alpha_E} \sin 2\alpha_{Mo} \qquad (7)$$

where G is the gravitational constant, M and r are the mass and distance, respectively, of the disturbing body (these can be M_E and M_{sat}, the masses of the Earth and the lunar satellite, respectively), f is the dynamic flattening of the Moon, H_E, ω, Ω, and α are the contemporary values of the dipole geomagnetic field at the equator, the angular velocity, and the velocity and amplitude of precession of the Earth, respectively, and ρ and σ are the densities and conductivity of the liquid cores of the Earth and the Moon.

Extrapolation of the lunar surface paleofield of $50 \pm 100\,\mu$T to the boundary of a core with a radius $R_c = 350$ km results in values of the poloidal field of $\sim 5-10$ mT. None of the solar system's planets is assumed to have a poloidal field of such intensity at the boundary of the magnetoactive region. This makes one presume that the region of the dynamo process in a young Moon 4.2–3.5 billion years ago had a radius in excess of 350 km. This assumption is corroborated by the following experimental facts and calculations:

1. The results of seismic sounding of the Moon show that the lower mantle of the Moon is partially melted. Its upper boundary is found at a depth of ~ 1000 km.

2. According to calculations by *Toksots and Johnston* [1975], the contemporary value of the Moon's momentum of inertia is consistent with a core with a radius of ~ 450 km if it consists of Fe, and with a radius of ~ 700 km, if it is of FeS.

3. The Moon is heated up by tidal friction, when it was closer to the Earth ($\sim 20R_E$ [*Kaula,* 1966]). With $R_c = 700$ km, the poloidal field in the young Moon could reach 0.5 ± 1 mT at the core boundary. Important inferences on the possible dimensions of the magnetoactive region of the young Moon can be made based on the model of the precession dynamo of the Moon.

With the parameters of the young Moon's core ($R_{cMo} = 400$ km and $\rho_{cMo} = 6$ g cm^{-3}) and ($R_{cMo} = 700$ km and $\rho_{cMo} = 4.5$ g cm^{-3}), (7) yields the following expressions of the dipole of the Moon's paleofield

$$H_{Mo} = 7.65 \times 10^{32} r^{-3} \sin 2\alpha \qquad (8.1)$$

$$H_{Mo} = 164.8 \times 10^{32} r^{-3} \sin 2\alpha \qquad (8.2)$$

According to estimates of *Zharkov and Trubitsyn* [1980], in the distant past, the Moon was found at distances $r = 10 - 20R_E$. At a minimum distance $r = 10R_E$, $R_{cMo} = 400$ km, and $\alpha = 20°$, the field generated under the action of the Earth's gravitational field could reach $H_{Mo} \simeq 2950$ nT. At $R_{cMo} = 700$ km, $H_{Mo} \sim 40,700$ nT. This magnitude is commensurate with the minimum magnitude of the Moon's paleofield (0.5×10^{-4} nT) reported by *Collinson* [1993]. However, if the minimum distance between the Moon and the Earth has never been less than $20R_E$ (as the same authors argue in one of their works), then it is impossible that a $\sim 0.5 \times 10^{-4}$ T paleofield be generated under the action of the Earth's gravitational field.

Following the arguments of *Runcorn* [1983] and *Reid* [1973] on the satellites orbiting the young Moon, we are in a position to explain the ancient magnetic field of the Moon by the most likely model for $R_c = 700$ km. According to *Reid* [1973], the satellite mass $M_{\text{sat}} = 10^{-3} M_{Mo}$. At $\rho_{\text{sat}} = 3.3$ g cm^{-3}, the satellite radius $R_{\text{sat}} = 430$ km. Such a satellite could exist intact provided its orbital radius $r_{\text{sat}} \geq 4345$ km (the Roche limit). But at $M_{\text{sat}} = 10^{-3} M_{Mo}$ and such an orbit, no field of $H_{Mo} = 0.5 \times 10^{-4}$ T can be generated. Bearing in mind that $M_{Mo} = M_E : 81.3$, formula (8) allows one to suggest that a field of $H_{Mo} = 0.5 \times 10^{-4}$ T could be generated by satellite debris with an effective minimum mass $M_{\text{sat}} = 2 \times 10^{-3} M_{Mo}$ orbiting with a radius $r_{\text{sat}} \sim 2000$ km until its impact upon the lunar surface. If the minimum mass $M_{\text{sat}} = 3 \times 10^{-3} M_{Mo}$, then the maximum initial orbit could reach $r_{\text{sat}} \sim 2300$ km. This arithmetic, however, is not void of physical meaning and is confirmed in the total action of isolated bodies in the generation of the fields of Jupiter and Saturn.

The disappearance of the satellite should entail a decay of the intense lunar field. Afterward, there could exist a moderate and a weak dynamo field of the Moon which was generated under the action of the Earth's gravitational field which decayed as the Moon moved away from the Earth, slowing down to synchronous rotation.

7. On the Energy of the Precession Dynamo Mechanism of the Earth's Magnetic Field

What is the planetary "dynamo machine" energy and what is its source? The Earth and its magnetic field have been thoroughly studied and the search for answers to those questions began at nearly the same time as the development of the dynamo theory. Estimates of the energy contribution of the likely motive forces of the geodynamo were to determine the choice of these.

Malkus [1968] and *Stacey* [1973] suggested the possible role of the Earth's precession in the generation of the geomagnetic field. As is mentioned above, more recent works of *Rochester et al.* [1975], *Loper* [1975], and *Lowes* [1984] ruled out the participation of precession in the dynamo process, mostly because of the energetics of the mechanism. But, in (4), the base planet is the Earth with its magnetic field.

The conclusions on the noneffectiveness and even nonparticipation of planetary precession in the generation of the planetary magnetic fields do not agree with the experimental data discussed above in sections 4 through 6. We will show that the initial postulates of the precession dynamo model (formula (3)) allows for progress to be made in this area.

7.1. Energy Balance of the Earth's Core

Nearly four decades ago, *Hide* [1958], in his work dealing with the magnetohydrodynamics of the Earth's core, reported formulae of the energy balance of the globe. A detailed solution of the problem can be found in the work by [*Hide*, 1958, appendix], so we restrict ourselves to the most generalized expressions, adopting the designations of *Hide* [1958]:

$$\frac{d}{dt}(M_i + M_E) = -J + N \tag{9}$$

where the difference of the overall internal magnetic energy

$$\frac{dM_i}{dt} = \frac{1}{4\pi}\int_\nu \mu H^2 d\nu \tag{9.1}$$

the difference of the external magnetic energy, equal to the Poynting flux, through the conducting core surface to the mantle

$$\frac{dM_E}{dt} = \frac{1}{4\pi}\int_S [\mathbf{E}\times\mathbf{H}]dS \tag{9.2}$$

the rate at which the magnetic energy changes to Joule heat

$$J = \int_\nu (i^2/\sigma)d\nu \tag{9.3}$$

the rate at which the magnetohydrodynamic processes cause the energy of motion liquid to change to magnetic energy (U is the liquid flow rate)

$$N = \frac{1}{4\pi}\int_\nu U(H\ \mathrm{rot}H)d\nu \tag{9.4}$$

No representation of integral (9.2) as dependent on specific parameters has been given. In subsequent years, the energy balance of the Earth's core has been addressed by *Backus* [1975], *Gubbins* [1977], *Gubbins and Masters* [1979], *Rochester* [1975], and *Smyle et al.* [1984]. In these papers, too, the component $\int[\mathbf{E}\times\mathbf{H}]dS$ remains undisclosed apart from the indication that it represents a flux of external magnetic energy. Integral (9.2) was dealt with easily: it was assumed to be zero at infinity [*Backus*, 1975].

The external magnetic energy of the Earth is determined simply from the Gaussian coefficients of analytical models of the field of different epochs. The internal magnetic energy, determined by the intensity of the toroidal field not detectable on the surface, differs in different models by a factor of 10^3.

We will show that integral (9.2) can be expressed using the parameters of the precession dynamo model and the parameters of the Earth's core.

7.2. Poynting Flux in the Precession Dynamo Model

In (3), the sign of proportionality can be replaced with a sign of equality; then, $H = KR_c^3\Omega\omega\sin\alpha/\eta_m$. The coefficient K is common for all the planets, and its dimension in cgs units is

$$[K] = \frac{\mathrm{cm}^{-1/2}\mathrm{g}^{1/2}\mathrm{s}^{-1}\mathrm{cm}^2\mathrm{s}^{-1}}{\mathrm{cm}^3\mathrm{s}^{-2}\mathrm{g\ cm}^{-3}} = \mathrm{cm}^{3/2}\ \mathrm{g}^{-1/2} = D^{-1}$$

where D is the electric induction ($D = \varepsilon E$; ε is the dielectric constant, and E is the electric field intensity). The dimension of $[\varepsilon] = \mathrm{cm}^{-2}\ \mathrm{s}^2$.

$$|HE| = \frac{R_c^3\rho v^2\Omega\omega\sin\alpha}{\eta_m} \tag{10}$$

Formula (10) takes into account only those processes described by the magnetohydrodynamic equation of induction. Meanwhile, the complete solution of the dynamo problem is described by the equation of convection allowing for other processes in the core. The conditions of changing from mechanical to magnetic energy are determined not only by the magnetic η_m but also the kinetic viscosity ν, and the temperature conduction X. All these parameters have the same dimension.

Introduction of these parameters into (10) makes it possible to take into account their important role in the dynamo process while keeping the dimensions of the whole formula and its physical meaning:

$$P = |HE| = \frac{R_c^3\rho v^2\omega\Omega\sin\alpha}{\eta + \nu + \chi}$$

There are grounds to assume that flow P is not isotropic and varies with latitude φ as is the case with the vertical component of the magnetic field $Z = Z_0\sin\varphi$:

$$\frac{dM_E}{dt} = \frac{2}{4\pi}\int_{\varphi=0}^{\pi/2}\int_{\lambda=0}^{2\pi}[E\times H]R_c^2\sin^2\varphi d\varphi d\lambda =$$

$$= \frac{\pi R_c^5 \rho v^2 \Omega \omega \sin\alpha}{2(\nu + \eta_m + \lambda)} \tag{11}$$

7.3. Geomagnetic Field External Energy and Variation Rate Thereof

Numerical values of energies of the dipole and multipole components of the geomagnetic field at the Earth's core boundary have been determined by *Verosub and Cox* [1971]. The calculations are performed from the values of the Gaussian coefficients g_n^m and h_n^m of analytical models of the geomagnetic field taking into account the series up to $n = m = 6$ for 16 epochs in the interval 1845–1965. According to the 1965 epoch analysis, the magnetic energy of the dipole field $M_1 = 5.09 \times 10^{25}$ erg, the total multipole energy $M_{2-n} = 1.44 \times 10^{25}$ erg, and the overall energy $M = 6.53 \times 10^{25}$ erg. Over the 120-year interval, the dipole field energy decreased at the average rate 5.1×10^{22} erg/year, and the energy of nondipole harmonics increased at a rate of 3.8×10^{22} erg/year. The magnetohydrodynamical processes in the core seemed to lead to a partial pumping of the dipole field energy into the multipole field energy. The decrease of the dipole field energy in the current epoch at the rate 1.3×10^{22} erg/year $= 4.5 \times 10^{14}$ erg/s can be associated with the losses to Joule heating in operation of the planetary dynamo machine. This amount of loss is 10^2 times less than that suggested for the most economical model of the geodynamo. It is natural to assume that the motor of the planetary dynamo machine must provide for compensation for the losses to Joule heating at a rate higher than 4×10^{14} erg/s. In this case, an adequate engine will provide for field generation within a time at least commensurate with the time of the field diffusion $\tau = R_c^2/\eta_m$ (for the Earth, $\tau = 2 \times 10^4$ year).

7.4. Numerical Value of Variations of External Magnetic Energy in the Earth's Precession Dynamo Model

We consider the numerical values of the parameters of (11) determining the Poynting flux, i.e., the rate of variation of the external energy for the Earth.

1. In the numerator, there is only one quantity, namely, the liquid flow rate in the core, that can be assigned differing values in different models. According to *Stewartson and Roberts* [1963], the velocity of a liquid confined in a spheroidal rotating and precessing container is $v = r\Omega \sin\alpha/2\xi$, where ξ is the dynamic flattening, and r is the distance from the center. Taking, for the Earth, $r_1 = 1230$ km, $r_2 = 3480$ km (the liquid core boundaries), $\xi = 2.45 \times 10^{-3}$, $\Omega = 8 \times 10^{-12}$ s^{-1}, and $\sin\alpha = 0.399$ yields $v_1 = 0.08$ cm s^{-1} and $v_2 = 0.22$ cm s^{-1}. The value of v_1 is nearly similar to the speed of westward drift of the nondipole part of the geomagnetic field (~ 0.06 cm s^{-1}).

2. The magnetic viscosity of the Earth's core $\eta_m = 2 \times 10^4$ cm^2 s^{-1}. In sections 4 and 5, the values of η_m of all the planets were compared to this value.

3. The kinematic viscosity of the core (ν) is the most important parameter in estimating the energy dissipation in a precessing Earth. Estimates of the kinematic viscosity in the core range vastly from $\nu = 10^{-3}$ cm^2 s^{-1} (molten metal viscosity data) to $\nu = 10^9 - 10^{10}$ cm^2 s^{-1} (data on minor absorption of longitudinal seismic waves in the core). The upper limit was assumed to reflect the properties of a very thin boundary layer [*Gans*, 1972].

As was mentioned in section 2, *Hide* [1971] analyzed the magnetic disturbances caused by large irregularities (± 6 km) of the lower boundary of the mantle and estimated $\nu \leq 10^6$ cm^2 s^{-1}. In kinematic models of the dynamo process, the flow of liquid in the core is attributed a turbulent character. One of the most important effects of precession on the character of currents of liquid in a rotating spheroidal container, experimentally discovered by *Malkus* [1968], is the occurrence of turbulence. This sets an upper limit to the viscosity. At $v = 0.08$ cm s^{-1} and $\nu = 10^5$ cm^2 s^{-1} and the liquid core characteristic dimension $R_c = 3480$ km, the kinematic Reynolds number $R_K = R_c v/\nu = 280$. This provides a favorable enough condition for turbulence to occur [*Leorat et al.*, 1983].

The assumption of a dominant role of the electromagnetic mechanism in the mantle-core system leads to the conclusion that energy dissipation under precession of the Earth occurs only in a narrow layer, at the boundary with the solid mantle [*Loper*, 1975]. At $\nu = 10^5$ cm^2 s^{-1}, however, the torque of the viscous coupling in the core-mantle system exceeds the torque of the electromagnetic coupling under the current intensity of the dipole field. In this case one must presume that energy dissipation in a precessing core takes place all over the magnetoactive region.

Arguments in favor of a dominant role for viscosity in the energy dissipation in the core and in transmitting the torque of the external forces to the liquid are as follows.

1. At the initial phase of the dynamo process, the magnetic field intensity can be arbitrarily low, and the process of enhancement of the field can proceed without electromagnetic coupling playing any significant role.

2. The precession dynamo model (4) agrees well with the magnetic fields of planets having solid and gas-liquid mantles.

3. The thermal conductivity in the core $\chi = 4.2 \times 10^{-2}$ cm^2 s^{-1}. There are different estimates of the temperature gradient in the core.

Taking into account the values of the above-mentioned parameters, let us evaluate the variations in the external magnetic energy in the Earth's precession dynamo model. The numerator of (11) characterizes the dynamics of a rotating and precessing liquid core, and its denominator characterizes the dissipative processes proceeding in the core under the action of external and internal forces.

At $R_c = 3.48 \times 10^8$ cm, $\Omega = 8 \times 10^{-12}$ s^{-1}, $\omega = 7.27\times 10^{-5}$ s^{-1}, $\rho = 10$ g cm^{-1}, sin $\alpha = 0.399$, and $v = 0.08$ cm s^{-1}, the numerator of (11) is 3.28×10^{27} dyn cm^{-3} s^{-2}.

How can one take account of the effect of the parameters of the denominator of (11) on the variation of the magnetic energy?

We already mentioned the predominant role of the viscosity mechanism in transmitting the torque of the external forces to the liquid of the core. This can be presumed to occur at least at the initial phases of the dynamo process. With increasing internal magnetic field and the formation of an external poloidal field, the electromagnetic coupling mechanism in the mantle-core system comes into operation, and the losses to Joule heating begin to grow. On the basis of this scenario of field generation, it is possible to successively take into account the effect of ν and η_m, assuming the condition of additivity of their actions. A successive division of the value of the numerator of (11) into values of $\nu = 10^5$ cm^2 s^{-1} and $\eta_m = 2 \times 10^4$ cm^2 s^{-1} leads to the value $dM_E/dt = 6 \times 10^{17}$ erg s^{-1}. The rate of variation of the external magnetic energy (6×10^{17} erg s^{-1}) in the precession dynamo model agrees well with the energy dissipation (10–11 W) [*Stacey*, 1973] in a rotating and precessing mantle-core system if the angular momenta of the mantle L_m and the core L_c do not coincide in direction and the angle between L_m and $L_c \varepsilon_d = 3.9 \times 10^{-5}$ rad.

The value $dM_E/dt = 6 \times 10^{17}$ erg s^{-1} obtained above by far exceeds the rate of variation of the energy of the Earth's dipole field in the current epoch (4.5×10^{14} erg s^{-1}). One possible reason for this is that we have made no allowance for the thermal conductivity χ. There is a notion in the recent literature that the temperature gradient in the core is very similar to its adiabatic value, which restricts the heat flow from the core to the mantle. This flow cannot accurately be distinguished from the background predominant heat flow from radioactive sources in the mantle.

The foregoing allows one to infer that the determinant role of external gravitational forces in the generation of planetary magnetic fields can be clearly seen within the scope of comparative planetology and geophysics. For the latter case, this is corroborated by the proof that the initial formula (3) of the precession dynamo model does express the density of the external magnetic energy flux through the surface of the magnetoactive region (Poynting flux).

A comparison of the rate of variation of the Earth's external magnetic energy (11) with the experimentally found rate of variation of geomagnetic field energy in the current epoch for reasonable values of viscosity and electric conductivity of the Earth's core shows that the energy of the precession dynamo mechanism is comparable to the energy of generation of the geomagnetic field if the precession dynamo efficiency is a mere 6–8%.

The geodynamo energy is derived from the kinetic energy stored in a rotating Earth which is extracted due to precession and tidal effects under the action of external gravitational forces.

The secular variations in the rotational speed and the inclination of the Earth's rotation axis are concrete astronomical parameters characterizing the variation of the Earth's kinematic energy. Which of these sources makes the largest contribution is a topic of future analysis.

8. Conclusions

In the seemingly chaotic picture of distribution of intensities of the magnetic fields of planets and the Moon, a pattern becomes apparent if these are described by the "numerical law" of the precession dynamo model. Some of the quantities included in the formula of the precession dynamo model are directly observable; others are determined from astronomical and geophysical observations and laboratory experiments. The physical meaning of the initial formula of the precession dynamo model is also clear: it defines the rate of variation of the external magnetic energy in the dynamo process, a quantity is determined from geomagnetic observations on the surface and from data on the liquid core parameters which, in turn, are determined, in a most substantiated manner, from magnetic and seismic data.

However, we are not equipped to find absolute values of dipole fields using the parameters of a model of the precession dynamo. This limitation is lifted due to the discovered features of a striking similarity of magnetic fields of otherwise markedly differing planets and using the widespread principles of the theory of physical similarity, having selected in the capacity of the "base" planet the Earth and its magnetic dipole field.

The striking agreement in the planetary dipole fields for reasonable values of the parameters included in the formula of the precession dynamo model indicates that the motive force of the planetary dynamos is due to the action of external gravitational phenomena that, in turn, lead to a turbulent convection of the conducting liquid in the magnetoactive regions of the planets.

Future planetary flybys will provide further experimental data for the dynamo problem. So, one can expect that the explorations from the Galileo spacecraft (1994) will make it possible to ascertain the secular variation of the dipole field of Jupiter which will allow one to determine, by the method suggested by *Hide* [1978], the upper boundary of the magnetoactive region of Jupiter: is it found in the internal molecular shell of Jupiter, as is suggested in this paper, or in the region of metallic hydrogen, as earlier investigators presumed? Again, of no less importance is the sign of the secular variation of the dipole field of Jupiter: does it coincide with the sign of the secular variation of the contemporary dipole of the geomagnetic field which is decreasing in intensity? Is there synchronism in these phenomena, which can be expected within the precession dynamo model?

It is likely that the Mars 96 spacecraft will provide information on the paleomagnetism of Mars [*Dolginov*, 1988].

It cannot be overlooked that over the last decade and a half, a number of authors concerned with the problem of the motive forces and energy source of the geomagnetic field favored the mechanism of gravitational separation of material in the core, the buoyancy force, in the capacity of the engine of the geodynamo. Even those investigators who did not categorically rule out the role of external forces and precession in the dynamo mechanism of the geomagnetic field still preferred the easier to postulate mechanism of convection, the gravitational chemical differentiation of material [*Gubbins and Masters*, 1979; H. K. Moffat, personal communication, 1991].

Meanwhile, this geophysically rooted model turned out to be difficult to reconcile with the results of measurements of the magnetic fields of the terrestrial and giant planets. *Stevenson et al.* [1983], who supported this model and believed that the presence or absence of a magnetic field at a planet is unambiguously associated with the presence or absence of an inner solid core in the planet, have discussed different scenarios of thermal evolution of the terrestrial planets enabling a planet to acquire a solid inner core or denying it such a possibility. Of course, these investigations are of independent significance. The model of precession dynamo has described the magnetic states of the terrestrial planets while providing no information on the presence or absence of inner solid cores in these planets.

The role of the light and heavy components of the model of gravitational separation, if one extends it to Jupiter and Saturn, is assigned to helium and hydrogen in the metallic shells of these planets [*Loper and Roberts*, 1979]. This assumption, however, is hard to realize if the magnetoactive regions of Jupiter and Saturn are found in the molecular shell, as we have argued.

The remarkable similarities of the fields of the Earth and Jupiter are hard to reconcile with the model suggested by *Stevenson* [1984] which postulates three types of dynamo in accordance with the external energy fluxes, wherein the dynamos of the Earth and Jupiter fall into different types. Also the results of measurements of the fields of Uranus and Neptune published in 1987–1991 do not agree with this model: the former has a field of a higher intensity with a hardly noticeable heat flux, and the latter, on the contrary, has a field of a lower intensity with a considerable heat flux.

It is tempting to attach a particular significance to the generality with which the precession dynamo model describes the entire array of experimental data available to data and to view this as a corroboration of the first principle of Newton's Principia [*Kapitsa*, 1973]: "Nature never does anything wastefully; and it would be wasteful to accomplish with much what could be done with less. Nature is artless and never luxuriates in superfluous causes of things."

At the same time, there is no denying that the precession dynamo model is open for numerical and experimental modeling, first, within geophysics. The processes of coupling in the core-mantle system should be addressed first in the course of revision.

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