Universal Thermal Radiation Drag on Neutral Objects

Vanik Mkrtchian,[1] V. Adrian Parsegian,[2] Rudi Podgornik,[2,3] and Wayne M. Saslow[4]
[1]*Institute for Physical Research, Armenian Academy of Sciences, 378410 Ashtarak-2, Armenia*
[2]*Laboratory of Structural and Physical Biology, National Institutes of Health, Bethesda, Maryland, USA*
[3]*Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia*
[4]*Department of Physics, Texas A&M University, College Station, Texas 77840-4242, USA*
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1 We compute the force on a small neutral polarizable object moving at velocity $\vec{v}$ relative to a photon gas equilibrated at a temperature T. We find a drag force linear in $\vec{v}$. Its physical basis is related to that in recent formulations of the dissipative component of the Casimir force, i.e., the change in photon momentum in emission and absorption between the moving body and the stationary thermal bath. We estimate the strength of this universal drag force for different dielectric response functions and comment on its relevance in various contexts, especially to radiation-matter coupling in the cosmos.

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The residual drag force on an atomic force microscope tip close to, but not in direct contact with, a substrate *in vacuo* raises an important and fundamental question on the origin of noncontact friction [1]. Other experimental techniques also are sensitive to noncontact friction [2]. Since it became clear that the Casimir effect, a noncontact phenomenon, can lead to a dissipative drag (see [3] and references therein), it has become a primary focus of theoretical research [4]. Such Casimir dissipative drag occurs when electromagnetic field fluctuations equilibrate in a specific reference frame, relative to which another system (e.g., a dielectric or a conducting body) is in uniform motion [3,5–7]. Such relative motion can involve *different* bodies, as for two conducting plates with relative motion in the parallel direction, for a neutral body moving relative to a conducting plate, or for two harmonic oscillators in relative motion [3,8–13]. In these cases, the radiation equilibrates within one of the bodies, with the friction depending upon the proximity of the other.

In the present work, we show that similar friction can also occur when a *single* body moves relative to a thermal bath of the electromagnetic field excitations, such as those between the walls of an oven or in the cosmic microwave background. However, in this case the coupling between field and matter, essential for the Casimir effect, is provided by the reference frame of the field excitations. The friction has no position dependence, i.e., it is spatially homogeneous. The consequence is a universal dissipative drag acting on all matter in relative motion with respect to a thermalized photon gas. To estimate the magnitude of this universal drag, we evaluate it as a function of the dominant frequency of the electromagnetic response of the body for dielectrics and conductors.

Consider the Lorentz force

$$\vec{F} = \int d^3\vec{r}(\rho\vec{E} + \vec{j} \times \vec{B}) \tag{1}$$

on a dielectric in fields $\vec{E}(\vec{r}, t), \vec{B}(\vec{r}, t)$. The charge and the current densities in the dielectric are set by the polarization, $\vec{P}(\vec{r}, t)$, such that $\rho(\vec{r}, t) = -\vec{\nabla} \cdot \vec{P}$ and $\vec{j}(\vec{r}, t) = \partial_t \vec{P}$, where $\rho, \vec{j}$ must obey the continuity equation $\partial_t \rho + \vec{\nabla} \cdot \vec{j} = 0$. Assume that the response of matter to the field is both linear as well as spatially local. Thus, we write in MKS units

$$\vec{P}(\vec{r}, t) = \epsilon_0 \int \chi_e(t - t')\vec{E}(\vec{r}, t')\, dt', \tag{2}$$

where the dielectric susceptibility in the frequency domain $\chi_e(\omega)$ is dimensionless. In condensed matter $\chi_e(\omega) = \epsilon(\omega) - 1$. For weakly interacting molecules or atoms $\chi_e(\omega) = \rho_N \alpha_m(\omega)/\epsilon_0$, with $\rho_N = N/V$ the atomic or molecular number density of the medium and $\alpha_m(\omega)$ the polarizability of a single molecule or atom.

With the polarization proportional to the field, the force is bilinear in the field. This bilinearity holds for objects moving at arbitrary nonrelativistic velocities relative to the frame of reference of the thermal bath. We require the thermal average of the force acting on a moving body in unbounded space filled with radiation at rest. Our reference frame is that where the particle is instantaneously at rest and the photon gas moves with velocity $\vec{v}$. The average force is obtained in terms of the thermal averages of the Fourier components of the field correlations. In unbounded space, the Fourier decomposition of the polarization and electric fields is

$$\vec{P}, \vec{E}(\vec{k}, \omega) = \int dt \int d^3r\, \vec{P}, \vec{E}(\vec{r}, t) e^{-i(\vec{k}\cdot\vec{r} - i\omega t)}. \tag{3}$$

Application of Maxwell's equations in Fourier space leads to the following form of the thermal average $\langle \cdots \rangle$ of the Lorentz force:

$$\langle \vec{F} \rangle = -i\epsilon_0 \int d^3\vec{r} \int \frac{d^3\vec{k}\, d\omega}{(2\pi)^4} \int \frac{d^3\vec{k}'\, d\omega'}{(2\pi)^4} e^{-i(\vec{k}+\vec{k}')\cdot\vec{r}+i(\omega+\omega')t} \times \chi_e(\omega) \left\langle \frac{\omega}{\omega'} \{\vec{k}'[\vec{E}(\vec{k},\omega)\vec{E}(\vec{k}',\omega')] - [\vec{k}'\cdot\vec{E}(\vec{k},\omega)]\vec{E}(\vec{k}',\omega')\} \right\rangle, \quad (4)$$

where we take note of the fact that the electric field has no sources in empty space. A slight generalization of the standard expression for the thermal average of the correlator of the electric field vectors (Eq. 77.12 of Ref. [14]) yields

$$\langle E_i(\vec{k},\omega)E_j(\vec{k}',\omega')\rangle = (2\pi)^4 \delta^3(\vec{k}+\vec{k}')\delta(\omega+\omega')\langle E_iE_j\rangle_{\vec{k},\omega}. \quad (5)$$

Here

$$\langle E_iE_j\rangle_{\vec{k},\omega} = \frac{2\pi^2\hbar}{\epsilon_0 k}\left(\frac{\omega^2}{c^2}\delta_{ij} - k_ik_j\right)\left[\delta\left(\frac{\omega}{c}-k\right) - \delta\left(\frac{\omega}{c}+k\right)\right] \times [1 + 2n(\omega,\vec{k})], \quad (6)$$

where $k = |\vec{k}|$ and

$$n(\omega,\vec{k}) = \frac{1}{e^{\beta\hbar(\omega-\vec{k}\cdot\vec{v})} - 1}$$

is the Bose occupation number for a photon distribution at temperature T, moving with velocity $\vec{v}$, and $\beta = (k_BT)^{-1}$. Equation (6) generalizes the fluctuation-dissipation theorem to a translationally invariant system, and takes a Gibbs distribution corresponding to a photon gas moving with velocity $\vec{v}$, just as is done for excitations in superfluidity [15]. From Eq. (6), we find the average of the Lorentz force

$$\langle \vec{F} \rangle = -i\epsilon_0 \int d^3\vec{r} \int \frac{d^3\vec{k}\, d\omega}{(2\pi)^4} \chi_e(\omega)\vec{k}\langle \vec{E}\cdot\vec{E}\rangle_{\vec{k},\omega}. \quad (7)$$

Substitution of Eqs. (5) and (6) into Eq. (4) with $\vec{v} = 0$ gives an integral over $\vec{k}$ that, by symmetry, is zero. To obtain the first nonzero term, it is convenient to eliminate first the delta functions by integrating over ω, and then to expand in powers of $\vec{v}$ (as is done to obtain the normal fluid density in the theory of superfluidity). We find

$$\langle \vec{F} \rangle = -i2\pi\hbar cV \int \frac{d^3\vec{k}}{(2\pi)^3} k\vec{k}\chi_e(ck)[1 + 2n(ck,\vec{k})] + i2\pi\hbar cV \int \frac{d^3\vec{k}}{(2\pi)^3} k\vec{k}\chi_e(-ck)[1 + 2n(-ck,\vec{k})], \quad (8)$$

where we have taken the dielectric to be homogeneous so that the integral over the volume of the moving body simply gives V. Alternatively, by following the arguments of Volokitin and Persson [11], one can start from the rate of dissipation of energy in the rest frame of the fluctuating field,

$$\frac{dW}{dt} = \int d^3\vec{r}(\vec{j} + \rho\vec{v})\cdot\vec{E} = \frac{dW_0}{dt} - \vec{F}\cdot\vec{v}. \quad (9)$$

Here we take into account that in the rest frame of the thermalized photon gas the total electric current is given via the linearized Lorentz form $\vec{j} \to \vec{j} + \rho\vec{v}$. Apart from the heat production $(dW_0)/(dt)$ in the frame of the body, Eq. (8) is reproduced immediately.

We now write, in a standard way, $\chi_e(\omega) = \chi_e'(\omega) + i\chi_e''(\omega)$, noting that the real part is an even and the imaginary part is an odd function of the argument. The expression for the force can now be written as

$$\langle \vec{F} \rangle = -i\frac{4\pi\hbar cV}{(2\pi)^3}\int d^3\vec{k}\, k\vec{k}\{\chi_e'(ck)[n(ck,\vec{k}) - n(-ck,\vec{k})] + i\chi_e''(ck) \times [1 + n(ck,\vec{k}) + n(-ck,\vec{k})]\}. \quad (10)$$

Expanding the Bose occupation number to the lowest order in velocity, we have $n(ck,\vec{k}) - n(-ck,\vec{k}) = \coth\frac{1}{2}\beta\hbar ck + \mathcal{O}(v^2)$ and $1 + n(ck,\vec{k}) + n(-ck,\vec{k}) = \frac{1}{2}\mathrm{csch}^2\frac{1}{2}\beta\hbar ck[\beta\hbar(\vec{k}\cdot\vec{v})] + \mathcal{O}(v^3)$. Placed in Eq. (10), the $\vec{k}$-space integral over the term in $\chi_e'(ck)$ is zero, by symmetry. The remaining term then yields

$$\langle \vec{F} \rangle = (8\pi\beta\hbar^2c)V \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{k\vec{k}(\vec{k}\cdot\vec{v})\chi_e''(ck)}{\sinh^2\frac{1}{2}\beta\hbar ck}. \quad (11)$$

Performing the angular integral over $d^3\vec{k}$ gives a factor of $4\pi/3$. Reverting again to ω by substituting $k = \omega/c$ gives

$$\langle \vec{F} \rangle = V\vec{v}\left(\frac{\beta\hbar^2}{3\pi c^5}\right)\int_0^\infty d\omega \frac{\omega^5\chi_e''(\omega)}{\sinh^2(\frac{1}{2}\beta\hbar\omega)}. \quad (12)$$

This is our fundamental result. EM field fluctuations exert a drag, proportional in the lowest order to the velocity, on a particle that moves with respect to the frame of reference in which the EM field fluctuations are thermalized.

Setting this force to $M\vec{v}/\tau$, where M is the total mass of the object and $1/\tau$ is the drag time, and using $\rho_M = M/V$, yields the result that

$$\frac{1}{\tau} = \left(\frac{\beta\hbar^2}{3\pi\rho_M c^5}\right)\int_0^\infty d\omega \frac{\omega^5\chi_e''(\omega)}{\sinh^2(\frac{1}{2}\beta\hbar\omega)}. \quad (13)$$

We now consider Eq. (13) in three different contexts: molecules, dielectric, and conducting condensed matter. First, assume that the dielectric response of the medium can be characterized by a single sharp absorption line at ω_0. Because $\frac{1}{\tau}$ is proportional to $\chi_e''(\omega)$, obviously each of the absorption lines for a molecule or a dielectric will contribute additively to the integral in Eq. (13). We set $\chi_e''(\omega) = \chi_0\delta(\omega/\omega_0 - 1)$, where $\chi_0 = \rho_N\alpha_m/\epsilon_0$

characterizes the strength of the absorption. This assumption, with $x = \frac{1}{2}\beta\hbar\omega_0$, gives

$$\tau = \left(\frac{3\pi\rho_M c^5 \hbar^4}{2^6 \chi_0 (k_B T)^5}\right)\frac{\sinh^2(x)}{x^6}. \quad (14)$$

In this form, the relaxation time depends strongly on the absorption frequency ω_0 as shown in Fig. 1. The relaxation time has a minimum at a temperature dependent frequency (see Fig. 1) that coincides with the minimum of the function $f(x) = [\sinh^2(x)]/x^6$, at $x_m = 2.98$, where $f(x_m) = 0.137$. Taking this minimum at x_m into account, the smallest possible relaxation time can thus be obtained from the above equation in the form

$$\tau = \frac{C}{T^5}, \quad (15)$$

where $C = \{[3\pi f(x_m)]/2^6\}[(\rho_M c^5 \hbar^4)/(\chi_0 k_B^5)]$. At this minimum, the absorption frequency $\hbar\omega_0 = 2x_m k_B T$ is proportional to the first Matsubara frequency. The temperature dependence of this minimal possible relaxation time is the upper curve in Fig. 2.

A different formula for τ is obtained for metals, which have constant conductivity at frequencies below the collision time of their charge carriers. In this case $\chi_e(\omega) \approx -[\sigma/(i\epsilon_0\omega)]$. Inserting this into Eq. (13), the inverse relaxation time for drag now takes the form

$$\frac{1}{\tau} = \left(\frac{\beta\hbar^2\sigma}{3\pi\rho_M c^5 \epsilon_0}\right)\int_0^\infty d\omega \frac{\omega^4}{\sinh^2(\frac{1}{2}\beta\hbar\omega)}, \quad (16)$$

so that

$$\tau = \frac{C'}{T^4}, \quad (17)$$

with $C' = (\epsilon_0/\sigma)[(45c^5\hbar^3\rho_M)/(16\pi^2 k_B^4)]$. For most common metals, the value of ϵ_0/σ is between 10^{-19}–10^{-17} s. For $\epsilon_0/\sigma = 10^{-18}$ s, the temperature dependence of the relaxation time is given in Fig. 2.

The times given in the figures are relatively long, corresponding to the general weakness of this universal thermal drag. Two circumstances under which such long times might be observable are ovens and the cosmos.

Tungsten ovens can operate at temperatures as high as 3000 K. If an atom or ion or molecular beam passes through such an oven, it will be subject to drag due to this mechanism, wherein a thermal photon is absorbed and then reemitted. Figure 2 shows that the drag time should be very long, and correspondingly difficult to observe. On the other hand, atom or ion traps might be instructive in this context. Presently, atoms can only be trapped at milli-Kelvin temperatures. However, ions with temperatures as high as keV can be trapped. Ba^+ has a resonance near 500 nm, or 2 eV, which is about 6 times the thermal energy associated with a 3000 K oven. For this resonance, the drag time would be near the minimum in Fig. 1, and corresponds to about a day ($\approx 10^5$ s). The associated quality factor Q is about 10^5 s$/10^{-16}$ s $= 10^{21}$, which is very large. In reality, an oven, or cavity, does not support a continuum of frequencies, but has a density of states with discrete resonances. Although the radiation is very low between the cavity resonances, on resonance the intensity can be higher than in free space by perhaps a factor of 10^8. However, in that case the ion line would have to be represented more accurately than by a δ function in frequency, and would yield different results than our Eq. (15).

For the cosmos, it is believed that hydrogen atoms condensed from protons and electrons when the radiation cooled to about 3000 K, and that the coupling of cosmic radiation and matter due to Compton scattering becomes

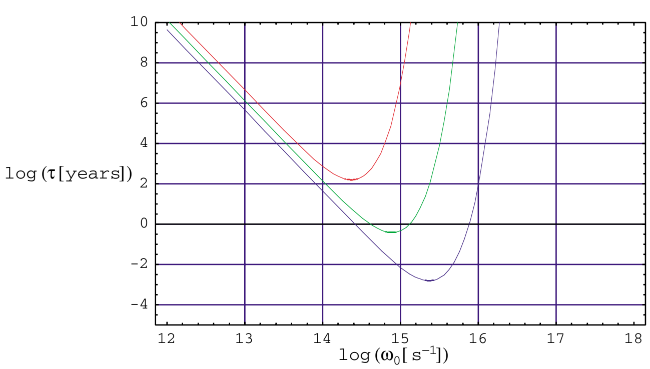

FIG. 1 (color online). Relaxation time (in years) vs the primary relaxation frequency [ω_0 in Eq. (14)] for three different temperatures, $T = 300$ K, $T = 1000$ K, and $T = 3000$ K (upper, middle, and lower curves). The frequency at the minimum is $\hbar\omega_0 = 5.9694 k_B T$. We take ρ_M to be the mass density of water and $\chi_0 \approx 1$. Numerically this value corresponds to a single particle of molecular polarizability $\alpha_m/\epsilon_0 \approx 1.0 \times 10^{-30}$ m^3 and a proton mass 1.67×10^{-27} kg.

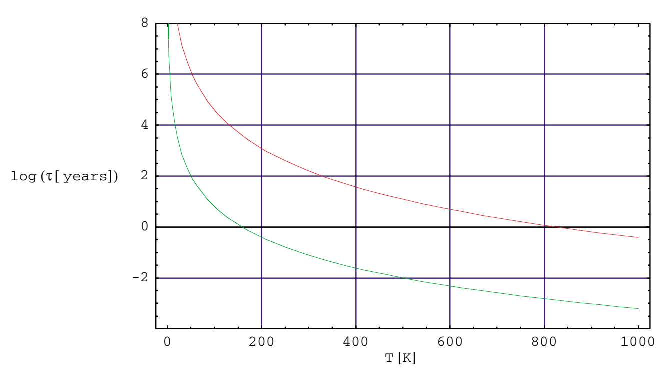

FIG. 2 (color online). Relaxation time (in years) vs temperature. For the dielectric response (upper curve), Eq. (15), we plot the minimal relaxation time obtained for the relaxation frequency, which is proportional to the first Matsubara frequency. The values χ_0 and α_m are the same as in Fig. 1. For metallic response (lower curve), Eq. (17), we take the characteristic time $\epsilon_0/\sigma \sim 10^{-18}$ s, well within the range of conductivities of simple metals.

ineffective below this condensation temperature [16]. However, as seen in Fig. 1, atoms, ions, and molecules with absorption in the appropriate frequency range should remain coupled to the cosmic radiation as its temperature drops from the 3000 K condensation temperature to perhaps 300 K or even a bit less. This coupling could influence the structure and anisotropies observed in recent experiments on the cosmic microwave background [17]. It could also influence the behavior of molecules formed from the residue of novas and supernovas, and then subject to drag from a still-hot cosmic microwave (i.e., electromagnetic) background. At much lower temperatures, macroscopic bodies can coalesce, in which case geometrically determined resonances may become relevant.

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