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NO SPLASH ON THE MOON

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ABSTRACT

The basic description of a droplet impacting a smooth dry substrate is introduced. The process of a drop falling, its oscillations and variations in size and shape during the fall is described, as well as the impact and the decomposition of the drop. An experiment, which demonstrates that a splashing can be completely suppressed if the pressure of a surrounding gas is decreased, is presented.

Contents

1 Introduction

On Earth, a splash can be observed when a liquid drop impacts with a smooth dry substrate. On impact, as the droplet reacts on the sudden increase in pressure, a thin sheet of liquid tends to jet radially outward and lift up off of the surface so that the corona is formed. The liquid than disintegrates into thousands of disconnect pieces. On the other hand, as the drop hits dry smooth surface on the Moon, no splash is observed. Why would a splash on the Moon be different from the one on the Earth? The planet and its satellite differ in many ways, but only one of them affects the falling drop. This difference is atmosphere, since an important effect on the process of splashing have the surrounding air, velocity of the impact and the size, shape and liquid surface tension of the drop. In this paper the mechanisms behind the falling, the impact and the violent shattering of the drop, when it hits the surface, are discussed, as well as what happens when the surrounding atmosphere is removed. It is shown that the corona splash is suppressed and can completely disappear by decreasing the pressure of the surrounding gas. As Moon has no atmosphere, and therefore no gas surrounding a falling drop, the drop on the Moon does not splash. This striking phenomenon can be described with a model in which compressible effects in the gas are responsible for splashing in liquid solid impacts.

2 Falling

2.1 The velocity

The velocity of the falling drop can be calculated, if we know viscosity, density and radius of the liquid drop. We assume that after the initial acceleration the drop reaches a constant velocity so that, the sum of the forces that act on the drop during its fall are in equilibrium. We can then write the following equations:

$$F_g - F_v - F_u = 0, \tag{1}$$

where F_g is the force of gravity, F_v is the lift force and F_u is drag force. We can rewrite equation 1 as:

$$mg - m_a g - b \cdot v = 0, \tag{2}$$

where m is the mass of fluid, m_a the mas of air, b the constant that depends on the properties of the fluid and the dimensions of the drop, and v the velocity of the falling drop. In special cases of small spherical drops moving slowly through a viscous fluid (and thus at small Reynolds number [1], where v_s is the mean fluid velocity, L characteristic length, μ (absolute) dynamic fluid viscosity, ν kinematic fluid viscosity: $\nu = \mu/\rho$ and ρ fluid density), an expression for the drag coefficient, $b = 6\pi\eta r$, was derived by George Stokes. Here r is the radius of the drop, and η is the fluid viscosity. If the drop is small, spherical and falls at a relatively low velocity, the falling velocity can than be calculated from equations (1,2):

$$v = \frac{2R^2 g}{9} \frac{\rho - \rho_a}{\eta}. \tag{3}$$

The ρ is the liquid density, ρ_a is the density of the surrounding air and η is the viscosity of the liquid.

At larger velocities Rayleigh's equation should be used:

$$\mathbf{F}_d = -\frac{1}{2}\rho v^2 A C_d \hat{\mathbf{v}}, \tag{4}$$

where F_d is the force of drag, ρ is the density of the fluid, v is the velocity of the object relative to the fluid, A is the reference area, C_d is the drag coefficient (a dimensionless constant), and $\hat{\mathbf{v}}$ is the unit vector indicating the direction of the velocity (the negative sign indicating the drag is opposite to that of velocity).

[1] Reynolds number is the ratio of inertial forces to viscous forces and consequently it quantifies the relative importance of these two types of forces for given flow conditions. It is used to identify different flow regimes, such as laminar or turbulent flow. Typical equation for Reynolds number is:

$$Re = \frac{\rho v_s L}{\mu} = \frac{v_s L}{\nu} = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

The velocity as a function of time is than for an object falling through a non-dense medium roughly given by a function:

$$v(t) = \sqrt{\frac{2mg}{\rho A C_d}} \tanh \left(t \sqrt{\frac{g \rho C_d A}{2m}} \right). \tag{5}$$

Velocity asymptotically approaches a maximum value called the terminal velocity: $v_t = \sqrt{\frac{2mg}{\rho A C_d}}$. For objects of water-like density (raindrops, hail, fog etc.) falling in air near surface of Earth at sea level terminal velocity is roughly equal $v_t = 90\sqrt{d}$. For example, for a fog droplet (d $\sim$ 0.0001 m) terminal velocity $v_t \sim$ 0.9 m/s [14].

2.2 The deformation and oscillation of fluid drops in the air

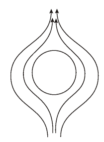
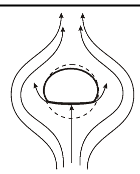
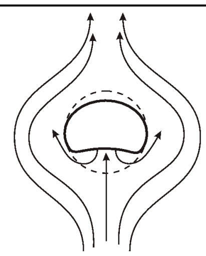
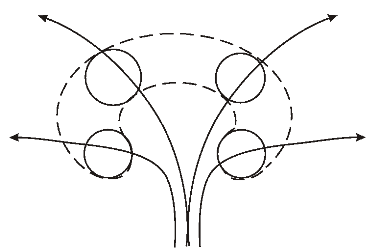
	Drop size	Characteristic shape
Electrostatic forces within the molecule are able to maintain the spherical shape against external forces.	0.14 mm	
A very slight shortening of the vertical axis and the drop is an "oblate spheroid". The vertical axis is about 98 % of the horizontal axis.	0.50 mm	
Flattening of bases begins.	1.4 mm	
Concavity of the flattened base begins.	2 mm	
At 5 mm the force of the air through which the drop is falling causes the drop to break up.	5mm	

Table 1: The above table presents the deformation of water drops in the air [bibli].

The shape of the drop at given size depends on the density, surface tension and viscosity of the liquid that forms the drop. Very small drops are spherical, because the electrostatic forces within the molecules that form the fluid are able to maintain this shape against the external forces. As the drop grows a slight shortening of the vertical axis appears and the drop becomes an oblate spheroid. With continuing growth, first the flattening of the bases begins. This is then followed by a concavity of the flattened base. Finally, when the drop is big enough that the electrostatic forces can no longer maintain the molecules together, the force of the air through which the drop is falling causes the drop to brake down.

As the drop falls it oscillates due to natural instabilities, turbulence in the air and perhaps also due to vortex shedding from the falling drop. Although surface tension pulls the distorted drop back to the spherical shape, the inertia of the liquid mass continues the motion of the liquid beyond the equilibrium spherical shape, and the drop distorts in a direction perpendicular to the initial perturbation. Oscillation occurs at the harmonic frequency of the drop. Since liquid is viscous, energy dissipates with each oscillation cycle so the drop eventually comes to rest in a spherical shape. The natural instabilities that drops undergo are the three modes of the fundamental harmonic of a drop. These three modes are referred to as axisymmetric, transverse and horizontal [12]. The oscillations of a falling drop are revealed in the following photographs.

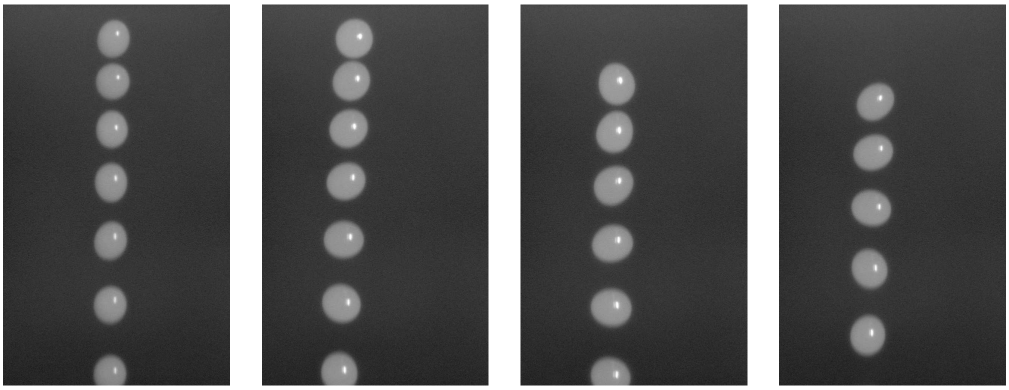


Figure 1: This images illustrate general oscillatory characteristics of a falling drop.

3 Impact

3.1 What happens during the impact?

The impact of a drop that falls onto a dry smooth solid surface in a low pressure environment is characterised by a very sudden transfer of momentum from vertical to the horizontal direction. On impact, as the droplet reacts to a sudden increase in pressure, a thin axisymmetric sheet of fluid begins to jet radially outward over the solid surface. This jet can travel even faster than the incoming drop. Soon after the impact, the leading edge of the thin sheet of fluid can become unstable, which leads to the emergence of undulations both in radial location of the contact line as well as the thickness of the expanding lamella. As the undulations continue to grow they begin to resemble fingers [2]. If the fingers continue to grow, the Rayleigh - Taylor instability [3] can cause the pinching off of secondary or satellite droplets [4]. There is not enough momentum normal to the surface and the kinetic energy necessary to overcome surface tension and gravity to form the corona dissipates during the deforming process.

When the same droplet impacts the dry smooth surface and the pressure is high enough, the expanding sheet of fluid tends to lift up off of the surface and the corona is formed and grows in time as the droplet fluid continues to feed the film. The time for the formation of corona is much larger than the time for the deformation of the primary droplet. Once the lower half of the droplet has undergone the deformation, the total volume flow rate into the wall film begins to decrease. Consequently, the corona, which has been

[2]The term 'fingering' refers to the presence of a perturbed leading edge, as the undulations usually resemble fingers.

[3]Rayleigh - Taylor instability is a fingering instability that occurs at an interface between two fluids of different densities, when the lighter fluid pushes the heavier fluid.

[4]The term 'splashing' refers to the formation of secondary or satellite drops.

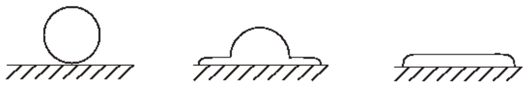


Figure 2: Sketch of the deformation process.

stretched in its radial expansion, has less fluid feeding its film and hence becomes thinner. An instability develops and leads to a rim, which propagates upwards in the corona. The rim then breaks up due to surface tension and the thin sheet compromising the corona surface disintegrates and rips into secondary droplets.

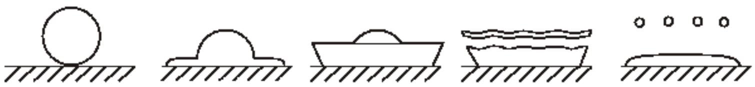


Figure 3: Sketch of splashing.

3.2 Theory

3.2.1 Deformation process

In this chapter it is assumed that a spherical drop hits the dry, smooth surface. The outcome of the droplet impact on a solid surface depends on a number of variables: the properties of the liquid (i.e. density ρ, viscosity μ), the surface tension conditions (surface tension γ) and the kinematic parameters (droplet parameter R_0, velocity v_0). The number of independent parameters can be reduced with nondimensionalisation. The usual choices are a non-dimensional average surface roughness $R_a^* = \frac{R_t}{d_0}$, a contact angle Θ and two of the Reynolds (Re), Weber (We) and Ohnesorge (Oh) numbers:

$$Re = \frac{\rho d_0 v_0}{\mu}, \tag{6}$$

$$We = \frac{\rho d_0 v_0^2}{\sigma}, \tag{7}$$

$$Oh = \frac{\mu}{\sqrt{d_0 \gamma \rho}} = \frac{\sqrt{We}}{Re}, \tag{8}$$

where ρ, μ and σ are the liquid density, the viscosity and the surface tension for a fluid-air interface. d_0 is the initial droplet diameter, R_t the mean (average) roughness height of the surface. The droplet initial velocity v_0 used in the Reynolds and Weber numbers is the component of velocity normal to the surface. They represent the relative magnitudes of inertial, viscous and surface tension forces. The Bond number (Bo):

$$Bo = \frac{d_0^2 \rho g}{\sigma} \tag{9}$$

is sometimes used as well in cases in which a film on the surface may have an influence on the impact event. Here g is the gravitational force. The contact angle Θ is defined using the tangent line at the liquid-gas interface at the point where meniscus begins, as sketched in figure 4.

The dynamics of droplet impact can be obtain by solving the Navier - Stokes equations, which fully describe the fluid in motion during the impact and the deformation process:

$$\frac{\partial V}{\partial t} + V \cdot \nabla V = -\frac{1}{\rho}\nabla p + \eta \nabla^2 V + \frac{1}{\rho}\mathcal{F}, \tag{10}$$

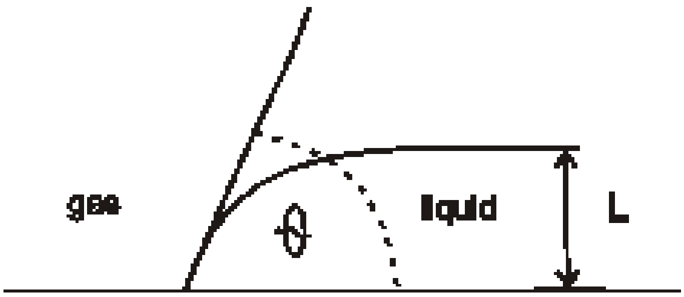


Figure 4: Contact angle Θ.

where V represents the velocity vector, p the pressure, $\eta = \frac{\mu}{\rho}$ kinematic viscosity, and $\mathcal{F}$ any body forces that act on fluid. Another equation:

$$\nabla V = 0 \tag{11}$$

also governs the flow within the liquid phase following impact. Boundary conditions [5], which are usually used in such numerical models, include:

- the surface tension induced pressure jump at the droplet surface: $p_s = \gamma\kappa$, where γ represents the surface tension, and κ the total curvature of the interface. κ is the geometric characteristic of the interface and is obtained from the volume fraction:

$$\kappa = -\nabla \cdot \hat{n}, \quad \hat{n} = \frac{\nabla f}{|\nabla f|}, \tag{12}$$

  where f is a scalar function which defines the interface and represents the fluid volume,
- a zero tangential stress condition at the surface,
- and the dynamic contact angle Θ imposed at the point where solid, liquid and gas phases meet (figure 4).

3.2.2 Deposition and splashing of the droplet

To describe the breakup or complete deposition of the droplet we must solve the equations of energy conservation [1]:

$$E_k + E_p + E_s = E_k{}' + E_p{}' + E_s{}' + E_d \tag{13}$$

and mass conservation:

$$m = m'. \tag{14}$$

Here, E_k, E_p, E_s and E_d are kinetic, potential, surface and dissipated energies, and m is the mass of the droplet. The left side of both equations describes the state before the impact and the right side the state after the impact. If

$$E_d{}' \approx E_k + E_p + E_s,$$

splashing does not occur and complete deposition of fluid mass takes place. The kinetic, surface and potential energy before the impact can be described by the following equations:

$$E_k = \frac{1}{12}\rho_L v_0^2 \pi d_0^3, \tag{15}$$

$$E_s = \pi d_0^2 \sigma, \tag{16}$$

$$E_p = mgh, \tag{17}$$

where ρ_L, σ and d_0 are the liquid density, surface tension and the diameter of the impacting droplet. The kinetic energy upon the impact is expanded into deforming the droplet. In the case when a full decomposition takes place , the corona is not formed. The kinetic energy becomes zero at the maximum extension of the liquid surface. The surface energy can be described as:

$$E_{s\prime} = \frac{\pi}{4} d_{max}^2 \sigma (1 - \cos\Theta), \tag{18}$$

at the maximum extension diameter d_{max}. Here Θ is the contact angle. The dissipation energy is hard to determine, because we do not know how the velocity is distributed inside the deforming droplet. Approximation is given as:

$$E_d' \approx \Phi V t_c. \tag{19}$$

Here Φ is the dissipation per unit mass of the fluid, which is given by:

$$\Phi = \mu \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \frac{\partial u_x}{\partial y} \approx \mu \left(\frac{v_0}{l} \right)^2, \tag{20}$$

where l represents the height of the disc. The volume of the fluid when it is flattened out in the shape of a disc is

$$V \approx \frac{\pi}{4} d_{max}^2 l. \tag{21}$$

t_c is the time of deformation and is estimated by:

$$t_c \approx \frac{d_0}{v_0}. \tag{22}$$

If we combine equations 13-22 and introduce the Oh number with $\beta_{max} = \frac{d_{max}}{d_0}$ and $E_p' = E_p$ the following equation can be obtained:

$$Oh = \sqrt{\frac{3(1 - \cos\Theta)\beta_{max}^2 - 12}{Re^2 - 4.5\beta_{max}^4 Re}}. \tag{23}$$

In equation 23 it shown that the splashing deposition boundary is determined only as a function of Oh and Re numbers, as the contact angle Θ and β_{max} are constant for a given fluid and surface material. Splashing occurs for an Oh number above the value given in equation 23. On the other hand, if the value of Oh is lower than the one given in 23 the complete deposition of the droplet occurs.

3.2.3 Evolution of the fingering pattern of an impacting drop

For low pressure regime the drop does not splash. Instead undulations in the thickness of the rim as well as radially of the contact line are observed. Where the undulations continue to grow they begin to resemble fingers. The number of fingers visible about the rim depends on the acceleration of the interface. Allen [13] estimated the acceleration of the interface as $a \approx -\frac{v_0^2}{d_{max}/2}$, where d_{max} was measured maximum fluid spread. He obtained for a number of fingers the following expression: $N = \pi \frac{d_{max}}{\lambda}$, whereas Bola and Chandra [4] got $N = \frac{K}{4\sqrt{3}}$, where K is the splash parameter [5]. They estimated acceleration of fluid interface as $s \approx -\frac{v_0}{d_0}$ and introduced an analytical expression for the maximum spread d_{max}. As the finger grows it widens and its tip can split into two peaks. As the drop edges undergo the splitting, the shapes of the fingers change dramatically. The shape depends on how close two adjacent fingers are. If the fingers are closely spaced, the two peaks on each finger can become equal in amplitude. This can lead into merging of fingers. When merging occurs in isolation, sufficiently far away from adjacent bumps, the remaining side bumps travel sharply away from the centre finger. The speed with which the merging occurs can be described in

[5]Splash parameter:

$$K = We^{0.5} Re^{0.25}$$

terms of capillary phase speed, which can be estimated from the dispersion relations for capillary waves on shallow layer of fluid:

$$c^2 = \left(g + \frac{\gamma k^2}{\rho}\right)\frac{tanh(kl)}{k}, \tag{24}$$

where k is the wave number, c the phase speed and l the fluid depth. It is assumed here, that the capillary forces are the one that drive these motion.

All undulations do not have the same wavelenght. The longer wavelenght is most likely the result of free oscillations of the drop in the transverse direction during the fall. The characteristic wavelenght does not change significantly as the front advances, which means that the instability cannot be the principal driving mechanism of the undulations. Studying the fundamental fingering wavelenght and radial evolution of the frontal undulations, can however help to illuminate the nature of fingering instability. It has been suggested by s.T. Thoroddsen and J. Sakakibara [3] that the instability mechanism is a Rayleig-Taylor instability.

4 Experiment and results of experiment

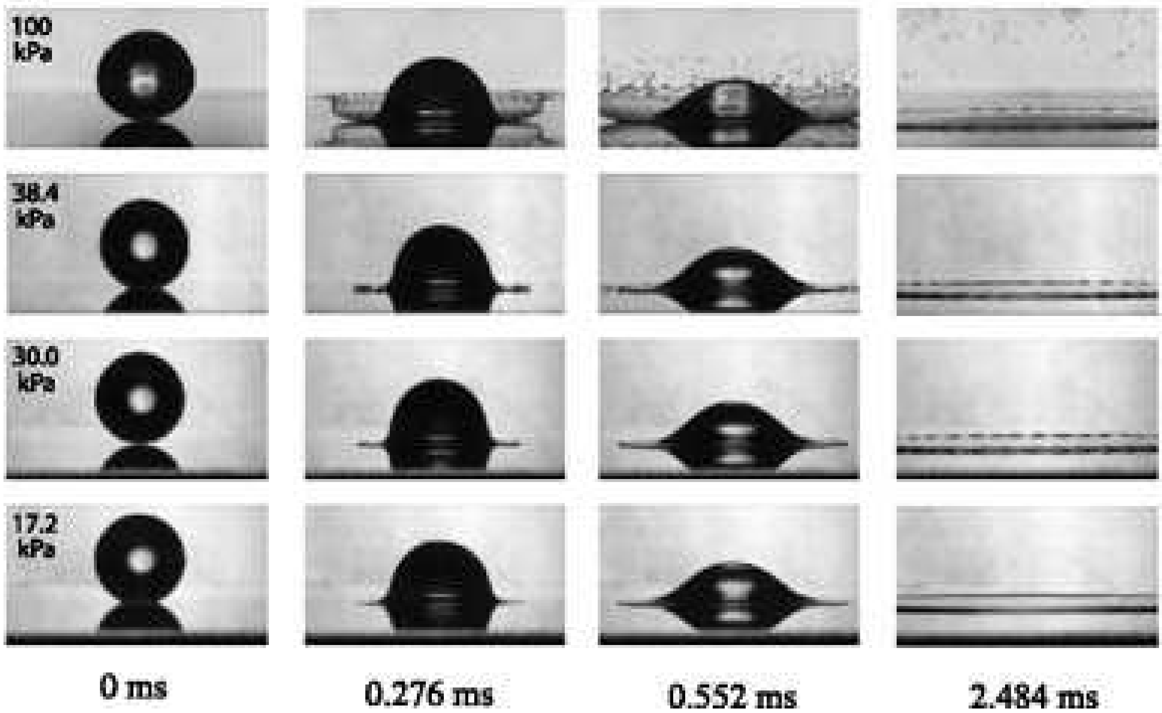


Figure 5: Photograohs of ethanol drop hitting a smooth dry substrate. Each row shows the drop at four times; just before it hits the surface, at 0.276 ms, at 0.552 ms and at 2.484 ms after impact. Pressure is varied from 100 kPa to 17. kPa.

In the experiment [10] reproducible drops were released from rest at different heights above glass microscope slide and recorded. The diameter of drops was $D = 3.4 \pm 0.1$ mm. The microscope slide was laid horizontally inside the transparent vacuum chamber. The height of the nozzle above the substrate as well as the pressure could be varied; the height between 0.2 m and 3.0 m and the pressure, P, between 1 kPa and atmospheric pressure (100kPa). The impact velocity was determined from films recorded with a high-speed video camera Phanton V7. After every measurement the substrate was replaced to avoid contamination of glass due to the possible residue of a previous drops. Because the drop oscillates during the fall, the height was carefully adjusted so that, the profile of the drop was nearly circular at the instant of the impact with the glass slide. Three different liquids (methanol, ethanol and 2-propanol) that all wet the substrate [6] were used as well as four different gases (helium, air, krypton and SF_6).

Figure 5 shows a 3.4 ± 0.1 mm diameter drop of ethanol hitting the substrate at a velocity $v_0 = 3.74\pm0.0.2$ m/s in the presence of different background air pressures. The images show the drop just before it hits the surface and its evolution after the impact. On the first row, at an atmospheric pressure the drop splashes. In the second row, taken at 38.4 kPa, there are only a few droplets emitted from the drop. In the third row, at a pressure of 30.0 kPa, the splash does not occur. However, the undulation in the thickness of the rim is observed. In the bottom row, at a pressure of 17.2 kPa, no splashing and no apparent undulations in the rim of the drop can be observed. As the pressure is lowered, fewer droplets are ejected from the surface. When the pressure is low enough, no droplets are emerged from the surface.

The experiment showed that the threshold pressure P_T, at which the splashing occurs, is a function of impact velocity v_0. Except for the region below V^*, the threshold pressure is a monotonic function of impact velocity.

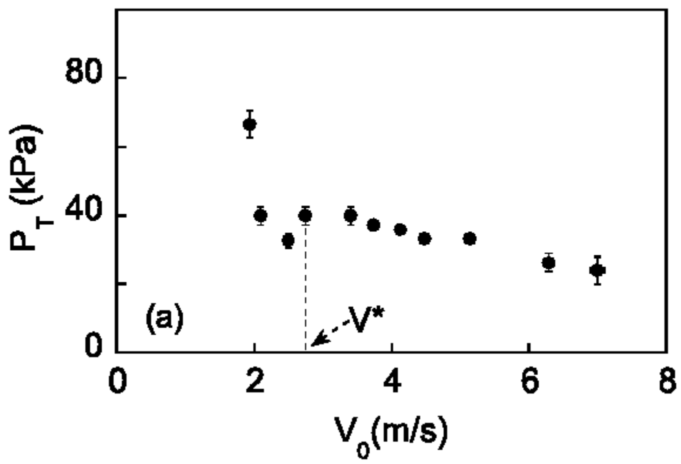


Figure 6: Threshold pressure is plotted versus impact velocity in a background atmosphere air.

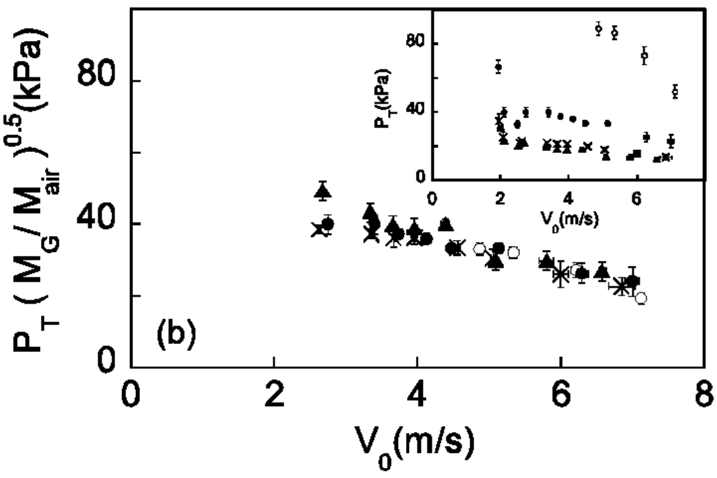


Figure 7: Threshold pressure is plotted versus impact velocity in a background atmosphere air for four gases: He (○), air (●), Kr (×) and SF_6 (▲).

[6]Liquids that wet the substrate do not rebound or retract after reaching the maximum diameter.

Pressure of the surrounding gas is essential for determining, if the drop will splash. However, it is not clear which physical property of the gas causes the splashing. In the experimental regime of [10], the dynamic viscosity of the air and the surface tension of the liquid do not vary with pressure. In order to understand the influence of gas, its composition was varied. Four different gasses with similar viscosity and very different molecular weights [6] were used. The results are plotted in figure 7. The inserted picture shows threshold pressure versus impact velocity. Although the the values of P_T are displaced from each other, the trends in data have relatively similar shape. The main panel shows data (P_T) scaled by factor $\left(\frac{M_G}{M_{air}}\right)^{0.5}$, where index G stands for each of the four gases. The best data collapse is thus obtained in the regime with $v_0 > V^*$.

5 Summary

The dynamics (falling and oscillations) as well as the size and shape of a falling drop are introduced. A basic description of splashing and decomposition of a droplet impacting a smooth solid surface has been presented. Mechanisms behind the process of splashing and the formation of corona have been introduced. Moreover, the disintegration of the droplet and fingering have been described.

In the presented experiment it has been shown, that the surrounding gas is essential for splashing to occur on a dry flat substrate. Gas provides means for a vertical component of momentum, which is responsible for the formation of the corona. It has been shown, that the splashing does not occur if the pressure is lower than 30.0 kPa. The connection between threshold pressure and impact velocity has been investigated.

The discovery that the compressible effects and the composition of gas can influence the occurrence of splashes should have an important effect on many technological processes where splashing is involved, such as in combustion of liquid fuels, spray drying, painting and coating, ink-jet printing, injection systems, thin film coatings, and industrial washing.

[6] Molecular weight values for He, air, Kr and SF_6: $M_{He} = 4$, $M_{air} = 29$, $M_{Kr} = 83.8$ and $M_{SF_6} = 146$.

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