SEMINAR II

Complexity in classical poetry

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Abstract

In this paper I'm trying to introduce the experiment that has been made on classical Greek and Latin poetry by mapping samples of some defining literary works from that era. As we will see, it's possible to prove that syntax of the poems increases in complexity from Greek to Latin poetry.
The technique or better function (mutual information function) that has been and is used for studying symbol series (e. g. DNA) is borrowed from the information theory. In this paper we will also look at the theory behind this function and some differences with the correlation function.

Contents

1. Introduction

What we want is to study a symbolic sequence. The study of DNA has been influential in symbolic sequences studies. More exactly it has been revolving around the long-range correlation (its power spectrum scales as $1/d^{\alpha}$, where d is the distance between symbols in the sequence and $\alpha \sim 1$) in DNA. So all this stimulated the study of written languages and music with similar techniques. Most of the previous studies of letter sequences in natural languges or nucleotide sequences in DNA polymers were focused on entropy. In some cases, the nearest-neighbor correlation using conditional probabilities were studied. It has been shown that [***3***] the Mutual information function M(d) is a more suitable way to study this kind of sequences as the more frequently used correlation function Γ(d). It is even proven that there exists a relationship between these two functions for binary sequences. But this is already another story. We really have to start at the beginning, because this theory is unknown to most of us.

After introducing Mutual information function we will introduce the classical poetry itself but it's maybe a good idea to tell at this point that the whole study revolves around the verse form of HEXAMETER, that we will introduce properly further on. The whole literary work will have its syllables replaced with only three different signs; long and short syllable and pause, with which we can still describe its rhythm. But it's still to early to start with classical poetry, since we should first define and explain the tool we will use - Mutual information function.

2. Mutual information function and correlation function

First we have to define the mutual information function and renew our knowledge of the correlation function. I'm going to use one of the interpretations that have been made for M(d) (2.2), but there are other, too. We can say that all the interpretations are related to the same notion of dependence and correlation. The mutual information function is a measure of the dependence between two variables. If the two variables are independent, the mutual information between them is zero. In case the two variables are strongly dependent, e.g. one of them is a function of the other, the mutual information between them is large.

The correlation function (2.1) is another frequently used method to measure dependence. The difference between them is the dependence they measure. Correlation function measures the linear dependence, while the mutual information measures the general dependence. This difference leads to different methods in choosing the independent variables. Another difference between the two functions is that the correlation can't be applied at symbolic sequences, but only to numeric ones, while the mutual information can be applied to both. This enables us to a more complete characterization of symbolic sequences.

Now we want to define the function in a more strict way for finite sequences $\{x_i\}$ (i = 1, 2,…, N), where $x_i \in \{a_\alpha\}$(α = 1, 2,…, K), the variable set. The correlation function is

$$\Gamma(d) \equiv \sum_{\alpha}\sum_{\beta} a_\alpha a_\beta P_{\alpha\beta}(d) - \left(\sum_{\alpha} a_\alpha P_\alpha\right)^2 \qquad (2.1)$$

Both the single-site probabilities $\{P_\alpha\}$ and the joint probabilities for two sites $\{P_{\alpha\beta}(d)\}$ are accumulated from the single sequence to be analized. The (site-to-site) mutual information function is defined as

$$M(d)^{[1]} \equiv \sum_{\alpha}^{K}\sum_{\beta}^{K} P_{\alpha\beta}(d)\log\frac{P_{\alpha\beta}(d)}{P_{\alpha}P_{\beta}} \tag{2.2}$$

The block-to-block mutual information is defined as the mutual information between two L-blocks (blocks of length L), separated by distances of d. It is similar to the site-to-site mutual information function except that P_α are the probabilities for L-blocks and $P_{\alpha\beta}(d)$ are the joint probabilities for two L-blocks:

$$M(d)^{[L]} \equiv \sum_{\alpha}^{K^L}\sum_{\beta}^{K^L} P_{\alpha\beta}(d)\log\frac{P_{\alpha\beta}(d)}{P_{\alpha}P_{\beta}}$$

In this paper, we will consider mostly the site-to-site M(d) (and the superscript is dropped). For better understanding at this point, we will make an illustrated comparison between the two functions and their effect on L-block. Fig.1 shows the $M(d)^{[L]}$ (L =1,2,3,4) and $\Gamma(d)$ for the binary sequences generated by nearest-neighbor cellular automaton rule 110. What shows up is that the $\Gamma(d)$ can have negative or positive values, while the M(d) remains always non-negative. We can even see a periodicity of 14 in the sequences (the peak at d = 14).

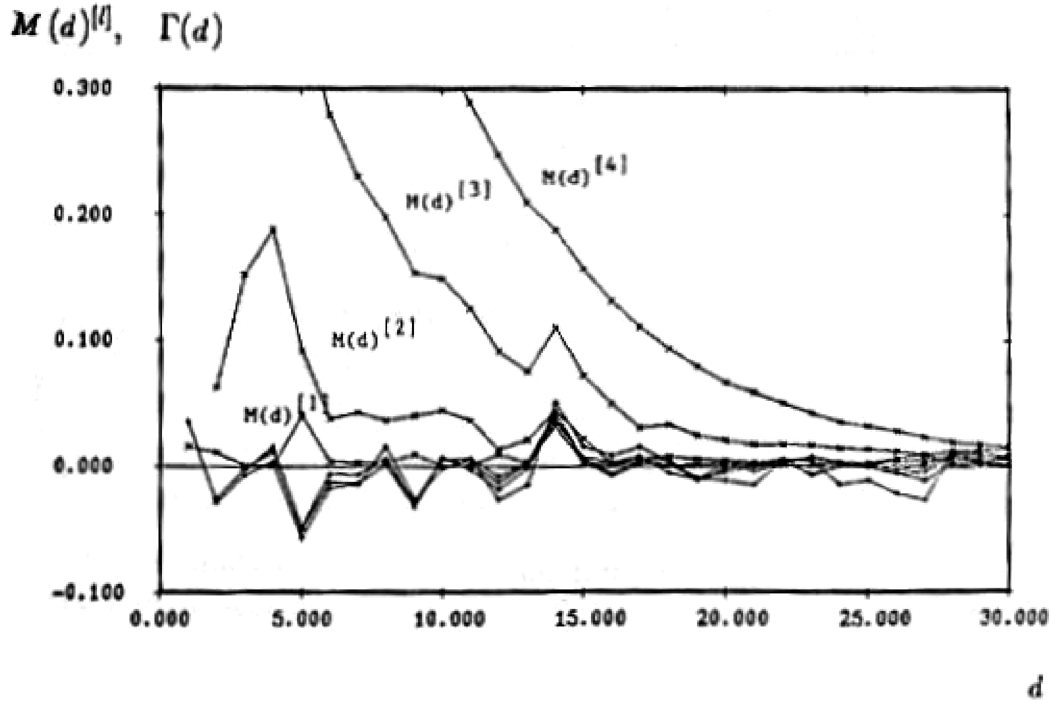


Fig.1. $\Gamma(d)$ and $M(d)^{[L]}$ (L=1, 2, 3, 4) for spatial sequences generated by cellular automaton rule 110 (or the following rule 000→ 0, 001→ 1, 010→ 1, 011→ 1, 100→ 0, 101→ 1 and 111→ 0). The sequences lenghts are N = 400, 1600, 6400 and 25,600, respectively for incresing L.

If we look at the definition of the correlation function we see that the probabilities are weighted by the variable values. The result of which is that correlation function generally is not directly related to mutual information function. As we saw before, it is possible that the value of the correlation function is equal zero at a given distance d, while the mutual information function can have a different value at that distance. But there is a special case where the joint ditribution is Gaussian and the two functions are directly related to each other.

2.1 Relation between M(d) and Γ(d) for binary sequences

In this section we will look at the case of a binary sequence and how the probabilities of the joint for two sites is reduced from four to two. The consequence is that we can directly relate the correlation function to the mutual one.
If we write the correlation function for a binary sequence:

$$\Gamma(d) = P_{11}(d) - P_1^2 ,$$

where $P_{11}(d)$ is a joint probability for having two symbol 1's separated by distance d and P_1 is the probability for having symbol 1.

If we don't take in account the different constraints, our mutual information has to be a function of all four joint probabilities. This constraints are a consequence of calculation for the two variables whose joint probabilities are extracted from the same stacionary sequence. First we can say that the sequence has no direction for the purposes of Mutal information function. And if this is true, this gives the symmetry constraint:

$$P_{\alpha\beta}(d) = P_{\beta\alpha}(d) ,$$

for $\alpha,\beta \in (0,1)$.

If we now consider the definition of the joint probability, we have

$$P_\alpha = \sum_{\beta=0}^{1} P_{\alpha\beta}(d)$$

The interesting thing about this formula is that the right-hand side of the equation is a function of distance d, while the left-hand side is not. The consequence is that the function form of the two expressions $P_{\alpha\beta}(d)$ should be such that they cancel each other's d-dependent form.

Then there is the normalization condition, but it turns out that it is equivalent to the condition $\Sigma_\alpha P_\alpha=1$ and will not provide more reductions.

If we sum up. The first constraint provides one reduction, and the second provides two reductions, so the number of independent joint probabilities is only one.

If we carry out the details, we get

$$P_{01}(d) = P_{10}(d) = P_1 - P_{11}(d)$$
$$P_{00}(d) = (1-2P_1) + P_{11}(d) .$$

In term of correlation function, these become

$$\begin{aligned} P_{11}(d) &= \Gamma(d) + P_1^2 \\ P_{00}(d) &= \Gamma(d) + P_0^2 \\ P_{01}(d) = P_{10}(d) &= -\Gamma(d) + P_0 P_1 \end{aligned} \qquad (2.3)$$

So if we now write the relation between mutual information function and the correlation function for binary sequences, we get the form:

$$M(d)=\Gamma(d)\log\frac{\left[1+\Gamma(d)/P_1^2\right]\left[1+\Gamma(d)/P_0^2\right]}{\left[1-\Gamma(d)/P_0P_1\right]^2}+P_1^2\log\left(1+\frac{\Gamma(d)}{P_1^2}\right)$$
$$+P_0^2\log\left(1+\frac{\Gamma(d)}{P_0^2}\right)+2P_0P_1\log\left(1-\frac{\Gamma(d)}{P_0P_1}\right).$$

It is possible to approximate the equation above for when the correlation function decays to zero at longer distances and both Γ(d) / ($P_\alpha P_\beta$) are small. Now if we look at this limit case, we found that only second-order terms remain:

$$M(d)\approx\frac{\Gamma(d)^2}{2}\left(\frac{1}{P_0^2}+\frac{1}{P_1^2}+\frac{1}{P_0\,P_1}\right)=\frac{1}{2}\left(\frac{\Gamma(d)}{P_0\,P_1}\right)^2.$$

We can notice from this equation that the mutual information function decays to zero faster than the corresponding correlation function. So for example, if $\Gamma(d) \sim 1/d^{\gamma}$, then $M(d) \sim 1/d^{2\gamma}$, where γ is characteristic of a spectrum. Which is important result in the study of symbolic noise.

If the sequences have more than two symbols, both functions receive additional contributions from more independent joint probability. So in these cases any relation between the two power law functions will depend on a particular assumption made about the joint probabilities.

2.2 Markov Chain and regular language

To better understand the dependence among the joint probability $P_{\alpha\beta}(d)$'s for binary sequences we will now look at some examples of Markov chain and a regular language. Logically before illustrating anything, we have to tell something about the Markov chain and its characteristics. A Markov chain is a special class of state model that includes different possible states and possible transitions from one state to another (in our examples marked with arrows). The weight assigned to each arrow is either the probability that something in the state at the arrow's tail moves to the state at the arrow's head, or the percentage of things at the arrow's tail which move to the state at the arrow's head. At each time step, something in one state must either remain where it is or move to another state. The sum of the arrows into (or out of) a state must be one. The state vector X(t) in a Markov model traditionally lists either the probability that a system is in a particular state at a particular time or the percentage of the system which is in each state at a given time. X(t) is the probability distribution vector and must sum to one.

If we now list all three properties which identify a state model as being the Markov chain: 1) The Markov assumption: the probability of one's moving from state *i* to state *j* is independent of what happened after moving to state *j* and how one got to state *i*. This probability is fixed p_{ij} and is called a transition probability. 2) Conservation: the sum of the probabilities out of a state must be one. 3)The vector X(t) is a probability distribution vector which describes the probability of the system being in each of the states at time n.

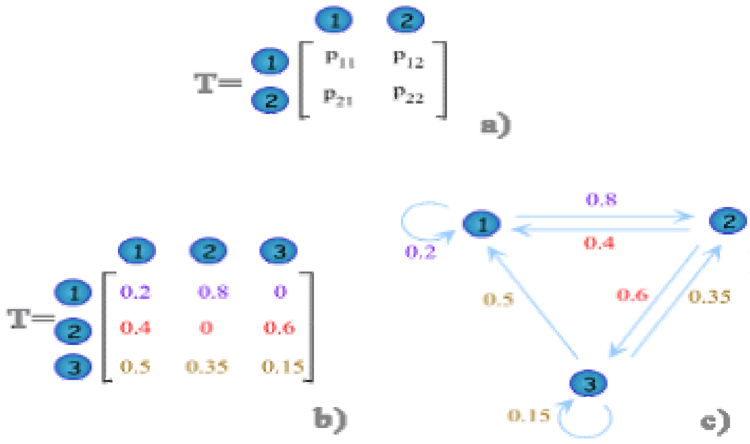


Fig. 2 a) General transition matrix for a bynary markov chain. b) Example of a transition matrix. c) The state diagram for the example of transition matrix.

A Markov system can be illustrated(Fig. 2) by a state transition diagram, which shows all the states and transition probabilities. There is also transition matrix T.

The example of Markov chain that we will need is called an Absorbing Markov chain. This kind of chain has states that are called »absorbing«. When the system enters an absorbing state, the system remains in it. We can identify an absorbing state from a state diagram in that they have loops with weight one. If we examine the structure of the transition matrix T of an absorbing chain (Fig. 3), we see it can be decomposed into blocks of the form $\begin{pmatrix} A_{2X2} & O_{2X2} \\ B_{2X2} & I_{2X2} \end{pmatrix}$.

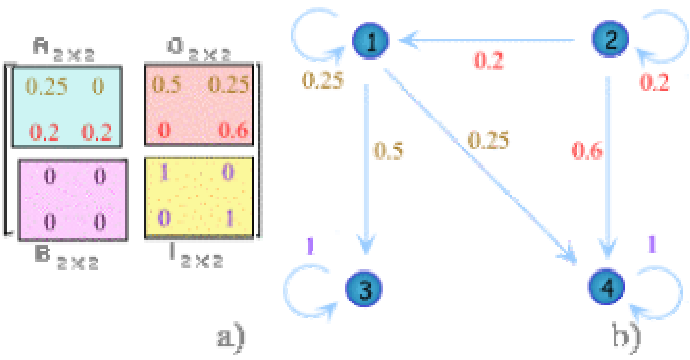


Fig. 3 a) Example of a transition matrix for an absorbing Markov chain b) The state diagram for the example (missing arrows indicate zero probability.).

In general, if a Markov chain has a absorbing states and b non-absorbing states, we can arrange the transition matrix to have the form $\begin{pmatrix} A_{bxb} & O_{bxa} \\ B_{axb} & I_{axa} \end{pmatrix}$.

Now let us look at a model of continuous stochasticity. The one we need is called »Service at the checkout counter«. For now let us start with a model of service for one customer at a check counter. At any time t there are only two possible states that the system can be in; let us them call them »being served« or »finished being served« and when an individual has finished being served, that person stays in that state (the absorbing state). Let N(t) be the number of individuals at the checkout at time *t* and here N(t) has only two possible values (0,1). The state »being served« corresponds to N(t)=1, while N(t)=0 corresponds to »finished being served«. Let us write the time-dependent transition probabilities for this system, let $X_t = \begin{pmatrix} p(t) \\ q(t) \end{pmatrix}$ be the time-dependent distribution vector for states »being served« p(t) and »finished being served« q(t); that is, $p(t) = P[N(t)=1]$ and q(t) = P[N(t)=0] .

It is because p(t) represents the probability of »being served«, we can expect that the service will be eventually completed so $\lim_{t\to\infty} p(t) = 0$, which gives $\lim_{t\to\infty} X(t) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

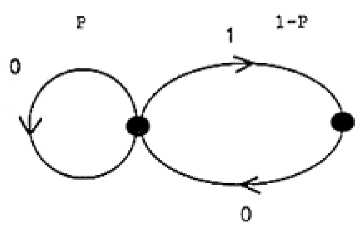
Let us now return to our examples of Markov chain in connection with the mutual information and correlation function. So we said that for Markov chains, the one-step transition probabilities $T_{\alpha\to\beta}$ are given and all the d-step transition probabilities can be derived from the one-step transition probabilities. The correlation function as well as the joint probabilities $P_{\alpha\beta}(d)$ decay exponentially with distance d.

Let us now look at a example of Markov chain with one-step transition probabilities :

$$\begin{aligned} T_{0\to 0} &= p \\ T_{0\to 1} &= 1 - p \\ T_{1\to 0} &= 1 \\ T_{1\to 1} &= 0 \end{aligned}$$

If we write this transition probabilities in a matrix, we get something similar, to what we saw before.

$$\underline{\underline{T}} = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} p & 1-p \\ 1 & 0 \end{pmatrix} \end{array},$$



(A)

(B)

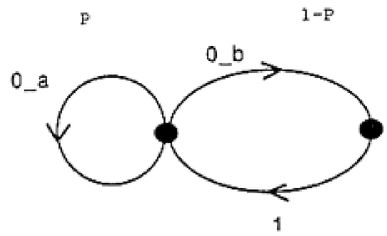


Fig.4 (A) A simple Markov chain with transition probabilities $T_{0\to 0} = p,\ T_{0\to 1} = 1-p,\ T_{1\to 0} = 1,\ T_{1\to 1} = 0$. (B) A regular language similar to the Markov chain in (A). And is an exact Markov chain by the original definition.

where $T_{\alpha\beta} = T_{\alpha\to\beta}$. The d-th power of this matrix gives the d-step transition probabilities. The joint pobabilities is then: $P_{\alpha\beta}(d) = P_\alpha(T^d)_{\alpha\beta}$. For the eigenvalue 1 the left eigenvector of the matrix gives the invariant probabilities of the symbols($P_0 = \frac{1}{2-p}, P_1 = \frac{1-p}{2-p}$). The form of

the dth power of the one-step transition matrix is :

$$T^d = \frac{1}{2-p}\begin{pmatrix} 1 & 1-p \\ 1 & 1-p \end{pmatrix} + \frac{(-1+p)^d}{2-p}\begin{pmatrix} 1-p & -(1-p) \\ -1 & 1 \end{pmatrix}$$

If we now multiply the first row with P_0 and the second with P_1, we get the joint probabilities $P_{\alpha\beta}(d)$ that we can write in one matrix:

$$P(d) = \begin{pmatrix} \frac{1}{(2-p)^2} + \frac{1-p}{(2-p)^2}(-1+p)^d & \frac{1-p}{(2-p)^2} - \frac{1-p}{(2-p)^2}(-1+p)^d \\ \frac{1-p}{(2-p)^2} - \frac{1-p}{(2-p)^2}(-1+p)^d & \frac{(1-p)^2}{(2-p)^2} + \frac{1-p}{(2-p)^2}(-1+p)^d \end{pmatrix}$$

From where the formula $P_{\alpha\beta}(d) = (-1)^{1-\delta_{\alpha\beta}}\Gamma(d) + P_\alpha P_\beta$ in Eq. (2.3) indeed holds ($\delta_{\alpha\beta} = 1$ if $\alpha = \beta$, and $= 0$ if $\alpha \neq \beta$).

It is possible to to consider the sequence in the Fig.(4B) as a sequence with three symbols: 0_A, 0_B and 1. On this case we have a 3-by-3 transition matrix instead of the 2-by-2 we had for the example above.

Until now we have been talking about the difference between MIF and Correlation function, introduced Markov chain (to better understeand to what kind of structure Mif can be applied) and we have seen the possible transition in between binary and ternary sequences and how the transition matrix changes form from 2-by-2 to 3-by-3. As we already said, the alphabet we will be dealing with in sampling classical poetry is composed of three symbols. What we want to know more about are ternary sequences and maybe even what happens in the special case when the Correlation function is equal to zero and MIF has a finite value.

2.3 Weak correlation in ternary sequences

Again we start with some definitions. Call two variables$\{a_\alpha\}$ and $\{b_\beta\}$ *linearly independent*, if $\Sigma_{\alpha\beta} a_\alpha b_\beta P_{\alpha\beta} = (\Sigma_\alpha a_\alpha P_\alpha)(\Sigma_\beta b_\beta P_\beta)$ for all α, β and *generally independent* if

$P_{\alpha\beta} = P_\alpha P_\beta$ all α, β. Where the linear independence is equivalent to the zero correlation function and generally independent to zero mutual information. Let us now examine ternary sequences and look at the constraints apply to M(d) when $\Gamma(d) = 0$. We will call two sites having zero correlation but non-zero mutual information weakly correlated, instead of using »linearly independent but generally dependent«.

If we apply the same logic (and symmetry condition) as we did before for the binary sequence, we see that our nine joint probabilities for two-site ternary sequences are reduced to three independent function densities. (Choose P_{00}(d), P_{11}(d) and P_{22}(d) as the three independent functions.)

It is easy to show that for $\alpha\neq\beta$ and as the third index $\gamma\neq\alpha\neq\beta$, the other joint probabilities become:

$$P_{\alpha\beta}(d) = \frac{1}{2}\left[P_{\gamma\gamma}(d) - P_{\alpha\alpha}(d) - P_{\beta\beta}(d)\right] + \frac{1}{2}\left(-P_\gamma + P_\alpha + P_\beta\right)$$

Now let us set the correlation function $\Gamma(d)$ equal to zero (as we said before):

$$0 = \Gamma(d) = P_{11}(d) + 2P_{12}(d) + 2P_{21}(d) + 4P_{22}(d) - \left(P_1 + 2P_2\right)^2$$
$$= 2\left[P_{00}(d) - P_0^2\right] - \left[P_{11}(d) - P_1^2\right] + 2\left[P_{22}(d) - P_2^2\right]$$

What we notice is that P_{11}(d) is no longer an independent function (now it's related to

P_{00}(d) and P_{22}(d)) and it has the form of

$$P_{11}(d) = 2\left[P_{00}(d) - P_0^2\right] + 2\left[P_{22}(d) - P_2^2\right] + P_1^2$$

But still two joint probabilities are independent function (P_{00}(d) and P_{22}(d))

$$P_{01}(d) = \frac{3}{2}\left[-P_{00}(d) + P_0^2\right] + \frac{1}{2}\left[-P_{22}(d) + P_2^2\right] + P_0P_1$$

$$P_{02}(d) = \frac{1}{2}\left[P_{00}(d) - P_0^2\right] + \frac{1}{2}\left[P_{22}(d) - P_2^2\right] + P_0P_2$$

$$P_{12}(d) = \frac{1}{2}\left[-P_{00}(d) + P_0^2\right] + \frac{3}{2}\left[-P_{22}(d) + P_2^2\right] + P_1P_2$$

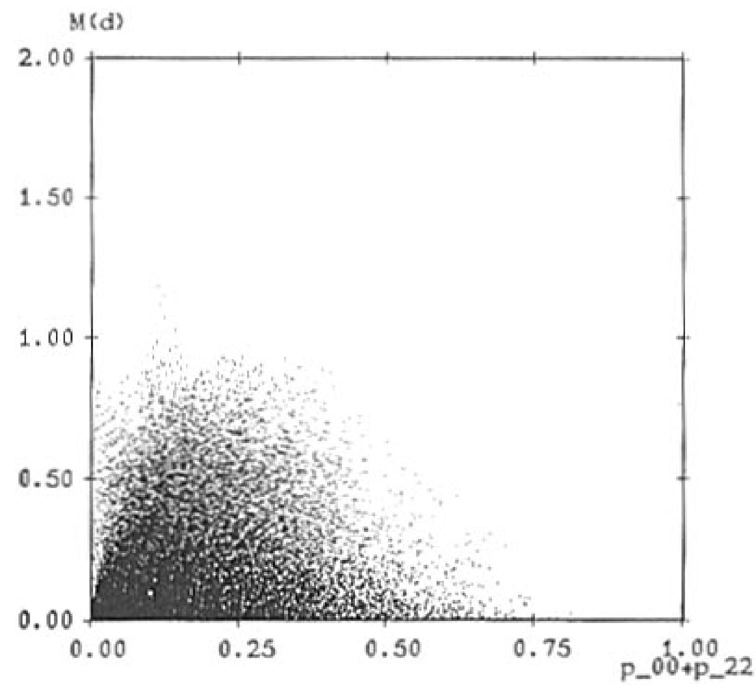


Fig.5 Mutual information function versus $P_{00}+P_{22}$ in ternary sequences when $\Gamma(d) = 0$.

There is an easy experiment to show what are possible mutual information values if $\Gamma(d) = 0$. First randomly choose P_0 and $0 < P_2 < 1 - P_0$, then randomly choose $0 < P_{00} < P_0$ and $0 < P_{22} < P_2$ and from that calculate the rest of the joint probabilities from equation above. And if all are non-negative it is possible to calculate the mutual information.

2.4 Finite-size effect: overestimation of M(d)

So if we want to calculate mutual information, we have to have the value of the joint probabilities $\{P_{\alpha\beta}\}$ or in other words the numbers of occurrence for the joint configuration $\{c_{\alpha\beta}\}$ devided by the total number of counting N, where N is the length of the sequence. But in the case of a finite number countings some problems arise. It is shown that in that particular case we get overestimation that is approximately K(K-2)/2N, where K is the number of the states for each variable.

If we leave out all the derivations we get that for the typical fluctuation of the countings is of the magnitude of the square root of a variable value ($\delta c_{\alpha\beta} \sim \sqrt{c_{\alpha\beta}}$,$\delta c_{\alpha} \sim \sqrt{c_{\alpha}}$ are defined as $\delta c_{\alpha\beta} = c_{\alpha\beta} - \overline{c_{\alpha\beta}}$ and $\delta c_{\alpha} = c_{\alpha} - \overline{c_{\alpha}}$) . Then

$$M(d) - \overline{M(d)} \approx \frac{K(K-2)}{2N}$$

where $K = \sum_{\alpha} 1$ is the total number of states for the variable. K is always grater than 2 and this is why the finite-size effect is always overstimation of the mutual information.

2.5 Symbolic noise

Dynamic systems with expended spatial degress of freedom naturally evolve into self-organized critical structures of states which are barely stable. There are suggestions that behind this is the occurrence of 1/f noise.

But let us continue slowly. As we already said correlation function does not apply to symbolic sequences, even if sometimes the correlation of a particular symbol is calculated (the numerical value is one if the symbol is present and zero if not). So we have K symbols, for K such correlation functions.This is equivalent to $\Gamma_{\alpha}(d) = P_{\alpha\alpha}(\mathrm{d}) - P_{\alpha}^{2}$, where $\alpha = 1,2,\ldots,K$. This method can't be used to calculate correlation between different symbols ($P_{\alpha\beta}$ (d)). We see that the mutual information function (MIF) for such calculation is equal zero if and only if two sites are generally independent or $P_{\alpha\beta} = P_{\alpha}P_{\beta}$ for all α,β.

The Fourier transformation (power spectra) and correlation function are important for characterizing and classifying numeric random sequences or *noise*. There are different noises, such as *white noise*, *brownian noise* and *1/f noise* (they can be distinguished by the form of their correlation function and power spectra). But we have to be aware that there is no standard way to measure correlation in symbolic sequences and that none of this measurements are important for application to DNA molecules and other biopolymers.

Wentian Li [**3**] proposed the name *symbolic noise* for those symbolic sequences with large value of single-site entropy but many possible forms of MIF. It was further suggested that if the MIF for a symbolic sequences decays to zero even at the nearest neighbor. This kind of sequences can be considered as the symbolic counterpart for *white noise*. In the case that the MIF decays very slowly the sequences might be something similar to the $1/f^{\alpha}$ *noise*, so we can call it *symbolic 1/f noise*. For the better understanding it's a good idea to look up at some examples in the real world. The simplest examples are letter sequences of natural language text, nucleotide sequences of DNA or RNA molecules and other. And the logical question that follows is what kind of noise are the sequences we are talking about.
From this calculation it is visible that the MIF in figure 4 somehow decays in a way in the middle of power low and exponential function (short distances). But it's still possible to approximate the function form by a power law form: $M(d) \sim 1/d^{3}$. Till now there was no good name for this kind of (symbolic) noise. The only thing we can be sure of is that it's not a symbolic *1/f* noise, because the correlation measure decays too fast.

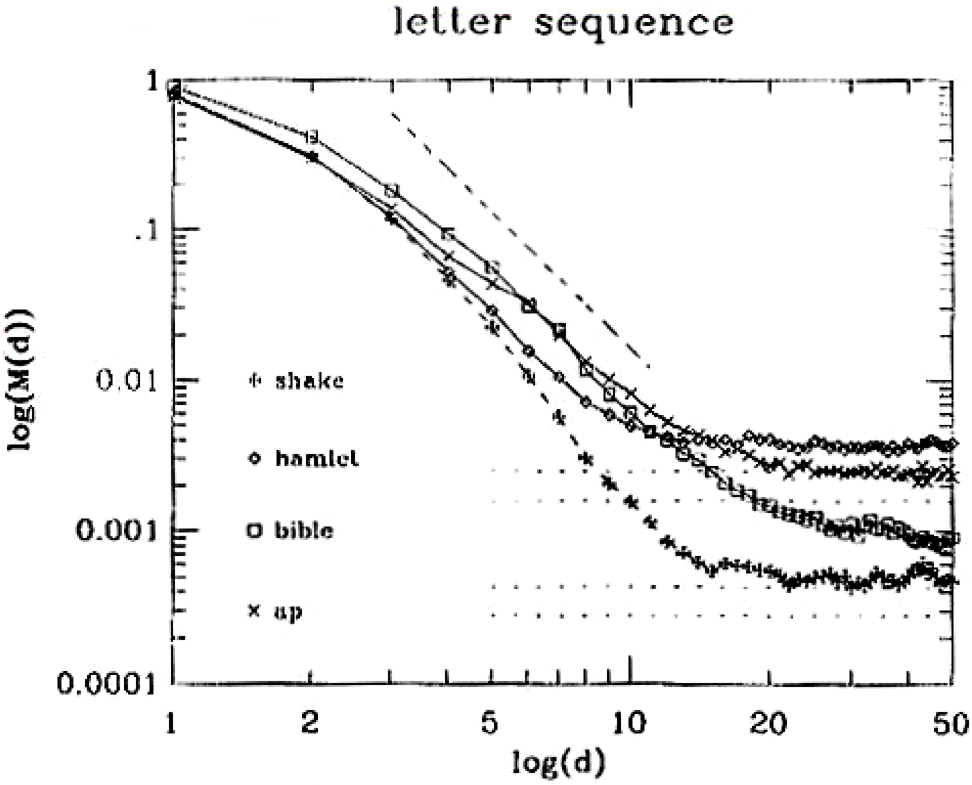


Fig.6 Site-to-site *M(d)* for letter sequences (28 symbols) from (1) Shakepeare's play *Hamlet*; (2) Associated Press news articles; (3) the five books of Moses from the Bible in German; (4) 11 plays by Shakepeare. The dashed line is an inverse power law function $1/d^3$. The dotted lines are estimated residual values of M(d) according to K(K - 2)/2N

3. Mutual information function and classical poetry

3.1 Introduction to classical poetry

For easier understanding of what we will be analizing it's a good idea to make a short overview of the ancient poetry. Of course we will also explain the methodology used to map the verses into a symbolic sequences.

Homer's *Iliada* and *Odysey* are two of the chosen works for representing classical greek poetry. Homer is said to be the first to have used the *HEXAMETER* (a verse of six meters or foots in the above listed works). If we look at these works linguisticaly we notice that there are a lot of formulae (strings of words that repeat themselves in both poems) and epithets (exp. »owl-eyed Athena«). This forms are used to help with the poems rhythm. It is important to say that the hexameter wasn't only used for epic poetry, but also for geners like didactic poetry. An example of such poet was Hesoid who used it in *Theogony* in an attempt to explain the origins of the universe and the »family tree« of the greek gods. The last example from greek literature that was used is Theocritus's *Idylls* with which he tried to draw a picture of the simple countryside lifestyle. This and other authors from Syracuse were part of Virgil's inspiration. Something interesting to point out is that both Homer and Hesiod have written in an artificial language that was reserved for poetry and Theocritus used a local dialect. It is also true that the older poetry had a tendency to take distance from the everyday speech because of the search for the rhythm.

As in greek poetry, we will now analize three latin poets: Lucretius, Vergil and Ovid. Lucretius in *On the nature of things* explains the order in the universe and human's relation to it (his use of the verse is similar to that of Hesiod), Vergil's epic poem Aeneida, which is somehow similar to Odyssey, but lacks formulae and epithets, and didactic Georgics, an agricultural advices book, and in the end Ovid's Metamorphoses that is a collage of myths about gods, semigods and heroes.

Hexameter is, as we said before, a verse of six meters whose basic feature is the *diactyl* (one long sylable followed by two short ones). Graphically it's represented by:

__ uu | __ uu | __ uu | __ uu | __ uu | __ u,

where __ indicates the long sylable and U the short ones. Another basic feature of the hexameter is a *spondee* (represented by __ __). Note that the different foots are separated by vertical bars. The first foot can be either of the two. So it can be

__ uu | __ uu | __ uu | __ uu | __ uu | __ u

or

__ __ | __ __ | __ __| __ __ | __ uu | __ u.

Every foot is formed by a biceps. This last is formed by the first half always long and second half being either short or long.

Another possible hexameter feature is the *caesura* (pausa). It's usual for the pause to be at the end of word and sometimes even coincides with the foot end.

Let us now analyze a verse from the first book in the *Aeneida*:

Arma virumque cano, Troiae qui primus ab oris

and is thus:

__ uu | __ uu | __ ⇑ u | __ uu | __ uu | __ u, where the ⇑ indicates the caesura and in the verse above coincides with the comma. Now we can write the above verse with arsis and thesis (Ta, ta), which are the so called up and down beats.

ARma virUMque caNO, TroiAE qui PRImus ab Oris

or

TA-ta-ta TA-ta-ta TA ⇑ ta TA-ta-ta TA-ta-ta TA-ta.

It shows that it is convient to map the hexameter into a symbolic time series using a trinary system (long 0, short 1, pausa 2). For this paper R. Mansilla and E. Bush [***1***] mapped the first 100 verses of the all listed works.

3.2 Mif and the alphabet for classical poetry

Now that we have became familiar with some basic concepts of both MIF and mapping poetry, we can write (for this work) the alphabet A = {0, 1, 2}, where $\alpha, \beta \in$ A. If we recall the MIF form:

$$M(d) \equiv \sum_{\alpha}^{K} \sum_{\beta}^{K} P_{\alpha\beta}(d) \ln \frac{P_{\alpha\beta}(d)}{P_{\alpha} P_{\beta}} .$$

It's obvious that our sequences are not infinite, but large enough to allow stable statistical estimations of $P_{\alpha\beta}(d)$, P_{α} and P_{β}.

Such function as the one above have often a periodic behavior. A widely used methode to study those kinds of behavior Fourier spectra (for time series analysis), which is represented in the frequency domain. This representation can easily reveal patterns that can indicate periodical behavior.

3.3 Result and discussion

After introducing the MIF and deciding the way to map the verses, let us now analyse the results Marsilla and Bush got from their analysis.

We'll start by analysing the behaviour of MIF related to each verse. In fig. 3.1.a and 3.1.b the change that happened from greek to latin poetry is noticable. The peak at d = 2 is much more pronounced in the Vergil's work. This peak is related to the long syllable common to the dactyl and spondee and the substitution between them. The next peak in Iliad (d=9) almost disappears in Latin poetry, because the verse there is more relaxed. The last big change is that the peak d = 17 in Iliade moves to d = 16 in the Aeneida. This shift is related to the use and the position of the pause. If we take a closer look to the structure of the verse, they used partial information function (PIF, $M_{\alpha\beta}{}^{P}(d) \equiv P_{\alpha\beta}(d) \ln \frac{P_{\alpha\beta}(d)}{P_{\alpha} P_{\beta}}$).

A remarkable property of function $M_{01}{}^{P}(d)$ and $M_{11}{}^{P}(d)$ is that their graphs are mirror images of each other with respect to the horizontal axis. The important consequence of the above property ($M_{01}{}^{P}(d) + M_{11}{}^{P}(d) = 0$) is that they don't contribute to MIF. This means that the peak noticed at the distance d = 2, 9 in the MIF of every poem is contribution to the remainder PIF.

We incounter another difference in Greek and Latin poetry, if we take a closer look at the relationship between the principes (the first part of a foot) and the caesurae. This contributes to the peak at d = 9, which is larger in Greek poems. Another peak which almost disappears from Greek to Latin poetry is the one at d =13. These facts suggest us that the use of more than one caesura in each verse is more often in Latin than Greek poems.

The distance between two consecutive pauses also contributes to the difference in complexity between Greek and Latin poetry. For pinpointing these differences it's a good idea to look a the $M_{22}^{P}(d)$; If we look at the fig 3.3. it has remarkable periodic behavior. It indicades a high correlation between pause at the distance of 170 verses in the Iliade.

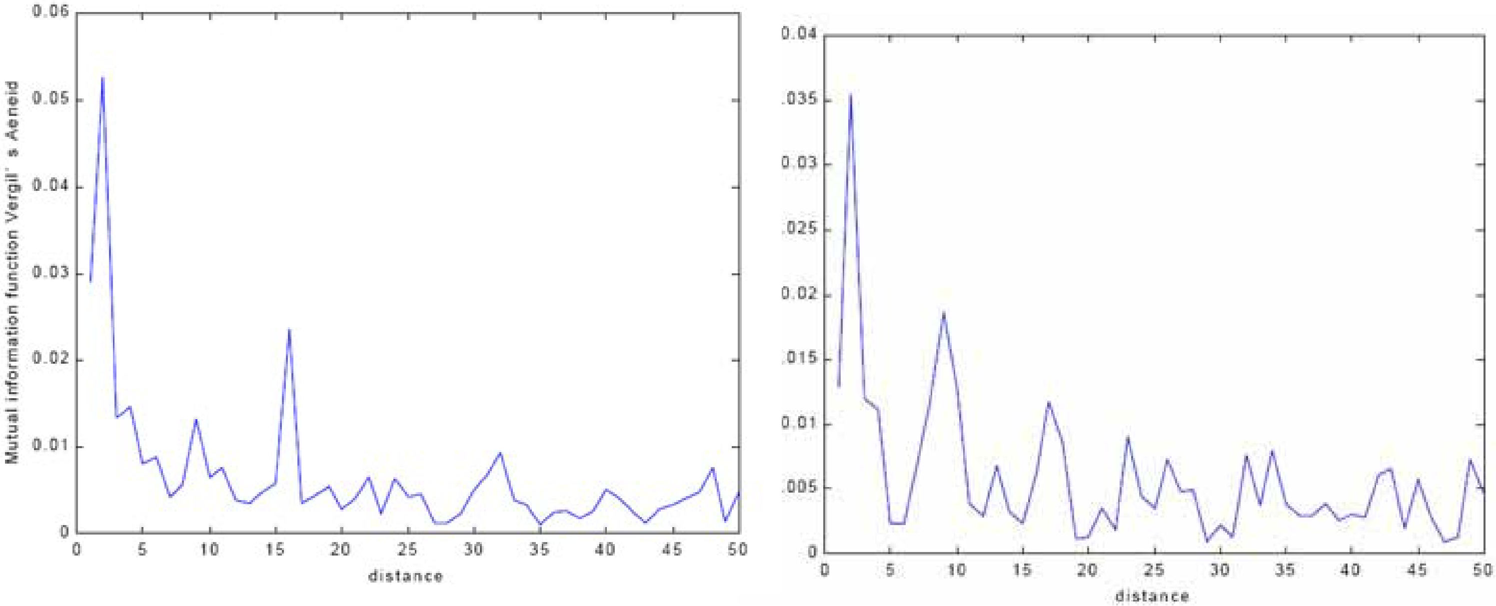


Fig.3.1 (A) M(d) for Homer's Iliad , (B) M(d) for Vergil's Aeneida.

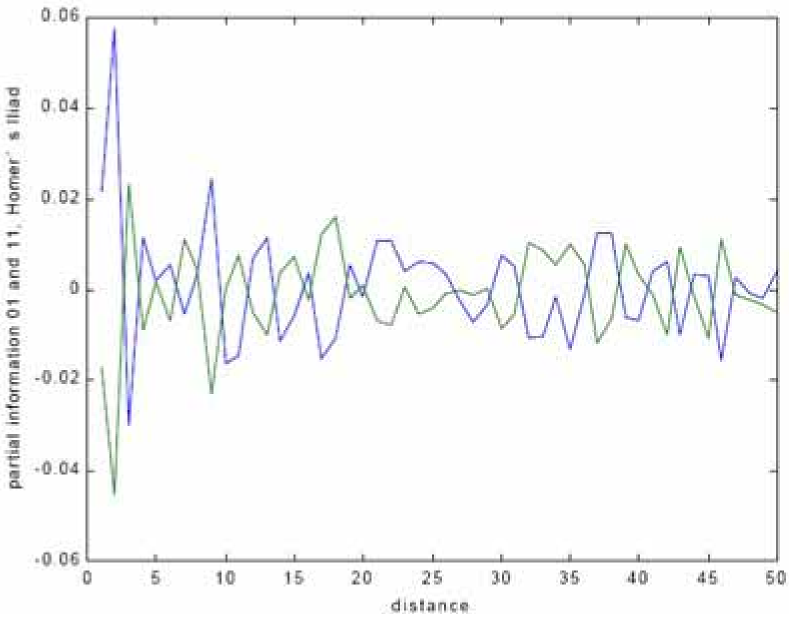


Fig.3.2 Partial information 01 and 11 for Homer's *Iliad* ;

In the Latin poetry the periodic structure persists, althought it's weakened. In Fig. 3.5 the averaged power spectrum of Greek and Latin poems is shown. The first large peak almost coincides in position, but is smaller in Latin poem, reflecting a lower influence of corresponding harmonic. A posible explanation is the fact that rhapsodies in harmonic times had to memorize large pieces of verses, because no written verses exited at that time and a strict rhythm facilitated memorization.

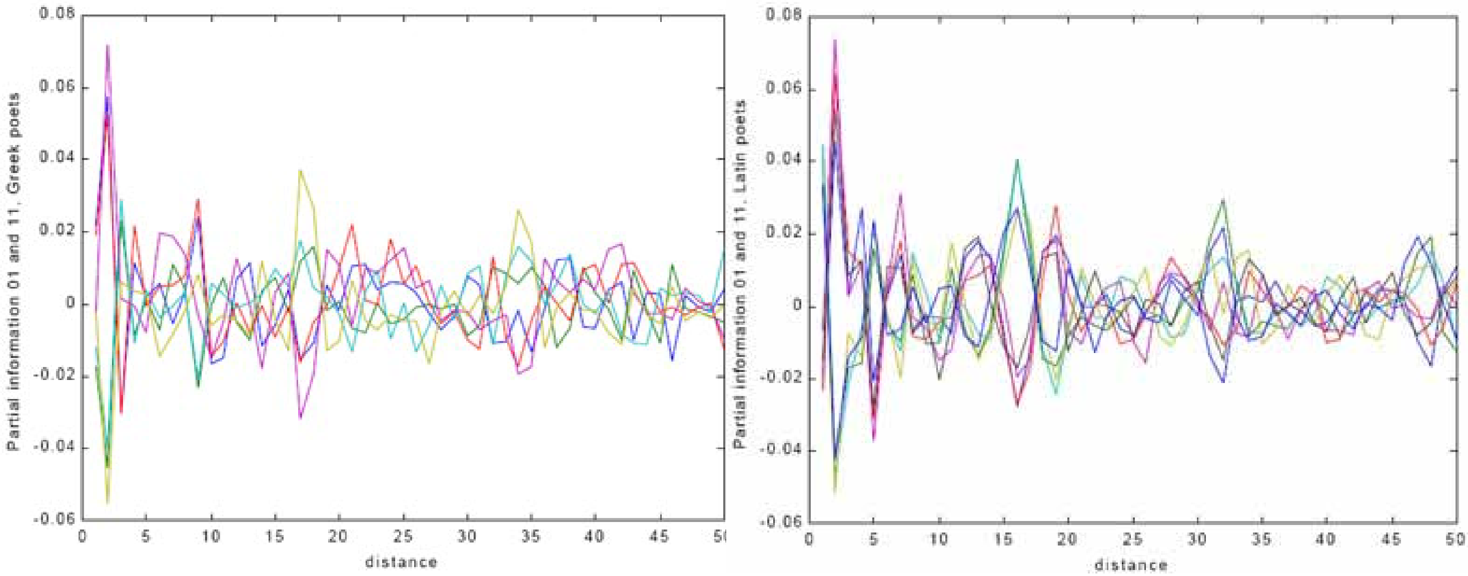


Fig.3.3 Partial information 01 and 11 for Greek and Latin poems.

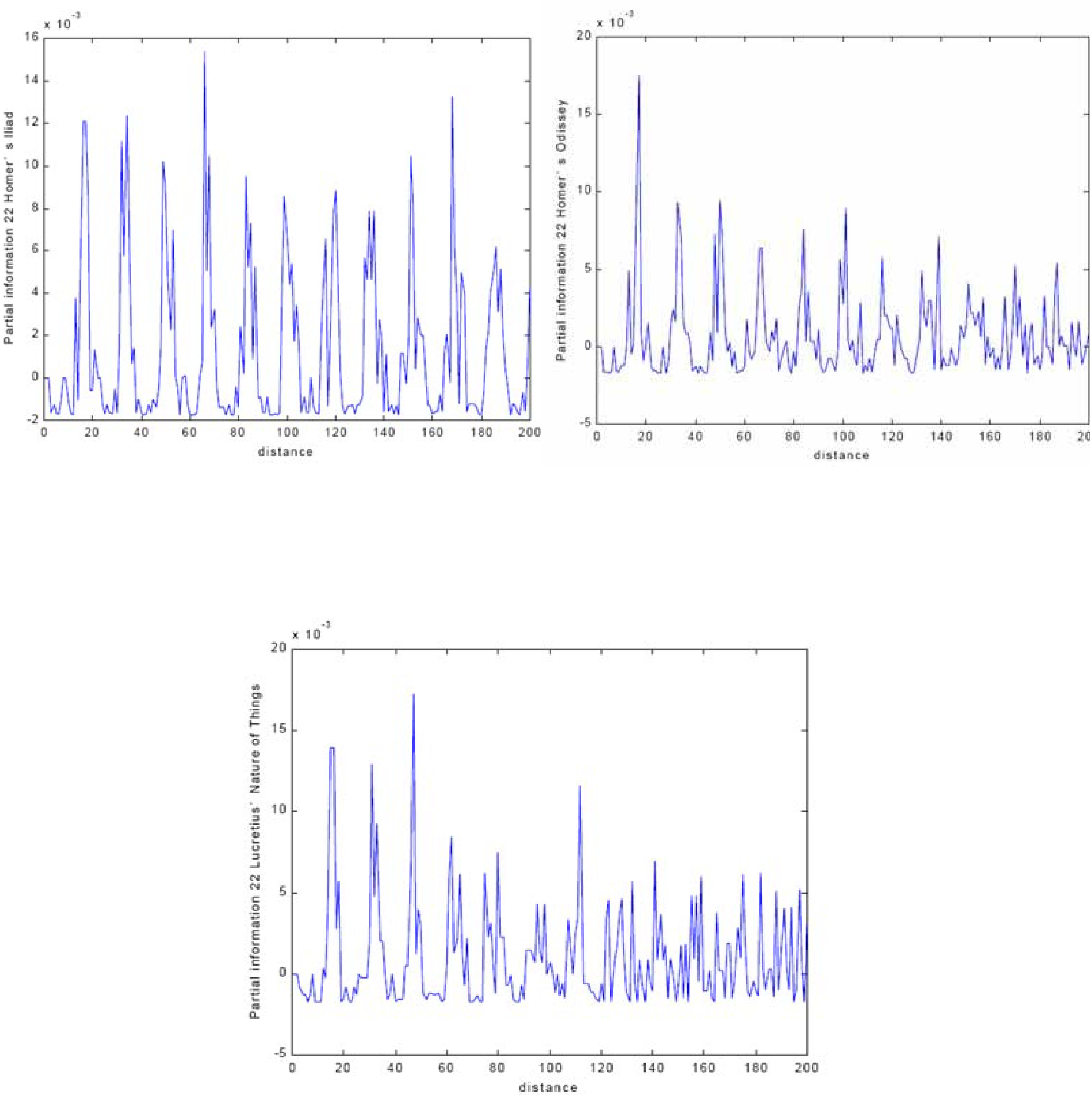


Fig.3.4 (A) Partial information 22 for Homer's Iliad , (B) Partial information 22 for Homer's Odissey and (C) Partial information 22 for Lucretiu's Nature of Things.

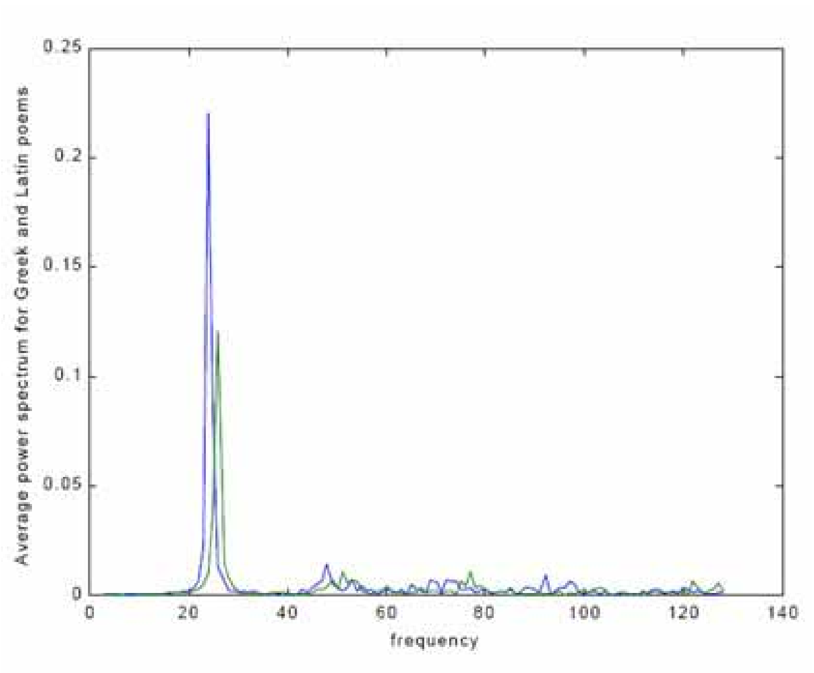


Fig.3.5 Avarage power spectrum for Greek and Latin poems; The Latin is green.

4. Conclusion

The use that was illustrated in this paper is only one possible use of the MIF. The results show that the structure changes or better increases in complexity from the Greek to Latin poetry. And we can notice some major differences even in the evolution of the Greek poetry itself. There are different explanation of why is so. One of the explanations for this increase is the introduction of writing in poetry instead of memorizing large parts of poems.
It is important to point out the MIF is mostly used for DNA researching. Nevertheless the study of natural languages opens the area of the »speech« of artificial intelligence and many other researches that were almost not possible before the start of use of this method for symbolic sequences analysis.

5. References

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