University of Ljubljana
Faculty for mathematics and physics
Department of physics

# Turbulence models in CFD

Jurij SODJA
Mentor: prof. Rudolf PODGORNIK

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## INDEX

# 1 INTRODUCTION

The abbreviation CFD stands for computational fluid dynamics. It represents a vast area of numerical analysis in the field of fluid's flow phenomena. Headway in the field of CFD simulations is strongly dependent on the development of computer-related technologies and on the advancement of our understanding and solving ordinary and partial differential equations (ODE and PDE). However CFD is much more than "just" computer and numerical science. Since direct numerical solving of complex flows in real-like conditions requires an overwhelming amount of computational power success in solving such problems is very much dependent on the physical models applied. These can only be derived by having a comprehensive understanding of physical phenomena that are dominant in certain conditions.[1], [8]

Why turbulence?

Whenever turbulence is present in a certain flow it appears to be the dominant over all other flow phenomena. That is why successful modeling of turbulence greatly increases the quality of numerical simulations.

All analytical and semi-analytical solutions to simple flow cases were already known by the end of 1940s. On the other hand there are still many open questions on modeling turbulence and properties of turbulence it-self. No universal turbulence model exists yet.

Further more the price tag for our ignorance is immense. That makes the area of CFD modeling also extremely economically attractive.

# 2 GENERAL REMARKS

## 2.1 Ideal turbulence model

Solving CFD problem usually consists of four main components: geometry and grid generation, setting-up a physical model, solving it and post-processing the computed data. The way geometry and grid are generated, the set problem is computed and the way acquired data is presented is very well known. Precise theory is available. Unfortunately, that is not true for setting-up a physical model for turbulence flows.

The problem is that one tries to model very complex phenomena with a model as simple as possible.

Therefore an ideal model should introduce the minimum amount of complexity into the modeling equations, while capturing the essence of the relevant physics.

## 2.2 Complexity of the turbulence model

Complexity of different turbulence models may vary strongly depends on the details one wants to observe and investigate by carrying out such numerical simulations. Complexity is due to the nature of Navier-Stokes equation (N-S equation). N-S equation is inherently nonlinear, time-dependent, three-dimensional PDE.

Turbulence could be thought of as instability of laminar flow that occurs at high Reynolds numbers ($\mathrm{Re}$). Such instabilities origin form interactions between non-linear inertial terms and viscous terms in N-S equation. These interactions are rotational, fully time-dependent and fully three-dimensional. Rotational and three-dimensional interactions are mutually connected via vortex stretching. Vortex stretching is not possible in two dimensional space. That is also why no satisfactory two-dimensional approximations for turbulent phenomena are available.

Furthermore turbulence is thought of as random process in time. Therefore no deterministic approach is possible. Certain properties could be learned about turbulence using statistical methods. These introduce certain correlation functions among flow variables. However it is impossible to determine these correlations in advance.

Another important feature of a turbulent flow is that vortex structures move along the flow. Their lifetime is usually very long. Hence certain turbulent quantities can not be specified as local. This simply means that upstream history of the flow is also important of great importance.

## 2.3 Classification of turbulent models

Nowadays turbulent flows may be computed using several different approaches. Either by solving the Reynolds-averaged Navier-Stokes equations with suitable models for turbulent quantities or by computing them directly. The main approaches are summarized below.

*Reynolds-Averaged Navier-Stokes (RANS) Models*

- **Eddy-viscosity models (EVM)**

  One assumes that the turbulent stress is proportional to the mean rate of strain. Further more eddy viscosity is derived from turbulent transport equations (usually $k$ + one other quantity).

- **Non-linear eddy-viscosity models (NLEVM)**

  Turbulent stress is modelled as a non-linear function of mean velocity gradients. Turbulent scales are determined by solving transport equations (usually $k$ + one other quantity). Model is set to mimic response of turbulence to certain important types of strain.

- **Differential stress models (DSM)**

  This category consists of Reynolds-stress transport models (RSTM) or second-order closure models (SOC). One is required to solve transport equations for all turbulent stresses.

*Computation of fluctuating quantities*

- **Large-eddy simulation (LES)**

  One computes time-varying flow, but models sub-grid-scale motions.

- **Direct numerical simulation (DNS)**

  No modelling what so ever is applied. One is required to resolve the smallest scales of the flow as well.

Extend of modelling for certain CFD approach is illustrated in the following figure Figure 2.1. It is clearly seen, that models computing fluctuation quantities resolve shorter length scales than models solving RANS equations. Hence they have the ability to provide better results. However they have a demand of much greater computer power than those models applying RANS methods. [2], [7]

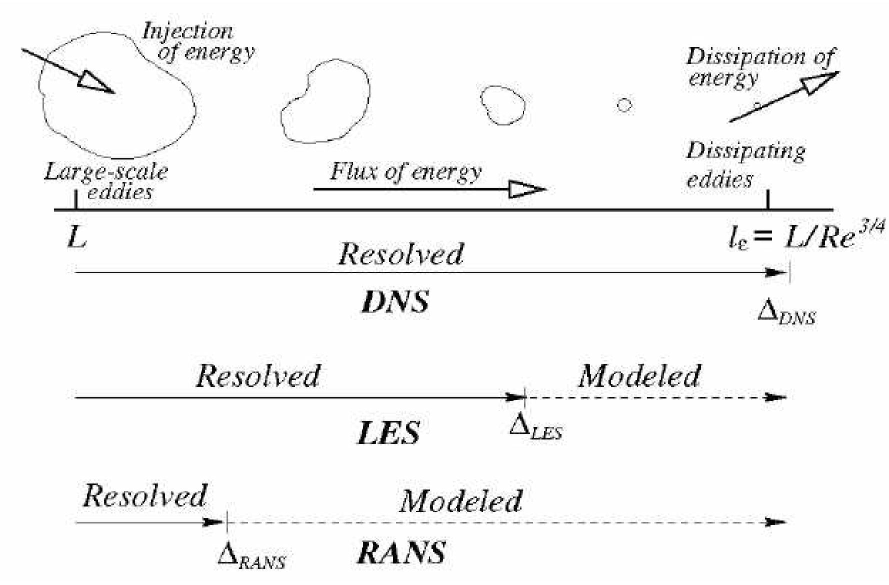


**Figure 2.1 Extend of modelling for certain types of turbulent models**

# 3 REYNOLDS-AVERAGED NAVIER-STOKES MODELS

The following chapter deals with the concept of Reynolds's decomposition or Reynolds's averaging. The term Reynolds's stress is introduced and explained briefly. Further on methods how to include these ideas into certain numerical models are presented. [1], [5], [8]

## 3.1 Reynolds's decomposition

### 3.1.1 Equations describing instantaneous fluid motion

For easier understanding of certain mathematical ideas it is convenient to briefly revise N-S equations describing instantaneous fluid motion at the beginning. All variables describing instantaneous flow are marked with a tilde. These variables are fluid's density ($\tilde{\rho}$), velocity components ($\tilde{u}_i$), pressure ($\tilde{p}$) and components of viscous stress tensor ($\tilde{T}_{ij}^{(v)}$). At this point it is also suitable to point out that these variables are al time and space dependent.

General N-S equations for both turbulent and non-turbulent flow run:

$$\tilde{\rho}\left(\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial \tilde{T}_{ij}^{(v)}}{\partial x_j} \quad \text{and} \tag{3.1}$$

$$\left(\frac{\partial\tilde{\rho}}{\partial t}+\tilde{u}_j\frac{\partial\tilde{\rho}}{\partial x_j}\right)+\tilde{\rho}\frac{\partial\tilde{u}_i}{\partial x_i}=0 \tag{3.2}$$

The firs equation (3.1) is called momentum equation (second Newtonian law for fluids). The second equation (3.2) is known as continuity equation. At this point I would also like to define viscous stress tensor $\tilde{T}_{ij}^{(v)}$ as follows:

$$\tilde{T}_{ij}^{(v)}=2\mu\left(\tilde{s}_{ij}-\frac{1}{3}\tilde{s}_{kk}\delta_{ij}\right), \tag{3.3}$$

where $\tilde{s}_{ij}$ means:

$$\tilde{s}_{ij}=\frac{1}{2}\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right) \tag{3.4}$$

Should one assume incompressible flow the previous equations simplify immensely. The continuity equation (3.2) is reduced to $\partial u_i/x_i=0$. Having this result in mind the momentum equation (3.1) can be rewritten as:

$$\left(\frac{\partial\tilde{u}_i}{\partial t}+u_j\frac{\partial\tilde{u}_i}{\partial x_j}\right)=-\frac{1}{\rho}\frac{\partial\tilde{p}}{\partial x_i}+\frac{\mu}{\rho}\frac{\partial^2\tilde{u}_i}{\partial x_j^2} \tag{3.5}$$

The factor $\mu/\rho$ is often regarded to as kinematic viscosity $\nu$. Viscous stress tensor simplifies as well:

$$\tilde{T}_{ij}^{(v)}=2\mu\,\tilde{s}_{ij} \tag{3.6}$$

### 3.1.2 Reynolds averaging

The concept of Reynolds averaging was introduced by Reynolds in 1895. One may consider Reynolds averaging in many different ways. There are three most common perceptions of this term: time averaging, space averaging or ensemble averaging.

Time averaging is appropriate when considering a stationary turbulence. That is when the flow does not vary on the average in time. In such cases time average is defined by:

$$F(\vec{r})=\lim_{T\to\infty}\left(\frac{1}{T}\int_t^{t+T}f(\vec{r},t)\,dt\right) \tag{3.7}$$

Space average is appropriate for homogenous turbulence. That is a turbulent flow that on the average does not vary in any direction. Space average is defined by:

$$F(t)=\lim_{V\to\infty}\left(\frac{1}{V}\iiint f(\vec{r},t)\,dV\right) \tag{3.8}$$

Ensemble average is the most general aspect of Reynolds average. It should be understood as an average of N identical experiments. Ensemble average is both time- and space-dependent. It is defined by:

$$F(\vec{r},t)=\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}f_n(\vec{r},t) \tag{3.9}$$

The main idea of Reynolds averaging is to decompose the flow to averaged and fluctuating component:

$$\begin{aligned}\tilde{u}_i &= U_i+u_i\\ \tilde{p} &= P+p\\ \tilde{T}_{ij}^{(v)} &= T_{ij}^{(v)}+\tau_{ij}^{(v)}\end{aligned} \tag{3.10}$$

This process is called Reynolds decomposition. The upper case letters represent the mean values; the lower case letters represent the fluctuating values on the right hand side in expressions (3.10). By inserting relations (3.10) into N-S equation (3.1) one obtains the following expression:

$$\rho\left(\frac{\partial(U_i+u_i)}{\partial t}+(U_j+u_j)\frac{\partial(U_i+u_i)}{\partial x_j}\right)=-\frac{\partial(P+p)}{\partial x_i}+\frac{\partial\left(T_{ij}^{(v)}+\tau_{ij}^{(v)}\right)}{\partial x_j} \tag{3.11}$$

This equation can now be averaged to yield an equation expressing momentum conservation for the averaged motion. At this point it is important to stress that the operations of averaging and differentiation commute. It is also assumed that the average of fluctuating quantities is zero. Therefore the averaged momentum equation reduces to:

$$\rho\left(\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j}\right)=-\frac{\partial P}{\partial x_i}+\frac{\partial T_{ij}^{(v)}}{\partial x_j}-\rho\left\langle u_j\frac{\partial u_i}{\partial x_j}\right\rangle \tag{3.12}$$

In similar manner continuity equation for incompressible flow can be decomposed. Such a continuity equation is linear therefore the original form for the instantaneous motion is preserved:

$$\begin{aligned}\frac{\partial U_i}{\partial x_i} &= 0 \\ \frac{\partial u_i}{\partial x_i} &= 0\end{aligned} \tag{3.13}$$

Using the second relation in equation (3.13) one can rework the last term on the right hand side of the equation(3.12). The result runs:

$$\rho\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right) = -\frac{\partial P}{\partial x_i} + \frac{\partial T_{ij}^{(v)}}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\rho\langle u_i u_j\rangle\right) \tag{3.14}$$

Term $\left(\rho\langle u_i u_j\rangle\right)$ has the same structure and dimension as the viscous stress tensor. However this term is not a stress at all. It is just a re-worked contribution of the fluctuating velocities to the change of the averaged ones. On the other hand as far as the motion of the fluid is concerned it acts as a stress. Hence its name, Reynolds stress.

## 3.2 The closure problem

The problem with the above concept of Reynolds decomposition and averaging is that it introduces additional variables ($\langle u_{i=1,2,3}^2\rangle, \langle u_1 u_2\rangle$, $\langle u_1 u_2\rangle$, $\langle u_2 u_3\rangle$), for which there are no available relations. Not in a general sense at least. [1], [8]

One could pretend that Reynolds stress is indeed a stress and try to write constitutive relations similar to those for viscous stress. However there is an important difference among these two stresses. Viscous stress is a property of a fluid. That is why separate experiments can be carried out in order to determine corresponding constitutive relations. These relations are valid then whenever a flow in that particular fluid is observed. On the other hand Reynolds stress is a property of the flow. Hence it is dependent on the flow variables them-selves. That is the reason why it changes from flow to flow and no general constitutive relations are available.

### 3.2.1 Laminar flow, infinitesimal fluctuations and superposition

One solution to the closure problem is to treat the flow as a laminar flow with fluctuations superimposed. One subtracts the averaged momentum equation from equation describing instantaneous motion. The result for fluctuating motion reads:

$$\rho\left(\frac{\partial u_i}{\partial t}+U_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\frac{\partial \tau_{ij}^{(v)}}{\partial x_j}-\rho\left[u_j\frac{\partial U_i}{\partial x_j}\right]-\rho\left(u_j\frac{\partial u_i}{\partial x_j}-\left\langle u_j\frac{\partial u_i}{\partial x_j}\right\rangle\right) \quad (3.15)$$

The equation (3.15) has a similar structure than the averaged N-S equation(3.14). The only difference is the last to terms on the right hand side. The first of them represents the production term. It describes the way fluctuating motion extracts momentum from the averaged motion. The second one is similar to Reynolds stress term in equation (3.14) except that its mean is zero. Should equation(3.15) be averaged its average is zero.

This approach requires fluctuations to be small. In the limit of infinitesimal fluctuations Reynolds stress terms are negligible. Therefore averaged N-S equation (3.14) yields a laminar flow. Furthermore the equation (3.15) reduces to linear PDE. As a result of this process one obtains a well-defined – closed, set of equations describing the observed flow.

## 3.3 Reynolds stress models

There were many different concepts and attempts to solve the turbulence closure problem in a general form in the past. Nowadays there are two concepts that underlie most of the Reynolds stress models.

One and the most obvious attempt was to describe Reynolds stress in a similar way viscous stress is described: the fluid is simply prescribed another property – turbulent viscosity. This model had been introduced by Boussinesq back in 1877 even earlier then Reynolds proposed his decomposition and averaging approach in 1895. There are many difficulties regarding this model. Probably the major problem is how to obtain this property without carrying out an actual experiment involving that particular flow.

Major breakthrough was done by Prandtl in 1925. He introduced the mixing length concept analogous to mean free path of the molecules in gas. He also prescribed an algebraic expression relating turbulent viscosity to the mixing length. That is why Prandtl is known as the founder of so called algebraic or zero-equation models. Zero-equation refers to the fact, that no additional transport equations besides to energy, mass and momentum equations are needed.

Another important breakthrough was done by Prandtl in 1945, by introducing a concept of turbulent viscosity as a function of turbulent kinetic energy. Major advantage of this concept over the previous one is that it already takes into account flows history. Hence it is a physically more realistic model. Prandtl used one additional transport equation to model turbulent kinetic energy. Models based on this concept are usually called one-equation models.

Still there is a need to specify a turbulence length scale, which is also a flow dependent property. Hence one still needs to have certain knowledge about the studied flow in advance. Therefore such models are called incomplete. Both zero- and one-equation models are incomplete.

On the other hand complete model would be characterized by the fact that no knowledge of the flow except the initial and boundary conditions is needed in advance.

First complete model was introduced by Kolmogorov in 1942. The basic idea of his model was to model turbulent kinetic energy ($k$) and the rate of energy dissipation ($\omega$) and then relate the missing information of length and time scales to these quantities. Since two additional equations are used to model $k$ and $\omega$ these kind of models are called two-equation models. They are also referred to as $k-\omega$ models. Variations of this concept are so called $k-\varepsilon$ models ($\varepsilon = k^n \omega^m$). Instead of $\omega$ $\varepsilon$ is modelled.

Another conceptually different attempt was to model Reynolds stress tensor directly. At first one tried to derive actual Reynolds stress equations. The idea was to re-work fluctuating momentum equation (3.15) in such a manner that it would describe Reynolds stress. Major problem with this attempt is that it introduces even more new unknown variables for which no constitutive relations are known

In 1951 Rotta managed to successfully model Reynolds stress tensor by using PDE. This model is concept is more realistic than the Boussinesq's turbulent viscosity model. However it introduces six additional equations describing Reynolds stress and one additional equation describing turbulence length scale.

In the field of RANS models no major conceptual break through was done ever since. There were many improvements mainly in a sense of adjusting certain models to particular flow cases.

# 4 COMPUTATION OF FLUCTUATING QUANTITIES

In the following section basic properties regarding direct numerical simulation (DNS) and large-eddy simulation (LES) are briefly summarized. [1]

## 4.1 Direct numerical simulation

DNS simply means numerical solving of N-S and continuity equation. When dealing with turbulent flow one tries to resolve all turbulent phenomena at all length and time scales simply by numerical solving of N-S and continuity equation. For a successful simulation one typically needs to know what the smallest length, time and velocity scales are. This information is crucial in order to set space grid and time steps of adequate scales. This data can easily be acquired by applying Kolmogorov turbulence theory in advance. What ones want to extract form these data typically is the number of grid point and time steps necessary.

Number of uniformly distributed grid points reads:

$$N_{uni} \approx \left(110\,\mathrm{Re}_T\right)^{9/4}, \quad \mathrm{Re}_T = \frac{u_T\,L}{\nu} \tag{4.1}$$

$\mathrm{Re}_T$ represents turbulent Reynolds number, $u_T$ represents frictional velocity, L is typical length scale, $\varepsilon = \mu/\rho$ is kinematic viscosity of the fluid. All quantities are defined at the integral turbulence scale. All can be derived solely by applying Kolmogorov turbulence theory.

Number of time steps is defined by:

$$N_{time} = \frac{\Delta t_{total}}{\Delta t}, \;\; \Delta t \approx \frac{0.003}{\sqrt{\mathrm{Re}_T}}\frac{L}{u_T} \tag{4.2}$$

The following table 4.1 lists numerical parameters for a certain flow. Figures handed under $N_{time}$ represents the number of time steps required in order to reach statistically steady flow. The figure handed under $CPU$ is the amount of time (in hours) required to obtain the solution using a standard Intel Core 2 Duo E6700 (12.53 gigaflops). Time step required to finish one time step is approximately 3.2$s$. [10][11]

As one can see the biggest problem regarding DNS is their overwhelming requirement for computer power in a sense of both processor's speed and a size of the memory for storing intermediate results.

**Table 4.1: Numerical requirements solving turbulent flow characterized by $Re_L$ and $Re_T$**

| $Re_L$ | $Re_T$ | $N_{DNS}$ | $N_{time}$ | *CPU [h]* |
|---|---|---|---|---|
| 12300 | 360 | $6.7*10^6$ | 32000 | 28 |
| 30800 | 800 | $4.0*10^7$ | 47000 | 42 |
| 61600 | 1450 | $1.5*10^8$ | 63000 | 56 |
| 230000 | 4650 | $2.1*10^9$ | 114000 | 101 |

DNS is of great importance. As computers develop one gains the capability to simulate flows at ever higher and higher Reynolds number. Nowadays results acquired by DNS are so good that one may consider them equivalent to data gained experimentally.

## 4.2 Large-Eddy simulation

LES is a computation where large vortexes (eddies) are computed directly, while small scale eddies are modeled. That is why space grid and time steps may be much longer than in DNS. Hence LES is much more economical in term of computational power required than DNS:

$$N_{LES} \approx \left( \frac{0.4}{\mathrm{Re}_T^{1/4}} \right) N_{DNS} \tag{4.3}$$

The following table 4.2 list numerical parameters regarding LES for the same flow that is discussed in paragraph 4.1. It seems that LES takes roughly 10% of the DNS CPU time to compute the solution. [10], [11]

**Table 4.2: Numerical requirements solving turbulent flow characterized by $Re_L$ and $Re_T$**

| $Re_L$ | $Re_T$ | $N_{DNS}$ | $N_{LES}$ | $N_{time}$ | *CPU [h]* |
|---|---|---|---|---|---|
| 12300 | 360 | $6.7*10^6$ | $6.1*10^5$ | 2913 | 2.5 |
| 30800 | 800 | $4.0*10^7$ | $3.0*10^6$ | 3525 | 3.15 |
| 61600 | 1450 | $1.5*10^8$ | $1.0*10^7$ | 4200 | 3.73 |
| 230000 | 4650 | $2.1*10^9$ | $1.0*10^8$ | 54285 | 4.87 |

The idea underlying LES is so called convergent evolution. Behavior of the large-scale eddies depends strongly on the forces acting on the flow and on initial and boundary conditions. They are flow-dependent On the other hand small-scale eddies are generally independent from what is happening on the larger scales. They are flow-independent. Hence large eddies are directly resolved while small eddies are modeled. One tries to find a universal model for small eddies.

Another important concept regarding LES is filtering. One applies filtering functions in order to remove sub-grid fluctuations from resolving. Sub-grid fluctuations are modeled. This is achieved by averaging. One of the simplest filtering functions is central-difference approximation it-self:

$$\frac{u(x+h)-u(x-h)}{2h}=\frac{d}{dx}\left(\frac{1}{2h}\int_{x-h}^{x+h}u(\xi)\,d\xi\right) \tag{4.4}$$

Length scales of order $h$ are still resolved, while length scales smaller than $h$ are modeled. They are called sub-grid scales (SGS).

# 5 RANS versus LES

Turbulent flow may be composed of many different features. Therefore it is very important for a CFD model to be able to predict as many of them as possible. Turbulent models are usually tested by simulating a flow past a bluff-body. In particular example flow past a square block is analyzed. [9]

An example of such a flow is shown in Figure 5.1.

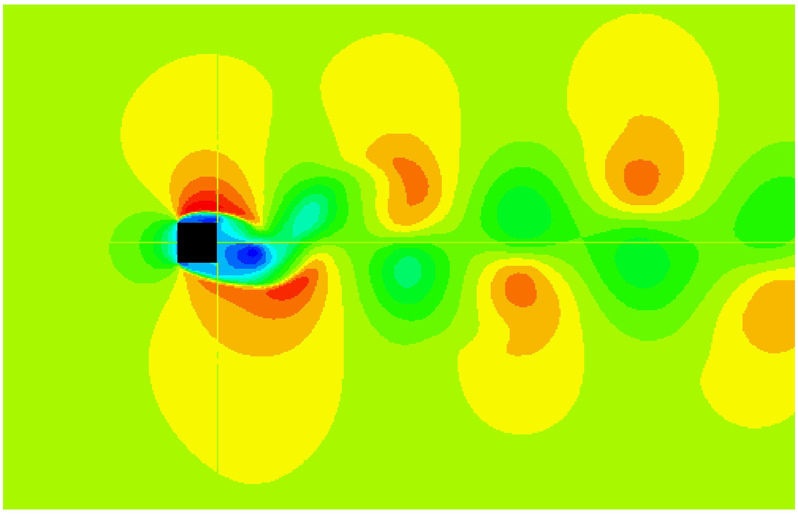

**Figure 5.1: Flow past a square block in 2D**

Streamlines predicted LES, EASM (a sophisticated RANS model) and RANS are shown in Figure 5.2. One can clearly see a strong influence of the model used in the RANS calculation. The EASM reveals the similar topological features as the LES, but differs in the extent of the recirculation zones. RANS predicts considerably larger recirculation zone and the wake region is much more stretched.

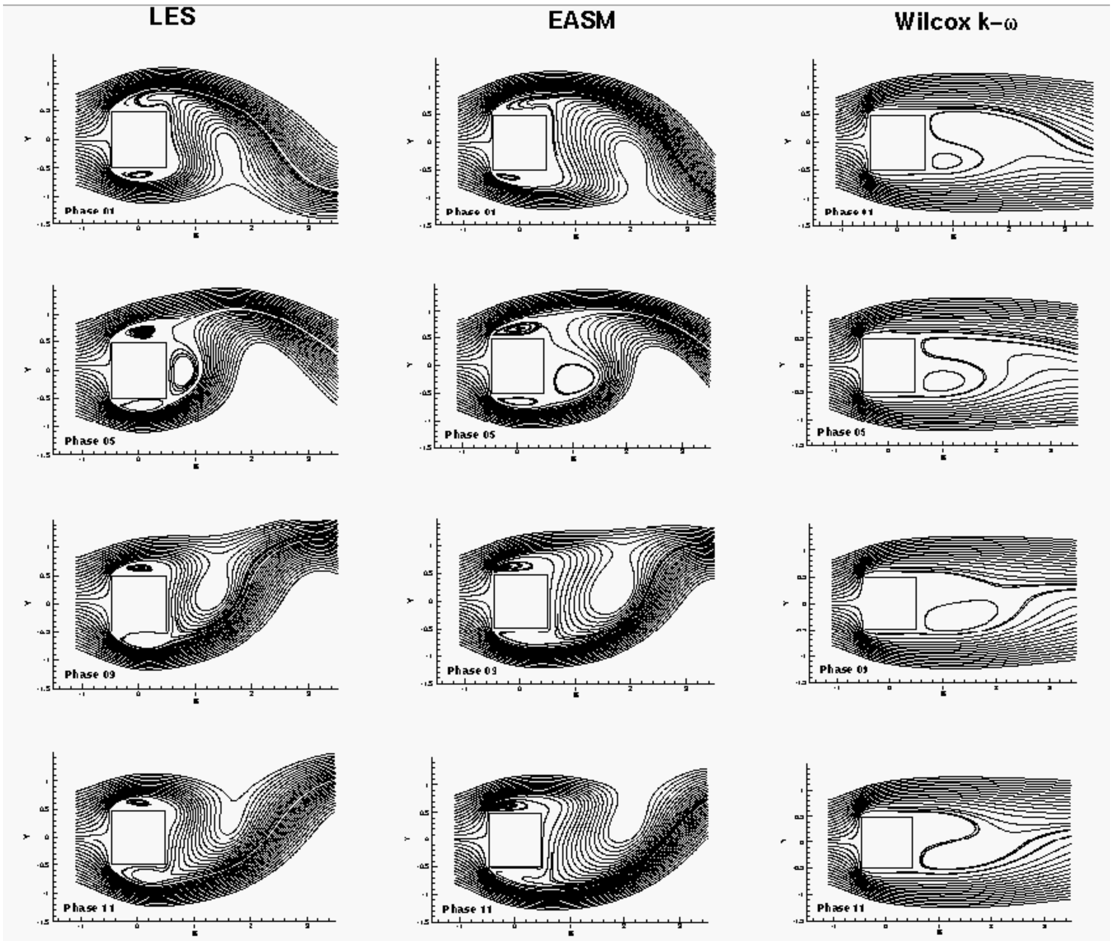


**Figure 5.2: Streamlines predicted by LES, EASM and RANS**

More quantitative difference among observed models is observed when comparing predicted information to experimental data. The comparison is shown in the following set of pictures shown in Figure 5.3 and Figure 5.4. One clearly sees that the simple RANS model of Wilcox fails terribly at predicting turbulent kinetic energy in the wake region. However more sophisticated RANS model of EASM is much more successful. Its results are of comparable quality to the results of LES model.

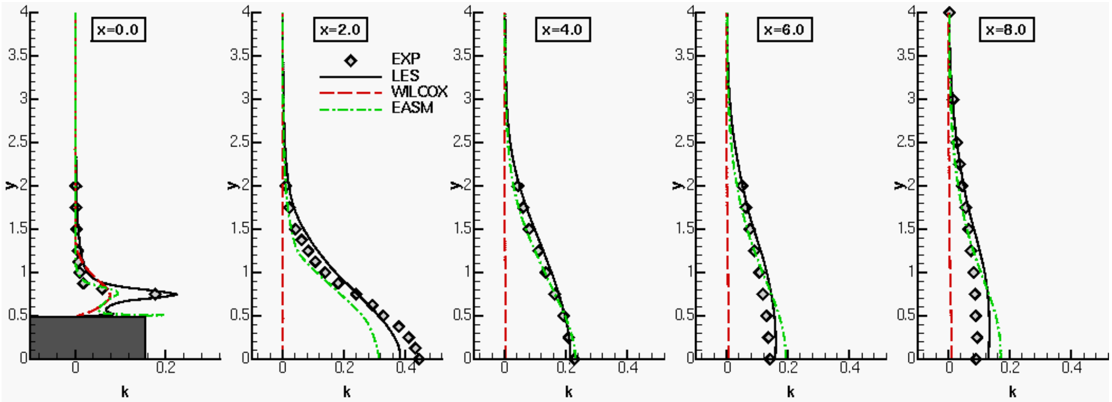


**Figure 5.3: Turbulent kinetic energy in the wake region**

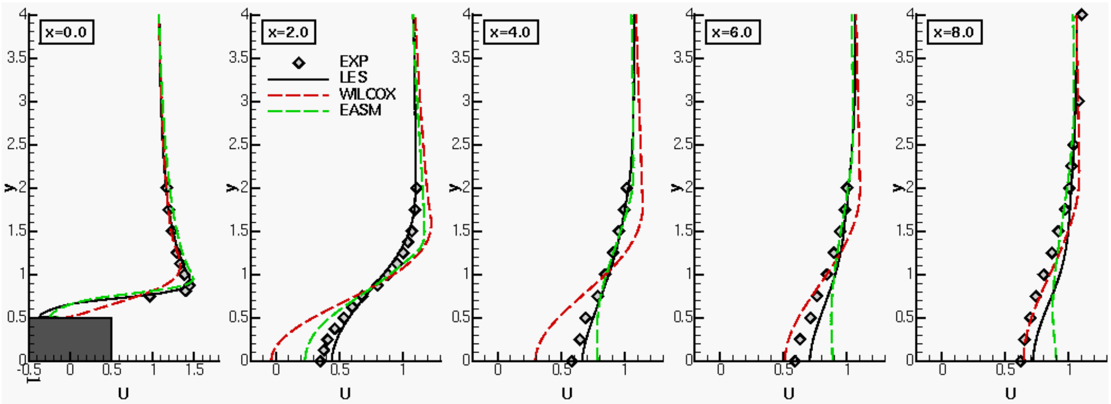


**Figure 5.4: velocity in the wake region**

There are certain examples when even simple RANS models outperform the sophisticated LES model. One example is shown in Figure 5.5, where flow past an airfoil (NACA 4412, alpha = 12°, Re = $1.6*10^6$) is observed.

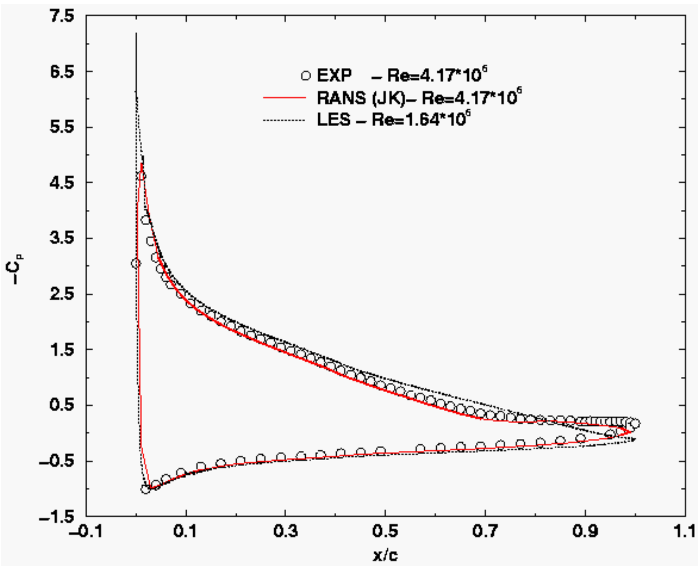


**Figure 5.5: Pressure distribution**

# 6 CONCLUSION

In the last decade CFD has become a major tool in engineering. Due to the progress in computer technology CFD seems now able to deal with industrial applications at moderate costs and turnaround times. The future relevance of CFD will therefore depend on how accurate complex flows can be calculated. Since many flows of engineering interest are turbulent, the appropriate treatment of turbulence will be crucial to the success of CFD.

The flow field of a Newtonian fluid is fully described by the Navier-Stokes equation. However, turbulent flows contain small fluctuations. The resolution of such small motions requires fine grids and time steps, such that a direct simulation becomes unfeasible for high Reynolds numbers.

Using RANS, the computational costs can be reduced by solving the statistically averaged equation system, which requires closure assumptions for the higher moments.

LES aims to reduce the dependence on the turbulence model. Hence the major portion of the flow is simulated without any models, and must be resolved by the grid. Only scales smaller than the resolution of the grid need a model. Consequently LES approach is computationally more demanding than RANS. RANS models have a computing time of only about 5% of the LES.

Sophisticated RANS models like EASM are able to capture important flow features correctly. At low computational costs that makes them already a useful tool in industrial design.

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