Faculty of Mathematics and Physics
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Seminar

# Predictability of financial crashes and other catastrophic events

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**Abstract**

Most previous models proposed for financial crashes have pondered the possible mechanisms to explain the collapse of the stock market index at very short time scales. In this seminar we show that the underlying cause of the crash must be searched years before it in the progressive accelerating ascent of the market price. First we show that large crashes are outliers, so different mechanism as in usual day-to-day price changes must be behind. Then complicated system of stock trading agents is modelled with spin system. The result is log-periodic oscillation of stock price prior to crashes. In the end real data that supports this theory is provided. Similar oscillations occur also in mechanical systems (ruptures, earthquakes). Obtained equation is useful also in forecasting mode (correct predictions of Nikkei and S&P500 indexes are given), which make the theory even more interesting.

# Contents

# 1 Introduction

Stock markets can exhibit very large motions, such as rallies and crashes. Should we expect these extreme variations? Or should we consider them as anomalous? In figure 1, we see

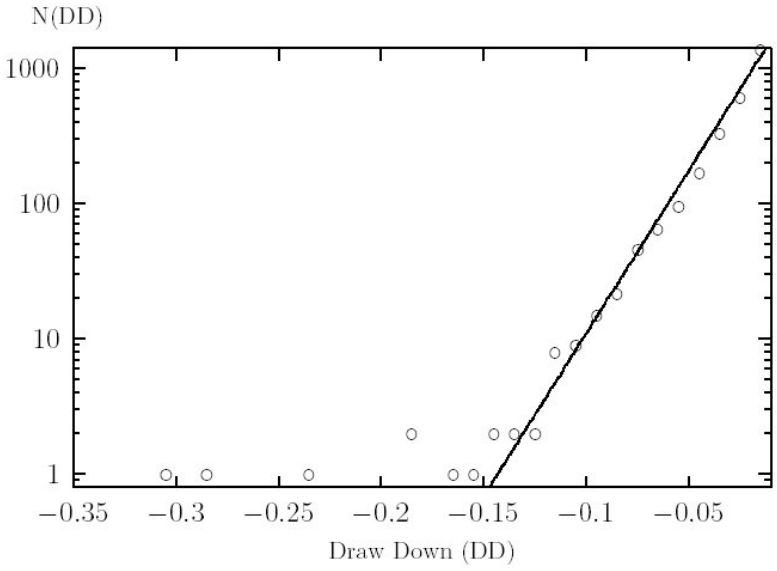


Figure 1: Number of times a given level of draw down has been observed in this century in the Dow Jones Average. The bin-size is 1%. Ref. [12]

the distribution of draw downs[1] (continuous decreases) in the closing value of the Dow Jones Average larger than 1% in the period 1900-94. The distribution resembles very much that of an exponential distribution while three events are standing out. If we fit the distribution of draw downs DD larger than 1% by an exponential law, we find

$$N(DD) = N_0 e^{-|DD|/DD_c}, \tag{1}$$

where the total number of draw downs is $N_0 = 2789$ and $DD_c \approx 1.8\%$. Ranked, the three largest crashes are the crash of 1987, the crash following the outbreak of World War I and the crash of 1929. To quantify how much the three events deviates from equation 1, we can calculate what accordingly would be the typical return time of a draw down of an amplitude equal to or larger than the third largest crash of 23.6%. Equation 1 predicts the number of such drawn downs per century to be $\approx 0.006$. The typical return time of a draw down equal to or larger than 23.6% would then be $\approx 170$ centuries. In contrast, Wall Street has sustained 3 such events in less than a century. The important point here is the presence of these three events that should not have occurred at this high rate. This provides an empirical clue that large draw downs and thus crashes might result from a different mechanism as normal price changes which are to a large extent governed by random processes.

[1]Drawdown is sum of consecutive daily drops until first price rise occurs, i.e. if daily ending price on the second day is 10% lower than on the first day, ending price on the third day is 20% lower than on the first day and at the end of fourth day price is higher than at the end of the third day, then the market has experienced 20% drawdown.

In this seminar this collective destabilizing imitation process will be analysed. The theory has been developed by D. Sornette and A. Johansen (Ref. [1]- [15]). They believe that largest crashes are signatures of cooperative behavior with long-term build-up of correlations between traders. They compare stock markets with systems close to critical point. This model leads to log-periodic oscillations of price. In the end of this paper real market data that confirms their theory will be given.

# 2 Dynamics of prices

In this section dynamics of prices from the rational expectation condition will be given. This is well known model in economics. In this simplified model, we neglect interest rate, risk aversion, information asymmetry, and the market-clearing condition.
Now an exogenous probability of crash will be introduced. If rational agents could somehow trigger the arrival of a crash they would choose never to do so, and if they could control the probability of a crash they would always choose it to be zero. In this model, the crash is a random event whose probability is driven by external forces, and *once this probability is given* it is rationally reected into prices.
Formally, let $j$ denote a jump process whose value is zero before the crash and one afterwards. The cumulative distribution function (cdf) of the time of the crash is called $Q(t)$, the probability density function (pdf) is $q(t) = dQ/dt$ and the crash hazard rate is $h(t) = q(t)/(1 - Q(t))$. The hazard rate is the probability per unit of time that the crash will happen in the next instant if it has not happened yet. Assume for simplicity that, in case of a crash, the price drops by a fixed percentage $\kappa \in (0, 1)$, say between 20 and 30%. Then the dynamics of the asset price before the crash are given by

$$dp = \mu(t)p(t)dt - \kappa p(t)dj \tag{2}$$

where the time-dependent drift $\mu(t)$ is chosen so that the price process satisfies the martingale condition [2]

$$E_t[p(t')] = p(t), \tag{3}$$

where $p(t)$ denotes the price of the asset at time $t$ and $E_t[]$ denotes the expectation conditional on information revealed up to time $t$, i.e. $E_t[dp] = \mu(t)p(t)dt - \kappa p(t)h(t)dt = 0$. This yields: $\mu(t) = \kappa h(t)$. Plugging it into Eq. 2, we obtain a ordinary differential equation whose solution

[2]Using the hypothesis of rational behavior and market efficiency, it is possible to demonstrate how $Y_{t+1}$, the expected value of the price of a given asset at time $t + 1$, is related to the previous values of prices $Y_0, Y_1, ..., Y_t$ through the relation $E\{Y_{t+1}|Y_0, Y_1, ..., Y_t\} = Y_t$. Stochastic processes obeying the conditional probability given are called martingales. The notion of a martingale is, intuitively, a probabilistic model of a 'fair' game. In gambler's terms, the game is fair when gains and losses cancel, and the gambler's expected future wealth coincides with the gambler's present assets. The fair game conclusion about the price changes observed in a financial market is equivalent to the statement that there is no way of making a profit on an asset by simply using the recorded history of its price fluctuations. Ref. [17]

is

$$\log \frac{p(t)}{p(t_0)} = \kappa \int_{t_0}^{t} h(t')dt' \tag{4}$$

before the crash. The higher the probability of a crash, the faster the price must increase (conditional on having no crash) in order to satisfy the martingale condition. Intuitively, investors must be compensated by the chance of a higher return in order to be induced to hold an asset that might crash.

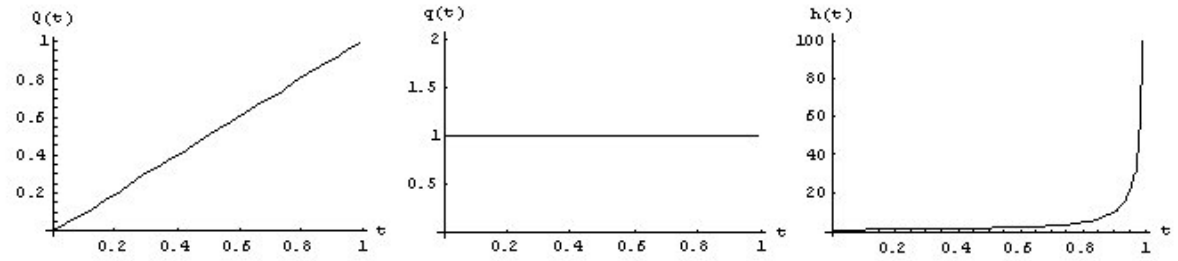


Figure 2: The cumulative distribution function of the time of the crash $Q(t)$, the probability density function $q(t) = dQ/dt$ and the crash hazard rate $h(t)$.

If the price during the crash drops by a fixed percentage $\kappa \in (0, 1)$ of the price increase above a reference value $p_1$. Then the dynamics of the asset price before the crash are given by similar equation as 2:

$$dp = \mu(t)p(t)dt - \kappa[p(t) - p_1]dj, \tag{5}$$

If we do not allow the asset price to fluctuate under the impact of noise, the solution for Eq. 3 is $p(t) = p(t_0)$. If we put Eq. 5 in Eq. 3 we obtain

$$\mu(t)p(t) = \kappa[p(t) - p_1]h(t). \tag{6}$$

In words, if the crash hazard rate h(t) increases, the return $\mu(t)$ increases to compensate the traders for the increasing risk. Final solution for the price before the crash is

$$p(t) = p(t_0) + \kappa[p(t_0) - p_1] \int_{t_0}^{t} h(t')dt' \tag{7}$$

These two different scenarios for the price drops raises a rather interesting question. If the second scenario is the correct one, then crashes are nothing but (a partial) depletion of preceding bubbles and hence signals the market's return towards equilibrium. Hence, it may as such be taken as a sign of economic health. On the other hand, if the first scenario is true, this suggest that bubbles and crashes are instabilities which are built-in or inherent in the market structure and that they are signatures of a market constantly out-of-balance, signaling fundamental systemic instabilities. Johansen and Sornette have shown that the second scenario is slightly more warranted according to the data.

# 3 "Microscopic" modelling

Consider a network of agents: each one is indexed by an integer $i = 1...I$, and $N(i)$ denotes the set of the agents who are directly connected to agent $i$ according to some graph. For simplicity, we assume that agent $i$ can be in only one of two possible states: $s_i \in \{+1, -1\}$. We could interpret these states as "buy" and "sell", "bullish" and "bearish", "calm" and "nervous"... When trader lacks information, it is optimal to imitate other traders (Ref. [1]). The state of trader $i$ is determined by:

$$s_i = sign\Big(K \sum_{j \in N(i)} s_j + \sigma\epsilon_i\Big), \tag{8}$$

where $K$ is a positive constant, and $\epsilon_i$ is independently distributed according to the standard normal distribution. This equation belongs to the class of stochastic dynamical models of interacting particles, which have been much studied mathematically in the context of physics and biology.

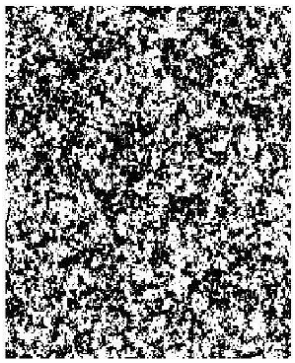

Figure 3: $K < K_c$: buy (white squares) and sell (back squares) configuration in a twodimensional Manhattan-like planar network of agents interacting with their four nearest neighbors. There are approximately the same number of white and black cells - the market has no consensus. The size of largest local clusters quantifies the correlation length, i.e. the distance over which the local imitations between neighbors propagate before being significantly distorted by the "noise" in the transmission process resulting from the idiosynchratic signals of each agent. Ref. [1]

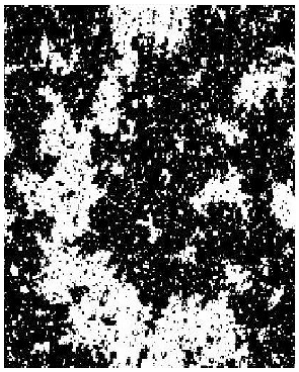

Figure 4: Same as Fig.3 for K close to $K_c$. There are still approximately the same number of white and black cells, i.e., the market has no consensus. However, the size of the largest local clusters has grown to become comparable to the total system size. In addition, holes and clusters of all sizes can be observed. The "scaleinvariance" or "fractal" looking structure is the hallmark of a "critical state" for which the correlation length and the susceptibility become infinite (or simply bounded by the size of the system.) Ref. [1].

In this model, the tendency towards imitation is governed by $K$, which is called the coupling strength; the tendency towards idiosyncratic behavior is governed by $\sigma$. Thus the value of $K$ relative to $\sigma$ determines the outcome of the battle between order and disorder, and eventually the probability of a crash. More generally, the coupling strength $K$ could be heterogeneous across pairs of neighbors, and it would not substantially affect the properties of the model. Some of the $K_{ij}$'s could even be negative, as long as the average of all $K_{ij}$'s was strictly positive.

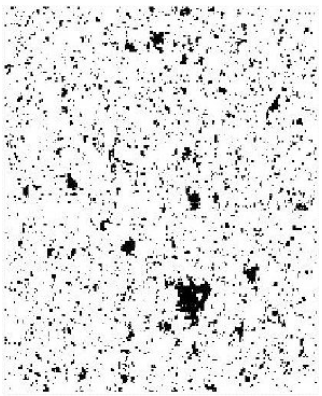

Figure 5: Same as Fig.3 for $K > K_c$. The imitation is so strong that the network of agents spontaneously break the symmetry between the two decisions and one of them predominates. Here, the case where the "buy" state has been selected. The collapse onto one of the two states is essentially random and results from the combined effect of a slight initial bias and of fluctuations during the imitation process. Only small and isolated islands of sellers remain in an ocean of buyers. This state would correspond to a bubble, a strong bullish market. Ref. [1]

Note that this equation only describes the state of an agent at a given point in time. In the next instant, new $\epsilon_i$'s are drawn, new influences propagate themselves to neighbors, and agents can change states. Thus, the best we can do is give a statistical description of the states. The one that best describes the chance that a large group of agents finds itself suddenly in agreement is the susceptibility of the system. To define it formally, assume that a global influence term G is added to Eq. 8:

$$s_i = sign\Big(K \sum_{j \in N(i)} s_j + \sigma\epsilon_i + G\Big) \tag{9}$$

This influence term will tend to favour state +1 (state -1) if $G > 0$ (if $G < 0$). Equation 8 simply corresponds to the special case $G = 0$: no global influence. Define the average state as $M = (1/I)\sum_{i=1}^{I} s_i$. In the absence of global influence, it is easy to show by symmetry that $E[M] = 0$ [3] : agents are evenly split between the two states. In the presence of a positive (negative) global influence, agents in the positive (negative) state will outnumber the others: $E[M] \cdot G > 0$. With this notation, the susceptibility of the system is defined as:

$$\chi = \left.\frac{d(E[M])}{dG}\right|_{G=0} \tag{10}$$

In words, the susceptibility measures the sensitivity of the average state to a small global influence. The susceptibility has a second interpretation as (a constant times) the variance of the average state $M$ around its expectation of zero caused by the random idiosyncratic shocks $\epsilon_i$. Another related interpretation is that, if you consider two agents and you force the first one to be in a certain state, the impact that your intervention will have on the second agent will be proportional to $\chi$. These reasons show that susceptibility correctly measures the ability of the system of agents to agree on an opinion. If we interpret the two states in a

[3] $E[Y]$ is expected value of $Y$, i.e. $E[Y] = 1/T \int_0^T Y(t)dt$

manner relevant to asset pricing, it is precisely the emergence of this global synchronisation from local imitation that can cause a crash. Thus, we will characterise the behavior of the susceptibility, and we will posit that the hazard rate of crash follows a similar process. We do not want to assume a one-to-one mapping between hazard rate and susceptibility because there are many other quantities that provide a measure of the degree of coordination of the overall system, such as the correlation length (i.e. the distance at which imitation propagates) and the other moments of the fluctuations of the average opinion. In the next section will be shown that all these quantities have the same generic behaviour.

## 3.1 Interaction networks

It turns out that, in the imitation model defined by Equation 8, the structure of the network affects the susceptibility. We will discuss two alternative network structures for which the behavior of susceptibility is well understood.

## 3.2 Two-Dimensional Grid

As the simplest possible network, let us assume that agents are placed on a two-dimensional grid in the plane. Each agent has four nearest neighbors. The relevant parameter is $K/\sigma$. It measures the tendency towards imitation relative to the tendency towards idiosyncratic behavior. In the context of the alignment of atomic spins to create magnetisation, this model is related to the two-dimensional Ising model. There exists a critical point $K_c$ that determines the properties of the system. When $K < K_c$, disorder reigns: the sensitivity to a small global influence is small, the clusters of agents who are in agreement remain of small size, and imitation only propagates between close neighbors. Formally, in this case, the susceptibility $\chi$ of the system is finite. When $K$ increases and gets close to $K_c$, order starts to appear: the system becomes extremely sensitive to a small global perturbation, agents who agree with each other form large clusters, and imitation propagates over long distances. These are the characteristics of critical phenomena. The hallmark of criticality is the power law, and indeed the susceptibility goes to infinity according to a power law:

$$\chi \approx A(K_c - K)^{-\gamma}, \tag{11}$$

where $A$ is a positive constant and $\gamma > 0$ is critical exponent of susceptibility.

| Ising 2-D | Stockmarket |
|---|---|
| $T$ | $K \approx const. \times t$ |
| $\chi(T)$ | $h(t)$ |

Table 1: Relation between 2-D Ising system and stock market.

We do not know the dynamics that drive the key parameter of the system $K$. We assume that it evolves smoothly, so that we can use a first-order Taylor expansion around the critical

point. Let us call $t_c$ the first time such that $K(t_c) = K_c$. Then prior to the critical date $t_c$ we have the approximation: $K_c - K \approx const \times (t_c - t)$. Using this approximation, the authors of the theory posit that the hazard rate of crash behaves in the same way as the susceptibility in the neighborhood of the critical point. This yields the following expression:

$$h(t) \approx B \times (t_c - t)^{-\alpha}, \tag{12}$$

where $B$ is a positive constant. The exponent $\alpha$ must lie between zero and one for an economic reason: otherwise, the price would go to infinity when approaching $t_c$. The probability per unit of time of having a crash in the next instant conditional on not having had a crash yet becomes unbounded near the critical date $t_c$.
Plugging Eq. 12 into Eq. 7 gives following law for the price:

$$p(t) \approx p(t_c) - \frac{\kappa B'}{\beta} \times (t_c - t)^{\beta}, \tag{13}$$

where $\beta = 1 - \alpha \in (0, 1)$ and $p_c$ is price at critical time. We see that the price before the crash also follows a power law. The slope of the price, which is the expected return per unit of time, becomes unbounded as we approach the critical date. This is to compensate for an unbounded probability of crash in the next instant.

### 3.3 Hierarchical Diamond Lattice

The stock market constitute an ensemble of inter-actors which differs in size by many orders of magnitudes ranging from individuals to gigantic professional investors, such as pension funds. Furthermore, structures at even higher levels, such as currency influence spheres exist and with the current globalization and de-regulation of the market structures on the largest possible scale, i.e., the world economy, are beginning to form. This means that the structure of the financial markets has features, which resembles that of hierarchical systems and with traders on all levels of the market. This means that the plane network used in the previous section is over-simplification. We will examine a slightly more complicated structure, that might be more realistic model of the complicated network of communications between financial agents. Start with a pair of traders who are linked to each other. Replace this link by a diamond where the two original traders occupy two diametrically opposed vertices, and where the two other vertices are occupied by two new traders. This diamond contains four links. For each one of these four links, replace it by a diamond in exactly the same way, and iterate the operation. The result is a diamond lattice. After $p$ iterations, we have and $2/3(2+4p)$ traders and $4p$ links between them. Most traders have only two neighbors, a few traders (the original ones) have $2p$ neighbors, and the others are in between.
The basic properties are similar to the ones described above: there exists a critical point $K_c$; for $K < K_c$ the susceptibility is finite; and it goes to infinity as $K$ increases towards $K_c$. The only difference is that the critical exponent can be a complex number because of nature of lattice. The general solution for the susceptibility is a sum of terms like the one in Equation

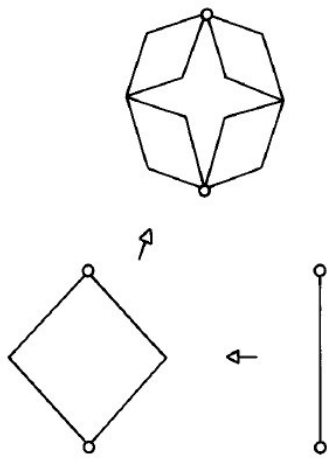

Figure 6: First three steps of building of a diamond lattice. Ref. [13]

11 with complex exponents. The first order expansion of the general solution is:

$$\chi \approx \Re[A_0(K_c - K)^\gamma + A_1(K_c - K)^{\gamma+i\omega} + ...] \quad (14)$$

$$\chi \approx A'_0(K_c - K)^\gamma + A'_1(K_c - K)^\gamma \cos[\omega \log(K_c - K) + \psi] + ... \quad (15)$$

where $A'_0, A'_1, \omega$ and $\psi$ are real numbers. We see that the power law is now corrected by oscillations whose frequency explodes as we reach the critical time. These accelerating oscillations are called "log-periodic" and $\frac{\omega}{2\pi}$ is called their "log-frequency". Following the same steps as in previous section, we can calculate the hazard rate of a crash and then evolution of the price before the critical time:

$$p(t) \approx A - B(t_c - t)^\beta + C(t_c - t)^\beta \cos(\omega \log(t_c - t) + \phi) \quad (16)$$

where $\phi$ is another phase constant. The key feature is that oscillations appear in the price of the asset just before the critical date. The local maxima of the function are separated by time intervals that tend to zero at the critical date, and do so in geometric progression, i.e. the ratio of consecutive time intervals is a constant

$$\lambda \equiv e^{\frac{2\pi}{\omega}} \quad (17)$$

This is very useful from an empirical point of view because such oscillations are much more strikingly visible in actual data than a simple power law: a fit can "lock in" on the oscillations which contain information about the critical time $t_c$. If they are present, they can be used to predict the $t_c$ simply by extrapolating frequency acceleration. Since the probability of the crash is highest near the critical time, this can be an interesting forecasting exercise.

# 4 Status of log-periodicity

Log-periodicity is an observable signature of the symmetry of discrete scale invariance (DSI). DSI is a weaker symmetry than (continuous) scale invariance. The latter is the symmetry of

a system which manifests itself such that an observable $\mathcal{O}(x)$ as a function of the "control" parameter $x$ is scale invariant under the change $x \to \lambda x$ for arbitrary $\lambda$, i.e., a number $\mu(\lambda)$ exists such that

$$\mathcal{O}(x) = \mu(\lambda)\mathcal{O}(\lambda x) \tag{18}$$

The solution of 18 is simply a power law $\mathcal{O}(x) = x^\alpha$, with $\alpha = -\frac{\log \mu}{\log \lambda}$, which can be verified directly by insertion. In DSI, the system or the observable obeys scale invariance only for specific choices of the magnification factor $\lambda$, which form in general an infinite but countable set of values $\lambda_1, \lambda_2, ...$ that can be written as $\lambda_n = \lambda^n$. $\lambda$ is the fundamental scaling ratio determining the period of the resulting log-periodicity. This property can be qualitatively seen to encode a lacunarity of the fractal structure. The most general solution of Eq. 18 with $\lambda$ (and therefore $\mu$) is

$$\mathcal{O}(x) = x^\alpha P\left(\frac{\ln x}{\ln \lambda}\right) \tag{19}$$

where $P(y)$ is an arbitrary periodic function of period 1 in the argument, hence the name log-periodicity. Expanding it in Fourier series $\sum_{n=-\infty}^{\infty} c_n \exp(2n\pi i \frac{\ln x}{\ln \lambda})$, we see that $\mathcal{O}(x)$ becomes a sum of power laws with the infinitely discrete spectrum of complex exponents $\alpha_n = \alpha + 2\pi i n \ln \lambda$, where $n$ is an arbitrary integer. Thus, DSI leads to power laws with complex exponents, whose observable signature is log-periodicity. Specifically, for financial bubbles prior to large crashes, we shall see that a first order representation of Eq. 19

$$I(t) = A + B(t_c - t)^\beta + C(t_c - t)^\beta \cos(\omega \ln(t_c - t) - \phi) \tag{20}$$

captures well the behaviour of the market price $I(t)$ prior to a crash or large correction at a time $t_c$.

# 5 Empirical tests

The interesting thing after all theoretical framework is test of theory with real data from stock markets. We have to fit data to a function that has three linear and four non-linear parameters. The last point used for all crashes is the highest value of the price before the crash, the first point used is the lowest value of the price when the bubble started. Data is fitted using the down-hill simplex algorithm. Measure of quality of fit is variance

$$Var(f) = \frac{1}{N}\sum_{i=1}^{N}(y_i - f(t_i))^2 \tag{21}$$

If one tries to fit data in period with no crash, Eq. 16 simply can not "lock-in" and meaningless results are obtained. Numerous stock crashes have been investigated as well as the collapses of currencies. Value of the scaling ratio is similar in all events ($2.2 \leq \lambda \leq 2.8$). This agreement cannot be accidental and constitutes one of the key tests of our framework.

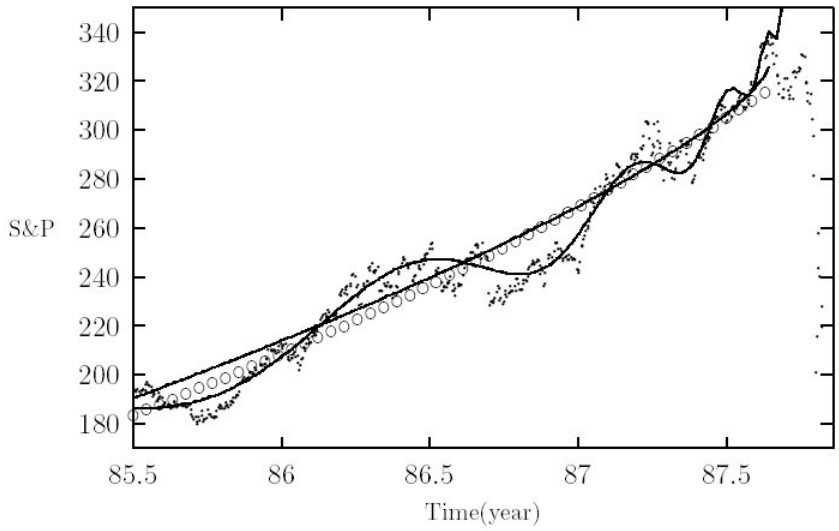


Figure 7: The New York stock exchange index S&P500 from July 1985 to the end of 1987. The $\circ$ represent a constant return increase - exponential function with a characteristic increase of $\approx 4$ years$^{-1}$ and var$(F_{exp}) \approx 113$. The best fit to a pure power-law gives $327 - 79(87.65 - t)^{0.7}$ and var$_{pow} \approx 107$. The best fit to Eq.16 gives $A \approx 412$, $B \approx -165$, $t_c \approx 87.74$, $C \approx 12$, $\omega \approx 7.4$, $\phi \approx 2.0$, $\beta \approx 0.33$ and var$_{lp} \approx 36$. Ref. [13]

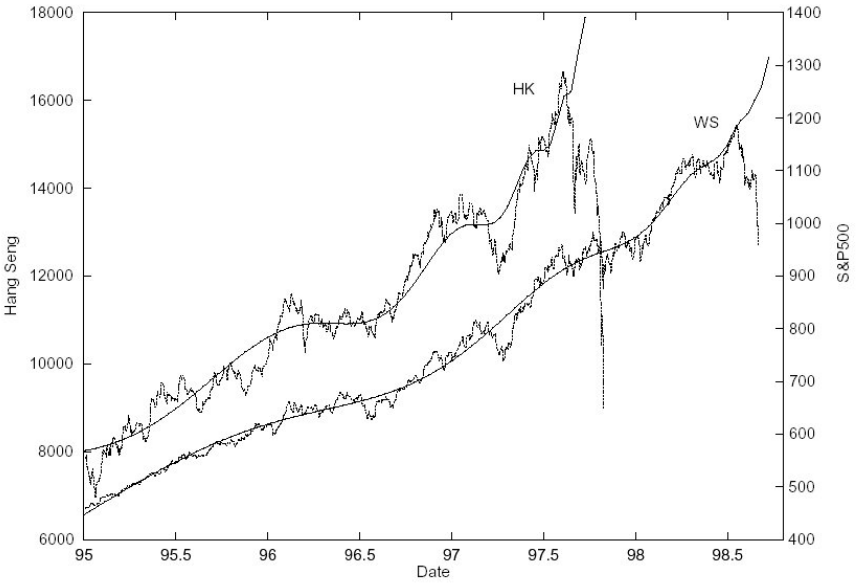


Figure 8: The Hang Seng index prior to the October 1997 crash on the Hong-Kong Stock Exchange and the S&P 500 stock market index prior to the crash on Wall Street in August 1998. The fit to the Hang Seng index is equation with $\beta \approx 0.34$, $t_c \approx 97.74$, $\omega \approx 7.5$. The fit to the S&P 500 has parameters $\beta \approx 0.60$, $t_c \approx 98.72$, $\omega \approx 6.4$. Ref. [2]

# 6 Prediction

An obvious question concerns the predictive power of Eq. 16. In almost all cases of fits of past crashes it turned out that the market crash started at a time between the date of the last point of the fit and the predicted $t_c$. However, since the development of this theory no crash

has been predicted before it had occurred. In the last few years only Nasdaq index crashed in April 2002. This was another example of log-periodicity in a speculative bubble ending in a crash.

Cooperative herding behaviour of traders produces market evolutions that are symmetric to the accelerating speculative bubbles often ending in crashes. This symmetry is performed with respect to a time inversion around a critical time $t_c$ such that $t_c - t$ for $t < t_c$ is changed to $t - t_c$ for $t > t_c$. There exist critical times $t_c$ at which the market culminates, with either a power law increase with accelerating log-periodic oscillations or a power law decrease with decelerating log-periodic oscillations. It is impossible to find a market for which both phenomena are simultaneously observed for the same $t_c$. The main reason is that accelerating markets with log-periodicity almost always end-up in a crash, a market rupture that thus breaks down the symmetry. The breakdown of local symmetry around the critical point $t_c$ is not unknown in thermodynamic phase transitions, for example $\lambda$-transition in $^4He$, so named because of the asymmetric shape of the specific heat around $T_\lambda$ with more abrupt decay above $T_\lambda$ than below qualitatively similar to a market price time series around a crash.

The largest practical success of this theory was correct prediction of behaviour of Nikkei index. If correction of higher order is added to Eq. 16 and term $t_c - t$ is replaced with $t - t_c$, new equation is suitable for accurate fits of "anti-bubbles", i.e. declines of stock indexes.

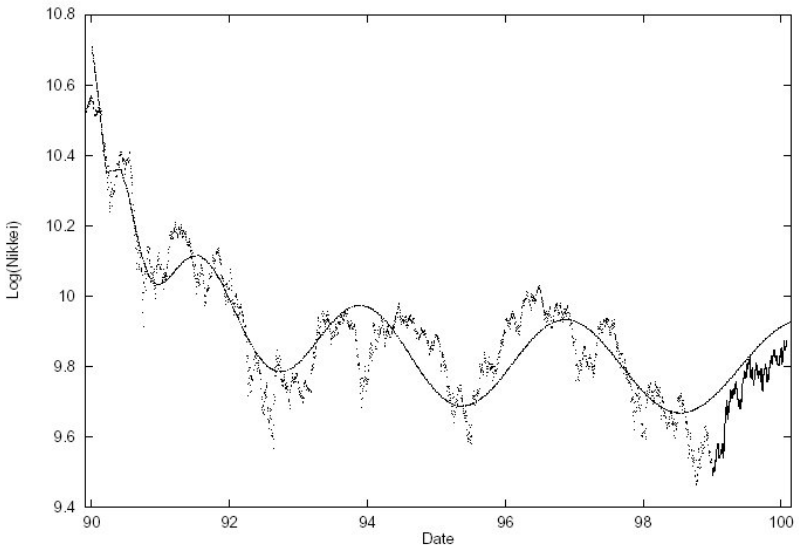


Figure 9: Logarithm of the Nikkei Index. The dots are the data used in the fit (ticked line) that covers the 9 year period from 1990 to 1999. The solid line is the actual behaviour of the Nikkei after the last point used in the fit and covers the period 1 Jan. 1999 to 28 Jan. 2000. The prediction was made public on the 25 Jan. 1999. Ref. [12]

# 7 Prediction of mechanical ruptures and earthquakes

Log-periodic oscillations are evidence for the existence of a special time scale in approaching crash. Such scales can spontaneously appear in other complex systems as well, for example

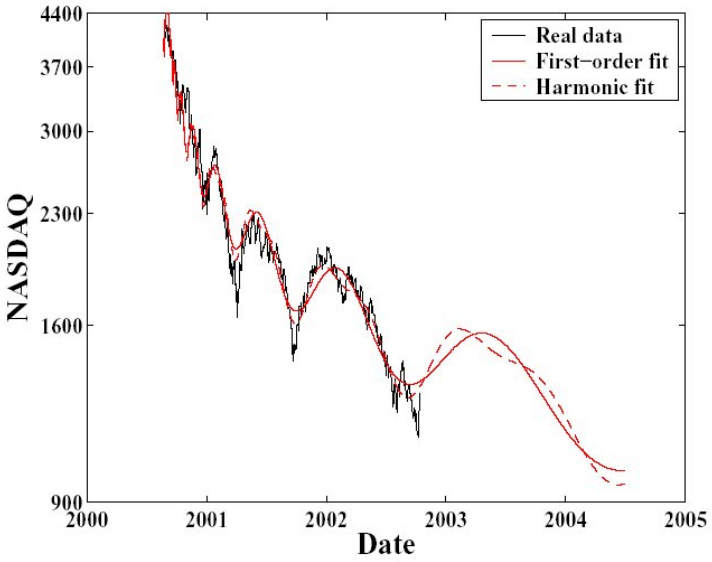


Figure 10: NASDAQ reached its all-time high of 5133 on the 10th of March 2000. "Bubble" turned into "anti-bubble". If $t_c - t$ is replaced with $t - t_c$ in Eq.16, "anti-bubble" can be fitted and price forecasted. This prediction was published on 1 Dec 2002. In the end of March 2003, NASDAQ is 1400. Ref. [15]

composite materials and tectonic plate movements. Explanations are beyond the scope of this seminar, only two cases are given.

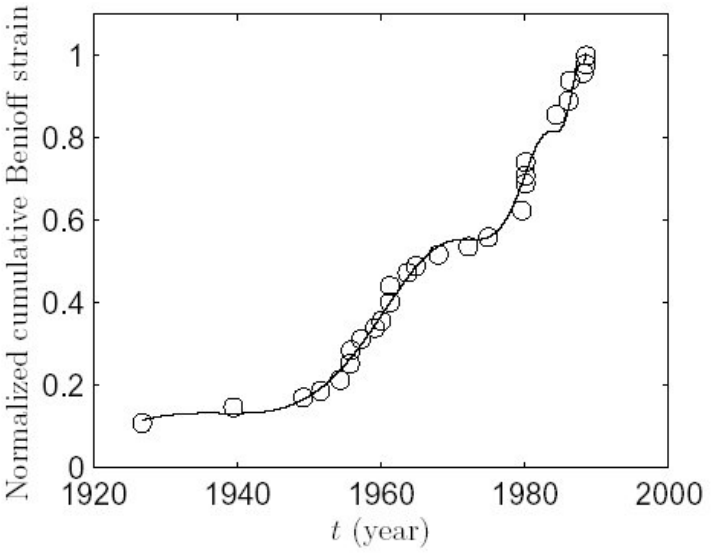


Figure 11: The fit of a power law with log-periodic oscillations to the normalized cumulative Benioff strain of the seismic precursors of the 1989 Loma Prieta earthquake. Ref. [16]

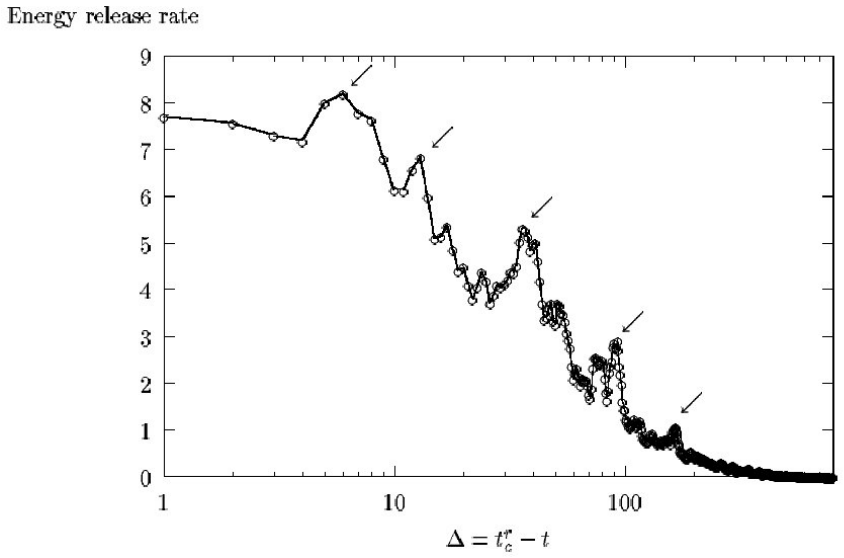


Figure 12: Log-log plot of the energy release rate of a mechanical system approaching rupture. Ref. [3]

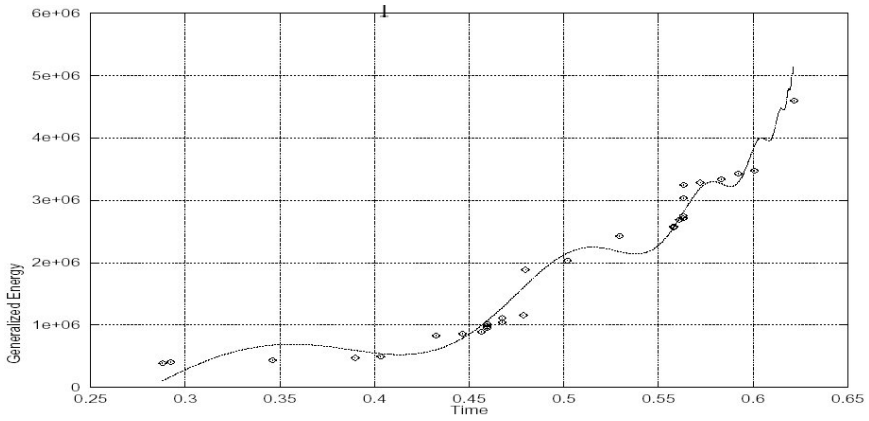


Figure 13: Cumulative seismic deformation rate prior to a large rockburst in a deep South African mine. Such fits are also used in a forecasting mode. Ref. [3]

# 8 Conclusion

In this paper it has been shown that large stock market crashes are analogous to critical points studied in the statistical physics in relation to magnetism, melting, and so on. Main assumption is the existence of a cooperative behavior of traders imitating each other. A general result of the theory is the existence of log-periodic structures decorating the time evolution of the system. The main point is that the market anticipates the crash in a subtle self-organized and cooperative fashion, hence releasing precursory "fingerprints" observable in the stock market prices. In other words, this implies that market prices contain information on impending crashes.

A fundamental remaining question concerns the use of a reliable crash prediction scheme. Assume that a crash prediction is issued stating that a crash will occur $x$ weeks from now. At least three different scenarios are possible:

- Nobody believes the prediction which was then futile and, assuming that the prediction was correct, the market crashes.
- Everybody believes the warning, which causes panic and the market crashes as consequence. The prediction hence seems self-fullfiling.
- Enough believe that the prediction may be correct and the steam goes off the bubble. The prediction hence disproves itself.

None of these scenarios are attractive. In the first two, the crash is not avoided and in the last scenario the prediction disproves itself and as a consequence the theory looks unreliable. This seems to be unescapable fate of scientific investigations of systems with learning and reflective abilities, in contrast with the usual inanimate and unchanging physical laws of nature. Furthermore, this touches the key-problem of scientific responsibility. Naturally, scientists have a responsibility to publish their findings. However, when it comes to the practical implementation of those findings in society, the question becomes considerably more complex.

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