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Airfoil boundary layer

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Abstract

The concept of a *boundary layer* is crucial to the understanding of the flow around an obstacle at large Reynolds numbers. Far from the object, as long as the incident flow is not turbolent, the terms corresponding to viscous forces, in the Navier-Stokes equation, are neglibile; the flow velocity profile is then effectivly that of an ideal fluid.
The transition between the solution corresponding to ideal fluid far from the boundry walls, and the zero-velocity condition at the walls themselves, occurs over a region know as a *boundary layer*. The seminar thus provides the necessary complement to the first seminar, which discussed the airfoil in ideal fluids.

Contents

1 Introduciton

In laminar flow at high Reynolds number around a solid object, the viscous terms in the equation of motion need only be taken into account in a narrow region close to the object, known as the *boundary layer*.
Vorticity created near the walls is then carried along downstream in the wake. We will see that downstream from a solid obstacle, the velocity gradients remain highly localized within a small region of the total voloume of the flow. We thus find the justification of for the extensive discussion of the flow of the ideal fluids (Reference [4]), since the coresponding flow profiles apply almost everywhere: the effects of viscosity are found to be significant only within the boundary layer near a solid wall or, downstream of an obstacle, in the wake.
The concept of a boundary layer thus provides the connecting link between two important domains of fluid mechanics: the study of the velocity field of ideal fluids in potential flow and the experimental determination of the flow of viscous fluids at finite Reynolds number.
This seminar is centered around incident laminar boundary layers. Therefore this seminar does not cover the boundary layer behaviour when the flow upstream of the object is already turbulent, but we do briefly examine the phenomenon of *boundary layer separation* which by itself does not fall into the category of laminar boundary layers. This seminar is written around the classical Prandtl - Titjens works (Reference [1] and [2]).

2 The Navier-Stokes equations

The Navier-Stokes equations describe how *velocity*, *pressure*, *temperature* and *density* of a moving fluid are related. The equations were derived independently by G.G. Stokes, in England, and M. Navier, in France, in the early 1800's. The equations are extensions of the Euler equations and include the effects if viscosity on the flow.
The equations are a set of coupled differential equations and could, in theory be solved for a given flow problem by using methods from calculus. But, in pratice, these quations are too difficult to solve analytically. In the past, engineers made further approximations and simplifications to the equation set, untill they had a group of equations that they could solve. Recently, high speed computers have been used to solve approximations to the equations using a variety of tehniques like finite difference, finite volume, finite element and spectral methods. This area of study is clalled Computional Fluid Dynamics or CFD.
The Navier-Stokes equations consist of time-dependent continuity equation for conservation of mass, three time dependent conservation of momentum equations and a time-dependent conservation of energy equation.
There are four independtent variables in the problem, the x, y and z spatial coordinates of some domain, and the time t. Thare six dependent variables; the pressure p, density ρ, and temperature T (which is contained in the energy equation through the total energy E_t) and three components of the velocity vector; the v_x component is in the x direction, the v_y component is in the y direction and the v_z component is in the z direction. All of the dependent variables are functions of all four independent variables.
The set of equations is:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0 \tag{1}$$

$$\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial \Pi_{ik}}{\partial x_k} = 0 \tag{2}$$

$$\frac{\partial \epsilon}{\partial t} + \nabla j_\epsilon = 0 \tag{3}$$

Where Π_{ik} is a momentum flow tensor ($\Pi_{ik} = \delta ik + \rho v_i v_k$) and ϵ equals $\epsilon = \rho^2 v/2$ and j_ϵ is the density of the energy flow. If we, for clarity rewrite these equations into component form the set is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0 \tag{4}$$

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial(\rho v_y^2)}{\partial x} + \frac{\partial(\rho v_x v_y)}{\partial y} + \frac{\partial(\rho v_x v_z)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) \tag{5}$$

$$\frac{\partial(\rho v_y)}{\partial t} + \frac{\partial(\rho v_y v_y)}{\partial x} + \frac{\partial(\rho v_y^2)}{\partial y} + \frac{\partial(\rho v_y v_z)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) \tag{6}$$

$$\frac{\partial(\rho v_z)}{\partial t} + \frac{\partial(\rho v_y v_z)}{\partial x} + \frac{\partial(\rho v_x v_z)}{\partial y} + \frac{\partial(\rho v_z^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) \tag{7}$$

$$\begin{aligned}
\frac{\partial(E_t)}{\partial t} + \frac{\partial(v_x E_t)}{\partial x} + \frac{\partial(v_x E_t)}{\partial y} + \frac{\partial(v_x E_t)}{\partial z} &= \\
&- \frac{\partial(v_x p)}{\partial x} - \frac{\partial(v_y p)}{\partial y} - \frac{\partial(v_z p)}{\partial z} \\
&- \frac{1}{Re \cdot Pr}\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) \\
&+ \frac{1}{Re}\left(\frac{\partial}{\partial x}\left(v_x \tau_{xx} + v_y \tau_{xy} + v_z \tau_{xz}\right)\right. \\
&\quad + \frac{\partial}{\partial y}\left(v_x \tau_{xy} + v_y \tau_{yy} + v_z \tau_{yz}\right) \\
&\quad \left. + \frac{\partial}{\partial z}\left(v_x \tau_{xz} + v_y \tau_{yz} + v_z \tau_{zz}\right)\right)
\end{aligned} \tag{8}$$

where Re is the Reynolds number, which is a similarity paramter that is the ratio of the scalling of the intertia of the flow to the viscous forces in the flow. The q variables are the heat flux components and the Pr is the Prandtl number which is a similarity parameter that is the ratio of the viscous stresses to the termal stresses. The τ variables are components of the stress tensor. Each component of the stress tensor is itself a second derivative of the velocity components.
The terms on the left hand side of the momentum equations are called the convective terms of the equations. Convection is a physical process that occurs in a flow of gas in which some property is transported by the ordered motion of the flow. The terms on the right hand side of the momentum equations that are multiplied by the inverse Reynolds number are called the diffusion terms. Diffusion is physical process that occurs in a flow of gas in which some property is transported by the random motion of the molecules of the gas. Diffusion is related to the stress tensor and to the viscosity of the gas. Turbolence,

and the generation of boundry layers are the result of the diffusion in the flow. The Euler equations contain only the convective terms of the Navier - Stokes equations and can not, therefore, model boundary layers. There is a special simplification of the Navier-Stokes equation that describe boundry layer flows.

3 The boundary layer near a flat plate in uniform flow

Consider a uniform flow, at velocity v, parallel to a semi-infinite flat plate $(y = 0, x > o)$ with its edge normal to the plane. if the velocity is sufficiently high, the pressence of the plate will not be noticable upstream of the plate's leading edge. In fact, velocity gradients will not have time to diffuse an appreciable distance from the edge of the plate before being carried downstream by the flow.
As a result, near the edge, the velocity gradinets, and the vorticity, are confined very close to the plate. The velocity gradients attenuate with time; indeed, the spatial distribution of vorticity spreads out by viscous diffusion starting from the wall over distances of the order of

$$\delta \approx \sqrt{\upsilon t} \tag{9}$$

where υ is the kinematic viscosity of the fluid. However the fluid is here simultaneously carried along parallel to the plate with a velocity of the order of magnitude of the external flow velocity v. The order of magnitude of the time t, elapsed in the reference frame of the moving fluid, while the later moves along a distance x_0 downstream from the edge of the plate, is thus given by

$$t \approx \frac{x_0}{v} \tag{10}$$

By substituting this value of t in the previous expression for δ, we find that, at a distance x_0 from the leading edge of the plate, the velocity gradients are confined within a distance from the wall of order

$$\delta(x_0) \approx \sqrt{\frac{\upsilon x_0}{v}} \tag{11}$$

$\delta(x_0)$ thus represents the thickness of the *boundary layer* over which a transition occous between ideal-fluid type flow far from the plate, where the vorticity has not had time to diffuse, and viscous flow near it.
In this region, viscosity effects dominate (requiring specifically a zero-velocity condition at the wall itself). We therefore have

$$\frac{\delta(x_0)}{x_0} \approx \sqrt{\frac{v}{\upsilon x_0}} \approx \frac{1}{\sqrt{Re_{x_0}}} \ll 1 \tag{12}$$

where Re_{x_0} is a local Reynolds number obtained by taking the distance x_0 to the plate edge at the local lenght scale. Thus, as Re_{x_0} tends towards infinity, the maximum thickness of the boundary layer becomes very small in comparission with the characteristic global extent of the boundaries.
This result explains why, with increasingly large Reynolds number, the external, non-turbolent-flow region of a viscous fluid behaves more and more closely like an ideal fluid. However as the thickness of the region where viscosity effects are significant tends toward zero, this should not be taken to imply that the effects of such a layer are neglibile; in fact the opposite is true.
Noteworthy is also the fact that the edge of the boundary layer is not a streamline, and that the rate of flow within the boundary layer increases as $\sqrt{x_0}$.
Moreover, if Re_{x_0} exceeds a certain value, the boundary layer itself becomes unstable and turbulent: the estimates made above are then no longer valid, since the momentum transport by turbulent convection causes δ to increase much more rapidly then in the laminar case.

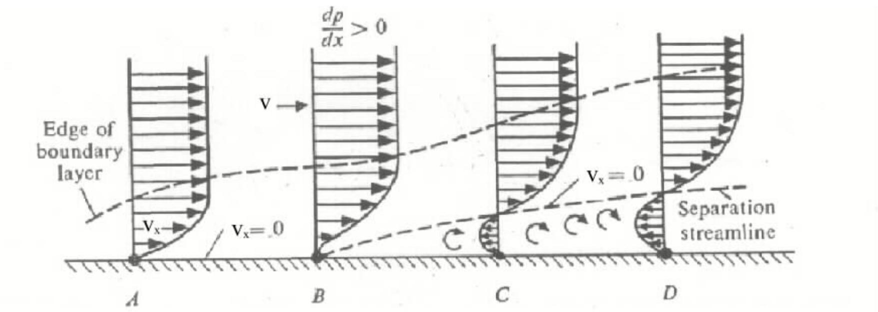

Image.1 The boundary layer near a flat plate.

We have shown that,in the presence of a flow with velocity v, the thickness of the boundary layer varies as $\sqrt{\upsilon x/v}$. In order to evaluate the constant of proportionality more precisly, we take the thickness of the boundary layer δ as equal to the value y for which the ratio v_y/v_x has a give value. Typically $\delta_{0.99}$ (corresponding to $v_y/v_x = 0.99$) is given by

$$\delta_{0.99} = 5\sqrt{\frac{\upsilon x}{v_x}} \tag{13}$$

4 Turbulent boundary layers

The stability of a laminar boundary layer is determined by the value of the local Reynolds number, evaluated by using the thickness of the boundary layer as the lenghtscale (similary, in order to determine whather the flow in a tube is laminar or turbulent, we take the diameter of the tube, not its lenght, as characteristic lenght for the corresponding Reynolds number). For example we define

$$Re_{\delta_{0.99}} = \frac{v\delta_{0.99}}{\upsilon} \propto \sqrt{\frac{vx}{\upsilon}} \tag{14}$$

This number increases as the square root of the distance x from the edge of the plate. Accordingly, even at high flow velocities, hydrodynamic instabilities appear only beyond an appreciable distance from the leading edge.
These first take the form of two-dimensional osciallations of the velocity, and later of three-dimensional fluctuations. Beyond a certain amplitude of the fluctuations, turbulent regions appear. At still higher velocities, the whole boundary layer displays rapid fluctuations of the instantaneous local velocity. Below a number of properties of these *turbolent* boundary layers are listed:

- The time-avaraged velocity profile varies as $\log(y)$, except very near the wall. The rate of change $(\partial v_y/\partial y)$ near the wall is much more significant than for the laminar flow (leading to higher stresses at the boundary), but the velocity attains much more gradually the limiting value, corresponding to the external flow.
- The drag coefficient varries much more slowly with Reynolds number (as $log(1/Re)$ instead of $1/\sqrt{Re}$).
- Momentum transport occurs by convection and results from turbulent fluctuations of the velocity througout the most of the thickness of the boundary layer (except very near the wall). Such a mode of transport is more effective than viscous diffusion, hence we have a much higher drag coefficient.
- The boundary layer thickness varies linearly with distance from the leading edge (as x instead as $\sqrt{x}$), because of external convection.

5 Boundary layer separation

Let us assume that the velocity of the external potential flow v_x decreases with the distance x downstream of the leading edge of the plate, as would be the case, for example, in a divergent flow. Outside the boundary layer, the pressure $p(x)$ increases with distance, since the pressure gradient $\partial p/\partial x$ in this direction obeys Bernoulli's equation:

$$\frac{\partial p}{\partial x} = \rho_f v \frac{\partial v}{\partial x} > 0 \tag{15}$$

Moreover, because variations in pressure in the tranvsverse direction are neglibile, we find an identical longitudinal pressure gradient within the boundary layer. Thus, in the low-velocity regions near the wall, the dynamics of the fluid elements is affected by two opposing effects; on one hand the positive pressure gradient $\partial p/\partial x$ slows down the their motion; on the other hand, momentum transfer by viscous diffusion from higher velocity regions tends to accelerate them.
If the velocity gradient $\partial v/\partial x$ is sufficiently large in magnitude there will be a reversal of the direction of the flow near he wall. This phenomenon characterizes *boundary layer separation*. In the opposite case of a positive, downstream, velocity gradient, the corresponding pressure gradient $\partial p/\partial x$ is negative, the fluid near the wall is accelerated, and

the boundary thins out.
In a number of real experiment cases, the boundary layer separates only at a certain, known as the *separation point*, beyond which a recirculation zone appears. Flow regions in which recirculation occurs are generally very unstable. The minimum value of Reynolds number at which instabilities can be easily amplified then decreses to values of a few tens. Therefore, a turbulent region of considerable width appears behind the separation point, in which there is significant energy dissipation.
The drag force then increases substantially; we observe such an effect for bodies that lack an 'aerodynamic´ profile. In contrast, the drag force is quite low for an aerodynamically shaped body from which the boundary layer does not separate, and for which the resultant wake is very narrow.
Turbulent boundary layers are much more effective in resisting external negative velocity gradients $\partial v/\partial x$, then are laminar ones: in fact, as we have already mentioned several times, momentum transport by convection is much more effective then that by diffsuion. The influx of momentum into low-velocity regions near the wall is thus much greater, and it delays the reversal of the direction of the flow.
Boundary layers can therefore be stabilised by causing them to become turbulent upstream of the normal separation point; for example, by placing a fine wire along the surface of the object. Separation of the boundary layer then occurs much further along then if it had remained laminar. The width of the turbulent wake behind the object, as well as the drag force, are siginificantly decreased (relative to what would have been the case in a laminar boundary layer). This reduction of the drag force decreases the energy dissipated as the body moved along, and thus of crucial significance for practical applications.

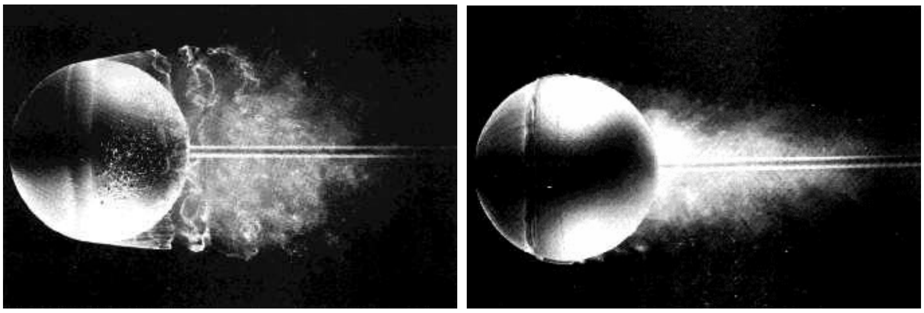

Image.2 The first image shows the flow around the sphere. Note the separated boundary layer. The second image upon close examination reveals a minute wire across the surface of the sphere. This wire causes the boundary layer to become turbulent and therefore delays the separation of the boundary layer as witnessed by the smaller wake. The same reasoning is behind the lines on the surface of the tennis ball. The lines prevent the separation and therefore enable the ball to follow a cuve path due to spin or *slice*

6 Applied boundary layer theory for airfoils

6.1 Starting vortex

As disscussed in the previous seminar the effects of viscosity are crucial for the generation of lift F_L. If the flow were to be potential througout the fluid, it would remain so at every instant of time, even after a change in the velocity of the wing, and the lift force on the wing would remain zero according to the Kutta-Joukowsky theorem

$$F_L = \rho v \Gamma \tag{16}$$

It is the presence of vorticity, concentrated in the boundary layer, that allows circulation to be created: the flow outside the boundary layer, nonetheless, remains potential. Consider the consequences of such an assumption: the inital circulation of the velocity v of the fluid is zero along a curve $C(t_0)$ surrouding the section of a wing; we assume that the wing is caused to move in a fluid at rest and that the $C(t_0)$ is located sufficiently far from the wall so that the flow can be considered as ideal everywhere along this curve.
Also, in accordance with Kelvin's theorem the circulation is zero at all later times around the curve $C(t)$, made up of elements orginally located on $C(t_0)$ and carried along by the flow. We therefore see a vortex appear at the trailing edge of the wing; the circualtion, $-\Gamma$, of the velocity of fluid around this *starting vortex* must be equal and opposite to the value Γ, around the wing, so that the total circulation along $C(t)$ remains zero.
Another aspect of the same phenomenon with the observation of wings of finite lenght. At the wingtips the pressure differences between the top and the bottom surfaces of the wing have to gradually dissapear. On account of the greater pressure below the wing surface then above it, some air will flow from the bottom to the top round the wingtips. Therefore a sidewise current exists over most of the wing surface, directed outward on the bottom side and toward the center on the top side. This causes a surface of discontinuity in the air leaving the wing, which is ultimately rolled up into two distinct vortices.

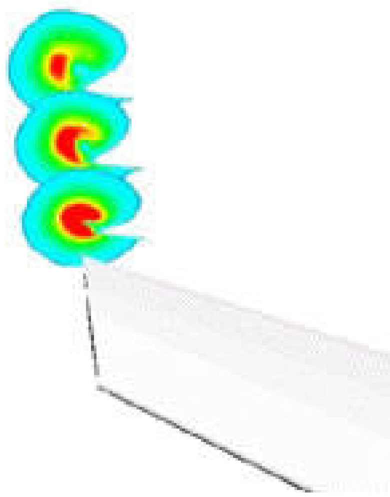

Image.3 The image above is the ONERA simulation of the wing-tip vortex separation.

According to Helmhlotz's theorem, these vortices always consist of the same air particles so that they leave the wing approximately with the velocity v in the form of two interwoven lines. As it was seen in the previous seminar the flow around an infinite long airfoil can be replaced by a flow due to a linear vortex in the wing. This is permisable also for a finite wing so that the simplest of picture of the situation is given by three linear vortices. As we also know that a linear vortex cannot terminate in the interiror of the fluid but only at infinity or at a surface. It is clear therefore that the ´bound‘ vortex of the wing cannot end at the tips but must be

continued into infinity as a ‘free´ vortex.
If the airplane has started some place, the starting vortex closes the two long free-vortex lines at the other end so that the total vortex picture consist of a very long rectangle.
However we should also know that every time the airspeed or the angle of incidence changes, a new vortex is shed off the trailing edge. Therefore the popular belief that only a starting vortex is present should be modified with the fact that the starting vortex is followed by a wake of later simillar vortices. But again the starting vortex is important for the formation of initial circulation dictated by the Kutta condition.

6.2 Wing stall

In the first seminar, we had shown how lift increases with the angle of incidence α, but we had also stipulated that this is true only up to a certain value of the angle of incidence. This angle is also known as the critical angle of incidence α_c. If we increase the angle of incidence beyond α_c experimental results show a very definite decrease of the lift.

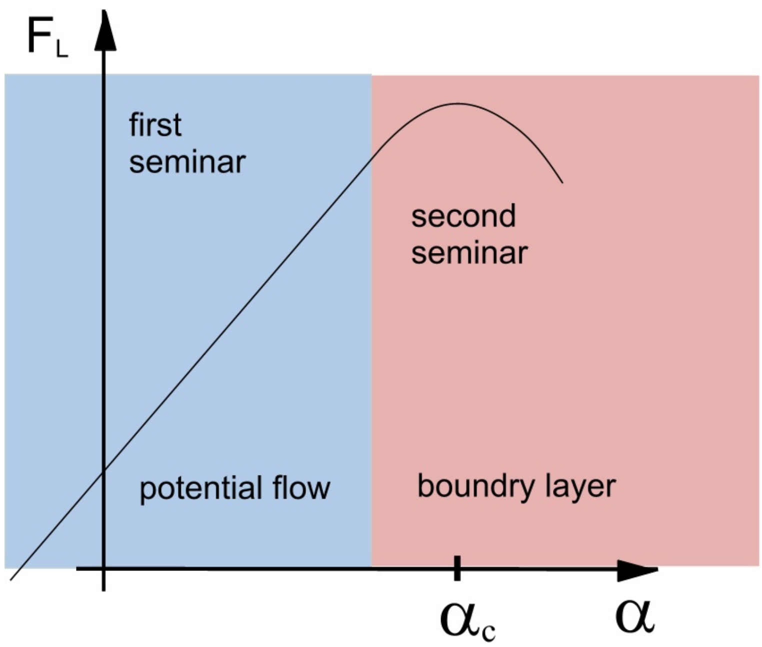

Image.4 Here presendeted is the lift F_L dependency for the angle of incidence *alpha*. We can see how the seminars are complementary in the exlpanation of lift formation.

We can now explain that beyond this critical angle of incidence, boundary layer separation occurs along the upper face of the wing, accompanied by the appearance of a wide, turbulent wake. The pressure on the upper face is accordingly increased and the lift rapidly decreases, with a corresponding increase in the drag; this situation, known as *stall*, leads to rapid loss of altitude and directional control of the airplane.

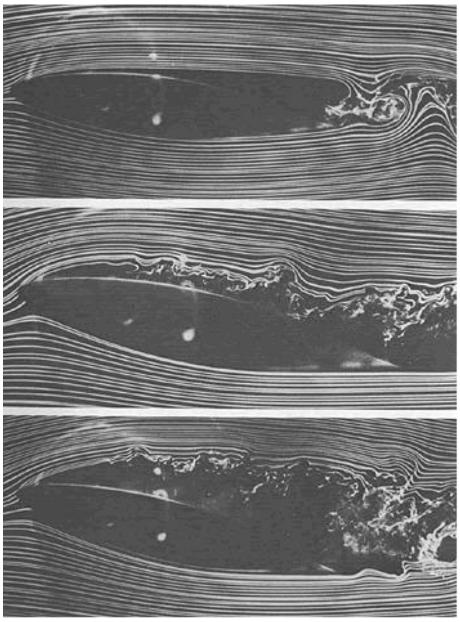

Image.5 Shown are three sequences of the air flow over an airfoil at a high angle of incidence. The bottom image shows the fully stalled wing. Lift is rougly inverse to the boundary layer separation, while drag is proportional to the width of the wake.

Hence, it is absolutely imperative to delay the appearance of the boundary layer separation, in order to maintain sufficiently high values of lift while the airplane is flying at a low velocity. However it should be noted that the boundary layer separation can only be delayed, but it is inevitable with the decrease of airspeed. The aim of delaying boundary layer separation with controlling the boundry layer is therefore to decrease the minimum airsped for which controlled flight can be maintained.

6.3 Controlling the boundary layer

Boundary layer separation is controlled for two reasons. The first is to reduce drag.
With the formation of the free vortices beyond the area where the boundary layer had separated non-trivial amounts of kinetical energy are wasted. Therefore a whole series of laminar airfoils were developed during the second world war. The basic idea was to create a profile, that would enable gradual reduction of the pressure gradient along the airfoil. The result would be a very gradual but steady increase of the flow velocity along the airfoil, which would prevent boundry layer separation. This was achieved in 1940 by NACA. These airfoils were the start of what we today know are the NACA six-digit airfoils. The dramatic reduction of drag was used to good effect on the famous North American P-51 Mustang, which was the first fighter with the laminar airfoil (this being the NACA/NAA 45-100).

Image.6 The North American P-51 Mustang full scale model being tested in the NACA wind-tunnel.(NACA)

This airfoil had a thickness ratio of 15.1% at the wing root at 39% of the chord. The tip ratio was 11.4% at the 50% chord line. These figures provided the maximum thickness area at 40% from the leading edge of the wing and resulted in a small negative pressure gradient over the leading $50\% - 60\%$ of the wing surface. The P-51 was able to escort the USAF heavy bombers even beyond Berlin due to the great cruise economy of the laminar wing. Such a feat would be impossibille with other contemporary fighters without the laminar airfoil.

Boundary layer separation is also controlled for increasing lift. Here separation is delayed by energizing the flow within the boundary layer. This can be done by wing design which incorporates either *leading-edge slats* or *trailing-edge flaps*.

The use of leading-edge wing flap results in increase of the critical incidence angle α_c, which appears in the dependance of the lift coefficient C_z on α. Due to the effect of the flap, air from the region below the wing is injected tangentially along the upper surfaces; it this regenerates the boundary layer across the wing by increasing the velocity of the fluid near the wall. At large angles of incidence, the effect of the inverse pressure gradient is thus reduced, and the critical angle of incidence α_c correspondingly increases.

A trailling-edge wing flap when extended, causes an increase, at a given velocity, in the circulation around the cross-section of the wing; as a result, the curve $C_z(\alpha)$ is translated upwards. On very large airliners, the flaps themselves have further flaps attached to them (up to three succesive stages on the Airbus A320 or the Boeing 747). We thus often see several stages of flaps extending one behind the other, with the rearmost one ending up almost vertical. It should be noted that these systems increase the drag on the wing considerably ; they are used exclusivly on take-off and landing.

Image.7 Boeing 747 shows several sections of Fowler flaps during its final approach.

The principle behind the action of the trailing-edge wing flaps is a combination of two different effects:

- The existence of a gap between the flap and the main portion of the wing allows the regeneration of the boundary layer, by inducing a flow from the lower face of the wing to the upper one (simmillar ot that mentioned with the leading-edge flaps). This avoids boundary layer separation at the flaps, in spite of their high angle of incidence relative to the main flow. All flaps don't have a gap between the flap and the wing. The type that does have the gap are usually termed the Fowler flaps.
- They create a significant downward deviation, δv, of the flow velocity at the rear of the wing and in the wake, resulting in a major increase in the circulation, and in the corresponding lift.

A third way of energizing the boundary layer is by suction. While this method does not directly add energy in the flow, it deletes the particles with low energy, therefore the avarage kinetical energy within the boundary layer is increased. The basic idea is that boundary layer separation is delayed by drawing in, by suction, the (low-velocity) fluid near the wall. This decreases the possibility that the negative pressure gradient, arising from the decrease of the external flow velocity, might reverse the direction of the flow of these elements of fluid. This principle is the one used in controlling the boundary layers along the turbosail of Malavard and Cousteau's catamaran *Moulin a Vent*. By means of suction, boundary layer separation can be inhibited even at large angles of incidence, leading to very high values of circulation of velocity around the turbosail. This results in a very significant Magnus force, with a large thrust component in the direction of motion. In airborne applications, this concept is still ony at an experimental stage, because of the difficulty to produce a mass and energy economical airborne source of low pressure. It should be noted here that it is often incorrectly stated that the process of sucking away the slow particles requires a lot of energy. This is not the case since the power consumed is very low, due to the fact that very small volumes are involved, which was experimentally proved already by O. Schrenk in 1926. However jet aircraft have an almost perfect source of high pressure already installed - the jet turbines. This leads to the most wide use of effective boundary layer separation delay - that is the use of *blown boundary layer* *mechanisms.*

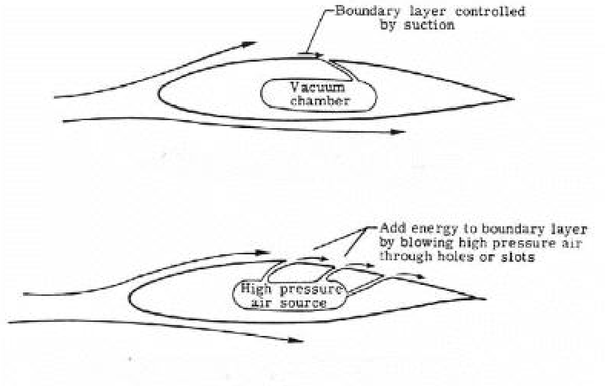

Image.8 Here are the schematics of the boundary layer mechanism control by suction and by high pressure.

Blown boundary layer is a system predominantly used on cold war era fighter jets. These jets had high wing-loading and very thin wings to optimise their supersonic flight capabilites. These two parameteres significantly decreased their low speed performance. For example the classical MiG-21 had the landing speed of $300km/h$ and its stalling speed not significantly lower. Therefore the idea proposed already in 1927 by Seewald is as simple as it is brilliant. As it was discussed in section 4 the cause of braking away of the boundary layer is the loss of kinetic energy due to viscosity. The high-pressure air coming from nozzles on the surface of the wing imparts fresh momentum to the flow particles that were slowing down. Owing to this regeneration of kinetical energy the boundary layer does not separate before reaching the trailing edge of the wing. On military grade jet turbines the source of high pressure is usually a bleed valve located in the turbine itself, typcially somewhere between the 9^{th} and the 15^{th} stage of the compressor. This valve is controlled as a parameter of angle of incidence. As the angle approaches the critical angle of incidence, the valve is progresivly opened and the nozzles on the wing start to blow high pressure air into the boundary layer. Typically the whole wing is not energized, but only the control surfaces and trailing-edge flaps to enable sufficient control of the aircraft during low-speed flight.

7 Conclusion

The whole of the airfoil theory can be parted into two regions. The first is the region of the potential flow, while the second is the region of the boundary layer. Again it must be stressed that the boundary layer is not defined by a set of stramlines, but by the velocity profile along these streamlines.
By studying the boundary layer we can explain the starting vortex which is crucial for the Kutta-Joukowsky theorem and the Kutta condition. Moreover we can fully explain the profile drag in both the induced and in frictional drag. We also saw that the boundary layer separation dictates the flight characteristics at high angles of incidence and we had shown that the whole airfoil is designed around the boundary layer separation condition

both for lift and for drag.
We can very simply stress that the whole design process of airfoils concentrates in improving the airfoil so that the smallest amount of energy is lost in the boundary layer and its wake.

References

[1] L. Prandtl, O.G. Titjens, *Fundamentals of hydrodynamics and aeromechanics* (Dover publications, New york, 1957).

[2] L. Prandtl, O.G. Titjens, *Applied hydrodynamics and aeromechanics* (Dover publications, New york, 1957).

[3] J.D. Anderson, *Fundamentals of Aerodynamics* (McGraw-Hill, Singapore, 2001).

[4] S. Knez, 2005, *Dvodimenzionalni modeli tvorbe vzgona* http://www-f1.ijs.si/ rudi/sola/knez.1.pdf

[5] J. Renaux, 2004, *Overview on drag reduction technologies for civil transport aircraft* (ONERA, Applied Aerodynamics Department, Chatillon Cedex, 2004).