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AN ACOUSTIC ANALOG OF CASIMIR EFFECT
SEMINAR

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Kazalo

1 Casimir effect

1.1 A history of Casimir force

The understanding of the nature of the force between molecules has a long history. We will start with Wan der Waals. It was early recognized, that the ideal gas laws of Boyle and Gay-Lussac could be explained by the kinetic theory of noninteracting point molecules. This laws were hardly exact. Van der Waals found in 1873 that signification improvements could be effected by including a finite size of the molecules weak forces between the molecules. At the time, these forces were introduced by placing two parameters in the equation of state.

$$\left(p + \frac{a}{V^2}\right)(V - b) = NkT$$

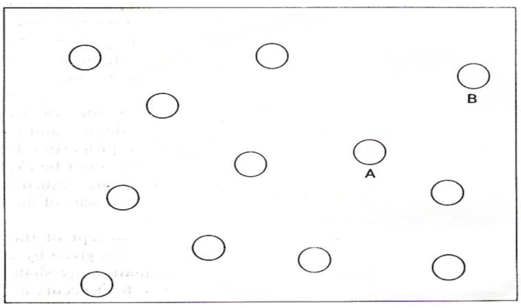

Fig. 1. The molecul B near the wall will have less kinetic energy then molecule A in the interior, because of attraction by other moleculec.
Cohesive forces between the wall and B are ignored

It required the birth of quantum mechanics to begin to understand the origin of atomic and interatomic forces. An early analysis of the force between pair of non–polar molecules was by Wang (1927) who used perturbation theory to solve the Schrödinger equation for two hydrogen atoms at large separation including the interactions between the elektrons and protons of the two atoms, and found that the interaction energy is given by

$$V(R) = -8,7\frac{e^2a^5}{R^6}$$

Eisenschitz and London (1930) analysed the same problem and came to the same kind of result, with a factor of the order of $6,5$ instead of $8,7$.

In 1930 London showed, that the force between molecules possessing electric dipol moments falls off with the distance r between the molecules as $1/r^6$. The simple argument goes as follows: The interaction Hamiltonian of a dipole momend $\vec{d}$ with an electric field $\vec{E}$ is $H = -\vec{d}.\vec{E}$. From this, one sees that the interaction energy between two such dipoles, labelled 1 and 2, is

$$H_{int} = \frac{(\vec{d}_1.\vec{d}_2)r^2 - 3(\vec{d}_1.\vec{r})(\vec{d}_2.\vec{r})}{r^5}$$

where $\vec{r}$ is the relative position vector of the two dipoles. Now in first order of perturbation theory, the energy is given by $< H_{int} >$, but this is zero because the dipoles are oriented randomly, $< \vec{d}_i >= 0$. A nonzero result first emerges in second order,

$$V_{eff} = \sum_{m\neq 0} \frac{\langle 0|H_{int}|m\rangle \ \langle m|H_{int}|0\rangle}{E_0 - E_m}$$

which evidently gives $V_{eff} \sim r^{-6}$. This is a short distance electrostastic effect. This London's analysis provide the framework for working out the force between two large dielectric materials by summing the London forces between the molecules in one and the molecules in the other.

In 1937 Casimir and Polder included retardation. They found that at large distances the interaction between the molecules goes like $1/r^7$. This result can be understood by a simple dimensional argument. For weak electric fields $\vec{E}$ the relation between the induced dipole moment $\vec{d}$ and the electric field is linear (isotropy is assumed for convenience),

$$\vec{d} = \alpha\vec{E}$$

where the constant of proportionality α is called the polarizabiility. At zero temperature, due to fluctuations in $\vec{d}$, the two atoms polarize each other. Because of the fallowing dimensional properties:

$$[d] = eL, \qquad [E] = eL^{-2}, \qquad \Rightarrow [\alpha] = L^3$$

where L represents a dimension of lenght, we conclude that the effective potential between the two polarizable atoms has the form

$$V_{eff} \sim \frac{\alpha_1\alpha_2}{r^6}\frac{\hbar c}{r}$$

while at high temperature the $1/r^6$ behavior is recovered

$$V_{eff} \sim \frac{\alpha_1\alpha_2}{r^6}kT, \qquad T \to \infty$$

The London result is reproduced by noting that in upper arguing, we implicitly assumed that $r \ll \lambda$, where λ is a characteristic wavelenght associated with the polarizabillity, that is

$$\alpha(\omega) \approx \alpha(0) \text{ for } \omega < \frac{c}{\lambda}$$

In the oposite limit is

$$V_{eff} \sim \frac{\hbar}{r^6}\int_0^\infty d\omega\alpha_1(\omega)\alpha_2(\omega), \quad r \ll \lambda.[2]$$

1.2 A derivation of the Casimir force

The origin of the dispersion force between two molecules is linket to a process which can be described as the induction of polarization on one due to the instantaneous polarization field of the other, and the value of the dispersion interaction energy is the exspectation value of the corresponding interaction term in the Hamiltonian. Since the interaction occurs through the electromagnetic field, it stands to reason that an alternativ viewpoint could be developed, accordind to which the dispersion interaction of a pair of molecules could be considered to be due to the effect of the pair on the energy of the electromagnetic field.

Historically, this approach was developed in a series of papers by Casimir (1949, 1949) and by Casimir and Polder (1946, 1948). An important result obtained by Casimir and Polder using quantum electrodynamics was that the dispersion interaction energy between a pair of molecules at a distance larger then the wavelenght of the radiation due to dipolar transitions in them falls off as $(1/R^7)$, according to the formula

$$E(R) = -\frac{23}{4\pi}\hbar c\frac{\alpha_1(0)\alpha_2(0)}{R^7} \tag{1}$$

The quantum elektrodynamic approach developed by Casimir and Polder (1948) was also formulated by Casimir in semi–classical terms, in which the interaction energy can be defined as the change in the zero–point energy of the electromagnetic field modes (obtained by solving Maxwell's equations) when the latter are perturbed by the molecules through coupling of the field with polarization currents induced on the molecules. This kind of semi–classical approach has attracted renewed interest lately in the study of problems involving interaction of radiation with matter (Scully and Surgent, 1972) with the development of lasers. We shall restrict our considerations here within the semi–classical approach, in view of the considerable simplification in the mathematical aspects of the framework over the approach based on quantum electrodynamics. A simple illustration of the method is in the derivation of the force between two perfectly conducting metallic plates by Casimir (1948). Consider two perfectly conducting plates separated by distance l along the z-direction, with the (x, y)-axes lying on one of them. The modes of the electric fields can be written as

$$\vec{E}(k_1, k_2, n) = \vec{E}_0 e^{i(k_1 x + k_2 y)} \sin \frac{n\pi z}{l} \tag{2}$$

where k_1 and k_2 are the wave numbers of propagating waves along the x- and y-directions respectively, and $(n-1)$ is the nubmer of standing wave modes along the z-direction. The corresponding frequency is

$$\omega(k_1, k_2, n) = c\Big(k_1^2 + k_2^2 + \frac{n^2\pi^2}{l^2}\Big)^{1/2} \tag{3}$$

In addition, there will be one mode for $n = 0$. For other integral values of n there will be two modes corresponding to two polarizations. The interaction energy per unit area between the plates can be defined as

$$\begin{aligned} E(l) &= \frac{\hbar}{2} \sum_{k_1, k_2, n} \Big(\omega(k_1, k_2, n) - \lim_{l\to\infty} \omega(k_1, k_2, n)\Big) \\ &= \frac{\hbar c}{(2\pi)} \int_0^\infty \kappa d\kappa \Big({\sum_{n=0}^{\infty}}' \sqrt{\kappa^2 + n^2\pi^2/l^2} - \int_0^\infty \sqrt{\kappa^2 + n^2\pi^2/l^2}\, dn\Big) \end{aligned} \tag{4}$$

where the prime is meant to indicate that in the sum the term for $n = 0$ is to be multiplied by $1/2$. The integral over n is an estimate of the sum for large l. The individual integrals diverge but their difference does not. To extract a meaningful result we can introduce a convergence factor $e^{-(\kappa^2+n^2\pi^2/l^2)\delta^2}$ with $\delta \to \infty$ ultimately. Then we can define a function

$$\begin{aligned} S(\delta, n) &= \int_0^\infty \kappa\ d\kappa e^{-(\kappa^2+n^2\pi^2/l^2)\delta^2} \sqrt{\kappa^2 + n^2\pi^2/n^2} \\ &= \frac{1}{2} \int_{n^2\pi^2/l^2}^{\infty} du \sqrt{u} e^{-u\delta^2} \end{aligned}$$

in terms of which the above sum can be expressed. We can now use the Euler–Maclaurin formula to simplify the sum in (4) as,

$$\sum_{n=0}^{\infty}{}' S(\delta, n) - \int_0^{\infty} S(\delta, n) dn = \frac{1}{6}\Big(S'(\delta, n = \infty) - S'(\delta, n = 0)\Big)$$
$$-\frac{1}{30 \times 24}\Big(S'''(\delta, n = \infty) - S'''(\delta, n = 0)\Big) + \cdots \tag{5}$$

The limits for $n = \infty$ and $n = 0$ are taken trivially and we get for $\delta \to 0$,

$$E(l) \cong -\frac{\hbar c}{(2\pi)} \frac{2\pi^3}{l^3} \frac{1}{30 \times 24} = -\frac{\hbar c \pi^2}{720 l^3} \tag{6}$$

This would lead to an atractive force per unit area between the plates, whose value is

$$\boxed{F = -\frac{\partial E(l)}{\partial l} = -\frac{\hbar\ c\ \pi^2}{240\ l^4}} \tag{7}$$

This force arises from the change in the zero–point energy per unit volume of the electromagnetic field from the free space value to what it is when the field is confined between the two plates separated by the distance l. If the distance l is measured in microns, the numerical value of F is,

$$\mid F \mid = \frac{0,013.10^{-5}}{l^4} \frac{\mathrm{N}}{\mathrm{cm}^2} \tag{8}$$

Since no metal in nature is an ideal conductor, this result may be expected to be valid as long as l is larger than the skin–depth, i.e., the penetration depth of electromagnetic waves, which for most metals is of the order of $0,1\ \mu$. We shall later show that a similar $(1/l^4$ force arises between two dialectric plates whwn l exceeds the characteristic absorbtion wavelenght of the media (Lifshitz, 1954, 1955).

This result is very different from the $1/l^3$ law was obtained for the force between slabs by pairwise summation of London $(1/R^6)$ force between the constituent molecules in the slabs. The dispersion force between a pair of molecules in this retarded region, where the finite velocity of propagation of electromagnetic interactions begins to be felt, has a different character from the London $(1/R^6)$ force. The force between two plates can also be derived by a different approach which emphasizes the surface current fluctuation as the source of radiation (Mitchell, Ninham and Richmond, 1972). [1]

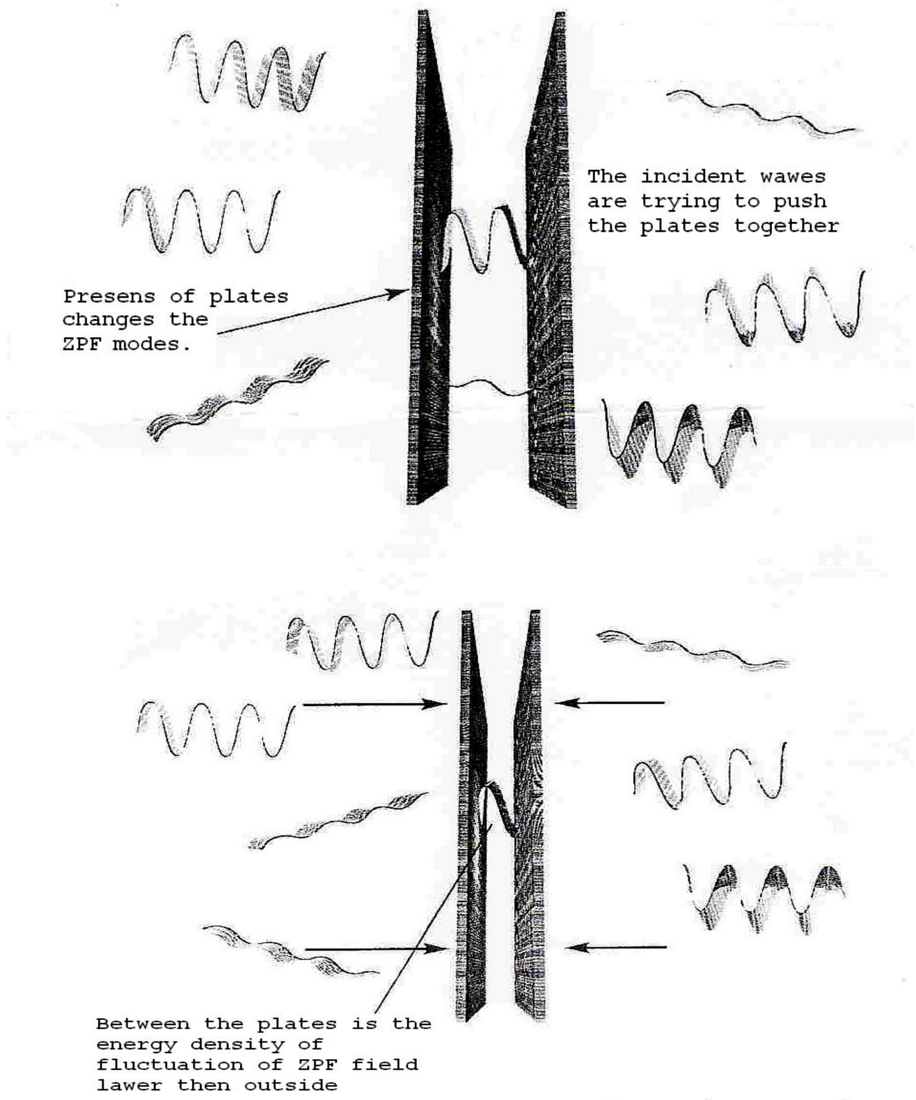

Fig.2-3. To the Casimir effect

There are many ways in which the Casimir effect may be computed. Perhaps the most obvious is to compute the zero–point energy in the presence of the boundaries. And the force between the two conducting rigid parallel plates is just one of the manifestation of Casimir effect. There are some others and his aplications:

- Casimir force between parallel dielectrics
- Casimir effect with perfect spherical bondaries
- Casimir effect of a dielectric ball
- Casimir effect in cylindrical geometries
- Casimir effect in two dimensions
- Casimir effect on a D- dimensional sphere
- Cosmological implications of the Casimir effect
- Sonoluminiscence and the dynamical Casimir effect
- Casimir force in nematic liquid crystals

Aboud this last, it was measured by our professors P.Ziherl, R. Podgornik, and S. Žumer and published in *Physical Review Letters* in 1999 under the title *Wetting–Driven Casimir Force in Namatic Liquid Crystals.*

1.3 Experimental Verification of the Casimir Effect

Attempts to measure the Casimir effect between the solid bodies date back to the middle 1950s. The early mesurements were, not surprisingly, somewhat inconclusive. The Lifshietz theory for zero temperature , was, however, confirmed accurately in the experiment of Sabisky and Anderson in 1973. So there could be no serious doubt of the reality of zero–point fluctuation forces.

New techological developments allowed for dramatic improvements in experimental techniques in recent years, and therebly permitted nearly direct confirmation of the Casimir force between parallel conductors. First, in 1996 Lameroux used a electromechanical system based on a torsion pendelum to mesure the force between a conducting plate and a sphere in the $0,6$ to $6\ \mu$m. The force per unit area is no longer of

$$F = -\frac{\pi^2}{240 l^4}\hbar c = -1,3.10^{-27}\ \mathrm{Nm^2 l^{-4}}$$

but may be obtained from that result by the proximaty force theorem, which here says the attractive force F between a sphere of radius $R$and a flat surface is simply the circumference of the sphere times the energy per unit area for parallel plates

$$F = 2\pi R E(l) = -\frac{\pi^3}{360}\frac{R}{l}\frac{\hbar c}{l^2}; \qquad R > l,$$

where l is the distance between the plate and the sphere at the point of closest approach, and R is the radius of curvature of the sphere at the point. Lameroux in 1996 claimed an agreement with this theoretical value at the 5% level. [2]

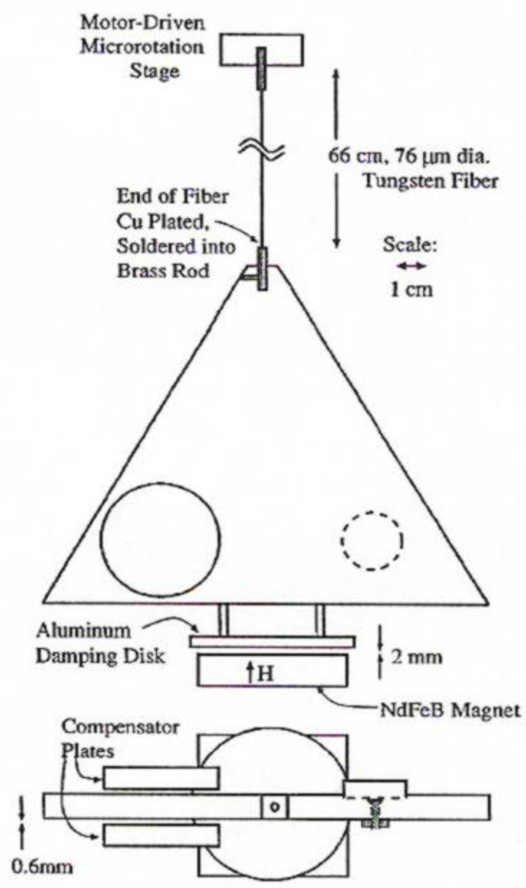

Fig.4. Details of the Lameroux pendulum. The bodi vas total mass 397 g. The ends of the W fiber were plated with a Cu cyanide solution; the fiber ends were bent into hairpins of 1 cm lenght and than soldered into a 0,5 mm diam, 7 mm deep holes in the brass rods. Flat–head screws were glued to the backs of the plates; a spring and nut held the plates firmly against their supports end ensured good el. contact [7].

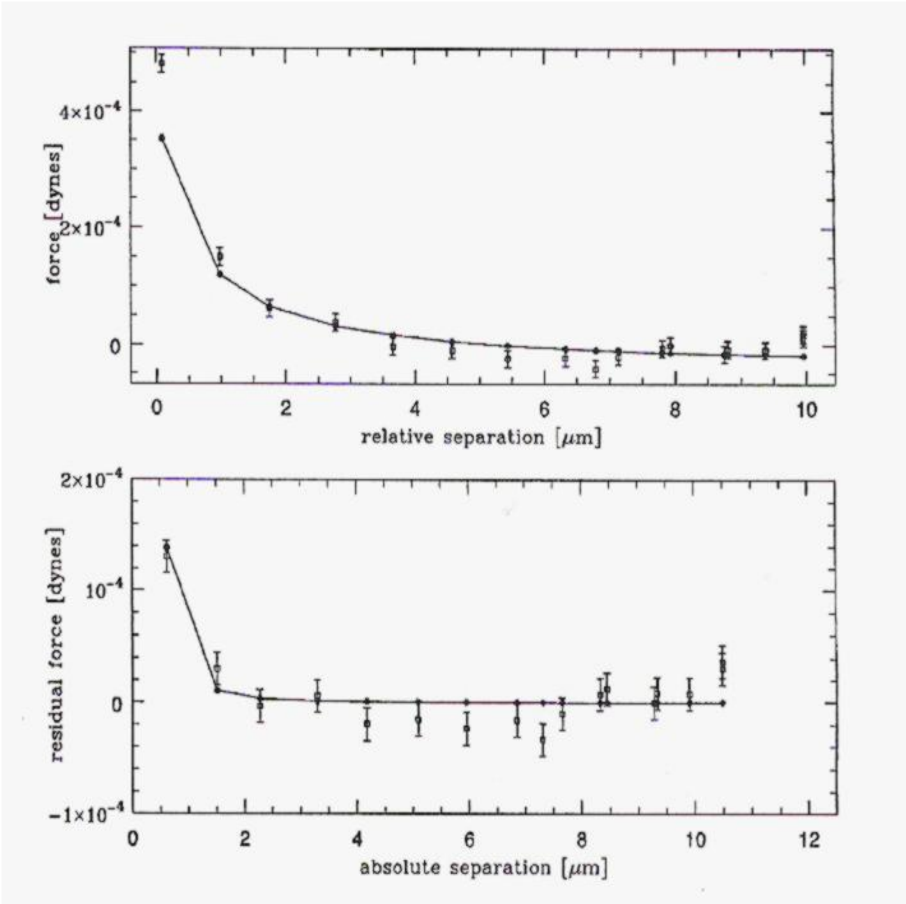

Fig.5-6. Top: Lameroux measured force as a function of relative position. Bottom: Measured force with electric contribution subtracted; the points connected by lines are as expected from the Casimir force [7].

An improved experimental mesurement was reported in 1998 by Mohideen and Roy, based on the use of an atomic force microskope in the range from $0,1$ to $0,9$ μm. They included finite conductivity, roughness, and temperature corrections, although the latter remains beyond experimental reach. Spectacular agreement with theory at the 1% level was attained.

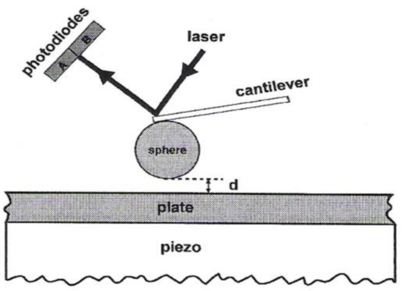

Fig. 7. Shematic diagram of ekperimental setup.

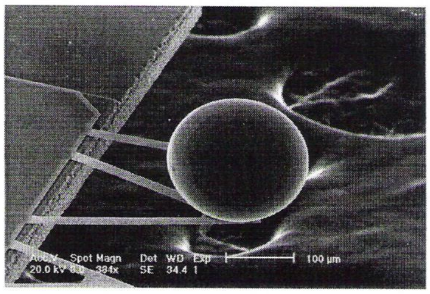

Fig. 8. Scanning electron microskope image of the metallized sphere

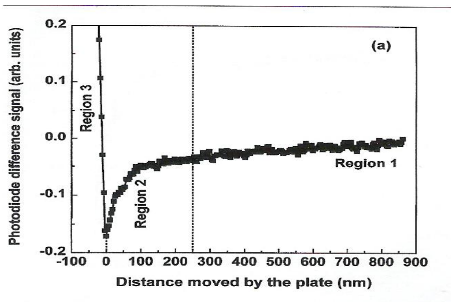

Fig. 9. A typical force curve as a function of the distance moved by the plate

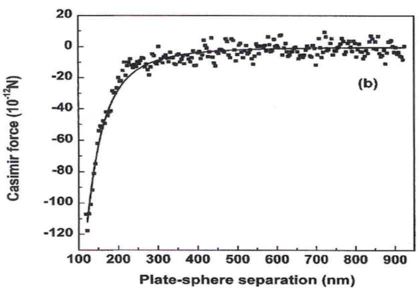

Fig. 10. The measured Casimir force as a function of sphere–plate surface separation.

Erdeth used template–stripped surfaces, and measured the Casimir forces with similar devices at separations of $20 - 100$ nm.

Very resently, a new measurement of the Casimir force has been announced by a group at Bell Labs, using a micromachined torsional device, by which they measure the attraction between a polysiliconplate and a spherical metallic surface. Both surfaces are plated with a 200 nm film of gold. The authors include finite conductivity and surface roughness corrections and obtained agreement with theory at better then 0,5% at the smallest separations of about 75 nm. Their experiment suggests novel nanoelectromechanical applications.

The recent intense experimental activity is very encouraging to the development of the field. Coming years, therefore, promise ever increasing experimental input into a field that has been dominated by theory for five decades [2].

2 An acoustic Casimir effect

2.1 Comparation between EM and acoustic Casimir effect

In the Casimir effect, two closely spaced uncharged parallel plates mutually attract because their presence changes the mode structure of the quantum electromagnetic zero point field (ZPF) relative to free space. If the plates are a distance d apart, the force per unit area is $f = \hbar/(240d^4)$, where $\hbar$ is the reduced Pnanck's constant and c is the speed of light in vakuum. Recently, Lamoreaux has provided conclusive experimental verification of the Casimir force. The force between two parallel plates can be understood in terms of the radiation pressure exerted by the plane waves that comprise the homogeneous, isotropic ZPF spectrum. In the space between the conducting plates, the modes formed by reflections off the plates act to push the plates apart. The modes outside the cavity formed by the plates act to push the plates together. The difference between the total outward pressure and the total inward pressure is the Casimir force per unit area.Because energy per mode has the of the zero point field has the same value $\frac{1}{2}\hbar\omega$ between and outside the plates, one can incorectly be led attribute the attractive character of the force as due to the fact that there are fewer modes between the plates. Surprisingly, as we show below, the force can be *repulsive* for band–limited noise.

Because the zero point field can be thought of as broadband noise of an infinite spectrum, it should be possible to use an acoustic broadband noise spectrum as an analog to at least some ZPF effects. An acoustic spetrum has several advantages. Because the speed of sound is six orders of magnitude less then the speed of light, the lenght and time scales are more mmanageable measurable. Also, in an acoustic field, the shape of the spectrum as well as the field intensity can be controlled. In this letter, i report theory and measurements of the force between two rigid parallel plates in an externally–generated band–limited noise field.

A simple calculation shows that for the same separation distance, the Casimir force, due to the electromagnetic zero point field is at least six orders of magnitude greater than the Casimir force due to zero point phonons, because the speed of light is six orders of magnitude greater then the speed of sound and because the phonon spectrum has a natural high frequency cutoff at the Debby temperature. To be distinguished from the effects of the zero point field, the Casimir–like effect due to thermal phonones and photons would require separation distances $d \gg \hbar c/2kT$ which would make the force extremely difficult to measure. Driven elecromagnetic white noise (composed of real photons) would yields forces much smaller then acoustically driven noise. The force we measure in this case are the equivalent of 30 mg, while the forces Lemeroux measured due to the actual Casimir effect are equivalent to 10 μg.

One of the key ideas in the derivation of the ZPF Casimir force is the fact that the energy per mode $\frac{1}{2}\hbar\omega$ is the same for modes both outside and between the plates, which can be understood with the adiabatic theorem. To this purpose, imagine that the plates are initially far apart so that the spectral intensity Of the ZPF is that of free space. If we now adiabatically move the walls towards each other, the modes comprising the ZPF will remain in their ground state; only their frequencies will be shiftet in such a way that the ratio of the energy per mode E to the frequency ω remains constant, or $E/\omega = \hbar/2$. Thus the main effect of the boundaries is to redistribute ground state modes of which there is an infinite number.

In the acoustic Casimir efect, in contrast to the ZPF Casimir efect, broadband acoustic noise outside two parallel rigid plates drives the discrete modes between the plates. The adiabatic theorem does not apply in this case both because of inherent losses in the system and because the spectrum can be arbitrary. In general, when the response and drive amplitudes are expressed in the same units, the response is approximately the quality factor Q multieplied by the drive ("Q amplification"). The energy per mode being the same in the ZPF Casimir effect therefore implies that the place between the plates cannot be considered as a resonant cavity unless the quality factor of each mode is unity. While external drivers can provide a steady state noise spectrum from which we can infer the energy per mode by dividing by the desity of states $\omega^2/2\pi^2c^3$, this energy may be different in the cavity formed by the plates as a result of Q amplification. However, for this open resonant cavity the quality factor is poor, so we may asume it to be equal to unity, which renders the energy per mode equal to its value in free space.

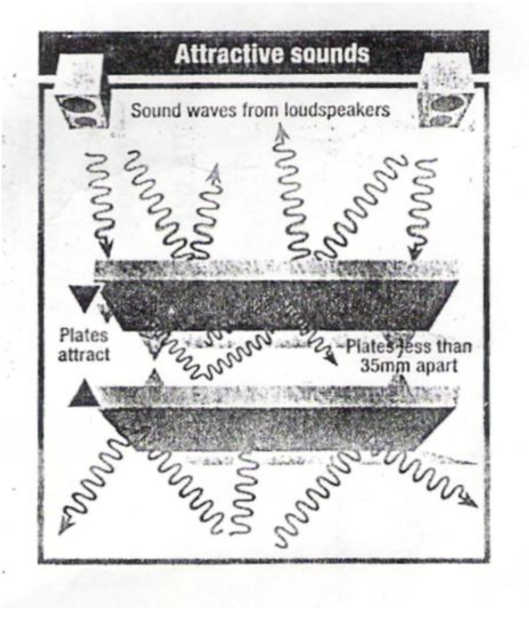

Fig. 11. The acoustic analog of Casimir force [6]

In general, the radiation pressure of the wave incident at angle θ on a rigid plate is

$$P = \frac{2I}{c} \cos^2 \theta \tag{9}$$

where I is the average intensity of the incident wave and c is the wave speed. The factor of 2 is due to perfect reflectivity assumed for the plate. Eq. (9) follows of the time–averaged second–order acoustic pressure, which equals the time–averaged potential energy density minus the time–averaged kinetic energy density. When the acoustic case is constrained to one dimension, mass conzervation yields an explicite dependence of the radiation pressure on the alasticity of the medium, characterized by γ, the ratio of specific heats, namely

$$P = (1 + \gamma)\frac{I}{c} \tag{10}$$

However, for the three–dimensional open geometry in our case, the constrained, due to the mass conservation does not apply and the acoustic and the electromagnetic expressions for the radiation pressure at a perfectly reflecting surface are identical.

With appropriate filters, one may shape the spectrum on an acoustic driver and obtain, in principle, different force law. For an isotropic noise spectrum with spectral intensity I_ω, (measured by a microphone) we can express the spectral intensity in the wavevector space of traveling waves. According to the formula

$$j_k 4\pi k^2 \; dk = j_\omega \; d\omega$$

and

$$k = \frac{\omega}{c} \qquad \frac{d\omega}{dk} = c$$

we get

$$I_k = \frac{I_\omega}{4\pi k^2}$$

We choose the z axis to be normal to the plate, so that $k_z = k \cos \theta$. From (9) the total radiation pressure due to waves that strike the plate is then

$$P_{out} = \frac{2}{c} \int dk_x dk_y dk_z I_k \cos^2 \theta \tag{11}$$

when the integration is over k values corresponding to waves that strike the plate.

Regarding the discrete modes between the plates, the for convenience we continue to deal with the traveling wave modes. We label these modes with wavevector components

$$k_x = \frac{n_x \pi}{L_x} \quad k_y = \frac{n_y \pi}{L_y} \quad k_z = \frac{n_z \pi}{L_z}$$

where n_x, n_y, n_z, are signed integers and L_x, L_y, and L_z are the dimenzions between the plates. As before, the z axis is chosen to be normal to the plates. As a result of the quality factor of the modes beeinf aproximately unity, the intensity $I_in(k)$ of each mode between the plates is expected to be approximately the same as the outside broadband intensity in a bandtwidth equal to the wavevector spacing of the inside modes:

$$I_{in}(k) = I_k \Delta k_z \Delta k_y \Delta k_z$$

where

$$\Delta k_i = \frac{\pi}{L_i}$$

In the limit of large dimensions, this expression for the inside intensity yields the correct wavevector spectral intensity I_k.

We assume that the dimensions L_x and L_y of the plates are sufficiently large that the coresponding components of the wavevectors are essentially continuous. Thus, in comparasion to (10), the total inside pressure is

$$P_{in} = \frac{2}{c} \sum \Delta k_z \int dk_x dk_y I_k \frac{k_z^2}{k^2} \tag{12}$$

where $\Delta k_z = \pi / L_z$ and the sum is over values of $n_z > 0$.

The difference between $P_{in} - P_{out}$ is the force f per unit area between the plates, which is a continuous and piecewise differentiable function of the separation distance between the plates.

$$P_{in} - P_{out} = \frac{2}{c} \int \int dk_x dk_y (\sum_n - \int dn) I_k \frac{k_z^2}{k^2}$$

It can be shown, that the force can alternate between negative (attractive force) and positive (repulsive force) values as the plate separation distance or the band–limiting frequencies are varied. On the other hand, if the lower in the band is zero, the force is always attractive.

In an experiment dealing with an acoustic analog to the Casimir effect, an important question is whether other nonzero time–averaged (dc) effects can play an omportant role. The only second order dc effects in acoustic are radiation pressure and streaming. Any other dc effect would be fourth order in the acoustic pressure, and at least 40 dB smaller in our case. Employing smoke in the apparatus descibed below, we detected no acoustic streaming when driving with broadband acoustic noise at the intensity level used in the experiment. Because the noise in our experiment can be thought as collection of monochromatic waves over a band of frequencies with randomly varying phases, we would expect very little or no streaming when the characheristic time of phase variations is less then that the diffusion time. Furthermore, because acoustic streaming is driven along the boudary from a pressure antinode to a pressure node, in the presence of broadband noise the pressure nodes and antinodes of the different noise components are dencely distributed along the boundary, thus reducing or eliminating the streaming [4].

2.2 About the apparatus

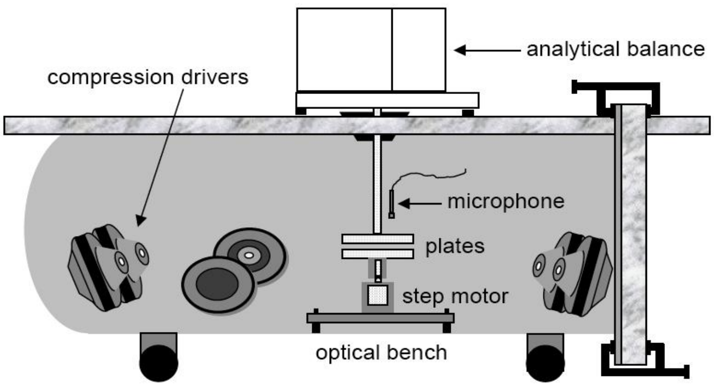

Fig. 12. The apparatus to measure the acoustic Casimir force

Two 15 cm diamet plates were used for the acoustic Casimir force measuremant. The bottom plate is 6, 35 mm thick aluminium, attachet to the top of a step motor mount. The top plate is 5, 57 mm thivk PVC which was vacuum–aluminized and hung beneath the analytical balance by an aluminium bar screwed into the plate and attached to the balance by a hook. The weight of the top plate, 168, 1653 g, is well within the 200 g maximum capacity of the balance, whose resolution is of 0, 01 mg. Both plates were grounded to a common ground to eliminate electrostatic effects, therebly minimizing fluctuations in the force measurements. The acoustical chamber was made from a 6, 35 mm thich steel propane tank. The amplified band–limited output of an analog noise source drives six compression drivers that provide the desired acoustic noise intensity within the acoustic chamber.

The step motor was mounted on a small alukinium optical bench with tree–point leg adjustments, centered in the acoustic chamber 46 cm from the tank access. The number of steps (1 to 255 steps) and direction (up and down) are varied with a microchip controller. Attached to a machined screw with 20/6, 35 threads/cm, the step motor mount yields displacement ranging from 6, 35 μm for one step (1, 8 degrees to 1, 27 mm for 200 steps (360 degrees). We employed plate spacing increments of 1, 27 mm through the full 6, 3 cm range of the step motor mount.

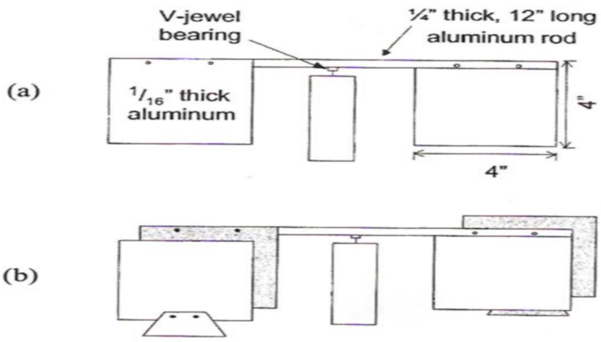

Fig. 13. The acoustic Casimir demonstration device

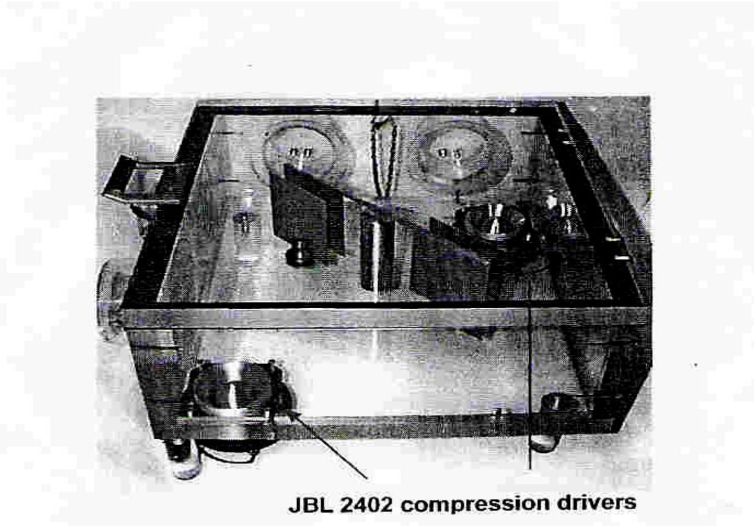

Fig. 14. The photografh of apparatus. The tank is made of 0,64 cm steel, lenght 1,5 m and diameter 0,5 m

The initialplate spacing was determined with a sparg plug gap gauge and the plate separation distance vas verified to be uniform by measurements at at different locations between the plates. Once, the balance reading in on the weight of the top plate, the balance was tared to zero and all force measurements were made relative to this zero. The acoustic noise field was turned on for each plate separation distance and the force measurement recorded once the balance readout first looked–in. The sound field was then turned off and the reading of the balance was verified to return to zero. We selected a uniform broadband noise spetrum between rougly 5 and 15 kH. The lower limit was selected well above the lower modes of the tank in order to excite the noise distribution as homogenius as possible. The upper limit was due to the compresson drivers rolling off 20 dB from 15 to 20 kH. Fig.3 shows the measured noise spectrum which, except for 5 dB variations throughout, is nearly flat within the spectral range of 4, 8 to 16 kHz. The total intensity is 133 dB (10^{-12} W/m^2) [5].

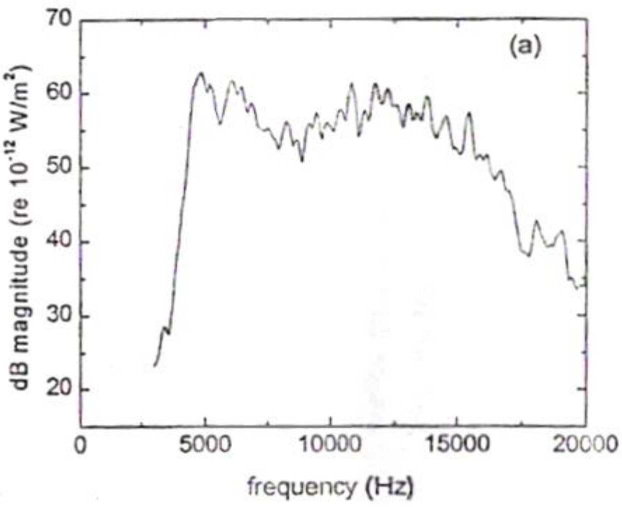

Fig. 15. The experimental spectrum

2.3 Measure resoults

Using a spark plug gouge, it was measured the initial plate separation distance to be 0, 76 mm. As shown in Fig. 16 and Fig. 18, throught a range of distances less then the smallest half wavelenght (no modes between the plates), the measured force is aproximately independent of distance, and agrees with the expected velue for an intensity level of 133 dB. When the half wavelenght of the highest frequency fits between the plates (10, 63 mm for 16 kHz), the force begins to decrease non–monotonically, and becomes repulsive at $30 - 40$ mm.

The repulsive force can be understood as follows. When the distance between the plates is comparable to the half wavelenght associated with the lower edge of the frequency band, the corresponding modes inside the plates have wavevectors that are nearly perpendicular to the plates. However, the modes outside the plates corresponding to the same frequencies are spread over all possible angles of incidence. Thus, for the same total intensity, the momentum transfer due to waves inside the plates is over a narrow cone while the momentum transfer due to waves outside the plates extends over all angles, leading to a repulsive force.

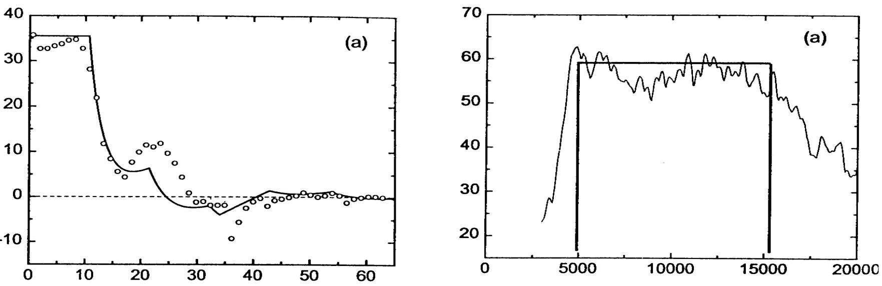

Fig. 16-17. Force between two parallel rigid plates as a function of the distance between them. The points are experimental data and the curves are from theory with no adjustable parameters for flat spectrum.

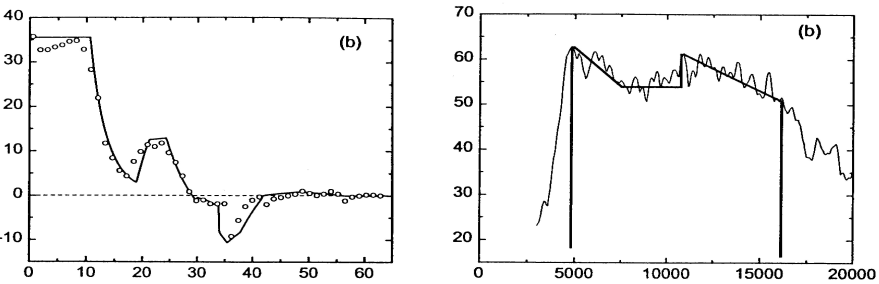

Fig. 18-19. Comparation between experimental data and the curves from theory with piecewise power–law spectrum [5]

2.4 A possible application

Experimnatl evidence of attractive and repulsive forces within a finite acoustic bandwindth suggests new means of acoustic levitation. The force between to objects can be malipulated by changing the distance between the objects or varying the spectrum. While the casimir force is small compared to the force of the Earth's gravity, in a low gravity environment a method of material control through the manipulation of an acoustic noise spectrum or plate geometry may be possible. The acoustic Casimir effect can also be a potential tool in noise transduction because of direct measurement of the force can determine the total intensity of background noise. The shape of the force over distance is effectively an instantaneous time average over all frequencies and may provide an alternative to measurements of background noise [5].

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