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Seminar

Magnetic Monopoles

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Abstract

In this work we explore the possibility of the existence of fundamental magnetic monopoles and the implications that come from them. We start by defining what a monopole is and continue by explicitly constructing a monopole in the classical and quantum regimes. We then give reasons for why monopoles have never been observed and provide context for their importance in modern theoretical physics. At the end, we also mention some experimental results on this topic.

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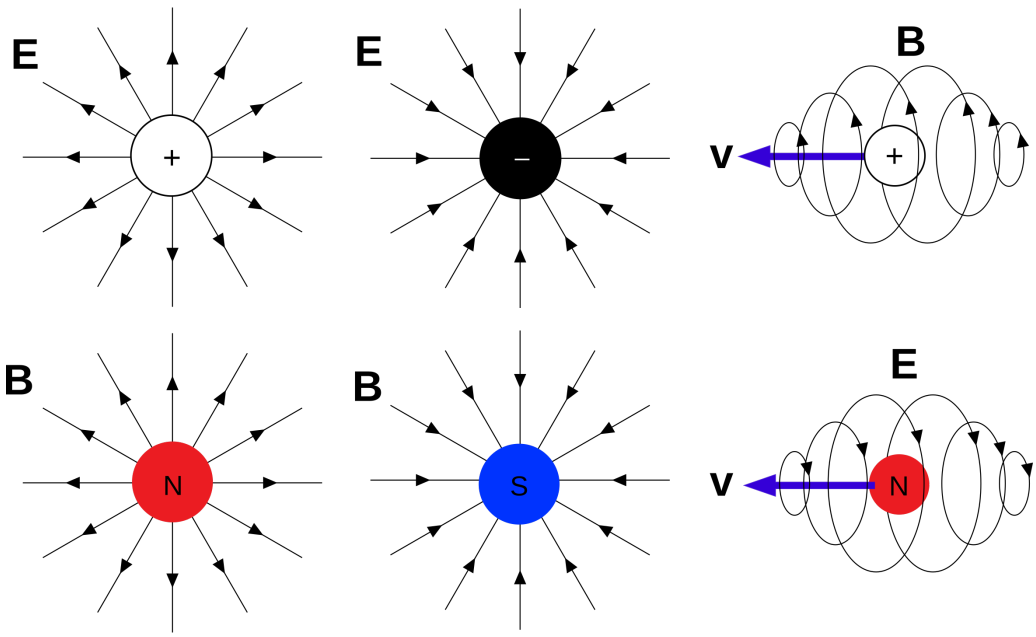

Figure 1: Comparison between the behavior of electric monopoles (here denoted by + and −) and magnetic monopoles (here denoted by N and S). Monopoles create radial fields in either case and a current of monopoles induces the opposite field. (CC0 1.0)

1 What is a Monopole?

To start discussing whether magnetic monopoles exist or what their significance is we need to have a clear idea of what they are. An electric monopole (i.e. a charge) e is an object which creates an electric field of the form

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{e}{r^2} \mathbf{e}_r, \tag{1}$$

where $\mathbf{r}$ is the distance from the electric monopole.

Similarly, a magnetic monopole is an object with a new type of charge, called magnetic charge and denoted by g, that has the same structure as an electric monopole, but for the magnetic field instead. Namely,

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{g}{r^2} \mathbf{e}_r, \tag{2}$$

where the only difference is that $1/\epsilon_0$ has become μ_0 as is usual for electromagnetism in SI units. Figure 1 shows a visual comparison of electric and magnetic monopoles and the electromagnetic fields they produce.

At this stage one might wonder why monopoles should even exist. A classical physics inspiration for this is the duality between the electric and magnetic fields in vacuum. There the Maxwell equations reduce to

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0, & \nabla \times \mathbf{E} &= -\partial \mathbf{B}/\partial t, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \, \partial \mathbf{E}/\partial t. \end{aligned} \tag{3}$$

What we mean by the duality of $\mathbf{E}$ and $\mathbf{B}$ is that under the simultaneous transformation of $\mathbf{E} \mapsto c\mathbf{B}$ and $\mathbf{B} \mapsto -\mathbf{E}/c$, where c — the speed of light — appears only because of dimensional reasons within the SI system of units, the set of Maxwell equations remains the same. Therefore, what we call $\mathbf{E}$ and what we call $\mathbf{B}$ is purely a convention.

In classical electrodynamics this duality gets broken by the addition of matter in the form of electric monopoles only, since this is the only type of charge that has ever been observed. Thus, since the breaking of symmetry has been a fruitful field for discoveries in theoretical physics, it's a natural question to ask why this happens. [1]

As a note, in this work, we will be discussing fundamental magnetic monopoles, i.e. fundamental particles that carry magnetic charge. We will not be discussing emergent monopoles [11, 14]. These non-fundamental particles are magnetic monopoles that can be observed in condensed matter systems as an emergent property of a lattice of underlying dipoles oriented in particular ways. Emergent monopoles don't carry the same implications for physics that fundamental monopoles do.

2 An Apparent Paradox

The moment we write down eq. (2), we should get worried; it clearly violates Maxwell's second law

$$\nabla \cdot \mathbf{B} = 0, \tag{4}$$

since even from the analogy with electric charge we know that the field lines of such a field flow outwards from the charge (or inwards if the charge is negative). So we have to face the breaking of this law head on.

At second glance, the breaking of this law is not as bad as it seems. After all, the Maxwell equations have been modified before, by Maxwell himself, to add the missing displacement current. And it's not surprising that if we

[1] In supersymmetric theories of physics, which we won't get into here, the duality between electric and magnetic fields persists even in the presence of matter. [17]

introduce a quantity that mirrors the electric charge then that quantity will appear in the Maxwell equations in a similar way to the electric charge — as a measure of the divergence of the respective field. Further requiring the local conservation of charge for magnetic monopoles $\nabla \cdot \mathbf{j}_M = -\partial \rho_M / \partial t$ as we do for electric monopoles we get the symmetrized Maxwell equations

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho_E}{\epsilon_0}, & \nabla \times \mathbf{E} &= -\mu_0 \mathbf{j}_M - \partial \mathbf{B}/\partial t, \\ \nabla \cdot \mathbf{B} &= \mu_0 \rho_M, & \nabla \times \mathbf{B} &= \mu_0 \mathbf{j}_E + \mu_0 \epsilon_0 \, \partial \mathbf{E}/\partial t. \end{aligned} \tag{5}$$

For completeness, Lorentz's force law also changes to

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + g \left(\mathbf{B} - \frac{1}{c^2} \mathbf{v} \times \mathbf{E} \right). \tag{6}$$

But quantum mechanics teaches us that what's really more fundamental than the fields $\mathbf{E}$ and $\mathbf{B}$ are the electric potential ϕ and the magnetic vector potential $\mathbf{A}$. And the relations that are of actual importance are

$$\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}. \tag{7}$$

The above definition is not enough to completely constrain the potentials because of the mathematical properties

$$\nabla \times \nabla \Lambda = 0, \qquad \nabla C = 0 \tag{8}$$

which hold for any scalar field Λ on a simply connected domain and any constant field C. This leads us to the concept of a *gauge transformation*, which is defined as the transformation

$$\mathbf{A} \mapsto \mathbf{A} + \nabla \Lambda, \qquad \varphi \mapsto \varphi - \frac{\partial \Lambda}{\partial t}, \tag{9}$$

which by the previous properties leaves the fields, and thus the physics, unchanged. Two pairs of potentials that are related by a gauge transformation are completely physically equivalent and should be thought of as the same thing.

One could consider modifying the Maxwell equations and the preceding two definitions by adding another set of potentials — something like a magnetic scalar potential and an electric vector potential — in addition to the well-known electric scalar potential and magnetic vector potential. This however runs into trouble when considering Quantum Field Theory because it turns out that along with monopoles one has to then add another boson particle (since we have more degrees of freedom) [34]. So taking the minimalistic route, we would like to try to find monopoles without modifying eq. (7).

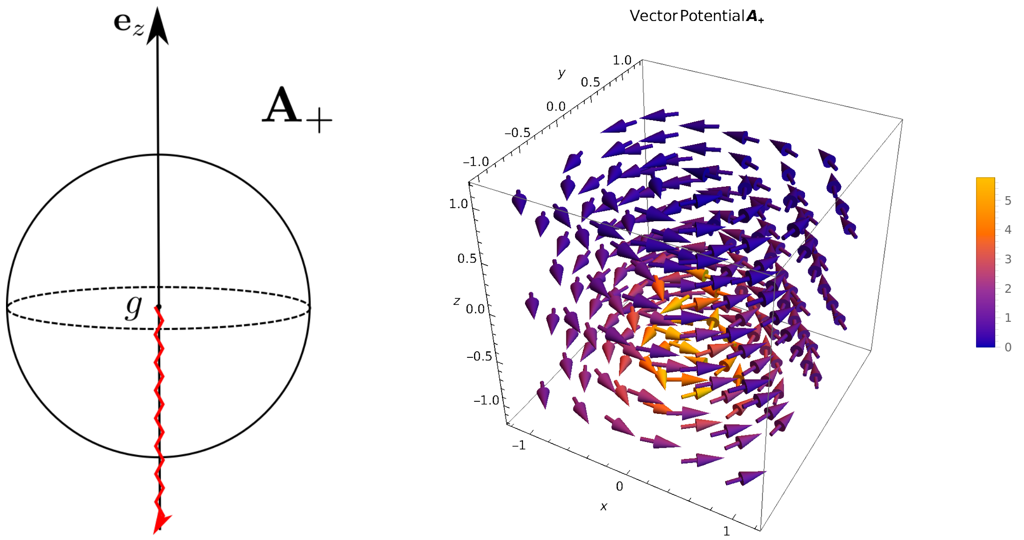

Figure 2: Representations of the potential $\mathbf{A}_+$; it has a singularity along the negative z-axis. Brighter colors on the right plot represent larger values of the potential.

The second one of these two definitions is the one that we run into trouble with. Isn't it always true that just by virtue of $\mathbf{B}$ being a curl of some vector field, its divergence has to be 0?

It looks like we have reached an impasse. Magnetic monopoles, it seems, can't be made to exist.

3 Constructing a Monopole

Consider however, the vector potential [13, 36]

$$\mathbf{A}_+ = \frac{\mu_0 g}{4\pi r} \frac{1 - \cos(\theta)}{\sin(\theta)} \mathbf{e}_\phi. \tag{10}$$

Remembering the formula for the curl in spherical coordinates, we can see that

$$\begin{aligned} \mathbf{B}_+ &= \nabla \times \mathbf{A}_+ \\ &= \mathbf{e}_r \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta\, A_+) - \mathbf{e}_\theta \frac{1}{r} \frac{\partial}{\partial r} (r A_+) \\ &= \frac{\mu_0}{4\pi} \frac{g}{r^2} \mathbf{e}_r, \end{aligned} \tag{11}$$

where A_+ is the component of $\mathbf{A}_+$ in the $\mathbf{e}_\phi$ direction.

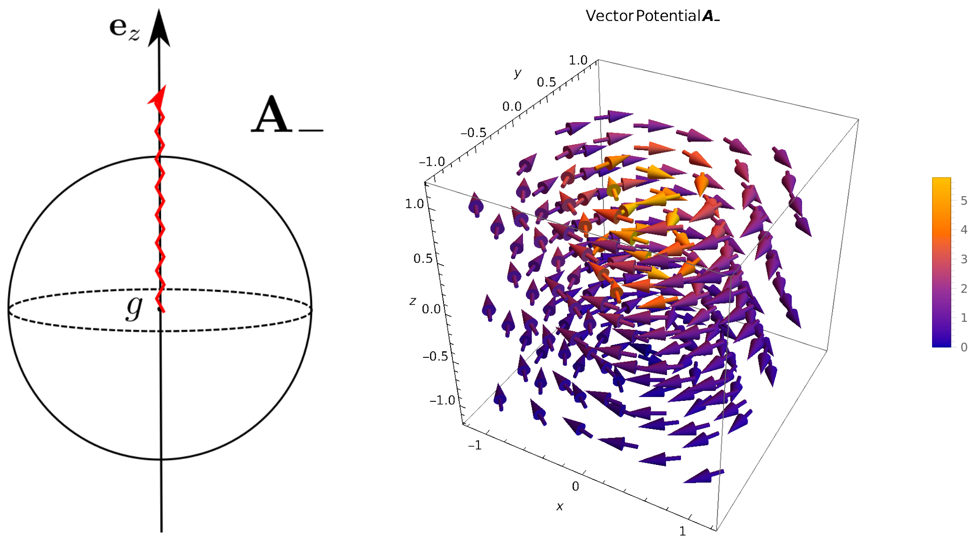

Figure 3: Representations of the potential $\mathbf{A}_-$; it has a singularity along the positive z-axis. Brighter colors on the right plot represent larger values of the potential.

So how did this happen? Didn't we say that the divergence of a curl always has to be 0? The answer lies in the fact that $\mathbf{A}_+$ is a divergent potential. For one thing it is divergent when $r = 0$, but the electric potential of an electric monopole is also divergent at the origin, so we agree that this divergence is allowed. The major divergence of $\mathbf{A}_+$ is that it's divergent along a whole half-line, from 0 to ∞ along the negative z-axis. This is the reason that $\nabla \cdot \nabla \times \mathbf{A}_+ \neq 0$ because mathematically this formula only holds when the domain is topologically trivial, i.e. when there are no "holes".

But this kind of singularity seems bad. How can we justify using a potential that is singular in this way?

The resolution lies in considering a related potential

$$\mathbf{A}_- = \frac{\mu_0 g}{4\pi r} \frac{-1 - \cos(\theta)}{\sin(\theta)} \mathbf{e}_\phi, \tag{12}$$

where the 1 has switched sign and which by exactly the same procedure as above gives *the same* $\mathbf{B}$ *field*

$$\mathbf{B}_- = \nabla \times \mathbf{A}_- = \frac{\mu_0}{4\pi} \frac{g}{r^2} \mathbf{e}_r. \tag{13}$$

However, crucially, the singularity now lies between 0 to ∞ along the *positive*

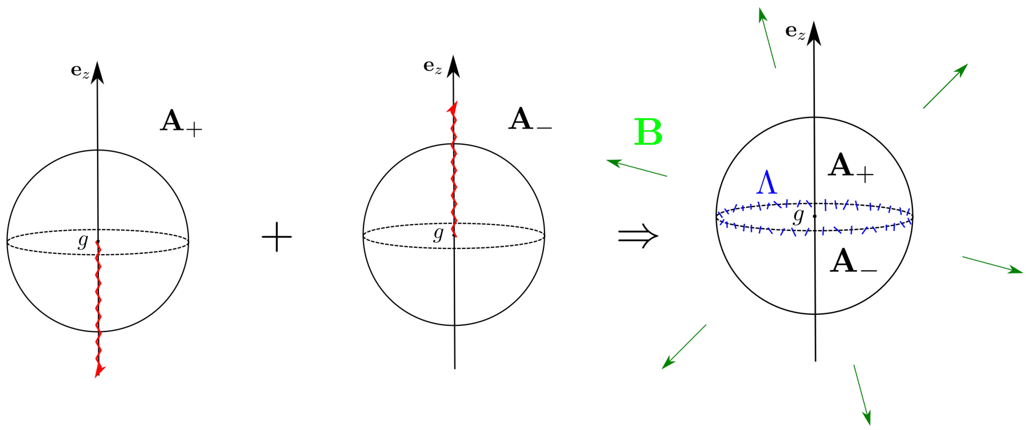

Figure 4: We glue the two potentials $\mathbf{A}_+$ and $\mathbf{A}_-$ along the equator using the gauge transformation Λ. This then gives a potential from which the monopole field can arise.

z-axis. And even more importantly,

$$\mathbf{A}_+ - \mathbf{A}_- = \frac{\mu_0 g}{2\pi r} \frac{1}{\sin(\theta)} \mathbf{e}_\phi = \nabla \left(\frac{\mu_0 g}{2\pi} \phi \right), \tag{14}$$

where we have used the definition of the gradient in spherical coordinates. *So $\mathbf{A}_+$ and $\mathbf{A}_-$ are related by the gauge transformation*

$$\Lambda = \frac{\mu_0 g}{2\pi} \phi \tag{15}$$

as long as we're away from the z-axis (i.e. $\theta \neq 0, \pi$).

At this point, classically, we're already done. If you ask what the potential that describes the monopoles is on the positive z-axis, the answer would be given by $\mathbf{A}_+$. If on the other hand you ask what the potential is on the negative z-axis, the answer would be given by $\mathbf{A}_-$. Anywhere in between, either of them works.

If you were sneakier you would ask: "What if I have a test particle that travels from the north pole of the sphere to the south pole of the sphere? Then you can't describe it at both ends.". This is where the gauge transformation comes to the rescue. One can simply do the gauge transformation with $-\Lambda$ when the test particle reaches the equator and switch from $\mathbf{A}_+$ to $\mathbf{A}_-$. This gluing of the two potentials is shown in fig. 4.

This might at first look weird, but it's analogous to changing coordinates. We know for example that polar coordinates have a slight problem in that the polar angle is undefined when $r = 0$, but if we switch to cartesian coordinates we see that the point $r = 0$ is nothing special. In a similar sense, here we can

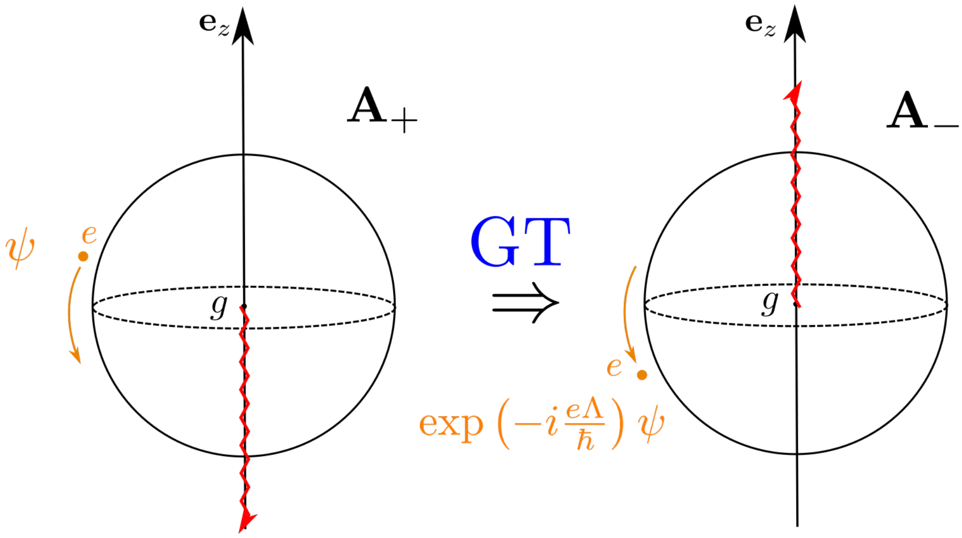

Figure 5: Under a gauge transformation at the equator the electrically charged particle gains a phase in its wavefunction; we must be careful to make to forbid phases that make the wavefunction multivalued.

shift the singularity of our potential to different lines in our space as long as it goes from the origin to ∞.

As classical physicists, at this point we declare victory; we have found two magnetic vector potentials that describe the field on each of the hemispheres and can be stitched together by a gauge transformation on the equator in between.

But quantum mechanically, we still haven't tied up all loose ends. The problem is that when you do a gauge transformation in quantum mechanics the wave function of any electrically charged particle, with charge e, that could be in the vicinity of our monopole gains a phase given by

$$\psi \mapsto \exp\left(i\frac{e\Lambda}{\hbar}\right)\psi, \tag{16}$$

as shown in fig. 5.

This is possibly very problematic because looking at our function $\Lambda = (\mu_0 g\phi)/2\pi$ we notice that it is multivalued because up to a constant it's the same thing as the polar angle. Namely, any position ϕ_0 on the equator can equivalently be described by $\phi_0 + 2\pi n$ ($n \in \mathbb{Z}$). This would in turn make the wavefunction of such an electric charge multivalued, which we know makes no sense. However, if it so happens that

$$\exp\left(i\frac{e\Lambda(\phi = 2\pi)}{\hbar}\right) = \exp\left(i\frac{e\Lambda(\phi = 0)}{\hbar}\right), \tag{17}$$

then we would have nothing to worry about, and we would have constructed a monopole that is allowed by quantum mechanics too. The equation above is exactly satisfied if

$$eg = \frac{2\pi\hbar}{\mu_0} n \qquad (n \in \mathbb{Z}). \tag{18}$$

This is called the *Dirac quantization condition.*

It says that if there exist one magnetic monopole then every electric charge in the universe has to be an integer multiple of some minimum electric charge, i.e. the existence of monopoles predicts charge quantization. Conversely, since we know that electrically charged particles exist, it also implies the quantization of magnetic charge and the quantum of magnetic charge is then

$$g_0 = \frac{2\pi\hbar}{\mu_0 e_0}, \tag{19}$$

where e_0 is the elementary electric charge. This minimum magnetic charge is sometimes referred to as the *Dirac charge.*

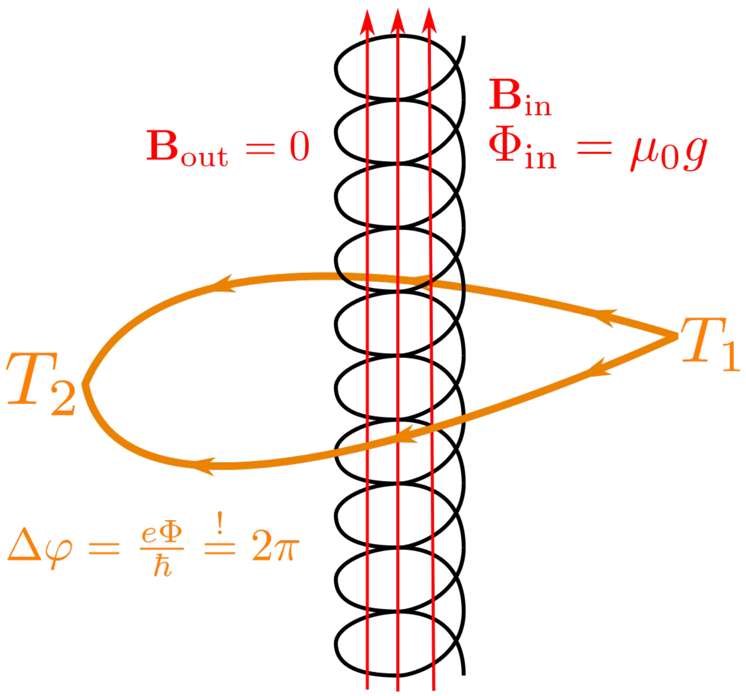

Figure 6: Trying to measure the Dirac string with an Aharonov-Bohm experiment results in an undetectable phase difference.

4 The Dirac String

There is another way to interpret the same vector potential with the half-line singularity we encountered above. This interpretation says that instead of trying to remove the singularity, we face it head on as an infinity in the math. Specifically, if we do the calculation[18] we get a peculiar situation. Anywhere that isn't on the singularity line the magnetic field is given by the monopole field as before. Only on the singularity itself is the field different from the previous construction and there it is given by

$$\mathbf{B} = \mathbf{B}_{\text{monopole}} + \mathbf{B}_{\text{solenoid}}. \tag{20}$$

This extra solenoid field turns out to be homogeneous (thus the name solenoid) and, as expected from a singularity, infinite; but exactly in such a way as to have the flux $\Phi_{\text{solenoid}} = \mu_0 g$. This infinitesimally thin solenoid is then called the *Dirac string*. There is an easy way to see why the flux in the sting must be exactly $\mu_0 g$. The singularity line we previously removed was exactly the reason why Maxwell's second law was modified. Here we don't remove the singularity so Maxwell's law can't be modified, and it tells us that the magnetic flux has to be 0 through a closed surface. Since the monopole has non-zero flux $\mu_0 g$, the solenoid has to pump exactly the same amount of flux through so as to compensate the monopole.

At this point we could ask how could we detect such a Dirac string? We mentioned before that the magnetic field due to the sting is 0 everywhere except right on top of it. Combined with the fact that it's infinitesimally thin this means that there is no way we could detect this string other than with a non-local experiment[2] i.e. the Aharonov-Bohm effect [6]. Here we only want to measure the string if at all possible and not the monopole, so since the monopole's field falls off as $1/r^2$ we can just go far away from the monopole and measure there. A pictorial representation of an Aharonov-Bohm experiment to measure the presence of a Dirac string is shown in fig. 6. But from the previous section we know that the magnetic charge g and subsequently the flux of the Dirac string have to be quantized according to the Dirac quantization condition, and it is exactly when this condition is fulfilled that the Aharonov-Bohm effect doesn't give a phase shift to the

[2]Here by non-local we mean an experiment that can detect magnetic fields even though every part of the experimental setup is in a region of space where the magnetic field is 0. This is of course just a description of the Aharonov-Bohm effect.

wavefunction

$$\psi \mapsto \exp\left(i\frac{e}{\hbar}\oint \mathbf{A}\cdot \mathrm{d}\mathbf{x}\right)\psi = \exp\left(i\frac{e\Phi_B}{\hbar}\right)\psi = \exp(i2\pi n)\psi = \psi \quad (n \in \mathbb{Z}). \tag{21}$$

So if we are physically forbidden from measuring the Dirac string in any conceivable experiment (which we could have expected since its location is gauge dependent), then can we say that it even exists as a physical object? Most physicists today would unequivocally say "No".

Thus, we have two choices of how to describe the math with words. Either we say that we remove the singularity from the potential and use the gauge freedom of **A** to patch together a non-singular vector potential. Or we can say that the potential is indeed singular with an infinite Dirac string with infinite magnetic field attached to every monopole, but that we're forbidden from ever measuring that this string exists.

5 A Brief Topological Perspective

From their beginning monopoles have been closely related to topology. Here we will only give a brief description of how the modern thinking about monopoles goes. There is a more abstract way of viewing this monopole that we have constructed above, and that is as a topological defect in the vector potential. Namely, when we did the gauge transformation $\Lambda = (\mu_0 g\phi)/(2\pi)$ we got acquainted with the phase shift

$$\exp\left(i\frac{\mu_0 eg}{2\pi\hbar}\phi\right). \tag{22}$$

In this abstract way of thinking we say that what is fundamentally important for the monopole and is encoded in this phase shift is *the amount of times we wind around the equator* (while coming back to the original position) to be able to glue the two halves of the potential together. This amount is given by $\phi/2\pi$ or up to a factor, through the complex analysis definition of the winding number of the phase in eq. (22) viewed as a function of ϕ. The winding number is, of course, an integer so that we exactly recover the charge of the monopole given by the Dirac quantization condition.

As an example, one could have required that the two halves of the potential are simply continuously connected without any kind of gauge transformation. This trivial transformation $\Lambda = 0$ corresponds to a winding number $n = 0$ and no magnetic monopoles, i.e. $g = 0$. Going around once ($n = 1$), on the other hand, corresponds to a monopole with one unit of Dirac charge and so on.

So the charge of the monopole is determined solely by the topological property of the number of twists necessary to glue the two potentials together. To give a more rigorous link between these two concepts one needs to use more advanced math from the field of differential geometry.

6 Importance of Monopoles

Historically, monopoles have been important across theoretical physics. Exploring the peculiarities of the vector potential, similar to the way that we considered in the construction above, was instrumental for the development of gauge theories, a backbone for almost all theories of theoretical physics today.

Along similar lines, working with domains with lines cut out of them and related constructions has introduced the use of the mathematical theory of topology into the everyday life of physics. This has been of major importance in a lot of areas, among which is the field of condensed matter physics. Work on topological phases of matter and transitions between them was awarded the 2016 Nobel Prize. [23, 24, 16]

More immediately related are the theories of grand unification (GUTs). These are theories that try to unify the strong nuclear force with the weak nuclear force and electromagnetism, just as the weak nuclear force was unified with electromagnetism in the 1960s and electricity and magnetism themselves before that. It has been observed that the existence of magnetic monopoles is a very robust prediction of these models under very mild assumptions that most physicists agree should hold. This is why the existence of magnetic monopoles has even been called "one of the safest bets that one can make about physics not yet seen". [29]

GUTs predict that monopoles would be created at energy scales close to the grand unification scale which is around $10^{15} - 10^{16}\,\mathrm{GeV}$ [25], obviously millions and millions of times larger than the energy scales that we can reach with even the best accelerators available today. So we can't hope to make magnetic monopoles in experiments we do on Earth, but there is one place where we can expect to find them. In the early stages of the universe's life it was very hot and very dense and there was enough energy for these monopoles to be created, meaning that we can possibly expect to observe monopoles in space.

This prediction of stable monopoles being created during the beginning of the universe are so theoretically generic that they're, in fact, observationally problematic. We have never observed magnetic monopoles on Earth or in space, which puts an upper limit on what their density could be. This was a huge problem for theoretical cosmologists in the 1970s since straightforward Big Bang models predicted high densities of magnetic monopoles that disagreed with the observational limits. This was called the *monopole problem.* [37, 31]

The search for a solution to this problem is what lead to the development of the inflationary model of cosmology, which solves the monopole problem, among others, by having the universe increase its size exponentially thereby diluting the monopole density to very small values. [30] In that sense, monopoles have been really important for the development of modern cosmology, since inflation is now widely accepted as a correct model of what the universe was like in that epoch of its existence.

7 Experimental Limits[3]

As discussed above, to date there have been no confirmed observations of a magnetic monopole.

There are a few experimental ways that probe for magnetic monopoles. The most commonly employed is probably the induction method. In this method, a magnetic monopole passing through a ring induces in it a current of characteristic magnitude and furthermore, this current doesn't decay by any means other than Joule heating. To prevent even this the ring detectors used are made superconducting; these are then called SQUIDs (Superconducting Quantum Interference Devices) [20] and are used for detecting monopoles in cosmic rays and matter.

We can calculate the current that we expect to see by following Maxwell's equations but being careful about using the symmetrized version. Namely, since we are talking about induction, we take the third Maxwell equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} - \mu_0 \, \mathbf{j}_M. \tag{23}$$

Carrying out a model calculation for the induced current in the superconducting loop detector when a monopole flies in we get

$$LI(t) = -\Phi(t) - \mu_0 g H(t), \tag{24}$$

where L is the self-inductance of the detector, Φ is the magnetic flux through the detector and H is the Heaviside step function. The simple setup as well

[3]For a more complete discussion see the review of Milstead and Weinberg for the 2020 Particle Data Group report [15].

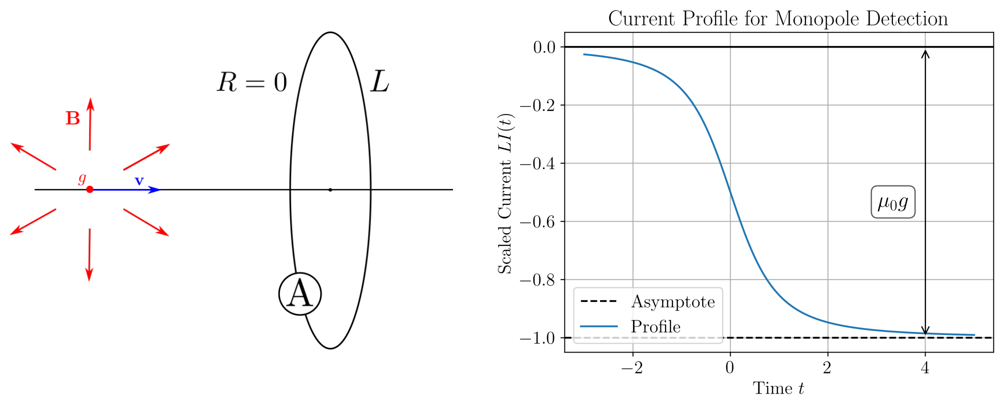

Figure 7: (left) Simple schematic of an induction method detector. (right) Theoretical characteristic current profile of a monopole detection event. The monopole leaves a constant current in the detector.

as the expected current profile are schematically shown in fig. 7 where we see the characteristic constant induced current.

Another approach uses the fact that, because of the Dirac quantization rule, magnetic monopoles are expected to lose energy (in the sense of radiating it out) much more quickly than normal electric monopoles. This is because the coupling constants, being proportional to the charges are related as

$$\frac{\alpha_M}{\alpha_E} = \frac{\mu_0}{4\pi}\frac{g_0^2}{\hbar c} \cdot 4\pi\epsilon_0 \frac{\hbar c}{e_0^2} = \left(\frac{g_0}{ce_0}\right)^2 \approx 4695. \tag{25}$$

This means that we can hope to detect them using scintillators, cloud chambers and nuclear track detectors (in which plastic is exposed to nuclear debris whose tracks can be studied for information about it).

A further method that's useful for colliders is studying particles' paths that aren't helical in a homogeneous magnetic field. This obviously means that the particle doesn't carry any electric charge. Combined with an observation that the particle is accelerating in the direction of the magnetic field, this would be a dead giveaway for the path of a magnetic monopole.

Searches in bulk matter, which can have magnetic monopoles if they were embedded in it through millions of years of cosmic radiation have been carried out on moon rocks, meteorites and seawater among others. The detecting itself is done by passing large amounts of the aforementioned rocks and water through a SQUID and looking for the characteristic current, i.e. by using the induction method as described previously. These types of searches express their results in monopoles per nucleon, and have put a stringent upper limit on the number of monopoles as [21]

$$\sim 10^{-29} \text{ monopoles/nucleon.} \tag{26}$$

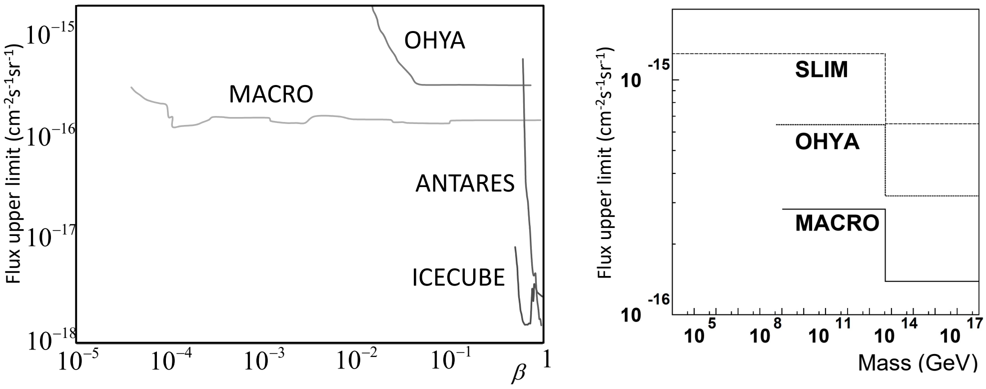

Figure 8: Flux upper limits for the detection of magnetic monopoles resulting from cosmic ray observations, presented as functions of the speed of the monopole and the mass of the monopole. (Figures taken from [15])

Searches in cosmic rays have produced two observed events consistent with magnetic monopoles [10, 32], however both of these have not been confirmed. In more recent times, the most extensive limits on monopoles with a single quantum of Dirac charge consistent with the quantization condition have been put through the work of the MACRO (Monopole, Astrophysics and Cosmic Ray Observatory) experiment [8] under Gran Sasso in Italy, which had an effective acceptance area of $\sim 10\,000\,\mathrm{m}^2$. Figure 8 shows the results of this experiment along with the OHYA experiment [27] in Japan and the SLIM experiment [9] in Bolivia and Pakistan. Relatively high β results from the ICECUBE [2] and ANTARES [7] experiments are also shown. Cosmic ray experiments like these express the abundance of monopoles in terms of their flux density in units of $\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$. The figure shows the upper limits on the flux of such particles as a function of (relativistic) speed β (since we expect a dependence of the flux on the speed of the monopoles — faster monopoles means more flux), and as a function of the mass of the particle. The RICE [19] and ANITA-II [12] experiments at the South Pole have dropped this limit even further to $\sim 10^{-19}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$ but only for more relativistic monopoles with $10^7 \lesssim \gamma \lesssim 10^{13}$.

Searches at colliders have also produced upper limits, but in their case the results are expressed as an upper limit of the cross-section of a detection event in units of picobarns ($1\,\mathrm{pb} = 10^{-36}\,\mathrm{cm}^2$), as a function of half of the center of mass energy of the collision. This is done because cross-sections are

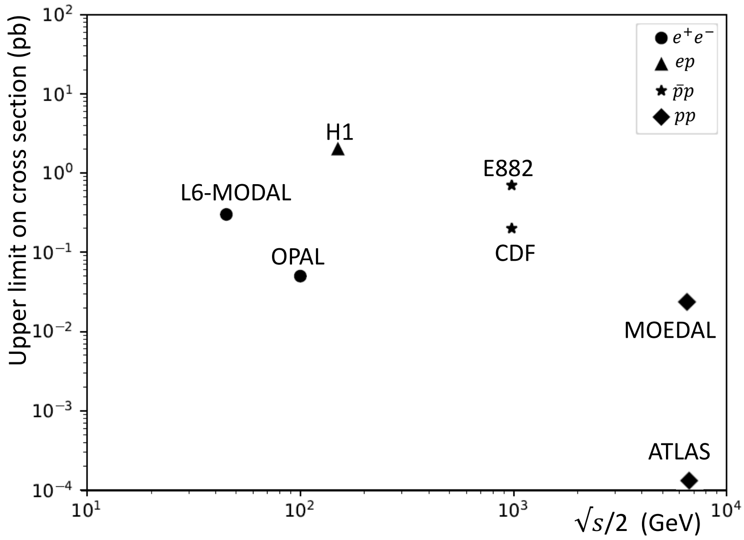

Figure 9: Cross-section upper limits for the detection of magnetic monopoles resulting from collider observations, presented as a function of half of the center of mass energy of the collision [1, 5, 4, 22, 3, 28, 26]. (Figure taken from [15])

quantities that are the least dependent on theoretical modeling; any other way of presenting the data would have to assume a theoretical model that predicts monopoles and since there are many of those kinds of models one would generically not be able to compare data between different experiments. Figure 9 compiles the results of a few collider experiments that have done searches in this fashion.

There is also the possibility of detecting monopoles through their effect on other process as virtual particles. This approach has been used in a few experiments but results from it are model dependent.

8 Conclusions

In this work we have looked at the theoretical and experimental evidence for and against the existence of monopoles. We have theoretically constructed the simplest kind of monopole and commented on its frequent appearance in Grand Unified Theories. We further discussed how the magnetic monopole, even though undiscovered, has been an inspiration for some groundbreaking theoretical work in physics in the past half a century or more.

At the end we also discussed some methods that are used for the detection of magnetic monopoles and the different experiments that put have upper limits on the abundance of magnetic monopoles in the universe using those detection methods.

If discovered, monopoles would certainly be one of the greatest discoveries in physics in the past half a century, on par with the discovery of the Higgs boson if not even greater.

References

[1] Georges Aad et al. "Search for Magnetic Monopoles and Stable High-Electric-Charge Objects in 13 Tev Proton-Proton Collisions with the ATLAS Detector". In: *Phys. Rev. Lett.* 124.3 (2020), p. 031802. DOI: 10.1103/PhysRevLett.124.031802. arXiv: 1905.10130 [hep-ex].

[2] M. G. Aartsen et al. "Searches for Relativistic Magnetic Monopoles in IceCube". In: *Eur. Phys. J. C* 76.3 (2016), p. 133. DOI: 10.1140/epjc/s10052-016-3953-8. arXiv: 1511.01350 [astro-ph.HE].

[3] G. Abbiendi et al. "Search for Dirac magnetic monopoles in e+e- collisions with the OPAL detector at LEP2". In: *Phys. Lett. B* 663 (2008), pp. 37–42. DOI: 10.1016/j.physletb.2008.03.057. arXiv: 0707.0404 [hep-ex].

[4] A. Abulencia et al. "Direct search for Dirac magnetic monopoles in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV". In: *Phys. Rev. Lett.* 96 (2006), p. 201801. DOI: 10.1103/PhysRevLett.96.201801. arXiv: hep-ex/0509015.

[5] B. Acharya et al. "Magnetic Monopole Search with the Full MoEDAL Trapping Detector in 13 TeV pp Collisions Interpreted in Photon-Fusion and Drell-Yan Production". In: *Phys. Rev. Lett.* 123.2 (2019), p. 021802. DOI: 10.1103/PhysRevLett.123.021802. arXiv: 1903.08491 [hep-ex].

[6] Y. Aharonov and D. Bohm. "Significance of Electromagnetic Potentials in the Quantum Theory". In: *Phys. Rev.* 115 (3 Aug. 1959), pp. 485–491. DOI: 10.1103/PhysRev.115.485. URL: https://link.aps.org/doi/10.1103/PhysRev.115.485.

[7] A. Albert et al. "Search for relativistic magnetic monopoles with five years of the ANTARES detector data". In: *JHEP* 07 (2017), p. 054. DOI: 10.1007/JHEP07(2017)054. arXiv: 1703.00424 [astro-ph.HE].

[8] M. Ambrosio et al. "Final results of magnetic monopole searches with the MACRO experiment". In: *Eur. Phys. J. C* 25 (2002), pp. 511–522. DOI: 10.1140/epjc/s2002-01046-9. arXiv: hep-ex/0207020.

[9] S. Balestra et al. “Magnetic Monopole Search at high altitude with the SLIM experiment”. In: *Eur. Phys. J. C* 55 (2008), pp. 57–63. DOI: 10.1140/epjc/s10052-008-0597-3. arXiv: 0801.4913 [hep-ex].

[10] Blas Cabrera. “First Results from a Superconductive Detector for Moving Magnetic Monopoles”. In: *Phys. Rev. Lett.* 48 (20 May 1982), pp. 1378–1381. DOI: 10.1103/PhysRevLett.48.1378. URL: https://link.aps.org/doi/10.1103/PhysRevLett.48.1378.

[11] Claudio Castelnovo, Roderich Moessner, and Shivaji L. Sondhi. “Magnetic monopoles in spin ice”. In: *Nature* 451N7174 (2008), pp. 42–45. DOI: 10.1038/nature06433. arXiv: 0710.5515 [cond-mat.str-el].

[12] M. Detrixhe et al. “Ultra-Relativistic Magnetic Monopole Search with the ANITA-II Balloon-borne Radio Interferometer”. In: *Physical Review D* 83 (2011).

[13] Paul Adrien Maurice Dirac. “Quantised singularities in the electromagnetic field,” in: *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character* 133.821 (1931), pp. 60–72. DOI: 10.1098/rspa.1931.0130. eprint: https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1931.0130. URL: https://royalsocietypublishing.org/doi/abs/10.1098/rspa.1931.0130.

[14] Alan Farhan et al. “Emergent magnetic monopole dynamics in macroscopically degenerate artificial spin ice”. In: *Science Advances* 5.2 (2019). DOI: 10.1126/sciadv.aav6380. eprint: https://advances.sciencemag.org/content/5/2/eaav6380.full.pdf. URL: https://advances.sciencemag.org/content/5/2/eaav6380.

[15] Particle Data Group et al. “Review of Particle Physics”. In: *Progress of Theoretical and Experimental Physics* 2020.8 (Aug. 2020). [The relevant review can be found under the category “Magnetic Monopoles”.] ISSN: 2050-3911. DOI: 10.1093/ptep/ptaa104. eprint: https://academic.oup.com/ptep/article-pdf/2020/8/083C01/34673722/ptaa104.pdf. URL: https://doi.org/10.1093/ptep/ptaa104.

[16] F. D. M. Haldane. “Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"”. In: *Phys. Rev. Lett.* 61 (18 Oct. 1988), pp. 2015–2018. DOI: 10.1103/PhysRevLett.61.2015. URL: https://link.aps.org/doi/10.1103/PhysRevLett.61.2015.

[17] Jeffrey A. Harvey. "Magnetic monopoles, duality and supersymmetry". In: *ICTP Summer School in High-energy Physics and Cosmology*. Mar. 1996. arXiv: hep-th/9603086.

[18] Ricardo Heras. "Dirac quantisation condition: a comprehensive review". In: *Contemp. Phys.* 59.4 (2018), pp. 331–355. DOI: 10.1080/00107514.2018.1527974. arXiv: 1810.13403 [physics.hist-ph].

[19] D. P. Hogan et al. "Relativistic Magnetic Monopole Flux Constraints from RICE". In: *Phys. Rev. D* 78 (2008), p. 075031. DOI: 10.1103/PhysRevD.78.075031. arXiv: 0806.2129 [astro-ph].

[20] R. C. Jaklevic et al. "Quantum Interference Effects in Josephson Tunneling". In: *Phys. Rev. Lett.* 12 (7 Feb. 1964), pp. 159–160. DOI: 10.1103/PhysRevLett.12.159. URL: https://link.aps.org/doi/10.1103/PhysRevLett.12.159.

[21] Hunmoo Jeon and Michael J. Longo. "Search for magnetic monopoles trapped in matter". In: *Phys. Rev. Lett.* 75 (1995). [Erratum: Phys.Rev.Lett. 76, 159 (1996)], pp. 1443–1446. DOI: 10.1103/PhysRevLett.75.1443. arXiv: hep-ex/9508003.

[22] G. R. Kalbfleisch et al. "Limits on production of magnetic monopoles utilizing samples from the D0 and CDF detectors at the Tevatron". In: *Phys. Rev. D* 69 (2004), p. 052002. DOI: 10.1103/PhysRevD.69.052002. arXiv: hep-ex/0306045.

[23] J M Kosterlitz and D J Thouless. "Long range order and metastability in two dimensional solids and superfluids. (Application of dislocation theory)". In: *Journal of Physics C: Solid State Physics* 5.11 (June 1972), pp. L124–L126. DOI: 10.1088/0022-3719/5/11/002. URL: https://doi.org/10.1088/0022-3719/5/11/002.

[24] J M Kosterlitz and D J Thouless. "Ordering, metastability and phase transitions in two-dimensional systems". In: *Journal of Physics C: Solid State Physics* 6.7 (Apr. 1973), pp. 1181–1203. DOI: 10.1088/0022-3719/6/7/010. URL: https://doi.org/10.1088/0022-3719/6/7/010.

[25] A. Masiero, S.K. Vempati, and O. Vives. "Course 1 - Flavour Physics and Grand Unification". In: *Particle Physics Beyond the Standard Model*. Ed. by Dmitri Kazakov, Stéphane Lavignac, and Jean Dalibard. Vol. 84. Les Houches. Elsevier, 2006, pp. 1–78. DOI: https://doi.org/10.1016/S0924-8099(06)80021-1. URL: https://www.sciencedirect.com/science/article/pii/S0924809906800211.

[26] P. Musset, M. Price, and E. Lohrmann. "Search for magnetic monopoles in electron-positron collisions at 34 GeV CM energy". In: *Physics Letters B* 128.5 (1983), pp. 333–335. ISSN: 0370-2693. DOI: https://doi.org/10.1016/0370-2693(83)90270-8. URL: https://www.sciencedirect.com/science/article/pii/0370269383902708.

[27] S. Orito et al. "Search for supermassive relics with 2000-m**2 array of plastic track detector". In: *Phys. Rev. Lett.* 66 (1991), pp. 1951–1954. DOI: 10.1103/PhysRevLett.66.1951.

[28] J.L. Pinfold. "Searching for exotic particles at the LHC with dedicated detectors". In: *Nuclear Physics B - Proceedings Supplements* 78.1 (1999). Advanced Technology and Particle Physics, pp. 52–57. ISSN: 0920-5632. DOI: https://doi.org/10.1016/S0920-5632(99)00522-8. URL: https://www.sciencedirect.com/science/article/pii/S0920563299005228.

[29] Joseph Polchinski. "Monopoles, duality, and string theory". In: *Int. J. Mod. Phys. A* 19S1 (2004). Ed. by H. Baer and A. Belyaev, pp. 145–156. DOI: 10.1142/S0217751X0401866X. arXiv: hep-th/0304042.

[30] J Preskill. "Magnetic Monopoles". In: *Annual Review of Nuclear and Particle Science* 34.1 (1984), pp. 461–530. DOI: 10.1146/annurev.ns.34.120184.002333. eprint: https://doi.org/10.1146/annurev.ns.34.120184.002333. URL: https://doi.org/10.1146/annurev.ns.34.120184.002333.

[31] John P. Preskill. "Cosmological Production of Superheavy Magnetic Monopoles". In: *Phys. Rev. Lett.* 43 (19 Nov. 1979), pp. 1365–1368. DOI: 10.1103/PhysRevLett.43.1365. URL: https://link.aps.org/doi/10.1103/PhysRevLett.43.1365.

[32] P. B. Price et al. "Evidence for Detection of a Moving Magnetic Monopole". In: *Phys. Rev. Lett.* 35 (1975), pp. 487–490. DOI: 10.1103/PhysRevLett.35.487.

[33] Jun John Sakurai and Jim Napolitano. "Modern Quantum Mechanics". In: Cambridge University Press, 1985. Chap. Quantum Dynamics, pp. 145–148. ISBN: 978-1-108-64592-8.

[34] John Mayo Lucien Scott. "U(1) x U(1) gauge theory applied to magnetic monopoles". MA thesis. Fresno State, Dec. 2016.

[35] David Tong. "Topics in Electromagnetism". In: *Gauge Theory.* [Last accesed 14.03.2021]. 2018, pp. 3–10. URL: http://www.damtp.cam.ac.uk/user/tong/gaugetheory/1em.pdf.

[36] Tai Tsun Wu and Chen Ning Yang. "Concept of nonintegrable phase factors and global formulation of gauge fields". In: *Phys. Rev. D* 12 (12 Dec. 1975), pp. 3845–3857. DOI: 10.1103/PhysRevD.12.3845. URL: https://link.aps.org/doi/10.1103/PhysRevD.12.3845.

[37] Ya.B. Zeldovich and M.Yu. Khlopov. "On the concentration of relic magnetic monopoles in the universe". In: *Physics Letters B* 79.3 (1978), pp. 239–241. ISSN: 0370-2693. DOI: https://doi.org/10.1016/0370-2693(78)90232-0. URL: https://www.sciencedirect.com/science/article/pii/0370269378902320.