Evidence for a doubly charm tetraquark pole with lattice QCD

$cc\bar{u}\bar{d}$

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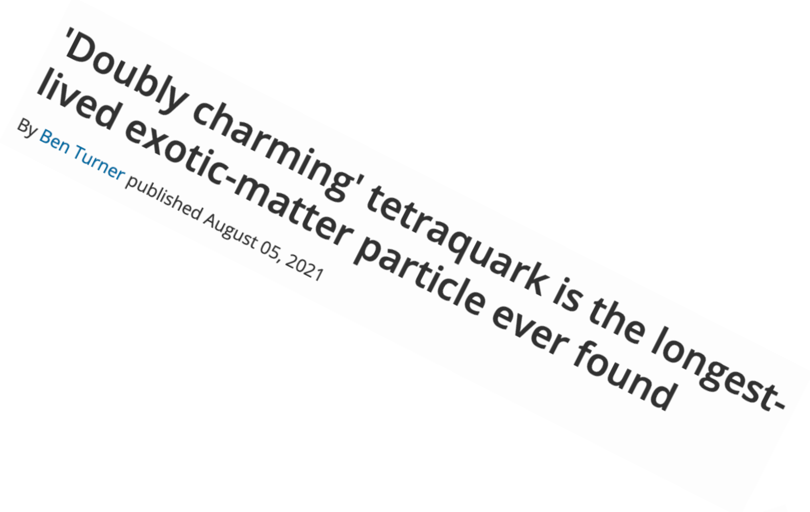
F1, IJS, 24th March 2022

based on **2202.101101**

done in collaboration
with **M. Padmanath**


'Doubly charming' tetraquark is the longest-lived exotic-matter particle ever found
By Ben Turner published August 05, 2021

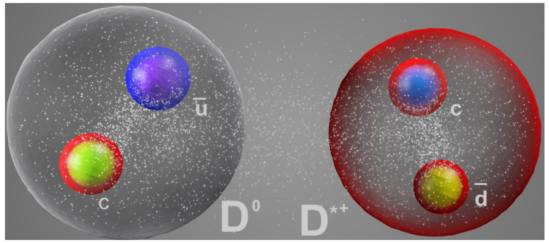

Twice the charm: long-lived exotic particle discovered
CERN Accelerating science

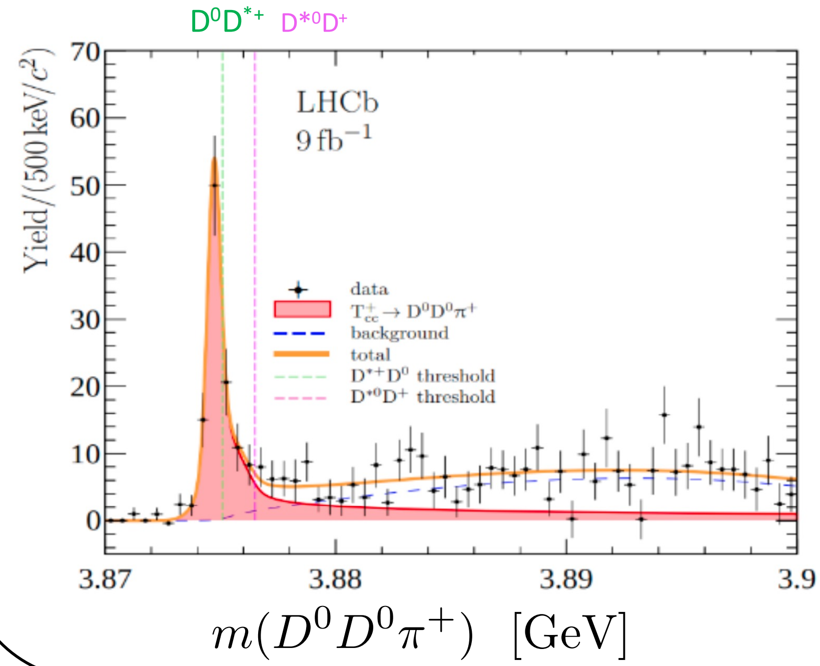
LHCb discovery of T_{cc}

$cc\bar{u}\bar{d}$

I=0, $J^P=1^+$

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$





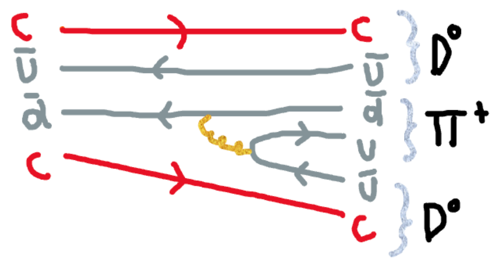
The longest lived discovered hadron with explicitly exotic quark content

LHCb July 2021, 2109.01038, 2109.01056

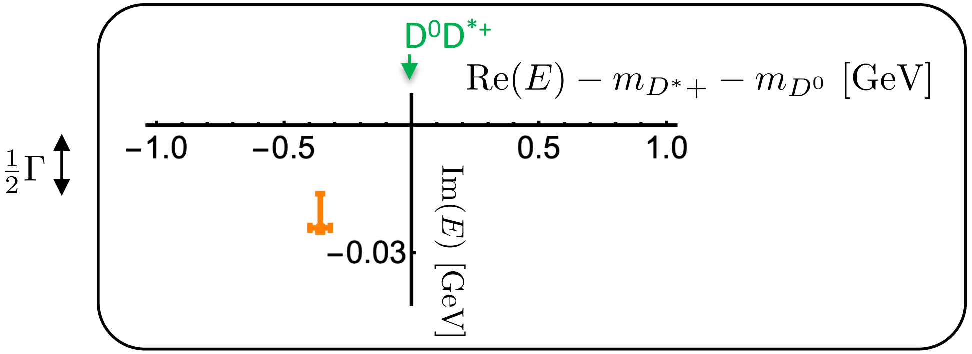
The doubly charmed tetraquark T_{cc}^+, $I=0$ and favours $J^P=1^+$.

No states observed in $D^0 D^+ \pi^+$: eliminates possibility of $I=1$.

Near-threshold state: Demands pole identification to confirm existence.



Pole in T(E) $\delta m = -0.36(4)$ GeV



$$\delta m_{\rm pole} = -360 \pm 40\,^{+4}_{-0}\ \ {\rm keV}/c^2\,,$$
$$\Gamma_{\rm pole} = 48 \pm 2\,^{+0}_{-14}\,{\rm keV}\,,$$

Theoretical predictions

❖ **Phenomenological approaches** →

* Janc & Rosina , Few Body Syst. 35, 175 (2004), hep-ph/0405208

one of the most sophisticated quark model predictions:

V_{ij} between all pairs of quarks, ground state energy of four-body problem

$$\delta m = -1.6 \pm 1.0 \text{ MeV}$$

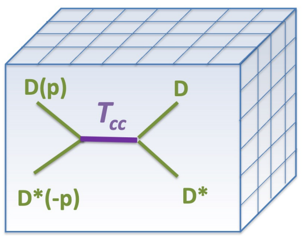
❖ **Lattice QCD**

only finite-volume eigen-energy $E_n(L)$ was extracted:

this does not suffice to establish a near-threshold state

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285
Hadron Spectrum, JHEP 11, 033 (2017), 1709.01417

To establish a near-threshold state: pole in T(E) needs to be found:



scattering amplitude T(E)

$$T(E) \propto \frac{1}{s-m^2} = \frac{1}{E^2-m^2}$$

$E=E_{cm}$

T(E) has not been extracted by lattice QCD before our study

Our study 2202.101101

first and still the only extraction of T(E) with lattice QCD

pole related to T_{cc} established for the first time with lattice QCD

δm [MeV] references at the back

Theoretical PREdictions

courtesy: Ivan Polyakov, EPS-HEP 2021

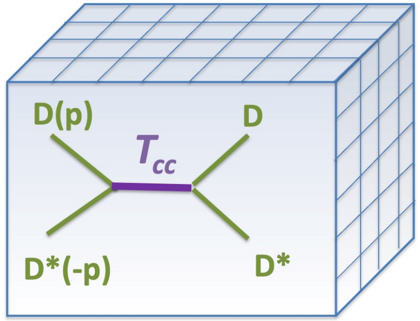
(references at the back)

Summary of our lattice results

$m_{u,d} > m_{u,d}^{phy}$

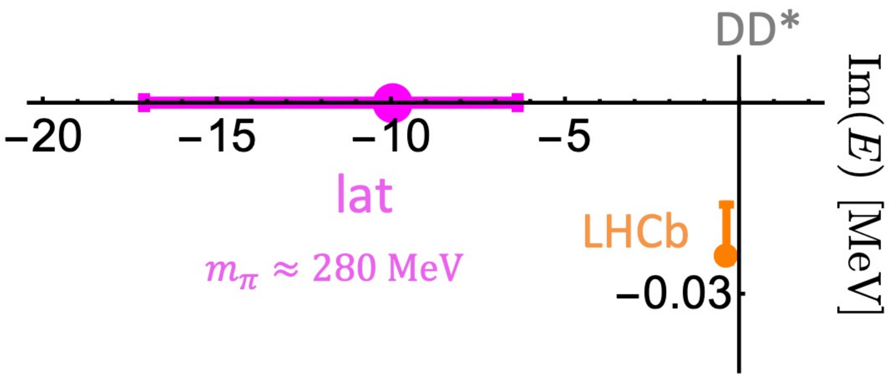
$m_\pi \approx 280$ MeV

$m_D \approx 1927$ MeV



Pole of T(E)

$$\delta m_{T_{cc}} = \mathrm{Re}(E) - m_{D^0} - m_{D^{*+}} \; [\mathrm{MeV}]$$



- T(E) extracted via the Luscher's method
- Evidence for pole related to Tcc
- For $m_{u,d} > m_{u,d}^{phy}$ one expects decreased attraction

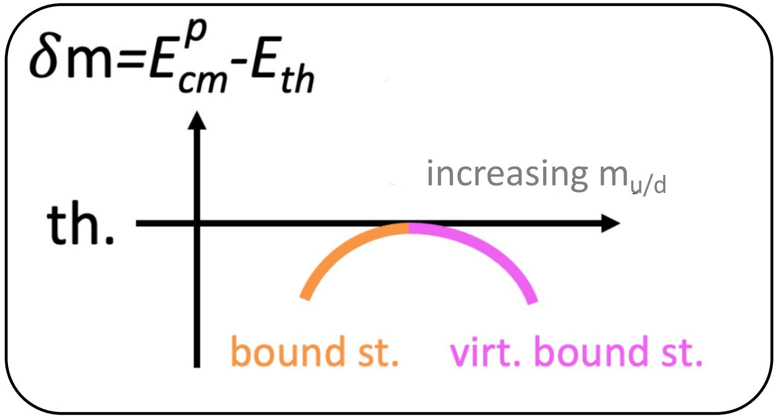
  T_{cc} : bound state becomse virtual bound state

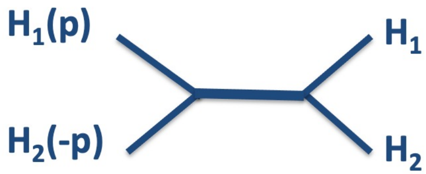
  indeed this is what we find

	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
LHCb	$-0.36(4)$	bound st.

omitting $D^0 D^0 \pi^+$

Sketch of expected binding energy

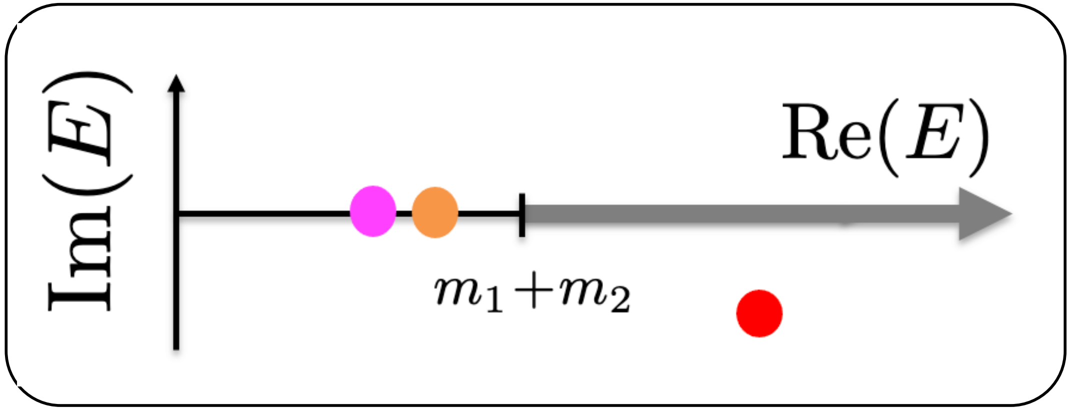




Definitions: bound state, virtual bound state & resonance

$$T(E) \propto \frac{1}{E^2 - m^2} \qquad T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$

Poles of *T(E)*, *E=E* $_{cm}$



Virtual bound st. — $p = -i\,|p|$

Bound st. — $p = i\,|p|$

Resonance

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + (-p)^2} \; < \; m_1 + m_2$$

How did we arrive at these lattice QCD result ?

eigen-energies on the lattice -> T(E)

Lattice QCD ensembles employed

CLS Consortium

with dynamical quarks: u,d,s

$m_u = m_d > m_{u,d}^{phy}$, $m_\pi \approx 280$ MeV

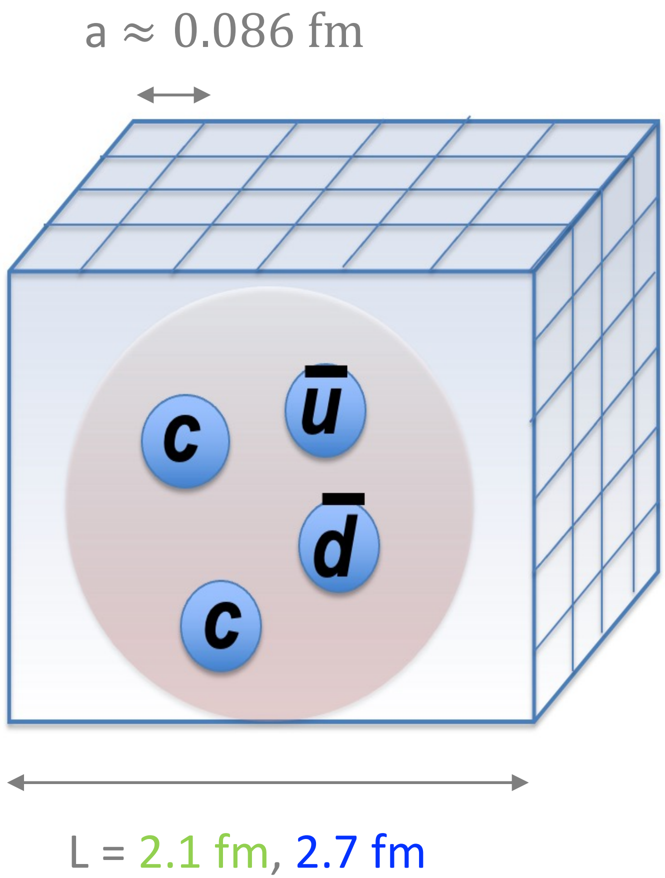
Clover Wilson fermions

$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$

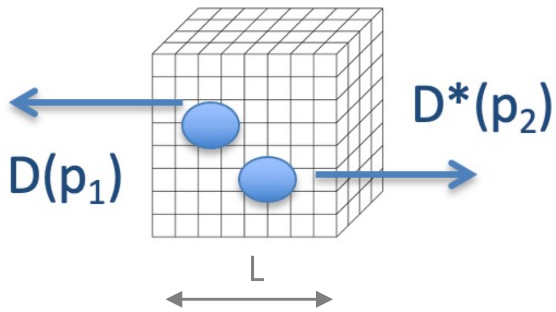
Eucledian space-time

$$S_{QCD}^E = \int d^4x_E\, \mathcal{L}_{QCD}^E(m_q, g_s)$$

strategy: $C \to E \to T(E)$



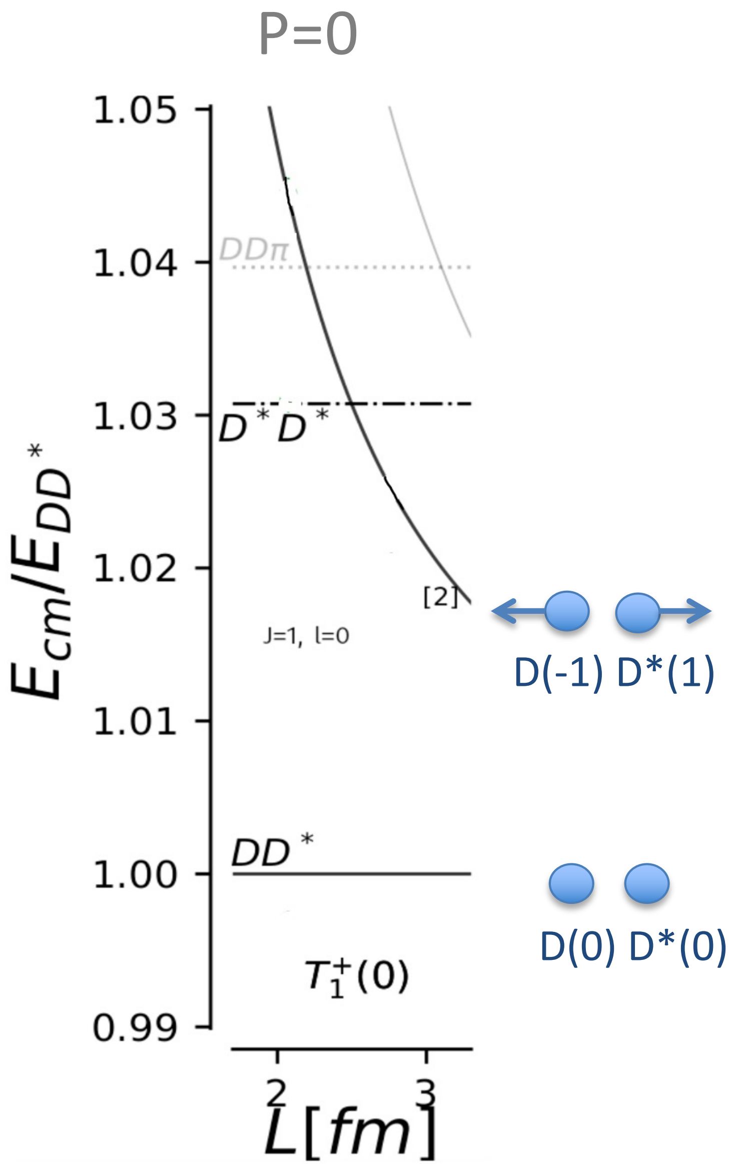
Energies of DD* in non-interacting limit



periodic bc in space

$$\vec{p}_{1,2} = \vec{n}_{1,2}\ \frac{2\pi}{L}$$

$$E = \sqrt{m_D^2 + \vec{p}_1^{\,2}} + \sqrt{m_{D^*}^2 + \vec{p}_2^{\,2}}$$

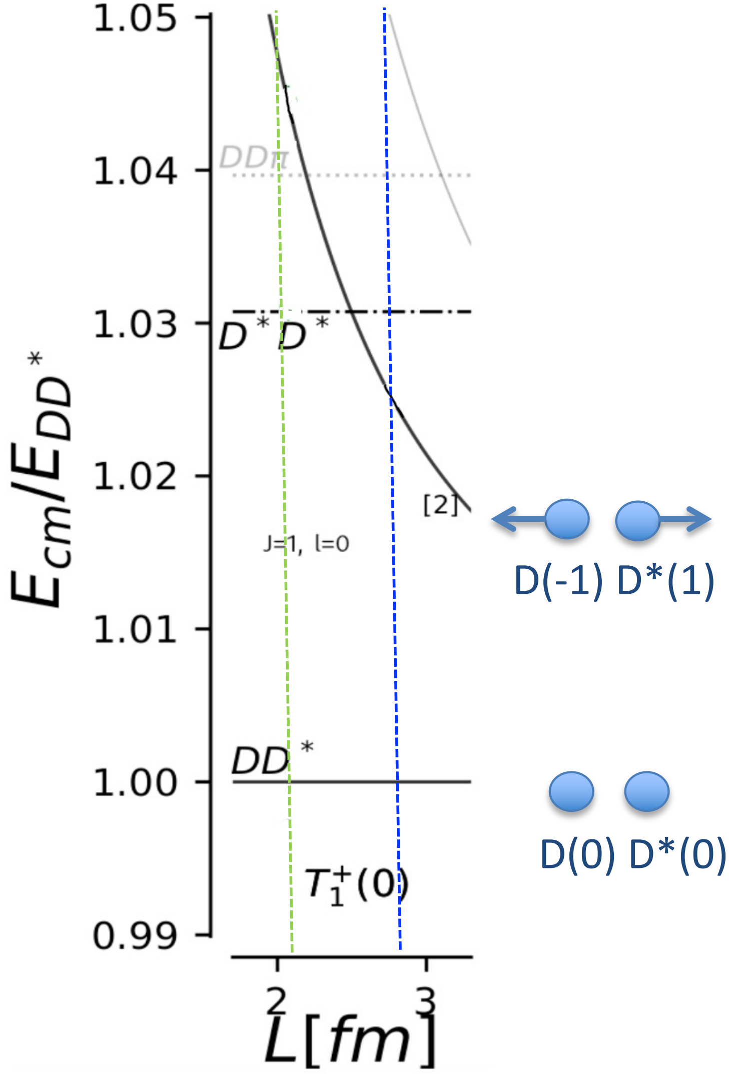


$$E = \sqrt{m_D^2 + (\tfrac{2\pi}{L})^2} + \sqrt{m_{D^*}^2 + (\tfrac{2\pi}{L})^2}$$

$$E_{DD^*} \equiv m_D + m_{D^*}$$

Energies of DD* in non-interacting limit

P=0



$$E = \sqrt{m_D^2 + (\tfrac{2\pi}{L})^2} + \sqrt{m_{D^*}^2 + (\tfrac{2\pi}{L})^2}$$

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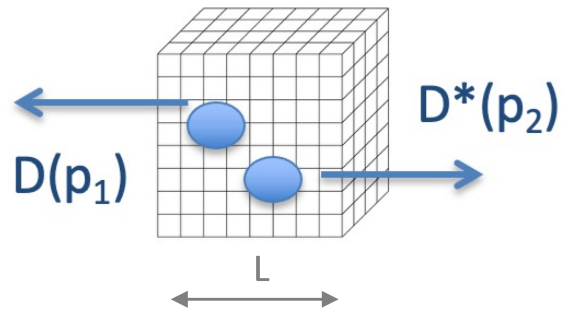


L = 2.1 fm, 2.7 fm

periodic bc in space

$$\vec{p}_{1,2} = \vec{n}_{1,2}\ \frac{2\pi}{L}$$

$$E = \sqrt{m_D^2 + \vec{p}_1^{\,2}} + \sqrt{m_{D^*}^2 + \vec{p}_2^{\,2}}$$

Extracting eigen-energies from correlation functions

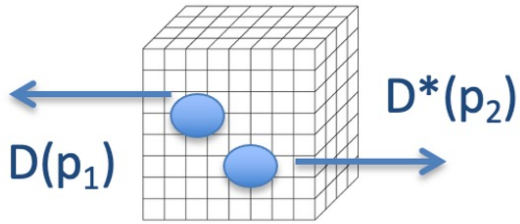
$cc\bar{u}\bar{d}$ $J^P=1^+$, $I=0$

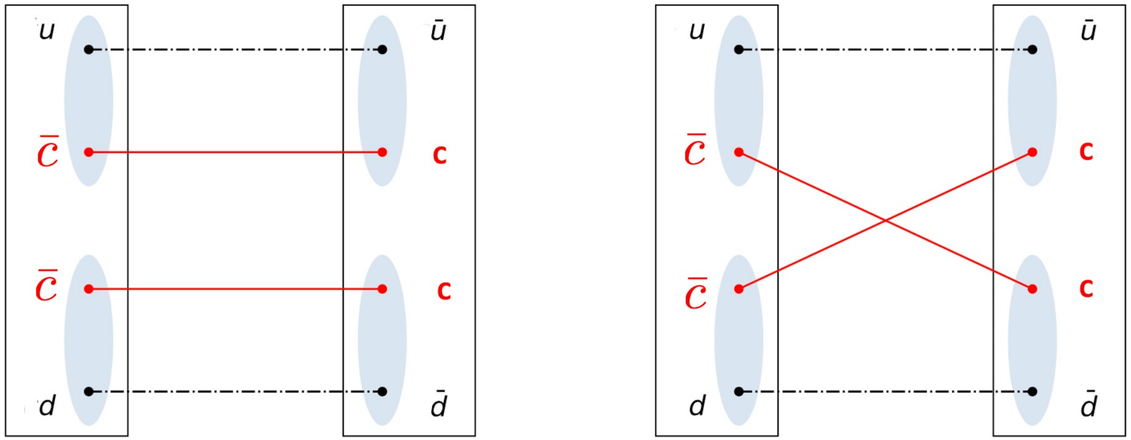
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^{+}(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\ e^{-E_n t}\ \langle n | \mathcal{O}_j^{+} | 0 \rangle$$

D   D*

$$\mathcal{O} = (\bar{u}\gamma_5 c)_{\vec{p}_1}\ (\bar{d}\gamma_i c)_{\vec{p}_2} - (\vec{p}_1 \leftrightarrow \vec{p}_2) \qquad \vec{p}_{1,2} = \vec{n}_{1,2}\ \frac{2\pi}{L}$$

$$(\bar{u}\gamma_5\gamma_t c)_{\vec{p}_1}\ (\bar{d}\gamma_i\gamma_t c)_{\vec{p}_2}$$



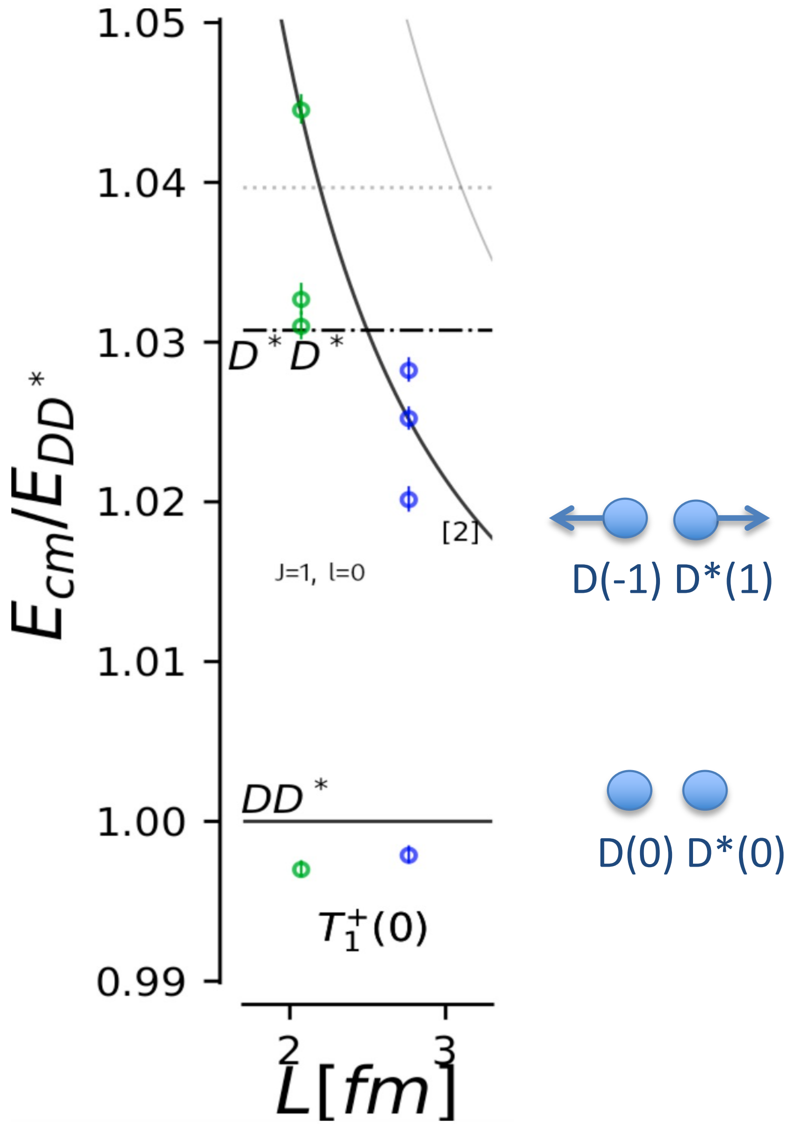


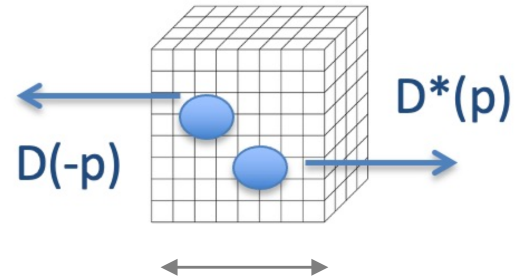
t   t=0

Diquark antidiquark operators [cc][ud] not incorporated: Emmanuel, Luka

Energies of DD* from lattice QCD

$P=0,\ J^P=1^+$





L = 2.1 fm, 2.7 fm

- energies shifted from non-interacting energies: renders info on T(E)
- focus on energy region near DD* threshold
- scattering in partial wave l=2 negligible

[2]: DD* with $J^P=1^+$ in l=0,2 :
both E degenerate in noninteracting limit

Luscher's relation $E \rightarrow T(E)$

Relation between E and δ(E), T(E): 1D quantum mechanics

V= 0: outside the region of potential

$$\psi(x) = A\cos(p|x|+\delta) = \begin{cases} A\cos(px+\delta) & x>R \\ A\cos(-px+\delta) & x<-\frac{R}{2} \end{cases}$$

- this form already ensures $\psi(L/2) = \psi(-L/2)$
- the other BC: $\psi'(L/2) = \psi'(-L/2)$

  this requires

  $$Ap\sin\left(p\left(\tfrac{L}{2}\right)+\delta\right) = -Ap\sin\left(-p\left(-\tfrac{L}{2}\right)+\delta\right)$$

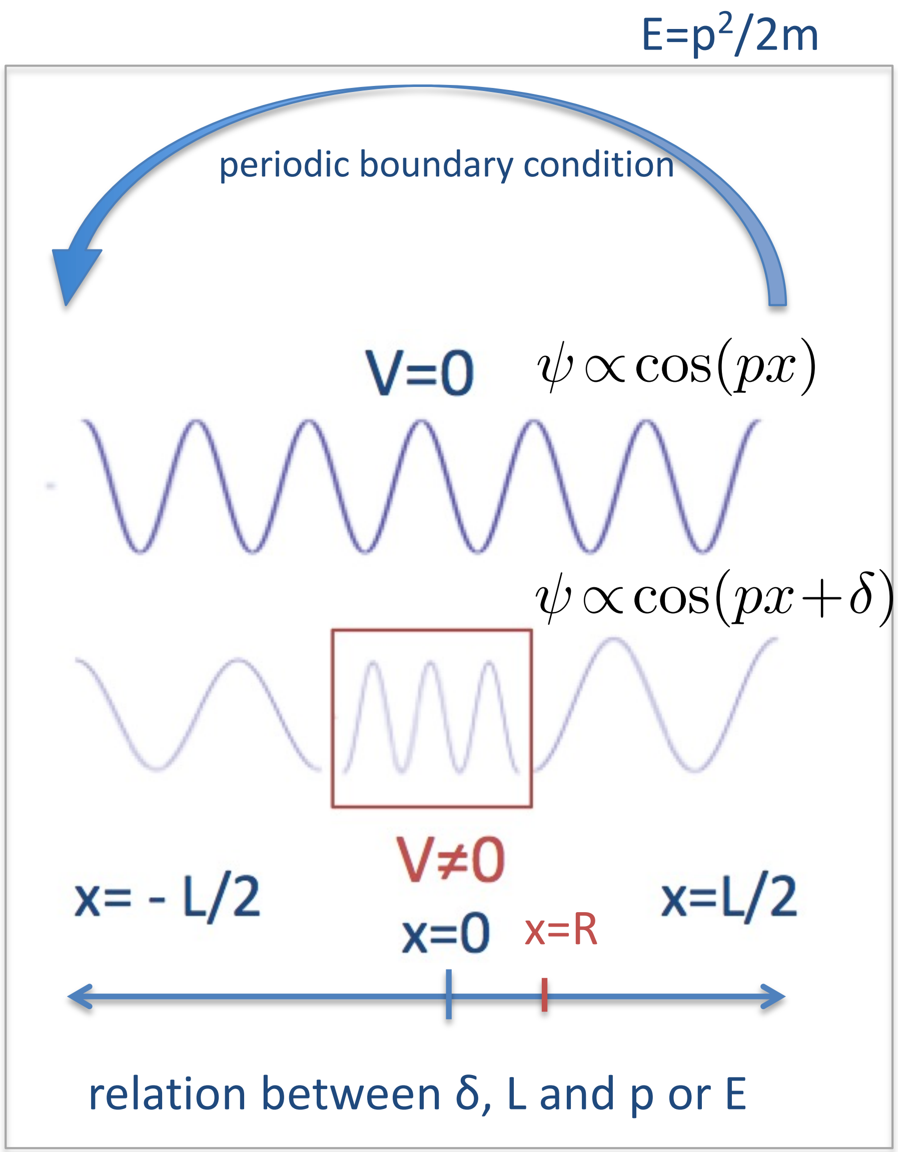
  $\rightarrow \psi'(L/2)=0$, $\sin\left(p\frac{L}{2}+\delta\right)=0$

  $p\frac{L}{2}+\delta = n\pi$ $\quad$ $p = n\frac{2\pi}{L} - \frac{2}{L}\delta$

  relation between δ, L

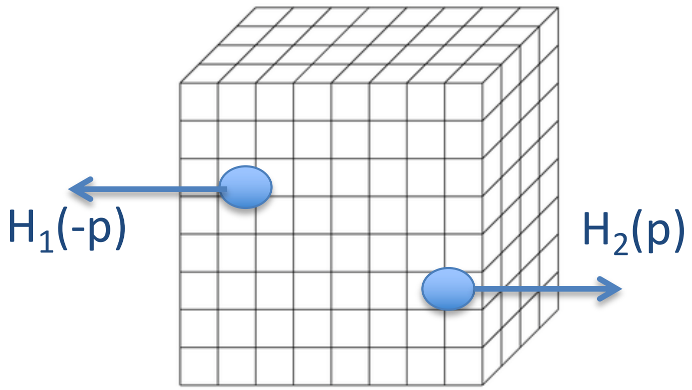
$$p = \frac{2\pi}{L}n$$

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta$$



Luscher 1991

Relation between E and $\delta(E), T(E)$



E = eigen-energy lattice from lattice in cmf

E = $\sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2}$

$$S = 1 + i\frac{4p}{E}T = e^{2i\delta}$$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

Luscher's relation :

$$E \;\rightarrow\; T(E),\;\delta(E)$$

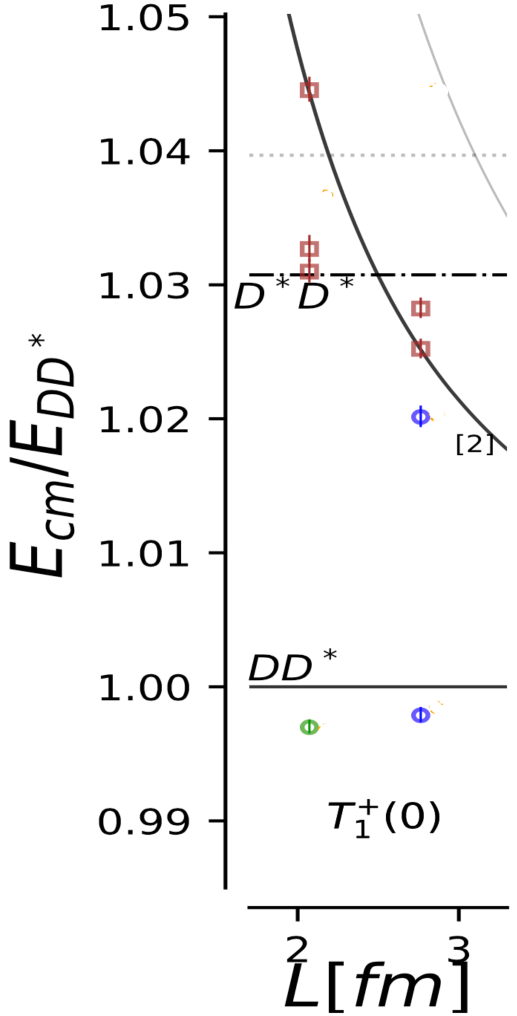
known function

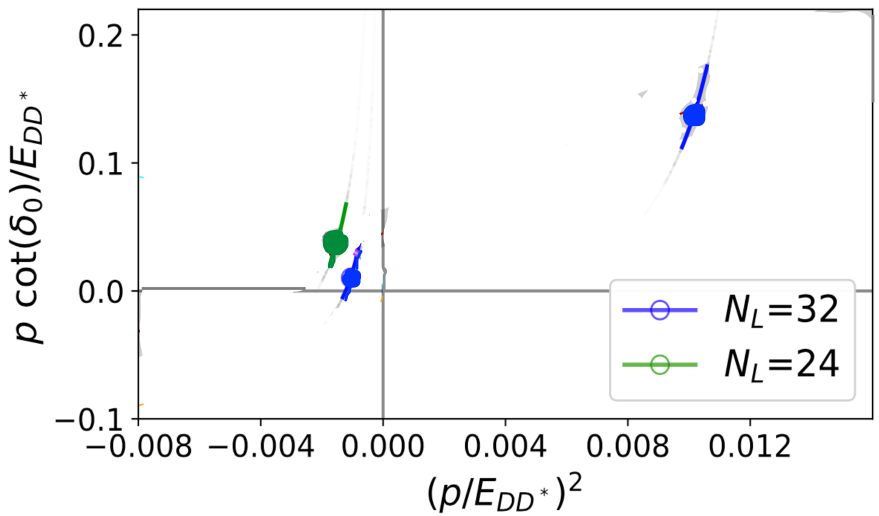
l=0: $$p\cot\delta(p) = \frac{2\; Z_{00}(1,(\frac{pL}{2\pi})^2)}{\sqrt{\pi}L}$$

DD* scattering amplitude with l=0

fit using just P=0

P=0, $J^P=1^+$





green and blue points

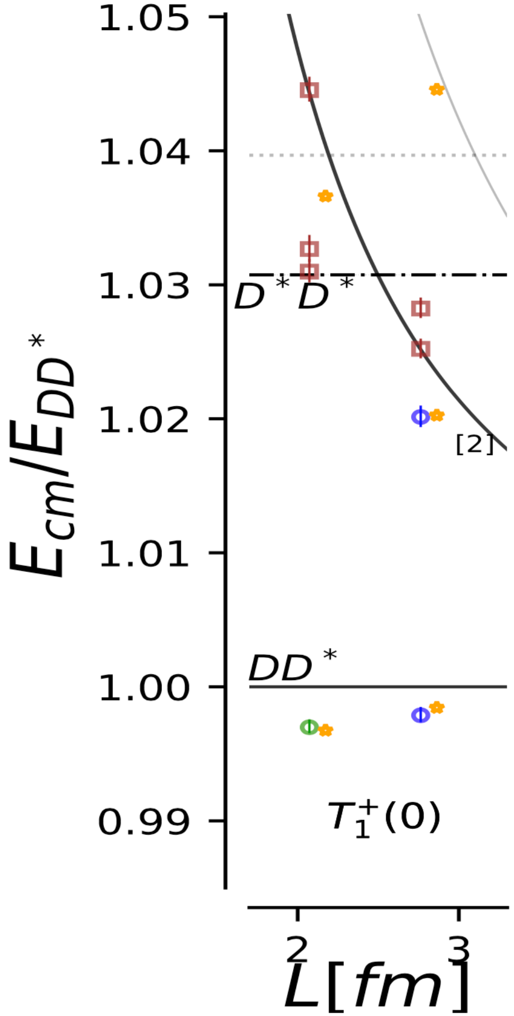
Luscher's relation

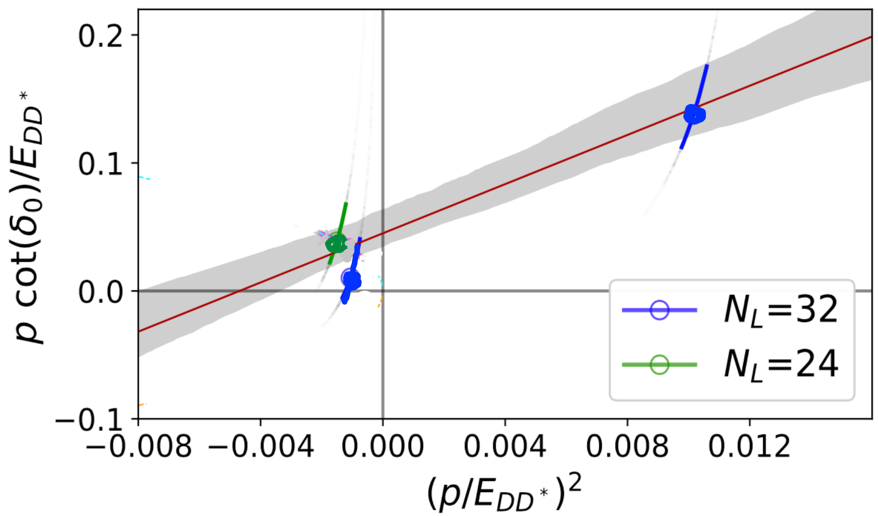
$$E \to T(E)$$

DD* scattering amplitude with l=0

fit using just P=0

P=0, $J^P=1^+$





$$T^{(J=1)}_{l=0}\ :\ p\cot\delta_0=\frac{1}{a_0^{(1)}}+\frac{r_0^{(1)}p^2}{2}$$

red line

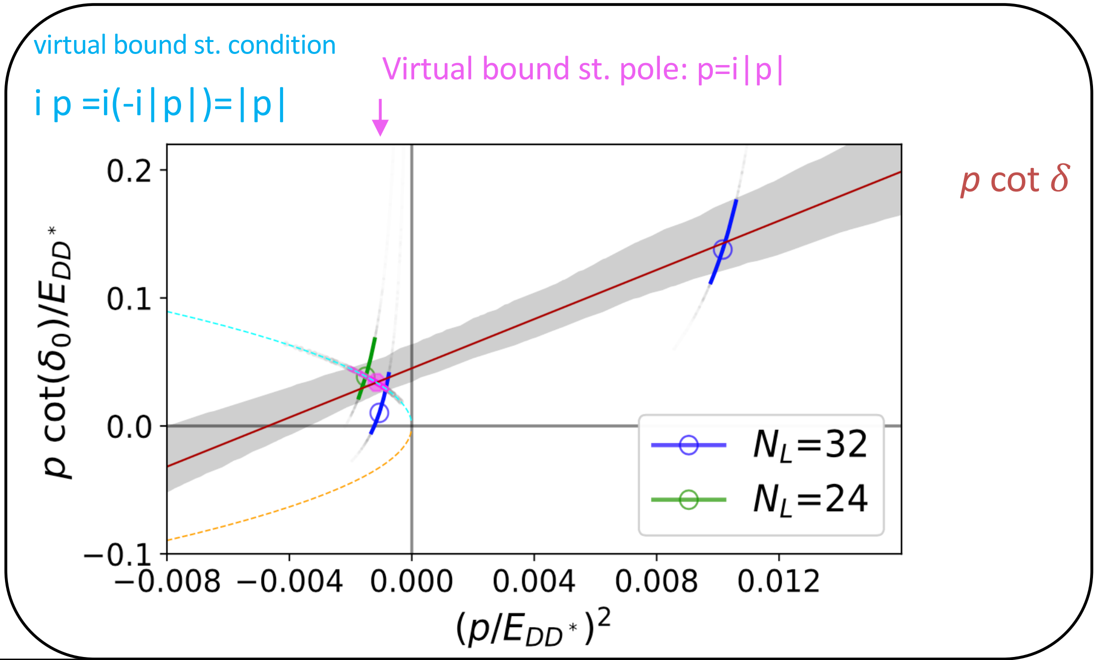
effective range expansion near threshold

- Fit parameters:
  $a_0^{(1)}=1.10(^{+0.34}_{-0.31})$ fm $\quad r_0^{(1)}=0.95(^{+0.24}_{-0.22})$ fm

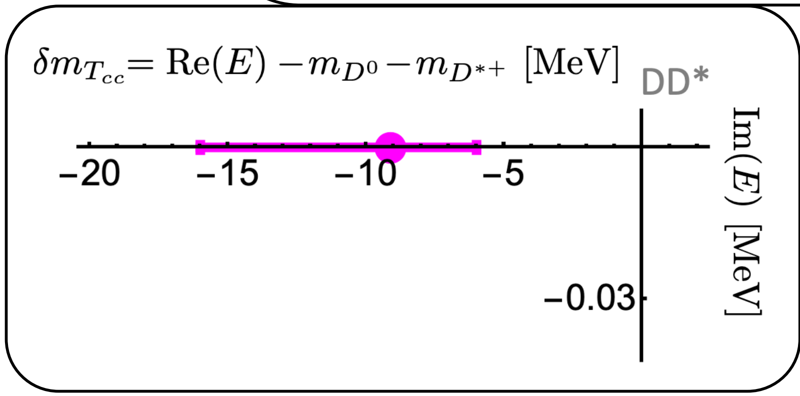
$E^{analytic}$ (fit params)

DD* scattering amplitude with l=0

$$T = \frac{E}{2} \frac{1}{p \cot \delta - ip}$$

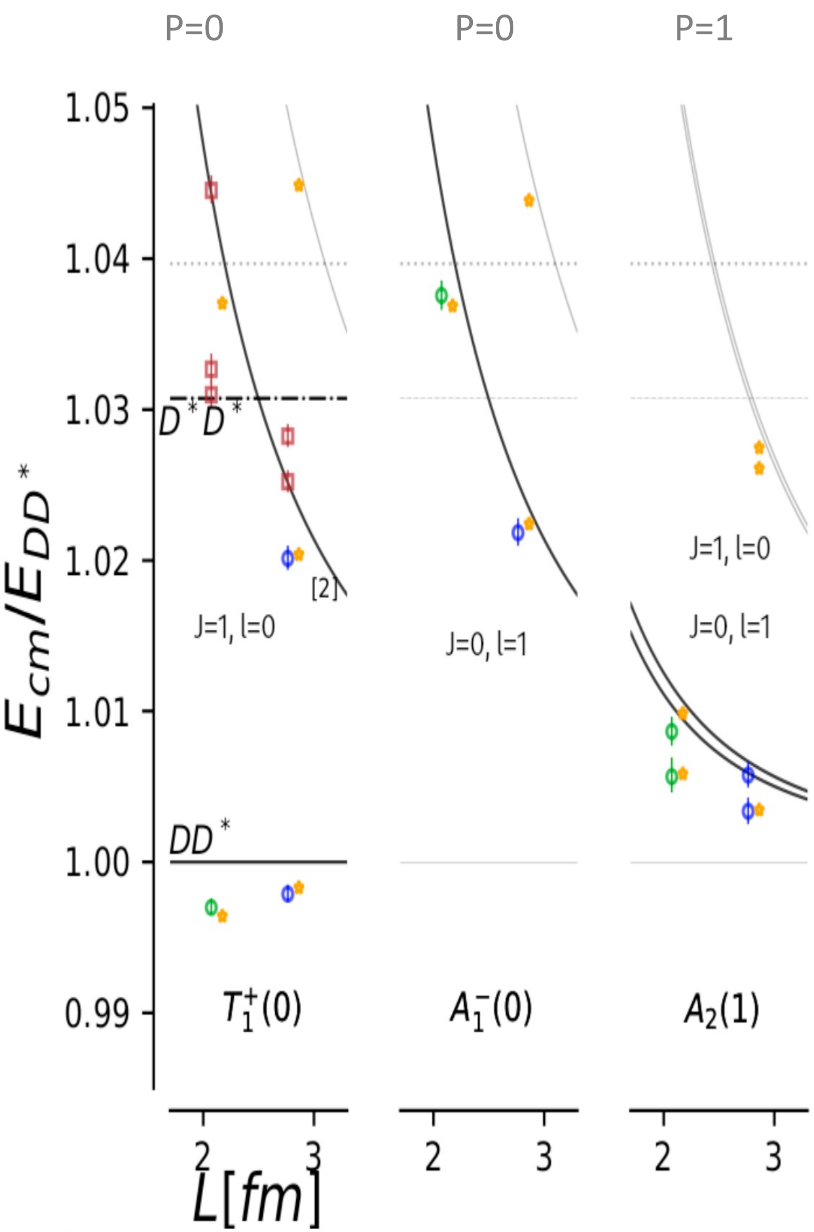


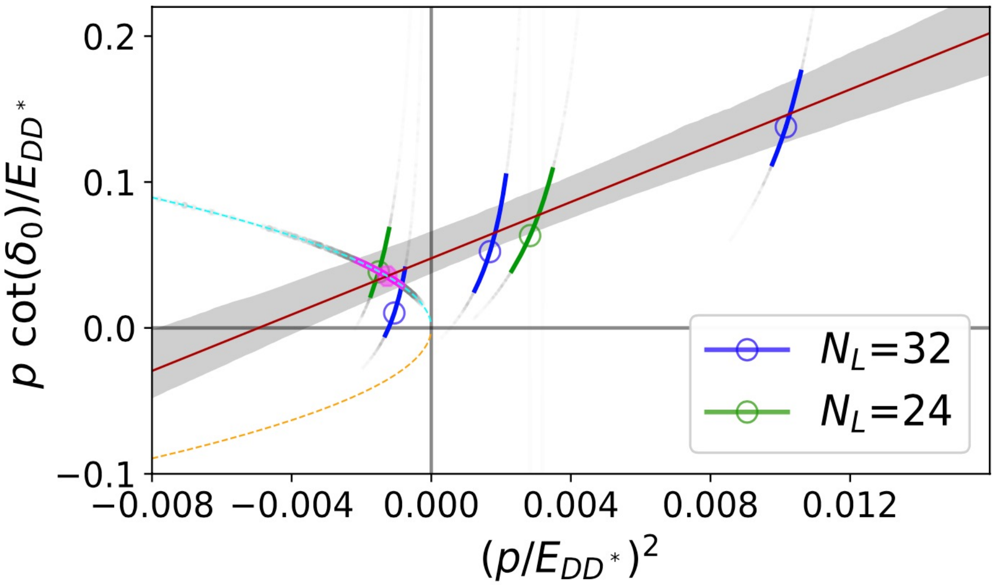
virtual bound st. pole of T(E)



- Binding energy:
  $\delta m_{T_{cc}} = -9.1(^{+3.1}_{-6.9})$ MeV.

DD* scattering amplitude with l=0,1



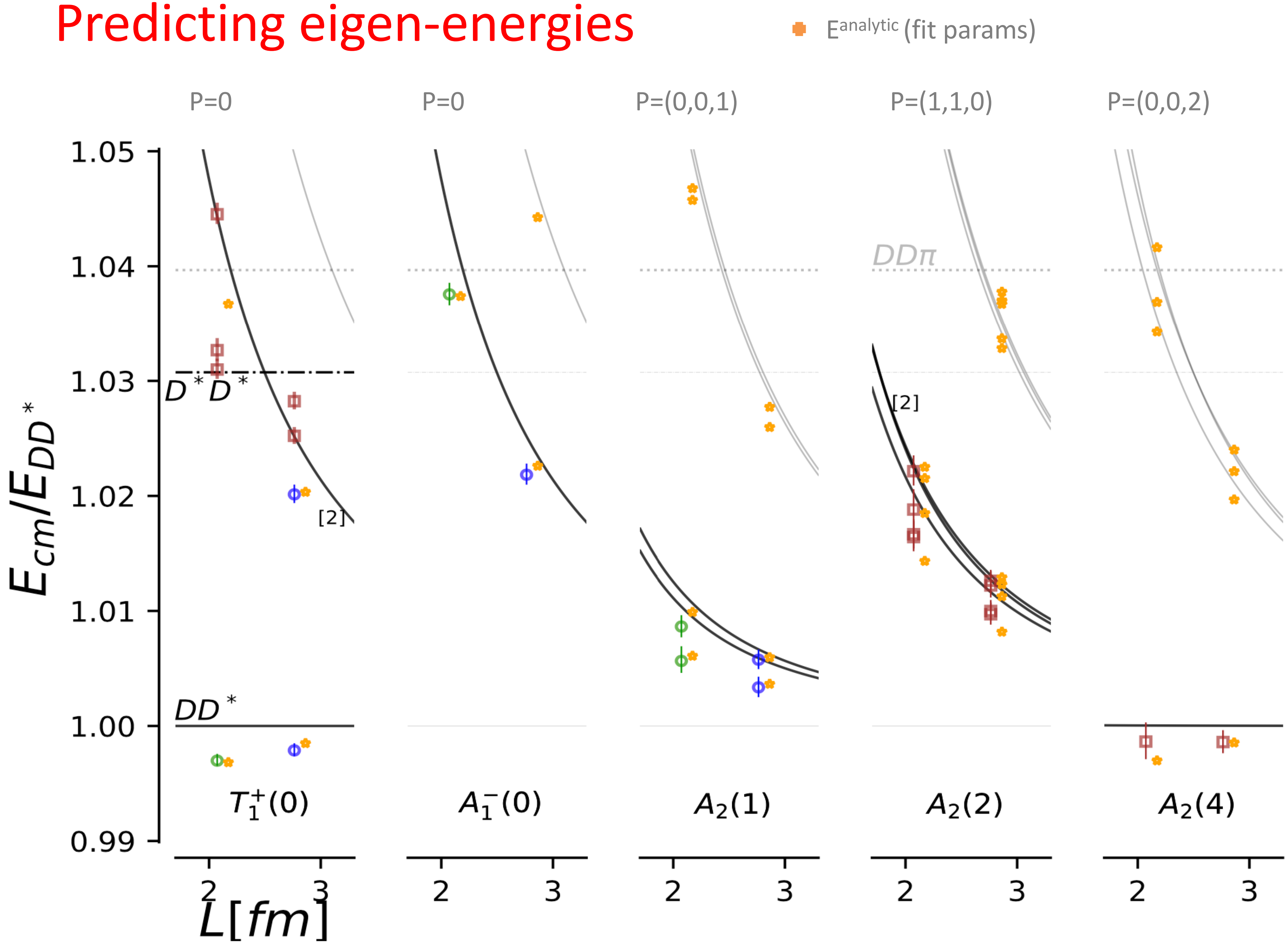


- $$p^{2l+1}\cot\delta_l^{(J)} = \frac{1}{a_l^{(J)}} + \frac{r_l^{(J)}}{2}\,p^2$$

- Fit parameters:
  $a_0^{(1)} = 1.04(0.29)$ fm & $r_0^{(1)} = 0.96(^{+0.18}_{-0.20})$ fm
  $a_1^{(0)} = 0.076(^{+0.008}_{-0.009})$ fm^3 & $r_1^{(0)} = 6.9(2.1)$ fm^{-1}

- Binding energy:
  $$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})\text{ MeV}.$$

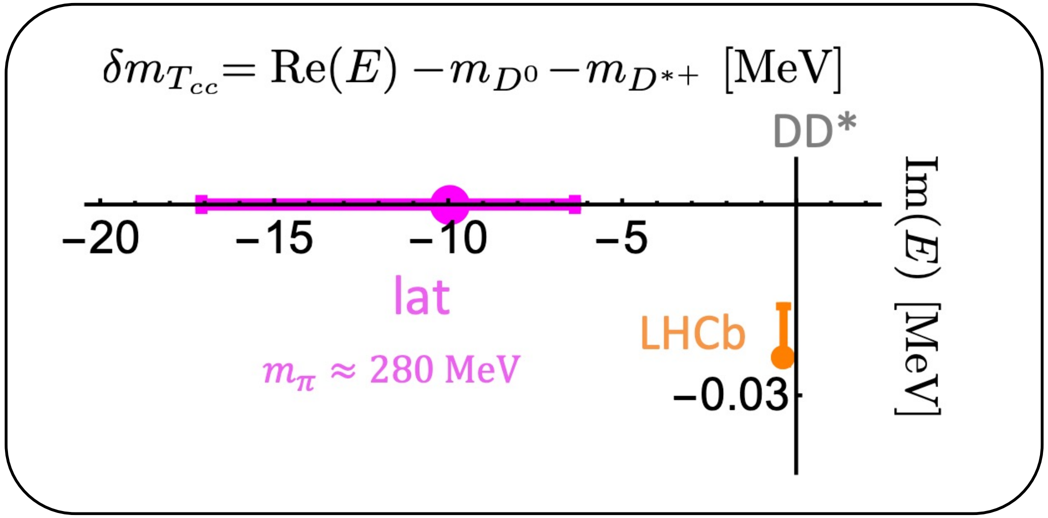
Predicting eigen-energies



Location of T_{cc} pole

$m_\pi \approx 280$ MeV
$m_D \approx 1927$ MeV
$m_{D^*} \approx 2049$ MeV

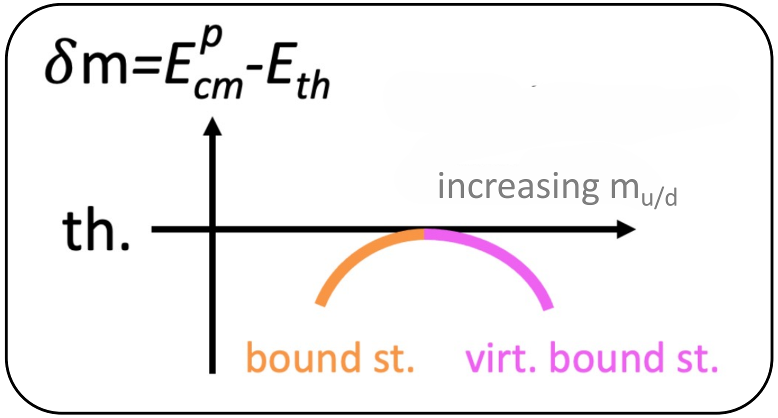
Pole of T(E)



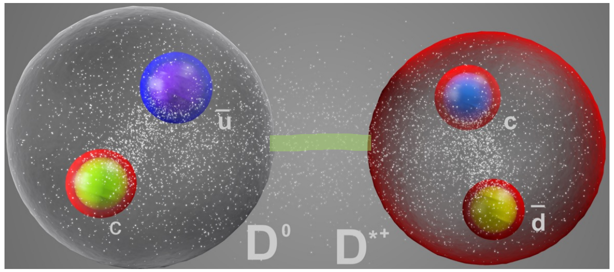
- lat: evidence for virtual bound state pole
- we expect this pole is related to Tcc bound state pole found by LHCb : arguments in Supplement of **2202.101101** and in the following slides

	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
LHCb	$-0.36(4)$	bound st.

Sketch of expected binding energy



Expected dependence of T_{cc} on $m_{u/d}$: simple QM arguments



exchanged particles:

light mesons $\pi, \rho, ..$

increasing $m_{u/d}$

increasing m_{ex}

decreasing attraction |V|

Yukava-like potential

$$V(r) \propto -\frac{e^{-m_{ex}r}}{r}$$

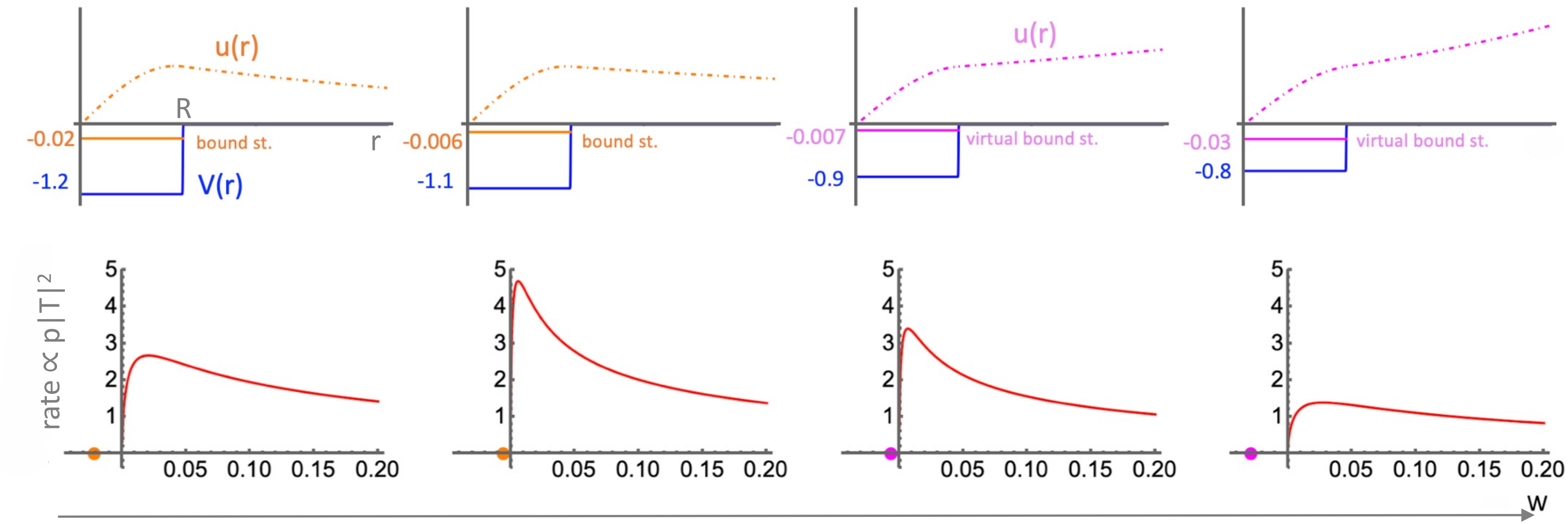
Simplest Example: scattering in square-well potential in QM

$\delta = \arctan[\tan(qR)\frac{p}{q}] - pR$

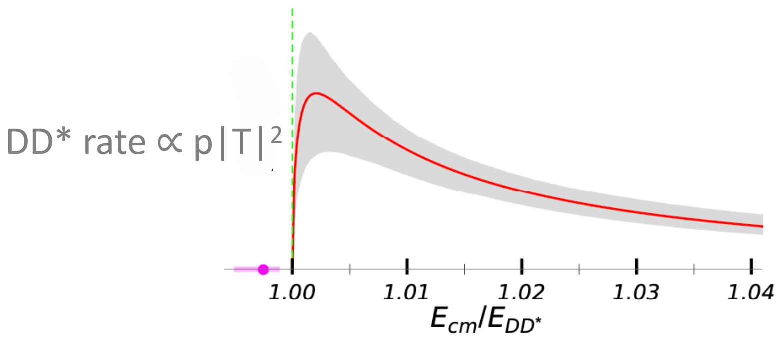
$u(r) = A\sin(qr)$ $\quad u(r) = B\sin(pr+\delta)$

p=i|p| $\quad e^{ipr} = e^{-|p|r}$

p=-i|p| $\quad e^{ipr} = e^{|p|r}$



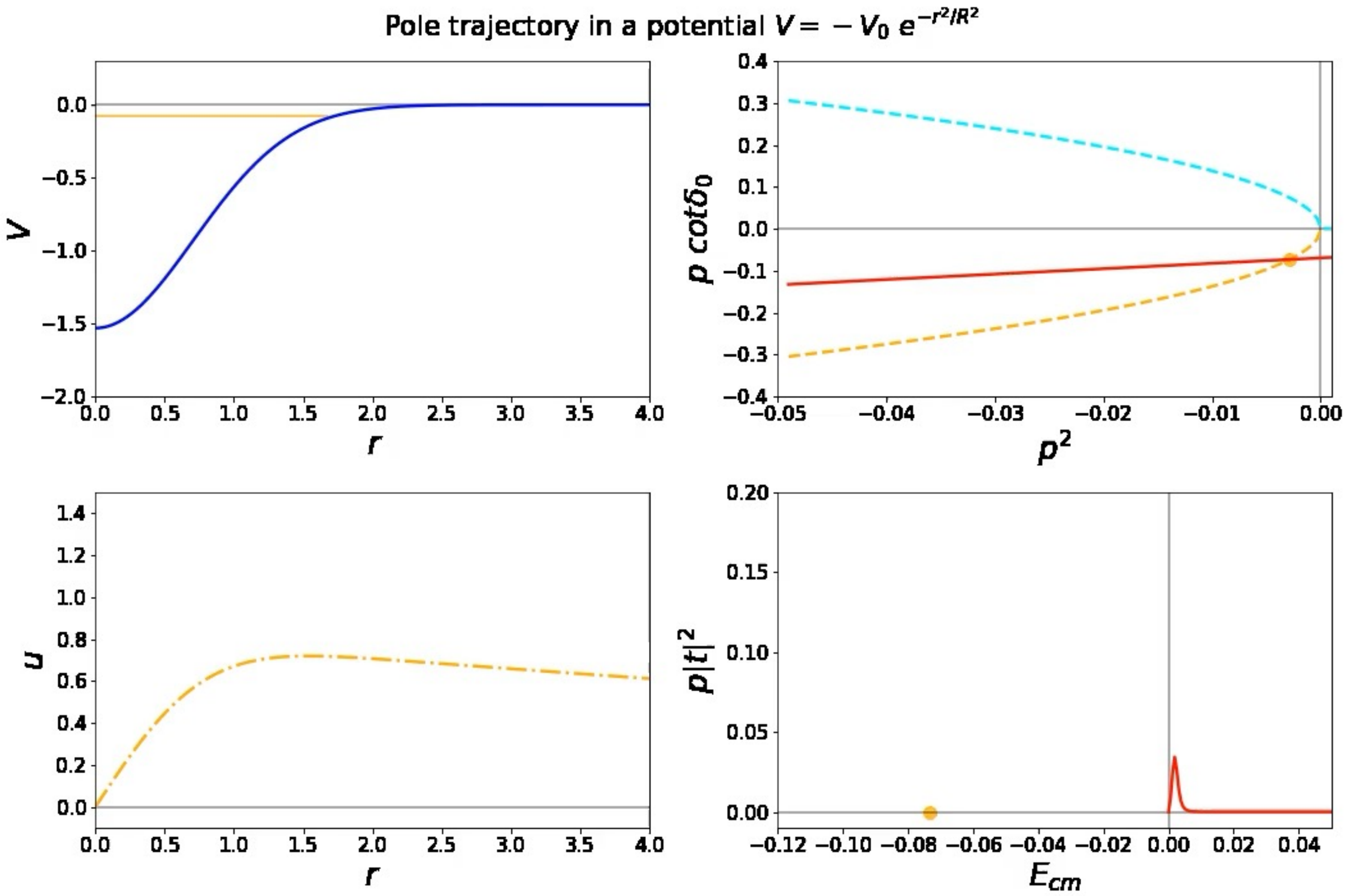
increasing $m_{u/d}$, decreasing attraction



for T_{cc} extracted on lattice

All fully attractive potentials lead to analogous conclusions

video: courtesy M. Padmanath



Dependence on the charm quark mass

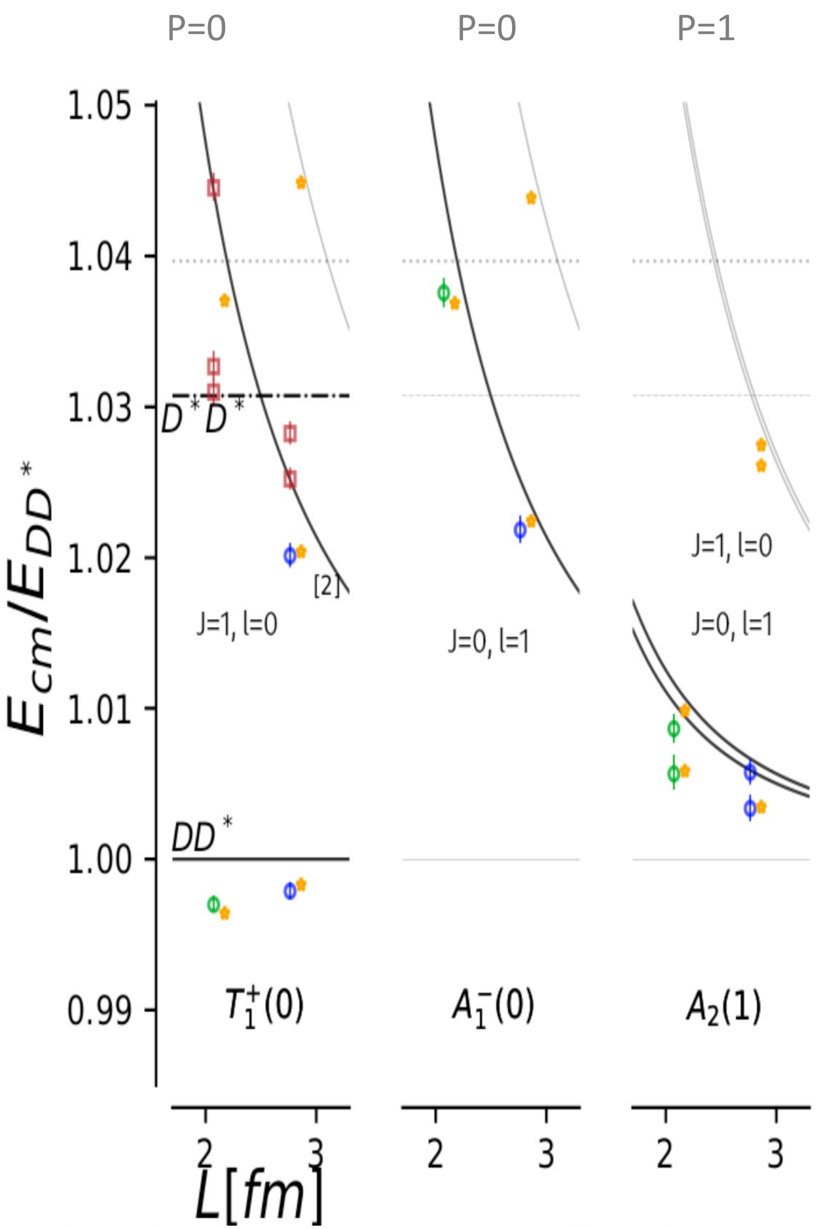
simulation at two charm-quak masses

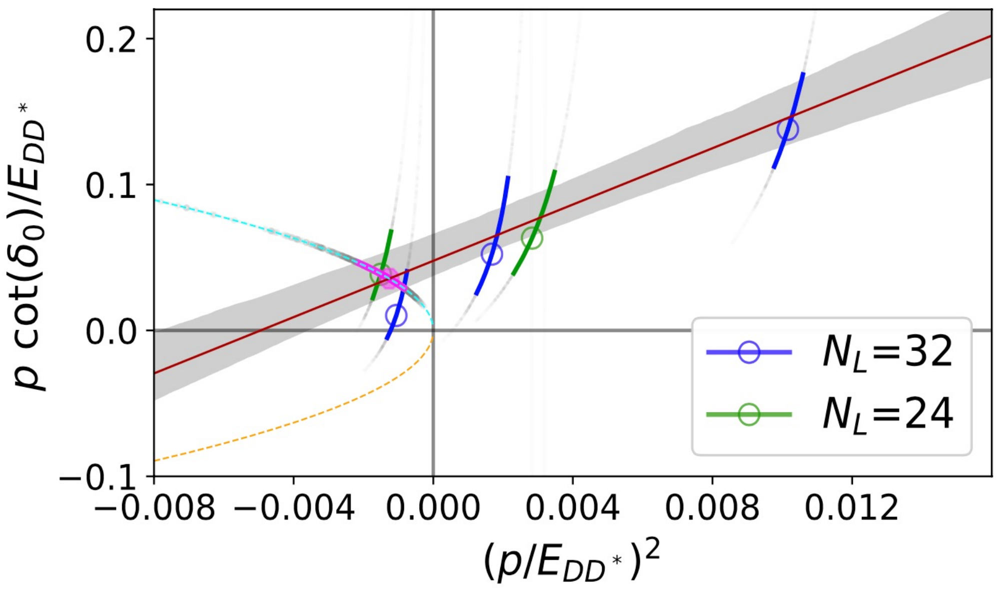
$$M_{av} \equiv \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$$

closer to physical (presented till now)

	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)
exp. [2, 37]	1864.85(5)	2010.26(5)	3068.6(1)

DD* scattering amplitude with l=0,1 at $m_D \approx 1927$ MeV





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- Fit parameters:
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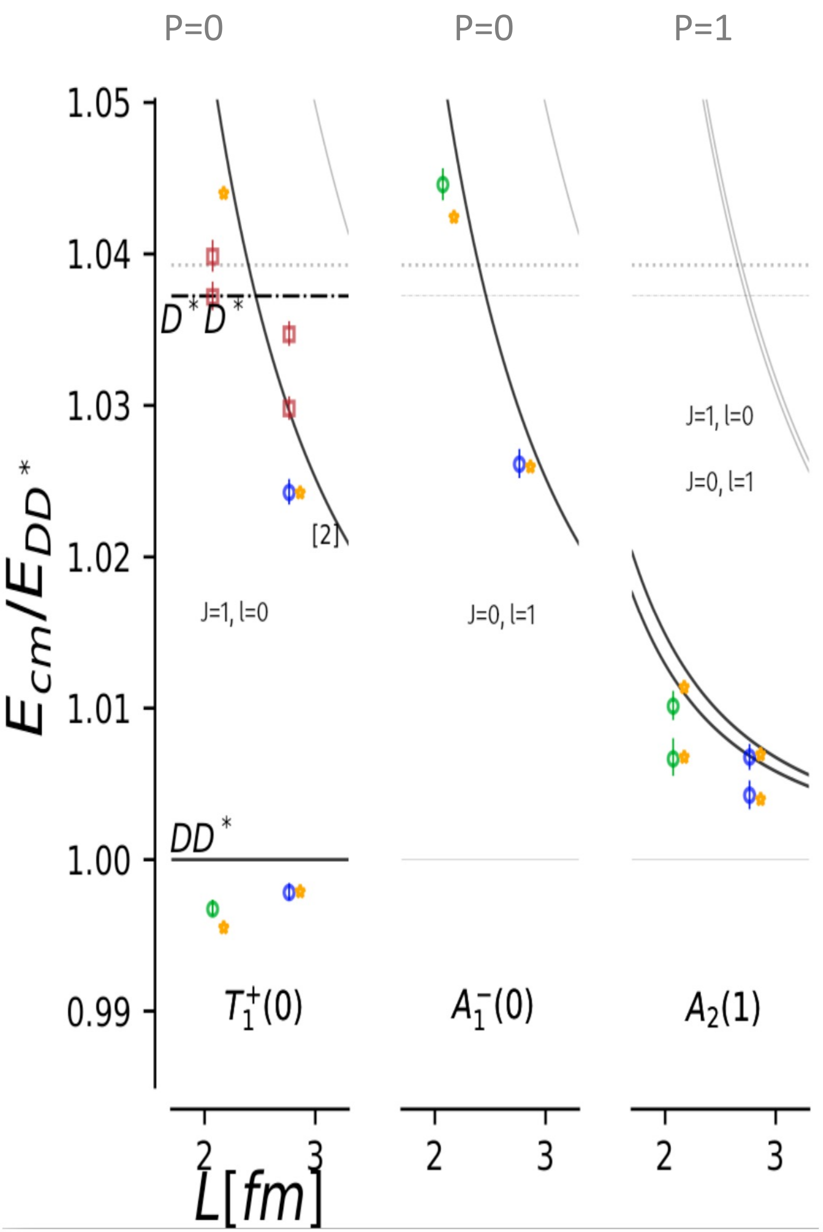
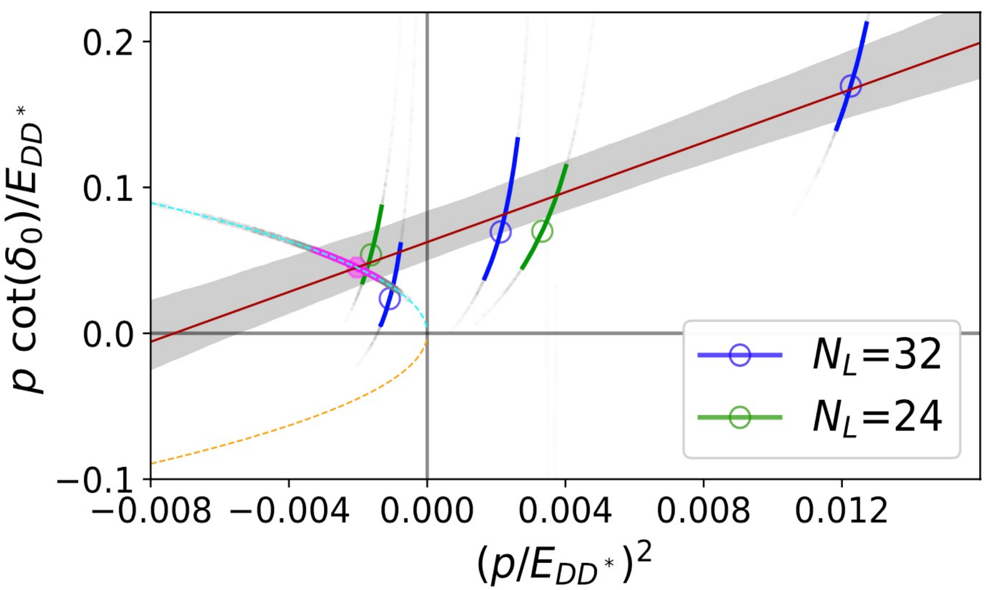
- Binding energy:
  $$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})\ \text{MeV}.$$

DD* scattering amplitude with l=0,1 at $m_D \approx 1762$ MeV (lighter charm quark mass)





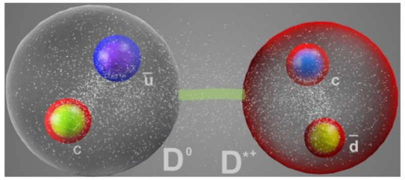
- $$p^{2l+1}\cot\delta_l^{(J)} = \frac{1}{a_l^{(J)}} + \frac{r_l^{(J)}}{2}\, p^2$$
- Fit parameters:
  $a_0^{(1)} = 0.86(0.22)$ fm & $r_0^{(1)} = 0.92(^{+0.17}_{-0.19})$ fm
  $a_1^{(0)} = 0.117(^{+0.013}_{-0.014})$ fm^3 & $r_1^{(0)} = 8.6(^{+1.5}_{-1.1})$ fm^{-1}
- Binding energy:
  $$\delta m_{T_{cc}} = -15.0(^{+4.6}_{-9.3})\ \text{MeV}.$$

Lattice results at two m_c

	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$r_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp. [2, 37]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	[-11.9(16.9),0]	$-0.36(4)$	bound st.

Observed m_c dependence in agreement with QM arguments for fully attractive potential

$$V(r) = -V_0\ f(r/R)$$

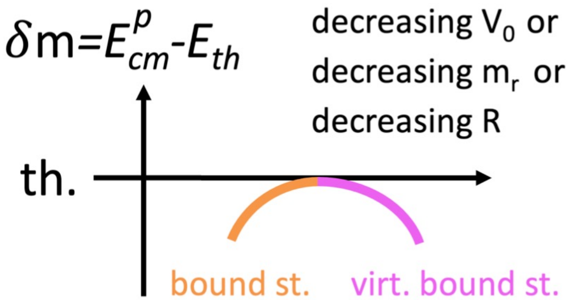


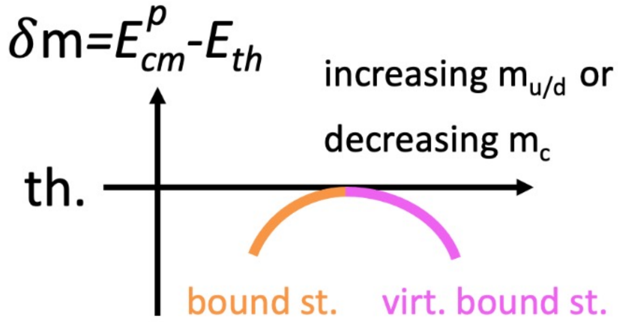
exchanged particles:

light mesons $\pi, \rho, ..$

V(r) independent on m_c ,

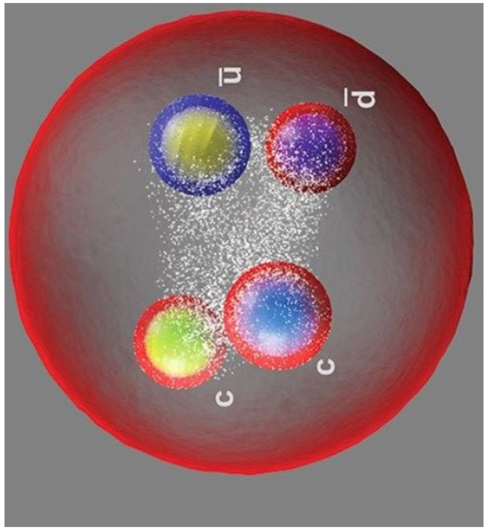
reduced mass m_r of D,D* system increases with m_c

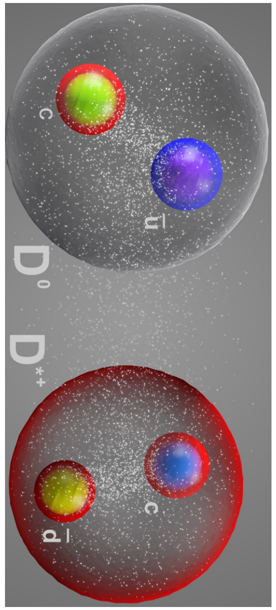


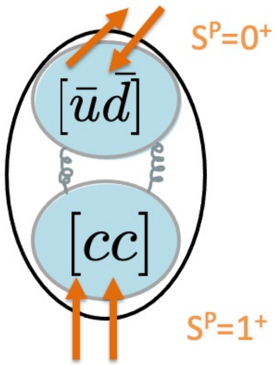


T_{cc} $(I=0,\ J^P=1^+)$

$cc\bar{d}\bar{u}$







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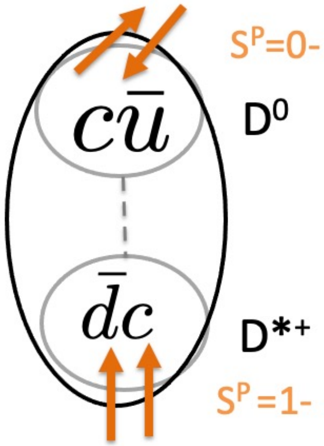
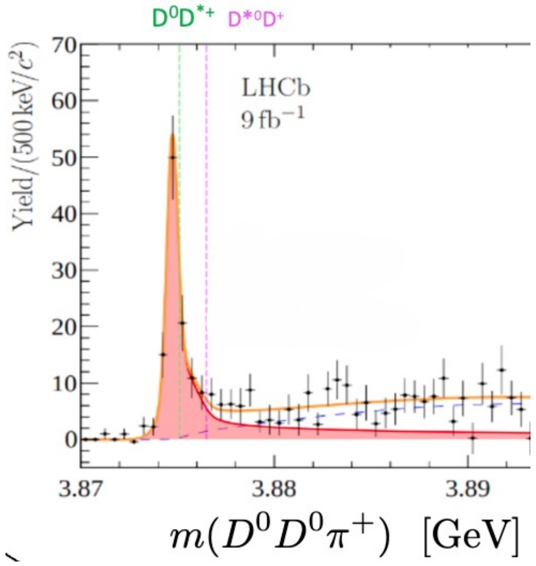


likely dominant

Conclusions on doubly charm tetraquark

$cc\bar{u}\bar{d}$

I=0, $J^P=1^+$



- The longest lived exotic hadron ever found
- It lies very close to DD* threshold
- Lattice QCD:

  to establish a state near threshold, scattering amplitude has to be extracted and pole identified

Our study **2202.101101** :

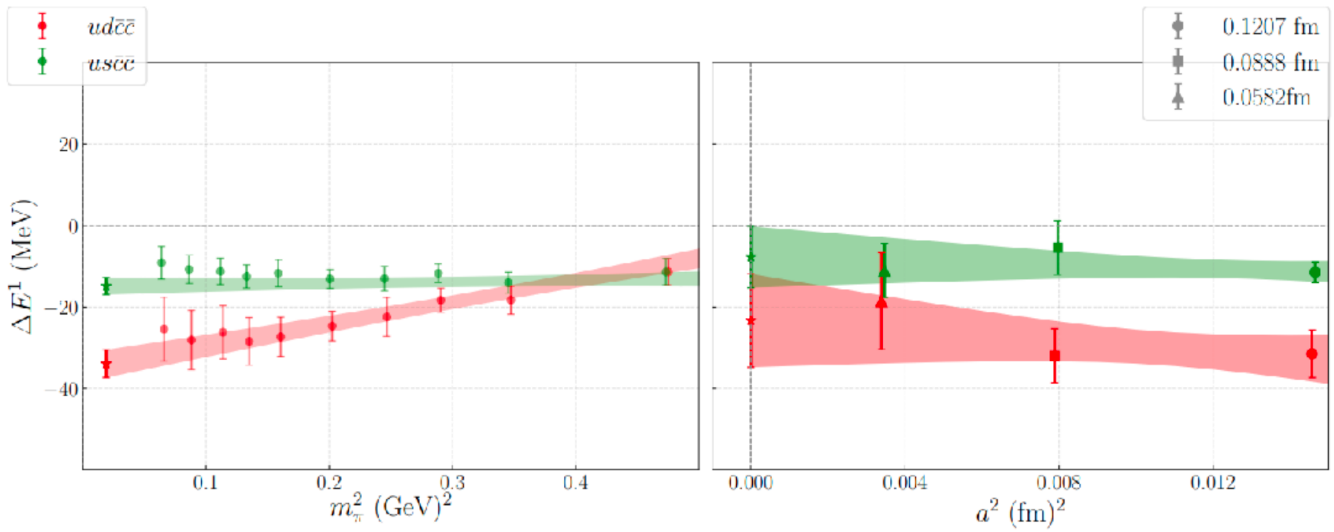
- the only extraction of DD* scattering amplitude
- virtual bound state pole found at $m_\pi \approx 280$ MeV
- likely related to Tcc found by LHCb

Many interesting questions and quantities still to be explored ...

Backup

Previous lattice QCD study of T_{cc} channel

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285



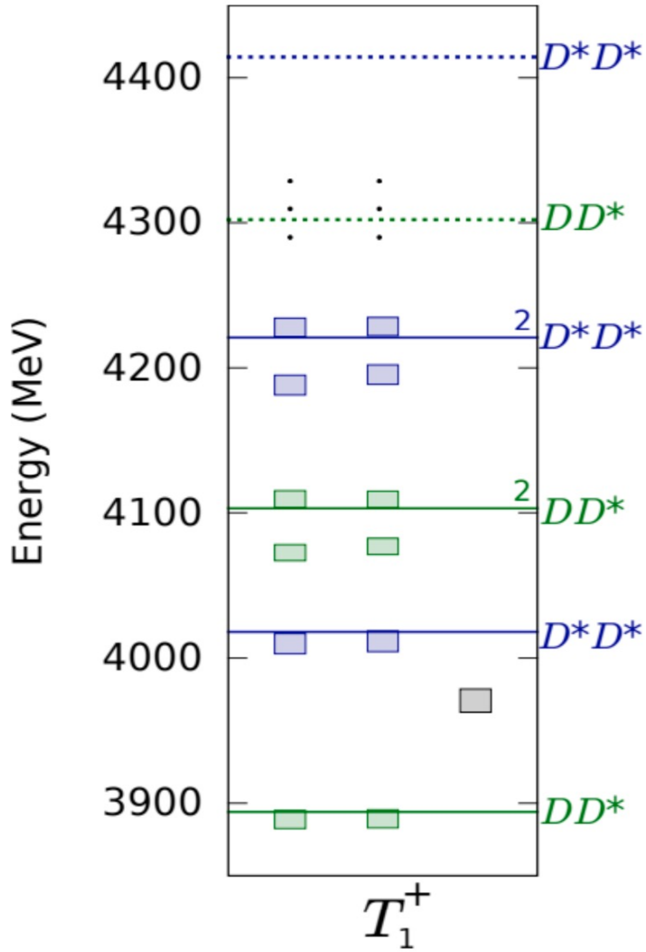
lowest finite-volume eigen-energy for P=0, $J^P=1^+$, I=0

- Study performed on LQCD ensembles with different lattice spacings. Single volume and only rest frame finite-volume irreps considered.
- Including a meson-meson and diquark-antidiquark interpolator. Diquark-antidiquark interpolators do not influence the low energy spectrum.
- The ground state energy subjected to chiral and continuum extrapolations.
- A finite-volume energy level 23(11) MeV below DD^* threshold. No rigorous scattering analysis and no pole structure determined.

Previous lattice QCD study of T_{cc} channel

Hadron Spectrum, JHEP 11, 033 (2017), 1709.01417

finite-volume eigen-energies for P=0, $J^P=1^+$, I=0

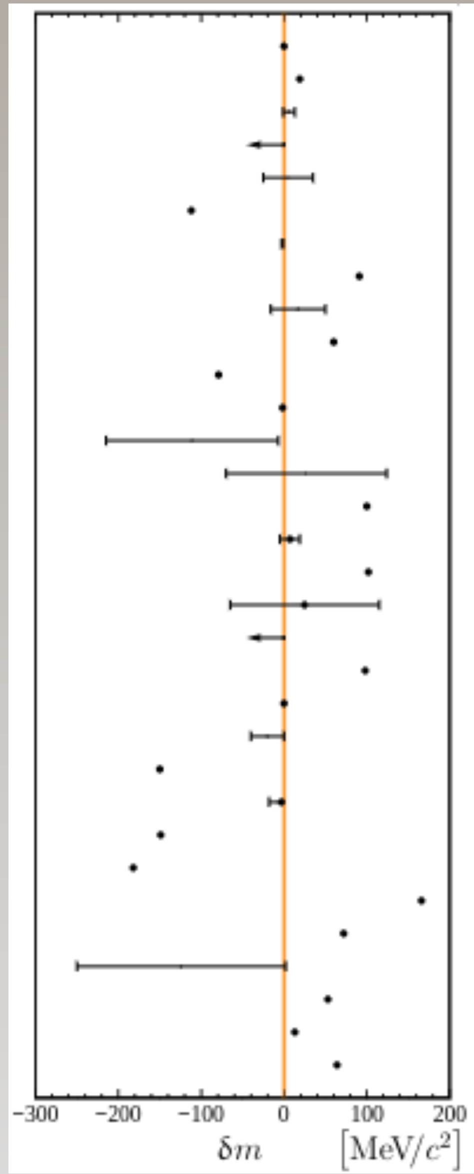


- Single volume rest frame study on a relatively coarse lattice ($a_s \sim 0.12$ fm).
- Large basis of meson-meson and diquark-antidiquark interpolators.
- Diquark-antidiquark interpolators do not influence the low energy spectrum.
- No statistically significant energy shifts observed near DD^* threshold.
  $\Rightarrow$ No scattering amplitude extraction.

Tcc theory predictions (taken from Ivan Polyakov, EPS meeting)

Theory predictions

Reference		Year	$\delta' m$ [MeV/c^2]
J. Carlson, L. Heller and J. A. Tjon	[36]	1987	~ 0
B. Silvestre-Brac and C. Semay	[37]	1993	$+19$
C. Semay and B. Silvestre-Brac	[38]	1994	$[-1,+13]$
S. Pepin, F. Stancu, M. Genovese and J. M. Richard	[39]	1996	< 0
B. A. Gelman and S. Nussinov	[40]	2002	$[-25,+35]$
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	[41]	2003	-112
D. Janc and M. Rosina	[42]	2004	$[-3,-1]$
F. Navarra, M. Nielsen and S. H. Lee	[43]	2007	$+91$
J. Vijande, E. Weissman, A. Valcarce	[44]	2007	$[-16,+50]$
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	[45]	2007	$+60$
S. H. Lee and S. Yasui	[46]	2009	-79
Y. Yang, C. Deng, J. Ping and T. Goldman	[47]	2009	-1.8
G.-Q. Feng, X.-H. Guo and B.-S. Zou	[48]	2013	-215
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	[49]	2013	$[-70,+124]$
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	[50]	2017	$+100$
M. Karliner and J. Rosner	[51]	2017	$7 \pm 12 \to 1$
E. J. Eichten and C. Quigg	[52]	2017	$+102$
Z. G. Wang	[53]	2017	$+25 \pm 90$
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	[54]	2017	$\lesssim 0$
W. Park, S. Noh and S. H. Lee	[55]	2018	$+98$
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	[56]	2018	~ 0
P. Junnarkar, N. Mathur and M. Padmanath	[57]	2018	$[-40,0]$
C. Deng, H. Chen and J. Ping	[58]	2018	-150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	[59]	2019	-3^{+4}_{-15}
G. Yang, J. Ping and J. Segovia	[60]	2019	-149
Y. Tan, W. Lu and J. Ping	[61]	2020	-182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	[62]	2020	$+166$
E. Braaten, L.-P. He and A. Mohapatra	[63]	2020	$+72$
D. Gao , D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	[64]	2020	$[-250,+2]$
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	[65]	2020	$+53$
S. Noh, W. Park and S. H. Lee	[66]	2021	$+13$
R. N. Faustov, V. O. Galkin and E. M. Savchenko	[67]	2021	$+64$



Ivan Polyakov, Syracuse University 17

Tcc theory predictions (taken from Ivan Polyakov, EPS meeting)

Refs. for theory predictions

[36] J. Carlson, L. Heller, and J. A. Tjon, *Stability of dimesons*, Phys. Rev. **D37** (1988) 744. 2

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Interpolators

Example: P=0

$J^P=1^+$ -> cubic irrep T_1^+

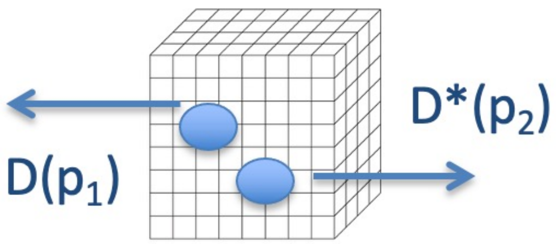
P=D, V=D*

$$O^{l=0} = P(\{0,0,0\})V_z(\{0,0,0\})$$

$$\begin{aligned} O^{l=0} = & P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\ & + P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\ & + P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \end{aligned}$$

$$\begin{aligned} O^{l=2} = & P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\ & + P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\ & -2[P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \end{aligned}$$

$$O^{l=0} = V_{1x}[0,0,0]V_{2_y}[0,0,0] - V_{1y}[0,0,0]V_{2x}[0,0,0]$$



Luscher 1991 + generalizations

Relation between E and $\delta(E), T(E)$

$$S = 1 + i\frac{4p}{E}T = e^{2i\delta}$$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

E = eigen-energy lattice from lattice in cmf

E = $\sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$

even and odd l contribute to

given irrep for nonzero mom.

Luscher's relation (l=0,1):

$$\det\left[\begin{pmatrix} p\cot\delta_0 & 0 \\ 0 & p^3\cot\delta_1 \end{pmatrix} - B(E,L)\right] = 0$$

known 2x2 matrix of kinematical functions (non-diagonal)

Luscher's relation (only l=0):

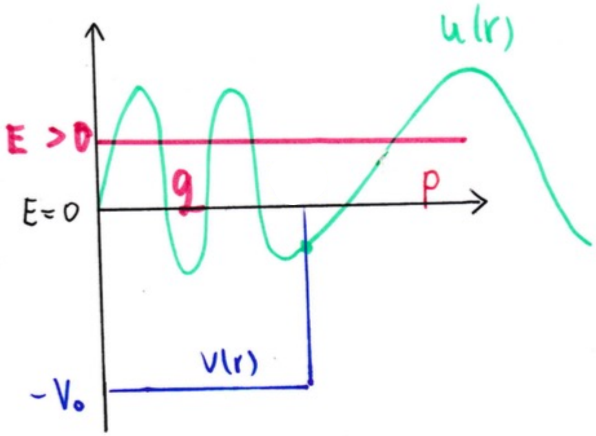
$$p\cot\delta_0 = B(E,L)$$

$$p\cot\delta_0 - B(E,L) = 0$$

known kinematical function

lattice eigen-energy

s-wave scattering on spherical potential well



$A \sin qr \qquad B \sin(pr + \delta_0)$

$$u(R) = A \sin qR = B \sin(pR + \delta)$$
$$u'(R) = q\, A \cos qR = p\, B \cos(pR + \delta)$$

dividing both eqs $\quad \frac{1}{q} \tan qR = \frac{1}{p} \tan(pR + \delta)$

$$\delta_0(p) = \arctan\left(\frac{p}{q}\tan(qR)\right) - pR + n\pi$$

$$q = \sqrt{2\mu(V_0 + E)} = \sqrt{2\mu V_0 + p^2}$$