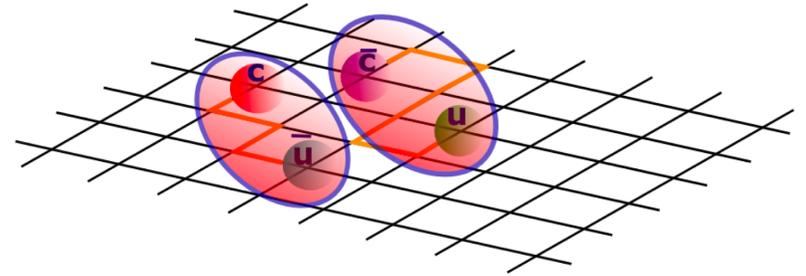


Lattice Results for New Hadronic States



Sasa Prelovsek

Jefferson Lab, Virginia, USA

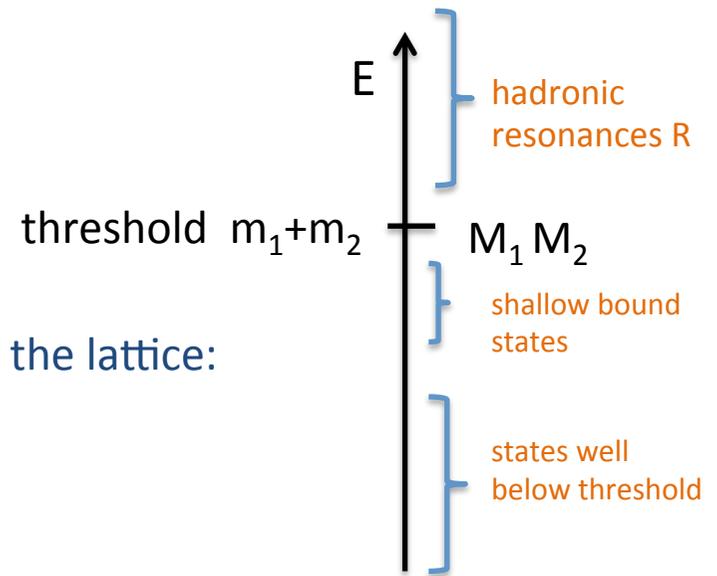
University of Ljubljana & Jozef Stefan Institute, Slovenia

APS 2015, 11th-14th April 2015, Baltimore, USA

Outline

Mesonic states from ab-initio simulations using QCD on the lattice:

- methods
- states well below threshold: *easy*
- states slightly below threshold and resonances above thresholds: *challenging*
 - single-hadron approximation
 - rigorous treatment by determining scattering matrix for two mesons $M_1 M_2$
- lattice searches for manifestly exotic hadrons (usually above several thresholds): *very challenging*
- conclusions



Hadrons

Conventional:

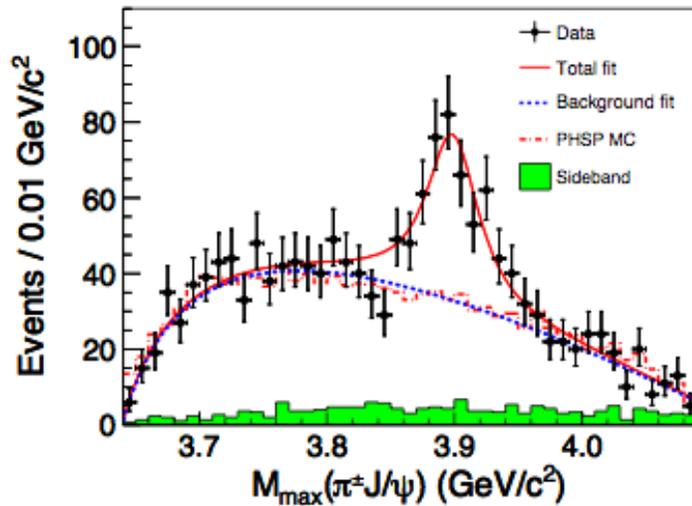


Normal baryon

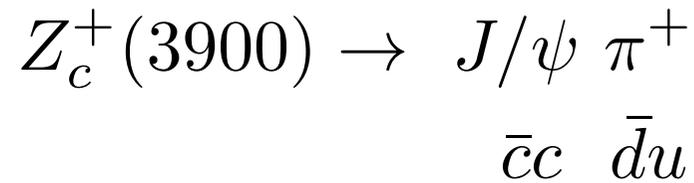


Normal meson

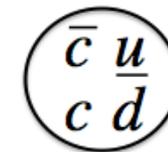
Exotic: example: charged charmonium-like state $Z_c^+(3900)$



[BESIII, 2013, 1303.5949, PRL]



state confirmed by
BeSIII, Belle, Cleo-c



DD* thr.

Theoretical problem

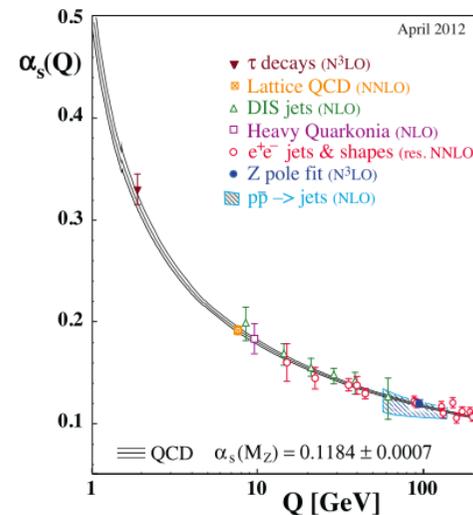
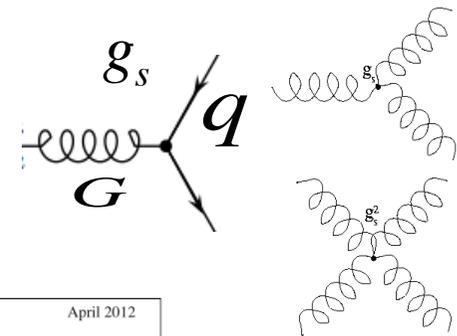
- Fundamental theory of strong interaction between **quarks** and **gluons** is known and reliable: chromodynamics (QCD)

$$L_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_b^\mu G_c^\nu$$

- But: $\alpha_s \equiv \frac{g_s^2}{4\pi} \approx O(1) \quad r \approx 1 \text{ fm}$

- Calculation of hadron properties (mass, ...) :
 - perturbative expansion in α_s not possible
 - non-perturbative method needed !



Non-perturbative method: QCD on lattice

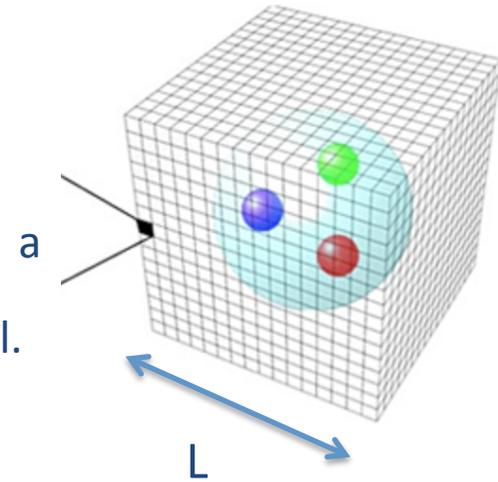
$$L_{QCD} = -\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}i\gamma_\mu(\partial^\mu + ig_s G_a^\mu T^a)q - m_q \bar{q}q$$

input: g_s , m_q

output: hadron properties

hadron interactions (if we are lucky)

precision cal.
 $a \sim 0.05$ fm
 $L \sim 4$ fm

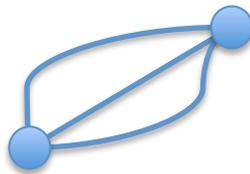


Evaluation of Feynman path integrals in discretized space-time

quantum mechanics

$$\int Dx e^{iS/\hbar}$$

$$S = \int dt L[x(t)]$$



quantum field theory in Euclidian space-time

$$\int DG Dq D\bar{q} e^{-S_{QCD}/\hbar}$$

$$S_{QCD} = \int d^4x L_{QCD}[G(x), q(x), \bar{q}(x)]$$

x, t (Minkovsky) \rightarrow $x, i t$ (Euclidean)

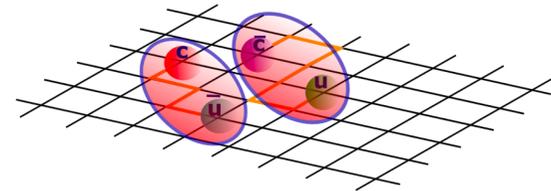
Discrete energies of eigenstates: E_n

Meson(like) system with given J^{PC} is created by a number of interpolating fields

$$J^{PC} \quad \mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)_{\bar{p}_1} (\bar{q}\Gamma_2 q)_{\bar{p}_2}, \quad [\bar{q}\Gamma_3 \bar{q}][q\Gamma_4 q], \dots$$

$$\rho(770), 1^{--} : \quad \bar{d}u, \quad (\bar{d}d)(\bar{d}u) = \pi\pi$$

$$X(3872), 1^{++} : \quad \bar{c}c, \quad (\bar{c}u)(\bar{u}c) = D\bar{D}^*, \quad [\bar{c}u][cu]$$



$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle e^{-E_n t} \langle n | \mathcal{O}_j^\dagger | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

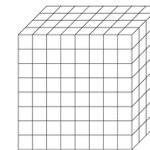
All physical states with given J^{PC} appear as E_n in principle (example: charmonium with 1^{++})

- single meson states χ_{c1} $m_{\chi_{c1}} = E_1$ for $P=0$ (after extrapolations)

$X(3872)$
- two-meson states $D\bar{D}^*, \dots$ E_n rigorously render two-hadron scattering matrix (for example $D\bar{D}^*$ scattering matrix)

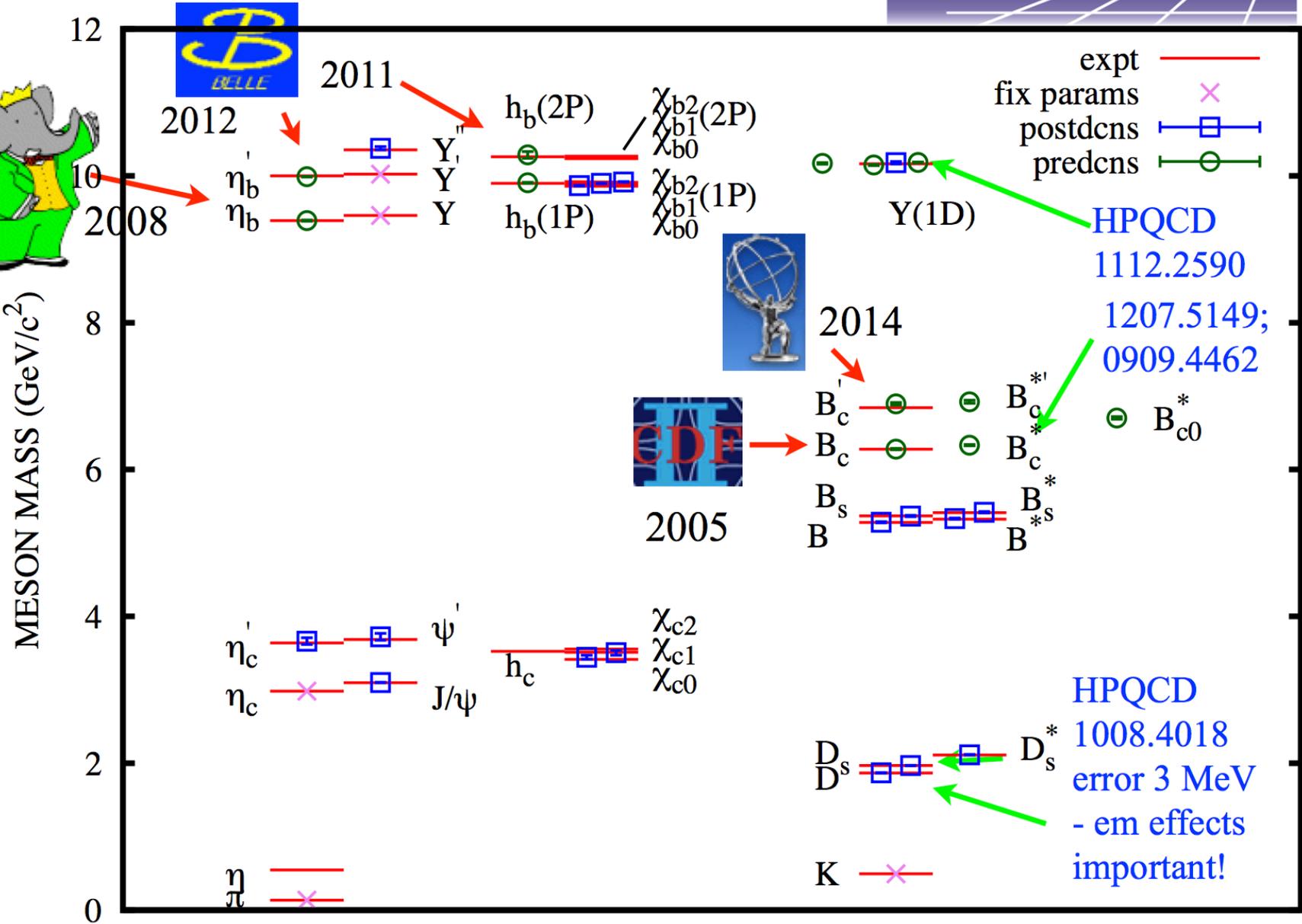
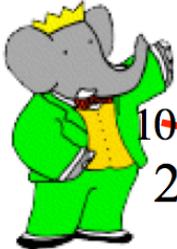
States well below threshold: precision spectrum

- $m=E$ for $P=0$ $a \rightarrow 0, L \rightarrow \infty, m_q \rightarrow m_q^{\text{phy}}$



- Available from a number of lattice QCD collaborations for a number of years
- Only one example shown on next slide

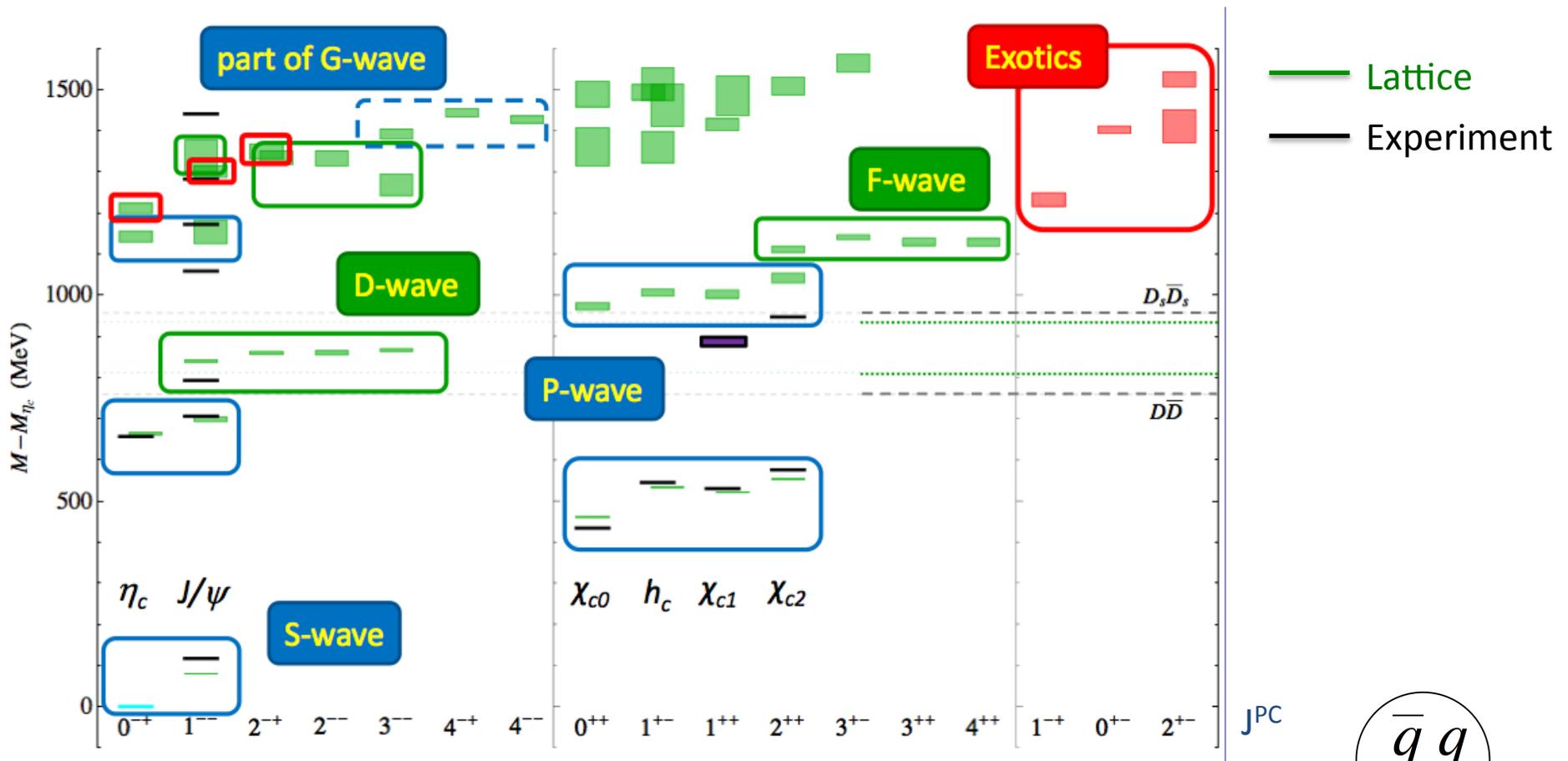
The gold-plated meson spectrum



Mesons near or above threshold: single-meson approximation

- only interpolating fields $\mathcal{O} \approx \bar{q} q$
- assumptions: all energy levels correspond to "one-meson" states
no two-meson state is seen
 $m=E$ (for $P=0$)
these are strong assumptions ...
but results still present valuable reference point

cc spectrum: single-meson approximation



[HSC, L. Liu et al: 1204.5425, JHEP]

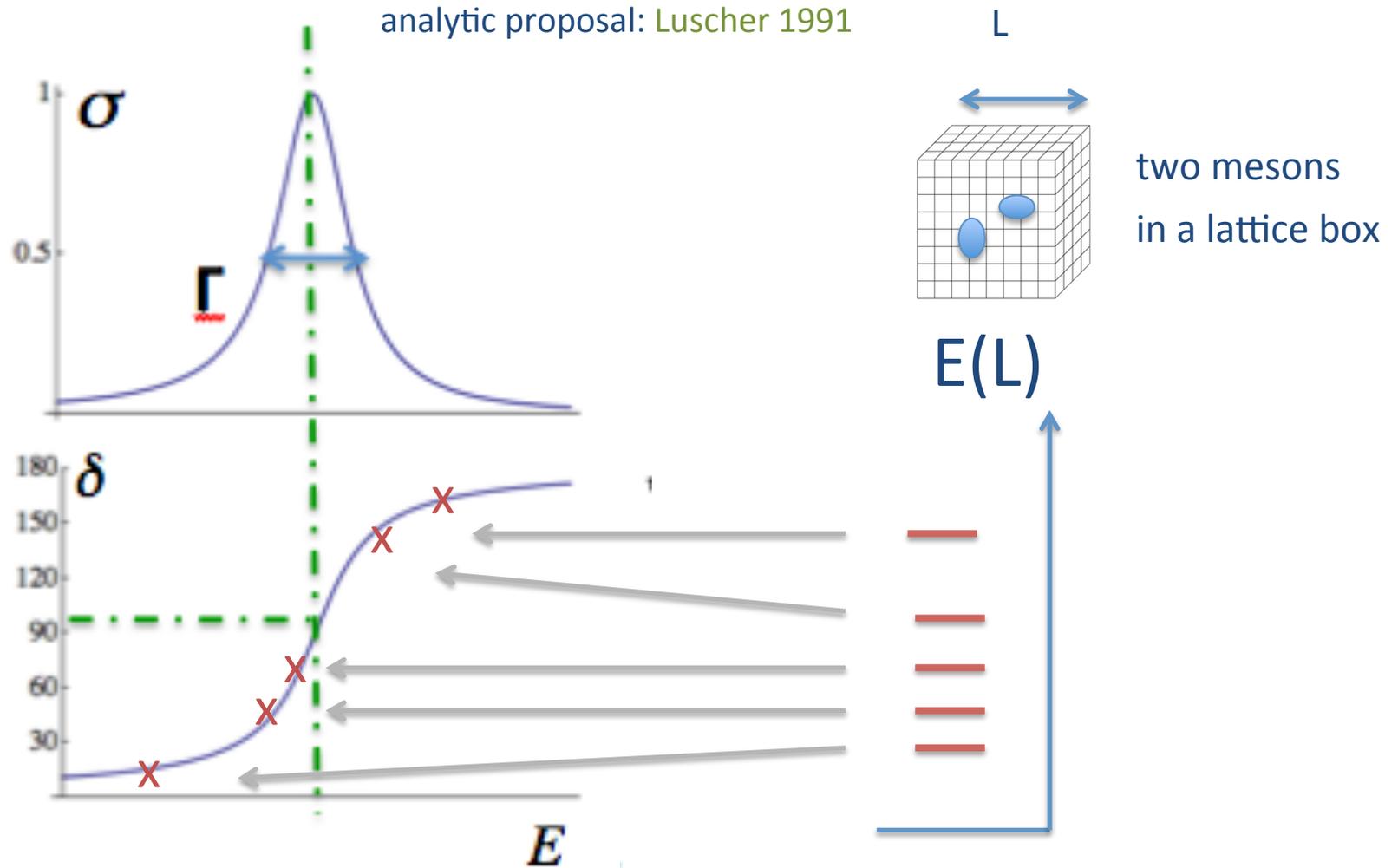
- $m_{\pi} \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:

some of them have exotic J^{PC}
 large overlap with $O = \bar{q} F_{ij} q$

Rigorous treatment: scattering info from spectrum

analytic proposal: Luscher 1991



scattering phase shifts
at infinite volume

$\delta(E)$

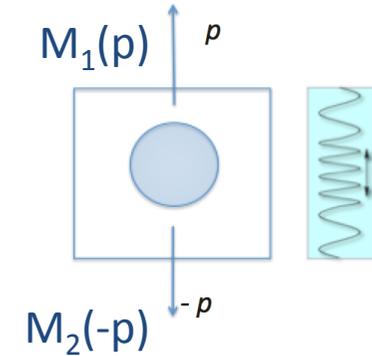
$E(L)$

energies from lattice
with spatial extent L

Scattering of two mesons

elastic scattering with total momentum $P=0$: $E=E_{cm}$

$$E_n(L) \xrightarrow{\text{Luscher's eq.}} \delta(E)$$



Scattering matrix for partial wave l :

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot \delta(E) - i}$$

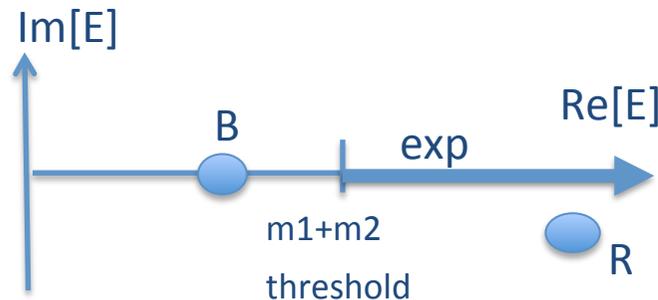
Bound state (B):

$$\cot[\delta(E_B)] = i, \quad E_B < m_1 + m_2$$

Resonance (R) (of Breit-Wigner type):

$$T(E) = \frac{-E \Gamma}{E^2 - m_R^2 + i E \Gamma}, \quad \Gamma(E) = g^2 \frac{p^{2l+1}}{E^2}$$

Locations of poles of $T(E)$
for res. and bound st.

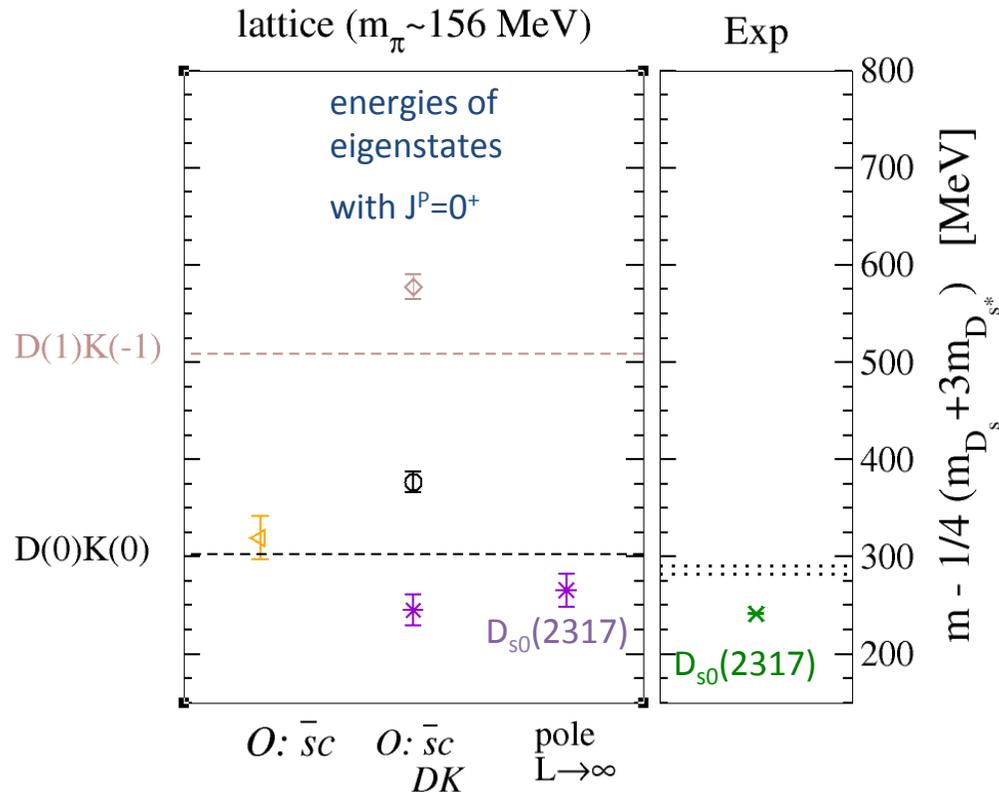


States slightly below threshold

D_{s0}^* , D_{s1} , B_{s0}^* , B_{s1} , $X(3872)$

deuterium-like systems in meson sector

Scalar meson D_{s0} below DK threshold



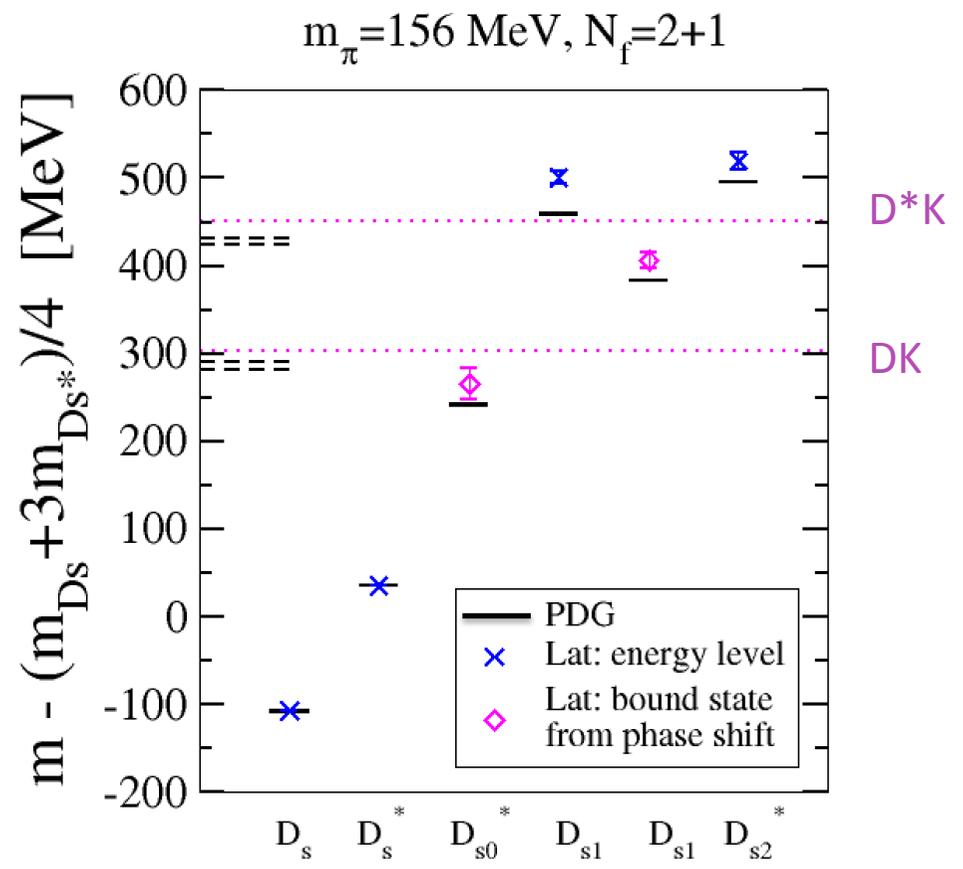
- phase shift for DK scattering
 $E \rightarrow \delta(E)$
- interpolation of $\delta(E)$ near th. using effective range expansion
- This renders scattering matrix in all region near threshold
- pole in $T(E)$ found slightly below th.

$$T \propto \frac{1}{\cot \delta - i} = \infty$$

- attributed to $D_{s0}(2317)$

[D. Mohler, C. Lang, L. Leskovec, S.P. ,
 R. Woloshyn, 1308.3175, Phys. Rev. Lett.]

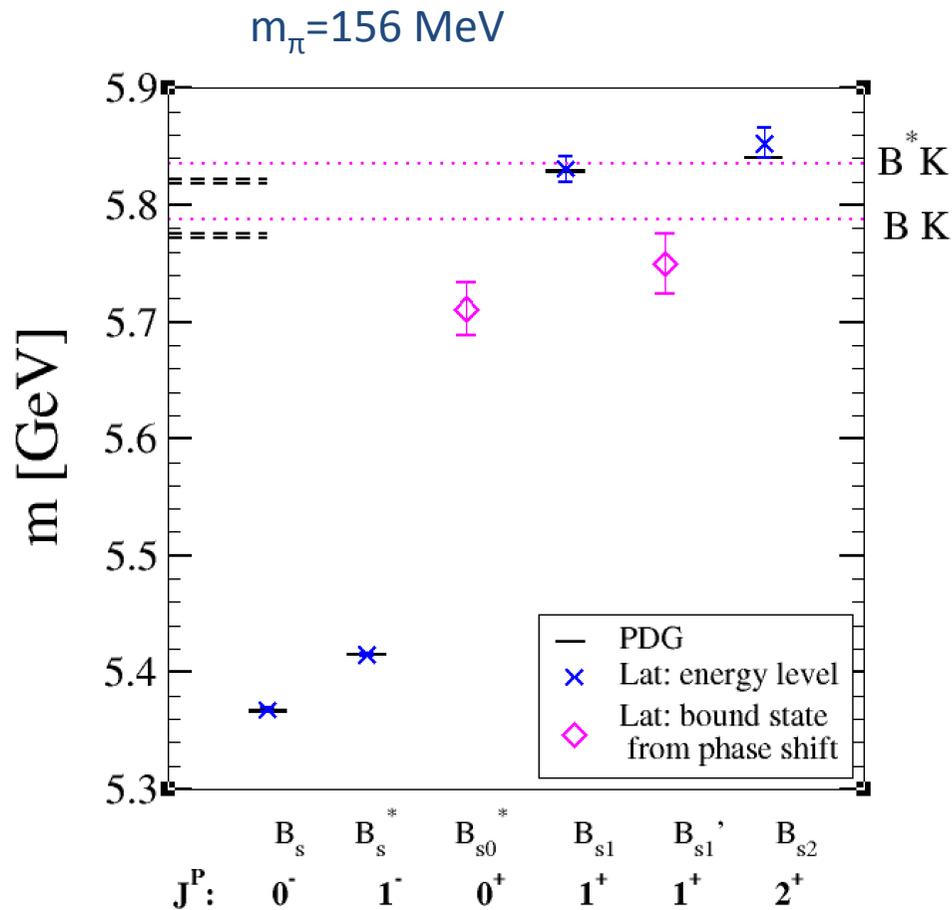
D_{s0} and D_{s1} below DK and D^*K thresholds



- In this case D_{s0} and D_{s1} have been observed experimentally
- Quark models expected them above thresholds but they were found below them
- Threshold effects lower their masses
- Our post-dictions agree with measured masses

[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn:
 1308.3175, Phys. Rev. Lett 2013
 1403.8103, PRD 2014]

Mass prediction for missing B_{s0} and B_{s1}



Quantities shown:

for two bound states :

$$m_B = (m_B - E_{th})^{lat} + E_{th}^{exp}$$

for other states :

$$m = (m - \bar{m})^{lat} + \bar{m}^{lat}$$

for dotted lattice thresholds :

$$E_{th} = (E_{th} - \bar{m})^{lat} + \bar{m}^{lat}$$

$$\bar{m} \equiv \frac{1}{4}(m_{B_s} + 3m_{B_s^*})$$

- B_{s1}' and B_{s2} agree well with exp

Predictions:

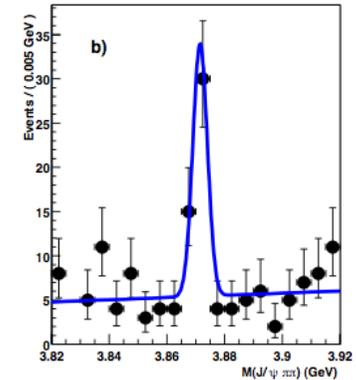
- B_{s0} bound state below BK th.
- B_{s1} bound state below B*K th.

X(3872), $J^{PC}=1^{++}$, charmonium-like

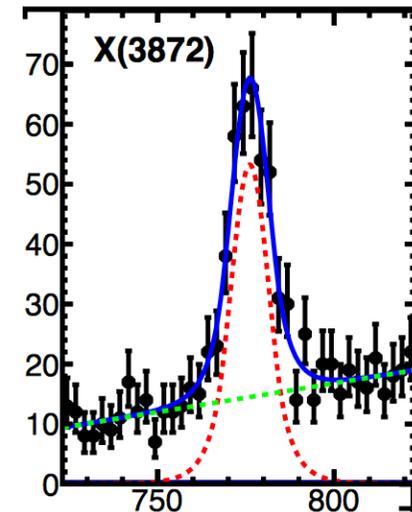
- First charmonium-like state discovered [Belle, PRL, 2003]
- sits within 1 MeV of $D^0\bar{D}^{0*}$ threshold
8 MeV below D^+D^{*-} threshold } isospin breaking effects may be important
- believed to have a large molecular $D^0\bar{D}^{0*}$ Fock component
- $\Gamma < 1.2$ MeV
- decays to $I=0, 1$ equally important

$$X(3872) \rightarrow J/\psi \omega \quad (I=0)$$

$$X(3872) \rightarrow J/\psi \rho \quad (I=1)$$

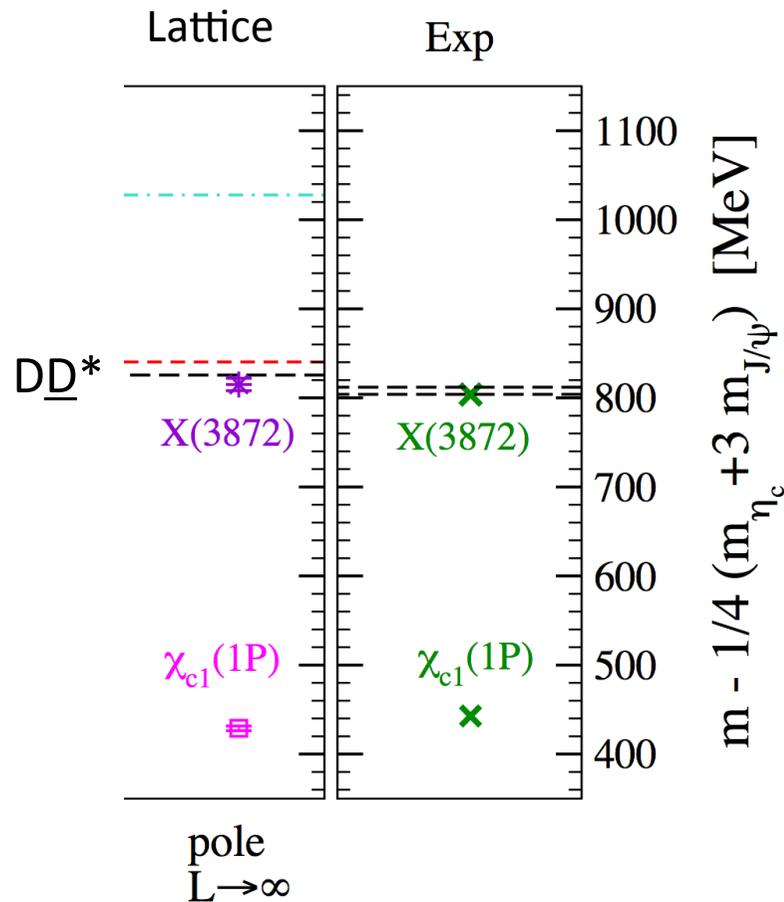


[LHCb, PRL 2013]



Evidence for X(3872) below $D\bar{D}^*$: $J^{PC}=1^{++}, I=0$

$\mathcal{O}: \bar{c} c, D\bar{D}^*$



- First lattice evidence for X(3872) found in [S.P. and L. Leskovec : 1307.5172, Phys.Rev.Lett. 2013] $m_\pi \approx 266$ MeV, $N_f=2$

- the ground state energy renders $\chi_{c1}(1P)$ mass
- $D\bar{D}^*$ scattering matrix near th. was determined
- A pole of T is found just below threshold and attributed to X(3872)

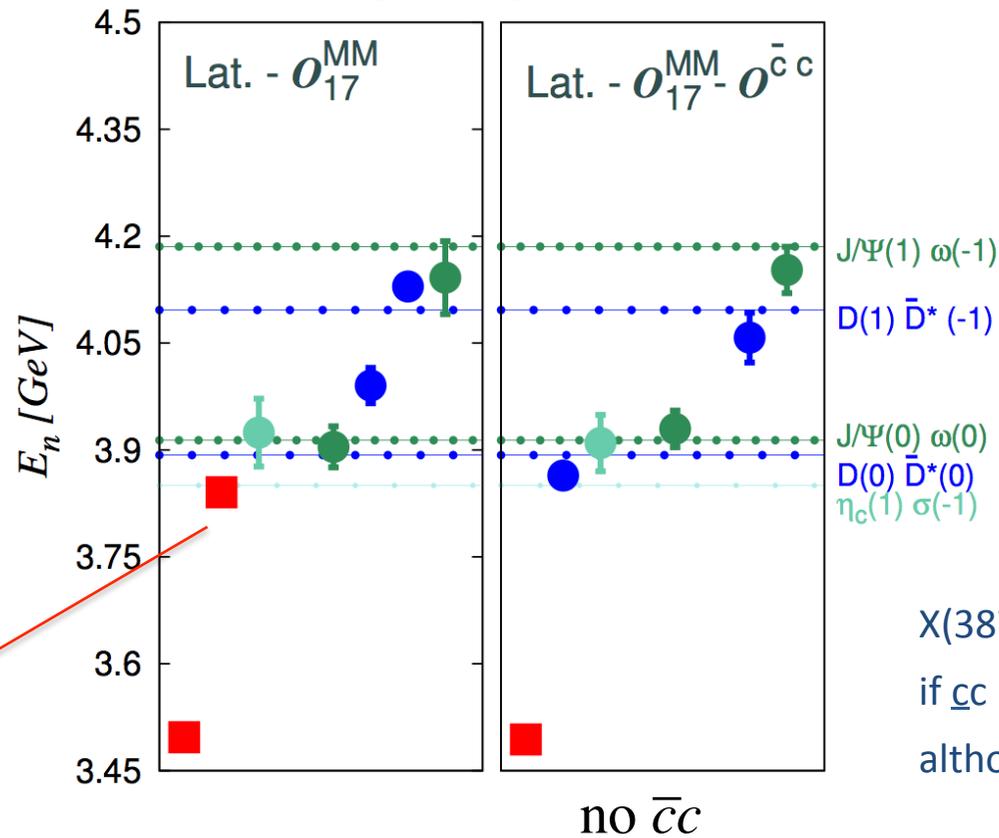
$$T \propto \frac{1}{\cot \delta - i} = \infty$$

- Similar conclusions from simulation [C. DeTar, Song-haeng Lee, et al., 1411.1389, Lat14 proceedings]

Which Fock components are essential for X(3872) with I=0 ?

$$\mathcal{O}: \underbrace{\bar{c}c, D\bar{D}^*}_{\text{essential}}, J/\psi\omega, \chi_{c1}\eta, \eta_c\sigma, \underbrace{[\bar{c}u]_{3c}[cu]_{3c}, [\bar{c}u]_{6c}[cu]_{6c}}_{\text{do not seem not essential}}$$

energies of eigenstates

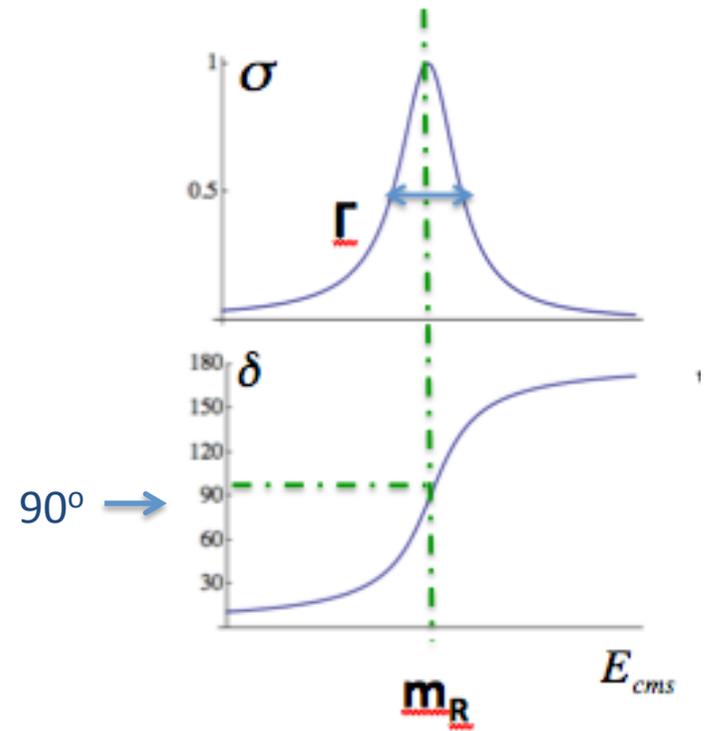
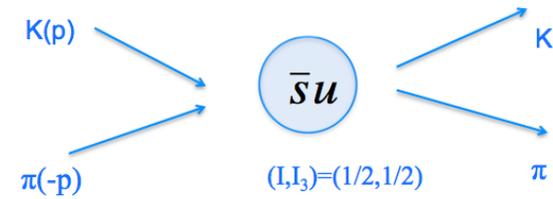


X(3872) found
only if $\bar{c}c$ in the basis

X(3872) not found
if $\bar{c}c$ not in the basis,
although $[\bar{c}u][cu]$ in the basis

[M. Padmanath, C.B. Lang, S.P.,
1503.03257]

Resonances above threshold

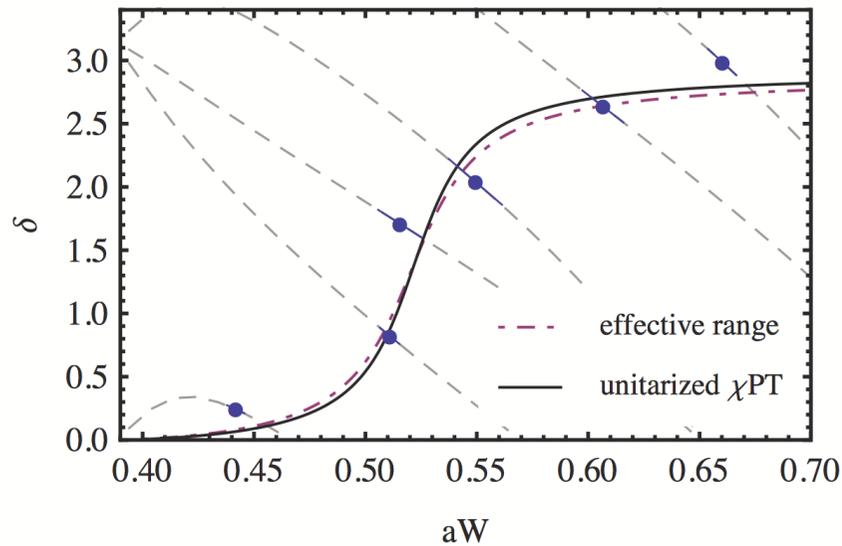
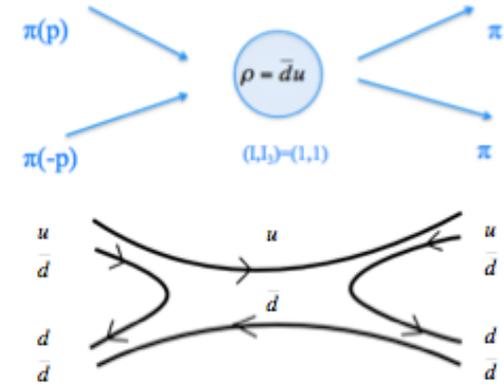


ρ resonance

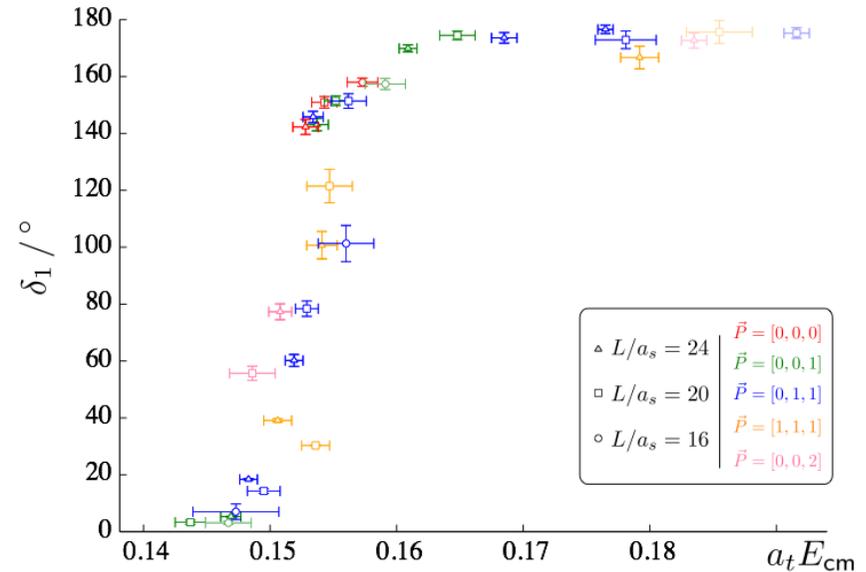
(the only “mature” hadronic resonance from lattice)

$\mathcal{O} : \bar{u}d, \pi\pi$

$$E_{\text{cm}} \rightarrow \delta(E_{\text{cm}})$$



[Pelissier, Alexandru, PRD 2013], $m_\pi \approx 300$ MeV



[HSC, PRD 2013], $m_\pi \approx 400$ MeV

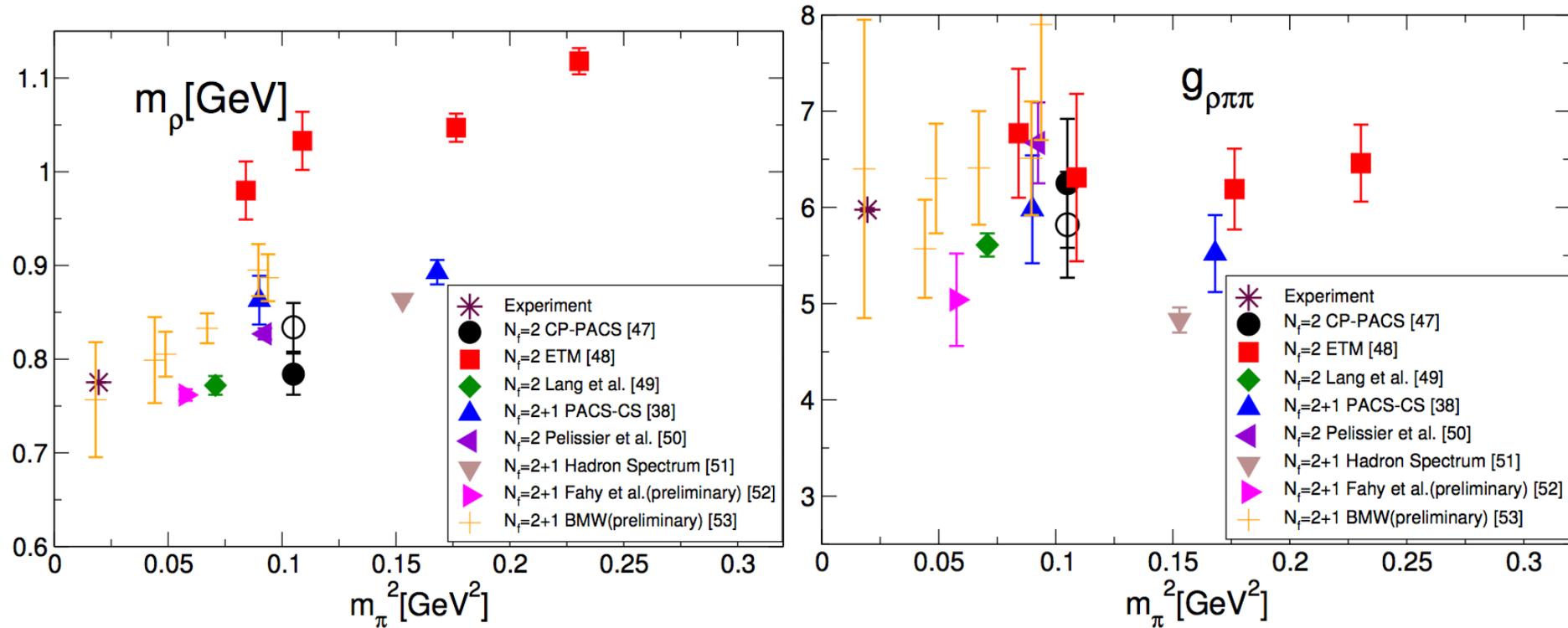
$$\text{BW: } \delta = \text{acot} \frac{m_R^2 - E_{\text{cms}}^2}{m_R \Gamma} \rightarrow m_R, \Gamma \text{ or } g_{\rho\pi\pi}$$

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{p^3}{E^2}$$

ρ resonance: lattice results

from T. Yamazaki [lat14 plenary, 1503.0867]

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{p^3}{E^2}$$



Other resonances simulated on the lattice

$\bar{c}c$	$\bar{u}u$	$\bar{s}u$	$\bar{c}u$
$\eta_c(1S)$	π^\pm	K^\pm	D^\pm
$J/\psi(1S)$	π^0	K^0	D^0
$\chi_{c0}(1P)$	η	K_S^0	$D^*(2007)^0$
$\chi_{c1}(1P)$	$f_0(500)$ or σ was $f_0(600)$	K_L^0	$D^*(2010)^\pm$
$h_c(1P)$	$\rho(770)$	$K_0^*(800)$ or κ	$D_0^*(2400)^0$
$\chi_{c2}(1P)$	$\omega(782)$	$K^*(892)$	$D_0^*(2400)^\pm$
$\eta_c(2S)$	$\eta'(958)$	$K_1(1270)$	$D_1(2420)^0$
$\psi(2S)$	$f_0(980)$	$K_1(1400)$	$D_1(2420)^\pm$
$\psi(3770)$	$a_0(980)$	$K^*(1410)$	$D_1(2430)^0$
$X(3872)$	$\phi(1020)$	$K_0^*(1430)$	$D_2^*(2460)^0$
$\chi_{c0}(2P)_{wa}$	$h_1(1170)$	$K_2^*(1430)$	$D_2^*(2460)^\pm$
$\chi_{c2}(2P)$	$b_1(1235)$	$K(1460)$	$D(2550)^0$
$X(3940)$	$a_1(1260)$	$K_2(1580)$	$D(2600)$
$\psi(4040)$	$f_2(1270)$	$K(1630)$	$D^*(2640)^\pm$
$X(4050)^\pm$	$f_1(1285)$	$K_1(1650)$	$D(2750)$
$X(4140)$	$\eta(1295)$	$K^*(1680)$	
$\psi(4160)$	$\pi(1300)$	$K_2(1770)$	
$X(4160)$	$a_2(1320)$	$K_3^*(1780)$	
$X(4250)^\pm$	$f_0(1370)$	$K_2(1820)$	
	$h_1(1380)$	$K(1830)$	
	$\pi_1(1400)$		
	$\eta(1405)$		
	$f_1(1420)$		
	$\omega(1420)$		
	$f_2(1430)$		
	$a_0(1450)$		
	$\rho(1450)$		

well below open charm decay th.

well below strong decay th.

pioneering simulations of resonances with rigorous treatment 2012-2015

$\bar{c}u$ [Mohler, S.P., Woloshyn: PRD 2013] $\bar{s}u$ [S.P., Lang, Leskovec, Mohler, PRD 2013; Wilson, Dudek, Edwards, Thomas, PRL 20114, PRD 2015] $\bar{c}c$ [Lang, Leskovec, Mohler, S.P., 1503.05363] a_1, b_1 [Lang, Leskovec, Mohler, S.P., JHEP 2014]

General result: m_R and Γ (or coupling) close to experimental values.

Exception: $\chi_{c0}(2P)$ - puzzling state

Searches for hadrons with exotic flavor

“XYZ”: Z_c^+ , charged X(3872), Y(4140)



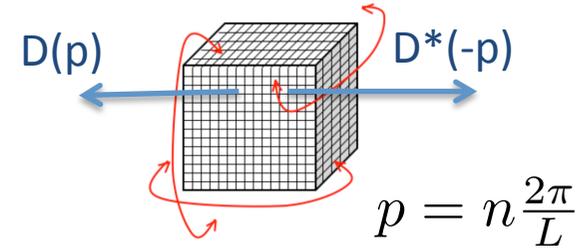
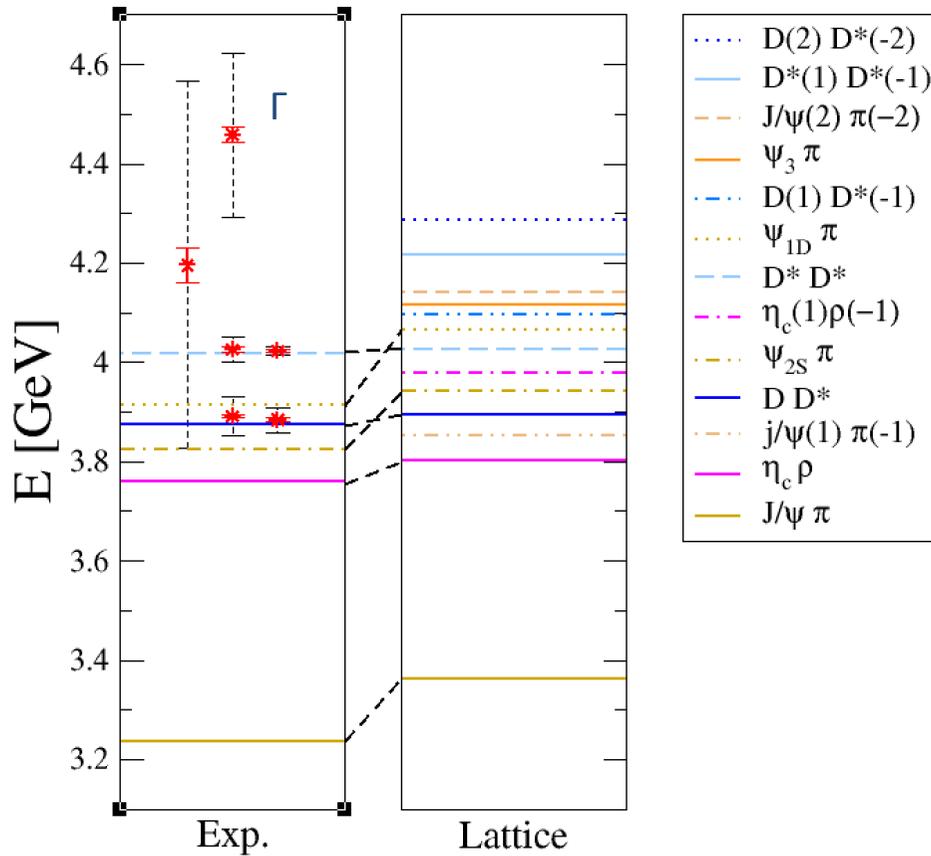
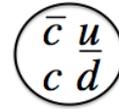
Even more challenging since most of experimental exotic “XYZ” states

- are above several thresholds and decay to several two-meson final states
- require simulation of coupled-channels

Coupled-channel scattering matrix extracted on lattice so far only in

- a-la Luscher: $K\pi$, $K\eta$ system [Willson, Dudek, Edwards, Thomas, HSC, PRL 2014]
- a-la HALQCD: H-dibaryon [HALQCD, 1504.01717]

Z_c^+ channel : $|^G=1^+, J^{PC}=1^{+-}$



Lattice:

Horizontal lines represent energies of 13 two-meson states in non-interacting case

$$E = E[M_1(p_1)] + E[M_2(p_2)]$$

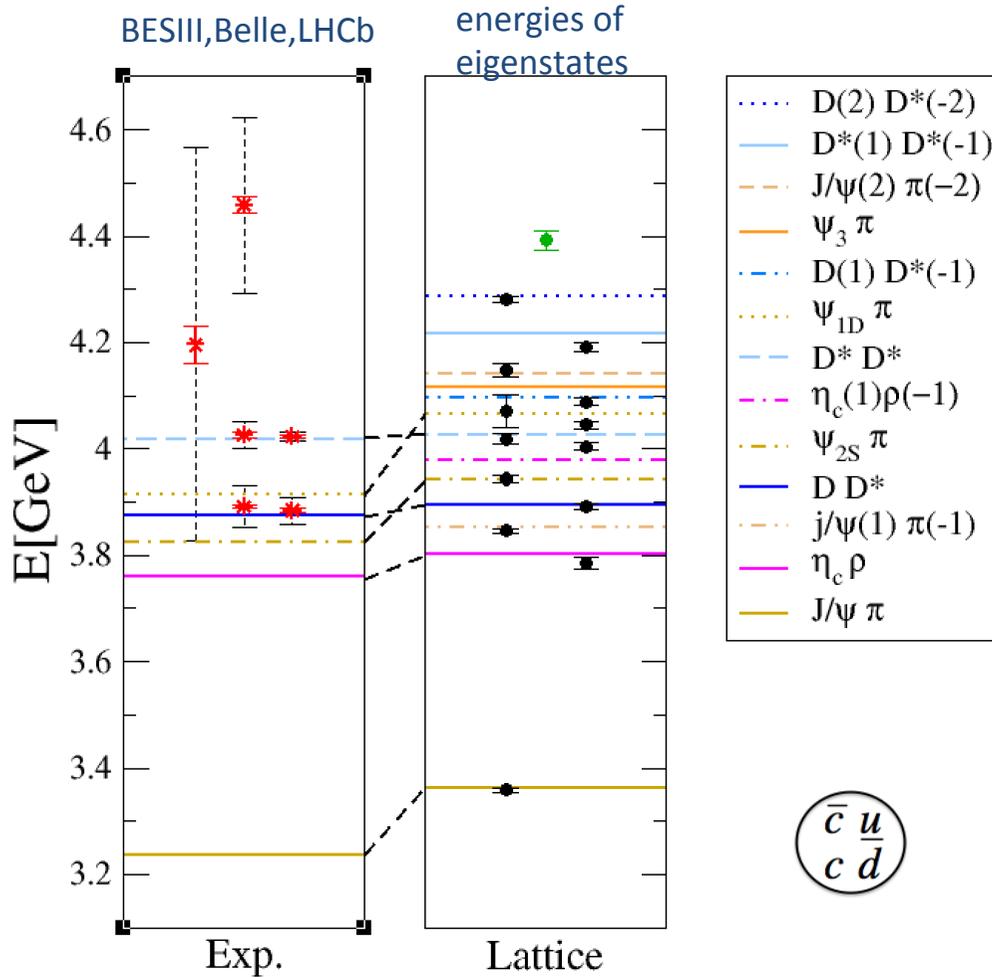
Extracting 13 two-meson states is a huge challenge!

$$\mathcal{O} : (\bar{c}u)(\bar{d}c), (\bar{c}c)(\bar{d}u), [\bar{c}\bar{d}][cu]$$

[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

Ensemble (2), $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$

Z_c^+ channel : $|^G=1^+, J^{PC}=1^{+-}$



$$\mathcal{O} : (\bar{c}u)(\bar{d}c), (\bar{c}c)(\bar{d}u), [\bar{c}\bar{d}][cu]$$

Lattice results:

- 13 expected two-meson eigenstates found as expected (black circles)
- no additional eigenstate below 4.2 GeV
- no candidate for Z_c^+ below 4.2 GeV

Exp:

- $Z_c^+(3900)$ confirmed by three exp

Puzzle:

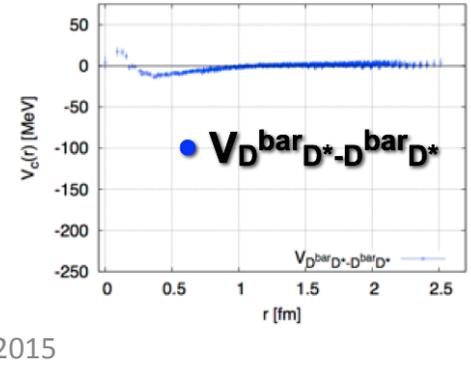
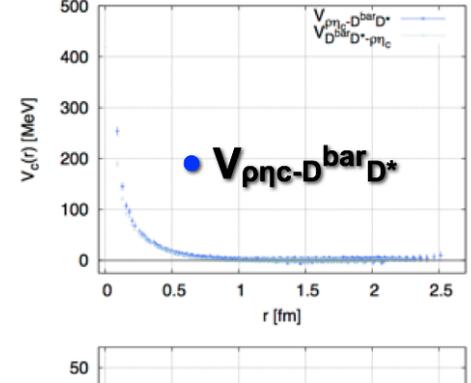
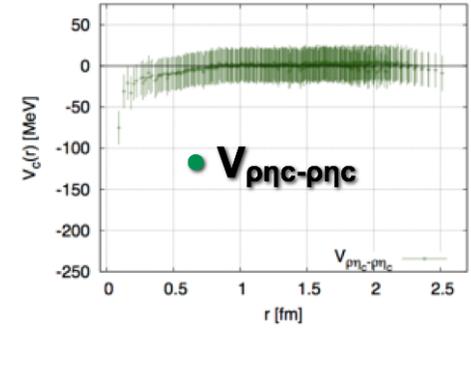
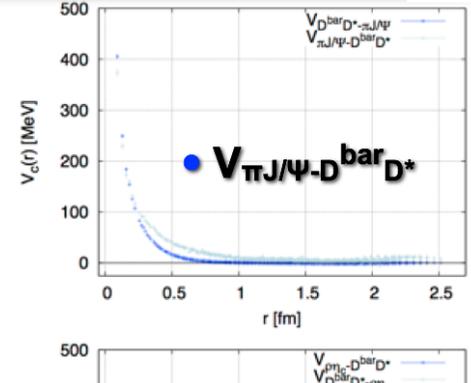
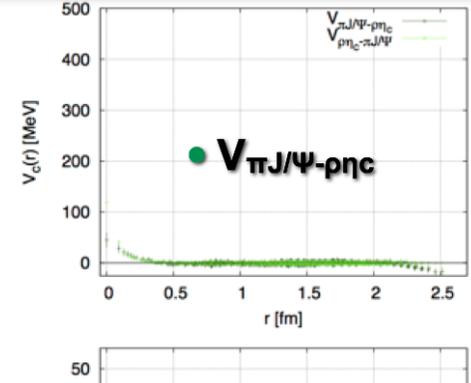
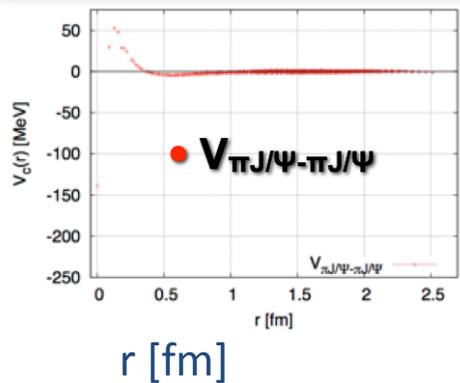
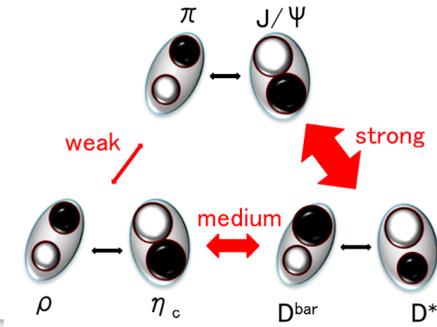
- why no eigenstate for $Z_c(3900)$?
- it would be naively expected if $Z_c(3900)$ related to a resonance pole
- is $Z_c(3900)$ of a different origin?
- perhaps coupled channel effect?

[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

similar conclusion [S.-H. Lee, C. DeTar, H. Na, 1411.1389]

Z_c^+ channel : $|^G=1^+, J^{PC}=1^{+-}$

- HALQCD method [application on H-dibaryon: HALQCD, 1504.01717]



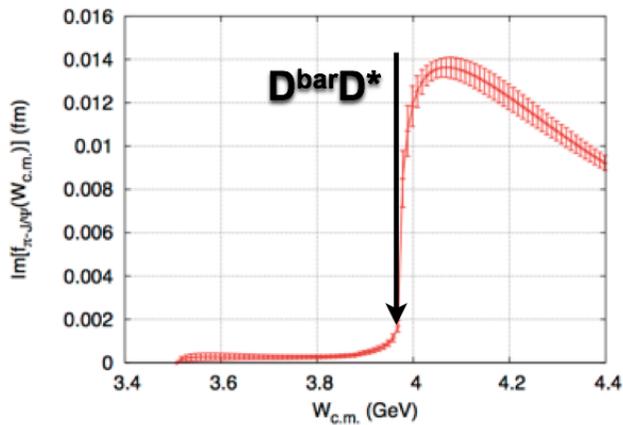
indication for coupled channel eff.

HALQCD coll, Y. Ikeda et al $m_\pi \approx 410$ MeV, $N_f=2+1+1$
 [private communication with Y. Ikeda]

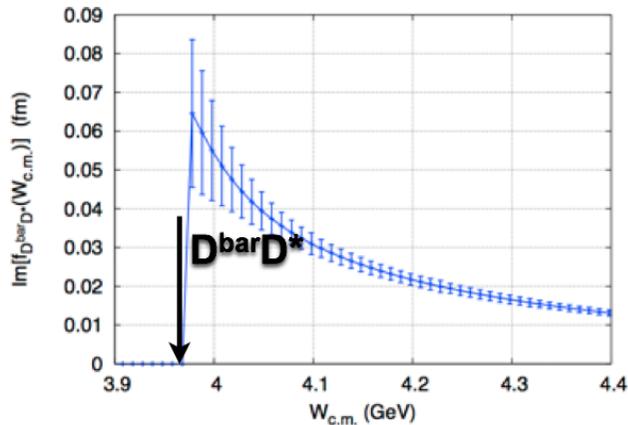
Z_c^+ channel: $|^G=1^+, J^{PC}=1^{+-}$

Lattice quantity related to cross-section

• $\pi J/\psi$ invariant mass ($m_\pi=410\text{MeV}$)

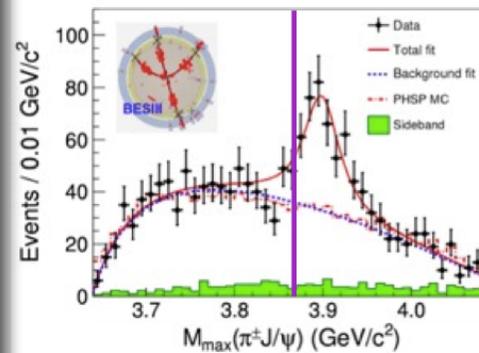


• $D\bar{D}^*$ invariant mass ($m_\pi=410\text{MeV}$)

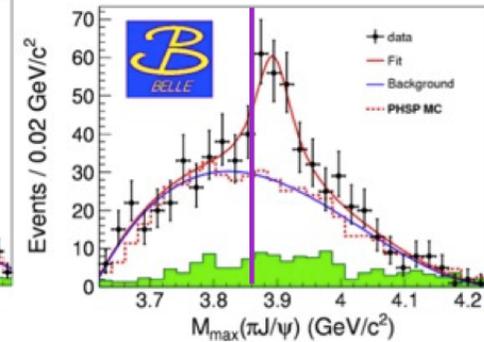


Experimental cross-section related to $Z_c^+(3900)$

• $e^+e^- \rightarrow \pi(\pi J/\psi)$ @ 4.26 GeV

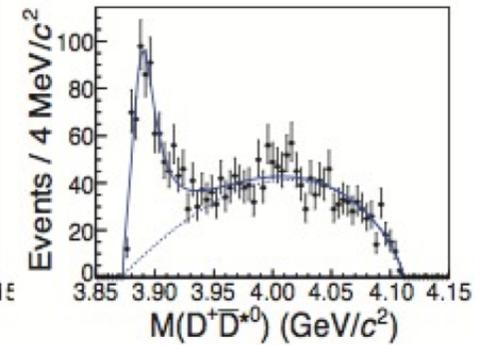
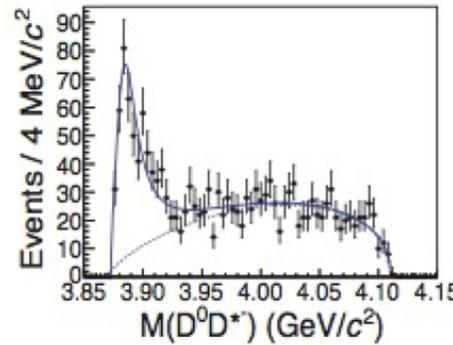


BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).

• $e^+e^- \rightarrow \pi^+ (D\bar{D}^*)^-$



BESIII Coll., PRL112 (2014).

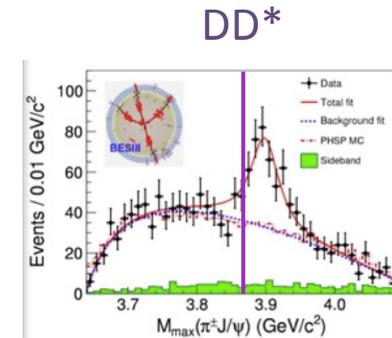
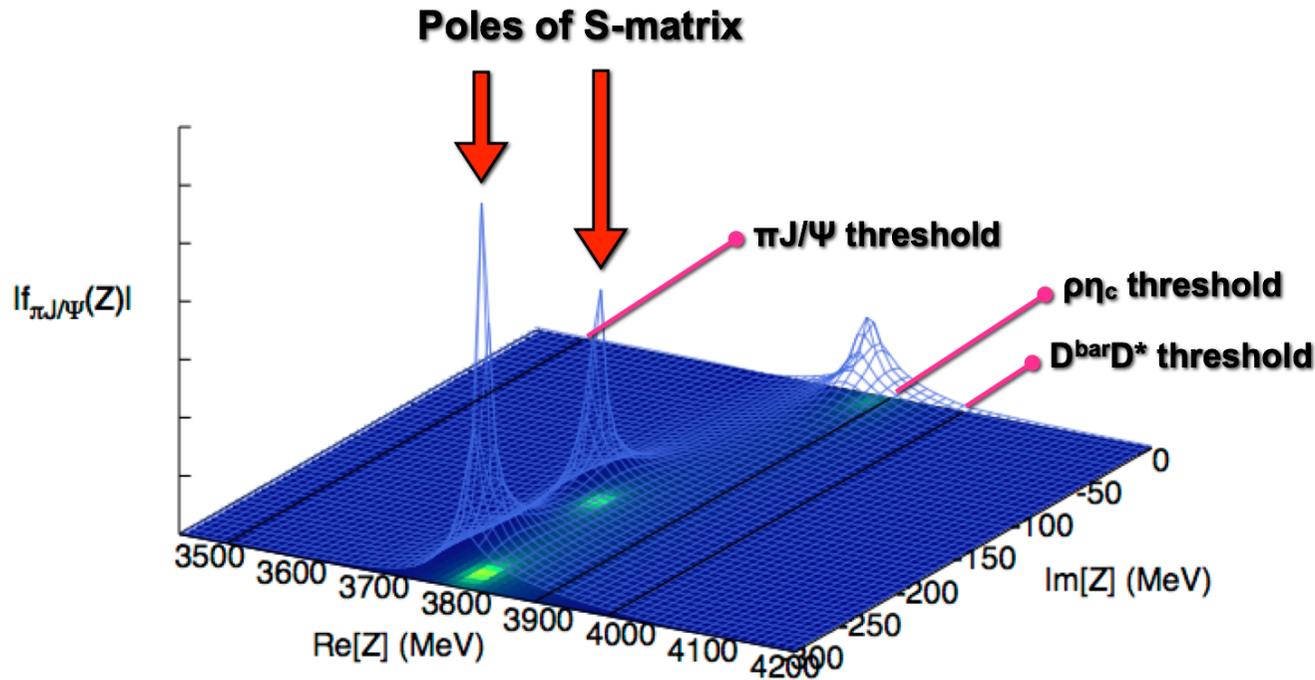
HALQCD coll, Ikeda et al $m_\pi \approx 410$ MeV, $N_f=2+1+1$

[private communication with Y. Ikeda]

S. Prelovsek, APS 2015

Lineshapes resemble
experimental $Z_c(3900)$.

Z_c^+ channel: $|G=1^+, J^{PC}=1^{+-}$

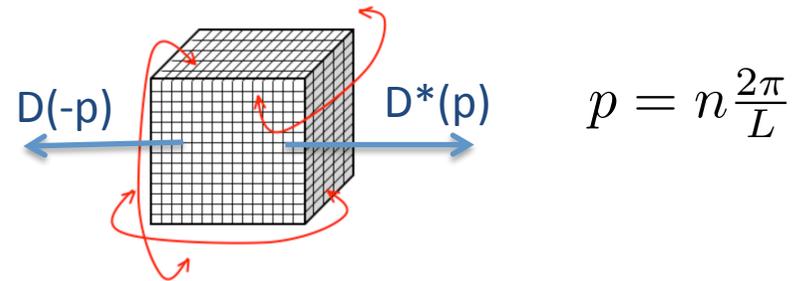
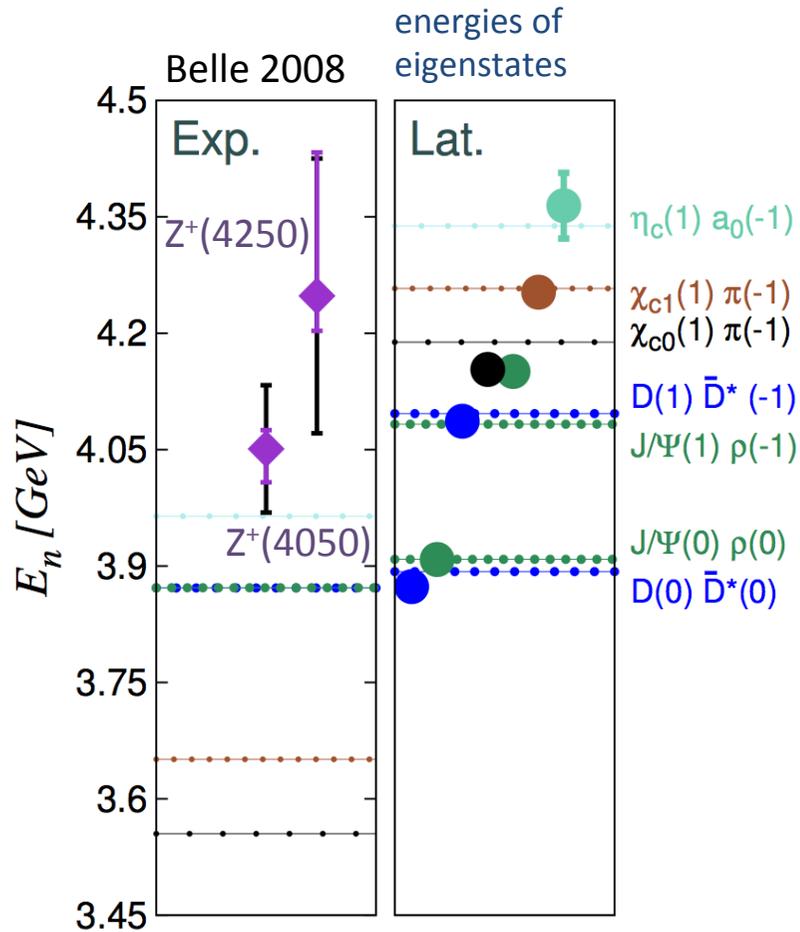


- exp: $Z_c^+(3900)$ peak appears above DD* th.
- HALQCD: poles found BELOW DD* th.
 pole NOT interpreted as a resonance (such a pole would be expected above DD*)
- Remains to be seen if HALQCD result is consistent with absence of Z_c eigenstate in S.P. et al, PRD 2015

HALQCD coll, Ikeda et al $m_\pi \approx 410$ MeV, $N_f=2+1+1$

[private communication with Y. Ikeda]

charged X(3872); channel $I^G=1^-, J^{PC}=1^{++}, \underline{c}\underline{c}\underline{d}\underline{u}$



$$\mathcal{O} : (\bar{c}u)(\bar{d}c), (\bar{c}c)(\bar{d}u), [\bar{c}\bar{d}][cu]$$

- Horizontal lines: energies of expected two-meson states in limit of no interaction: $E = E[M_1(p_1)] + E[M_2(p_2)]$
- Circles: energies of eigenstates from latt
- Only expected two-particle states observed.
- No lattice candidate for charged X(3872). In agreement with absence of such state in exp.
- No lattice candidate for other charged state. Two Belle 2008 states are exp. unconfirmed.

[M. Padmanath, C.B. Lang, S.P., 1503.03257]

Y(4140), $J^{PC}=?^{?+}$, CCSS

Experiment:

peak in $J/\psi \Phi$ just above $J/\psi \Phi$ threshold

found: CDF 2009, CMS 2012, D0 2013, Babar 2015

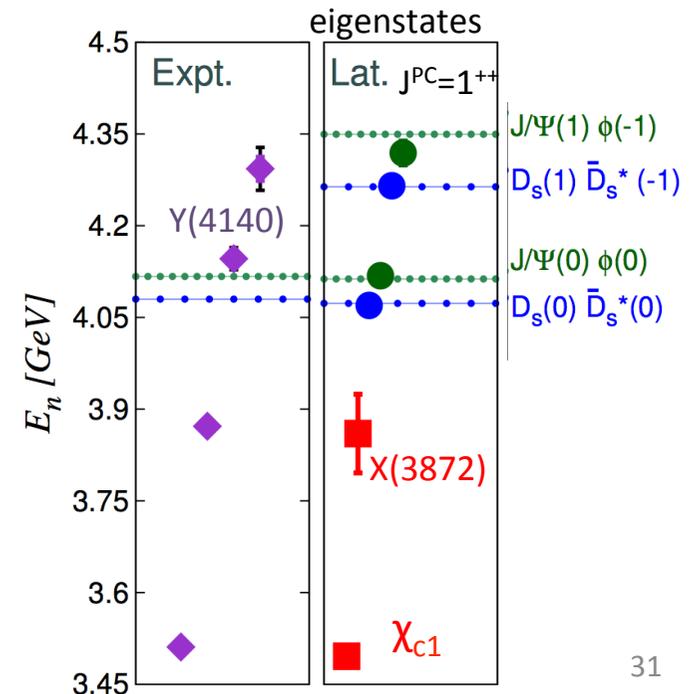
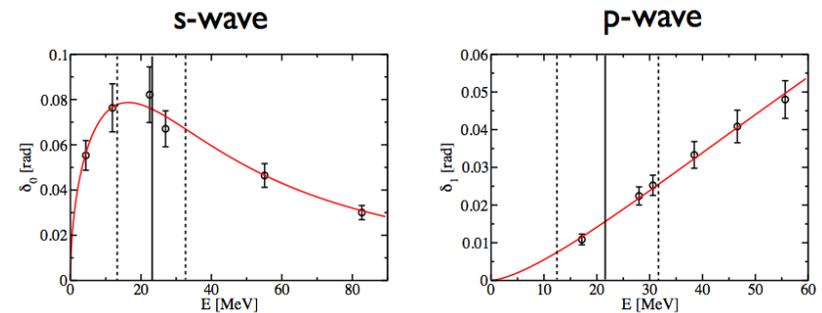
not found: Belle 2010, LHCb 2012

Lattice:

- S. Ozaki and S. Sasaki, 1211.5512, PRD
strange quark annihilation neglected
no resonant structure found for
- M. Padmanath, C.B. Lang, S.P., 1503.03257
 $\mathcal{O} : \bar{c}c, (\bar{c}s)(\bar{s}c), (\bar{c}c)(\bar{s}s), [\bar{c}\bar{s}][cs]$
channel $J^P=1^+$ considered only: expected two-particle
eigenstates found and χ_{c1} , X(3872) but not Y(4140)

$$Y(4140) \rightarrow \begin{matrix} J/\psi & \phi \\ \bar{c}c & \bar{s}s \end{matrix}$$

$J/\psi \Phi$ scattering phase shift [rad]



Conclusions

Status of meson spectrum from lattice QCD simulations (in brief):

- Evidence found for states with **non-exotic** flavor:
 - states well below th. : charmonium , D, π , K ... and all the others
 - shallow bound states : D_{s0} , D_{s1} , B_{s0} , B_{s1} , X(3872) with I=0
 - resonances via BW : ρ , K^* , $K_0^*(1430)$, K_2 , D_0^* , D_1 , a_1 , b_1 , $\Psi(3770)$

All these manifest themselves via an additional energy eigenstate (in vast experience so far) !

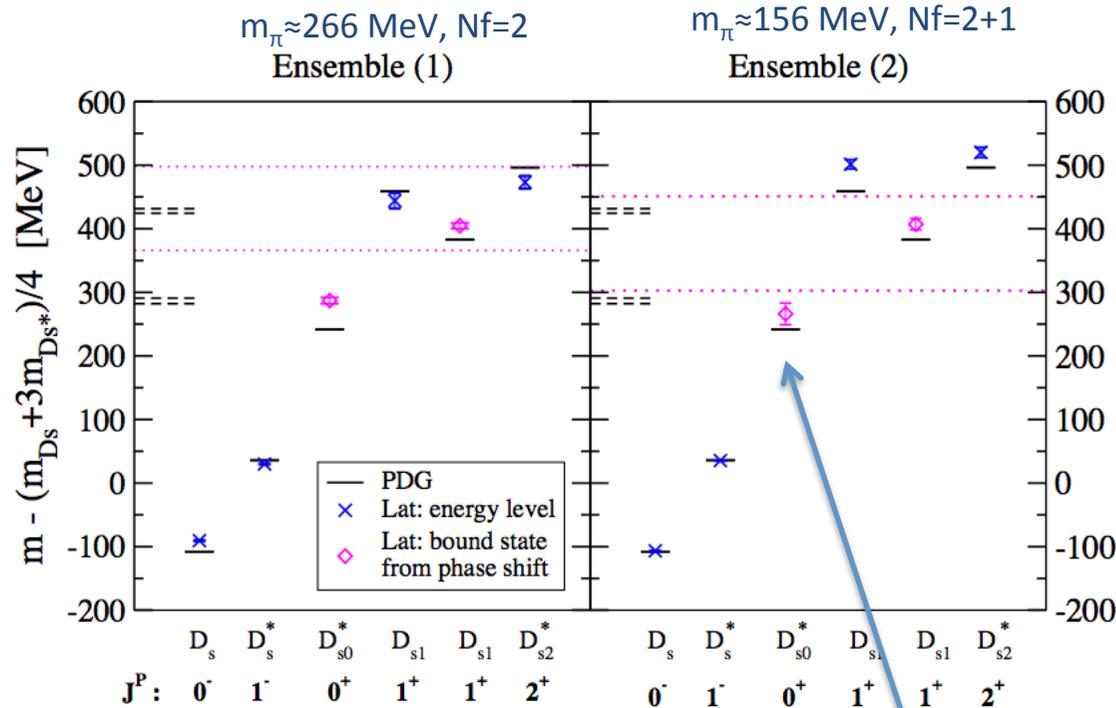
- No reliably evidence for manifestly **exotic** states yet (at least by searching an additional energy level)
 - Z_c^+ = $c\bar{c}u\bar{d}$, $J^{PC}=1^{+-}$ (HALQCD result possibly explains exp enhancement related to $Z_c(3900)$)
 - $X^+(3872)$ = $c\bar{c}u\bar{d}$, $J^{PC}=1^{++}$ (not present in exp either)
 - $Y(4140)$ = $c\bar{c}s\bar{s}$, $J^{PC}=1^{++}$

Theory is facing a serious challenge to establish whether exotic states arise from QCD or not.

Only after establishing them, one can discuss their nature (tetraquarks, mesonic molecules, coupled channel eff ..)

Backup slides

D_s and D spectrum



D_s mesons (near-threshold)

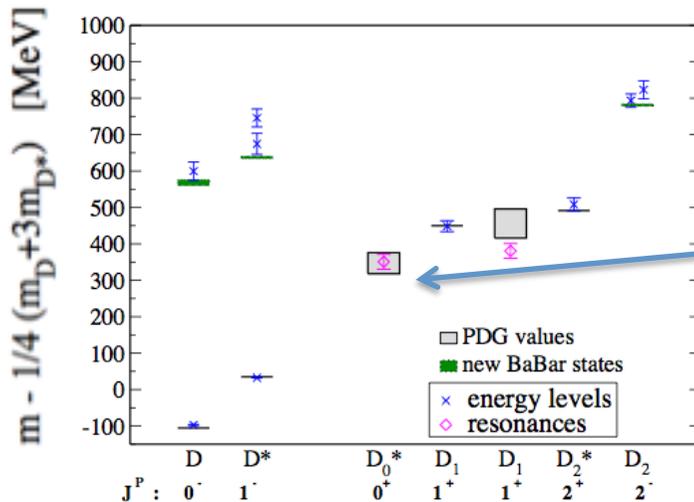
[C. Lang, L. Leskovec, D. Mohler, S.P., R. Woloshyn: PRL 2013, 1403.8103]

these results

C. B. Lang, Monday, 17h50

preliminary results for DK and Dπ

S. Ryan, Thursday 16h15



D mesons

(resonances)

[D. Mohler, S.P.,

R. Woloshyn:

PRD 2013]

Charmed scalar meson "puzzle" revisited

• why do these scalar partners have mass so close ?

$D_0^*(2400)$: $M \approx 2318$ MeV $\Gamma \approx 267$ MeV $\bar{c}u$ or $\bar{c}u\bar{s}s$?

$D_{s0}(2317)$: $M \approx 2318$ MeV $\Gamma \approx 0$ MeV $\bar{c}s$ or $\bar{c}s[\bar{u}u + \bar{d}d]$?

1) is D_0^* mass pushed up : valence $\bar{s}s$ pair ?? ✗

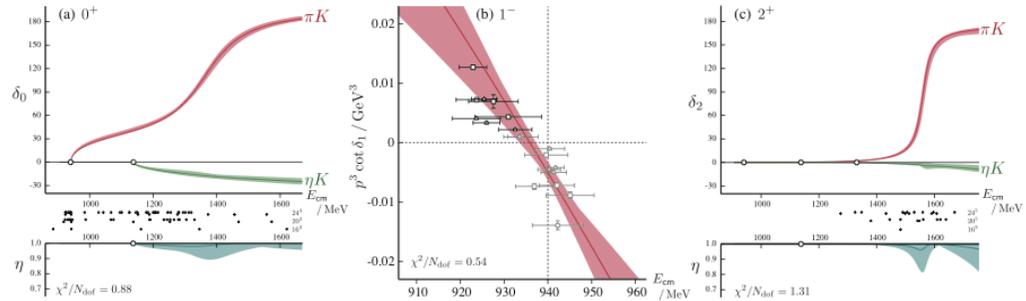
2) is D_{s0} mass pushed down : effect of DK threshold ?? ✓

Resonances in $K\pi$, $K\eta$ coupled channels

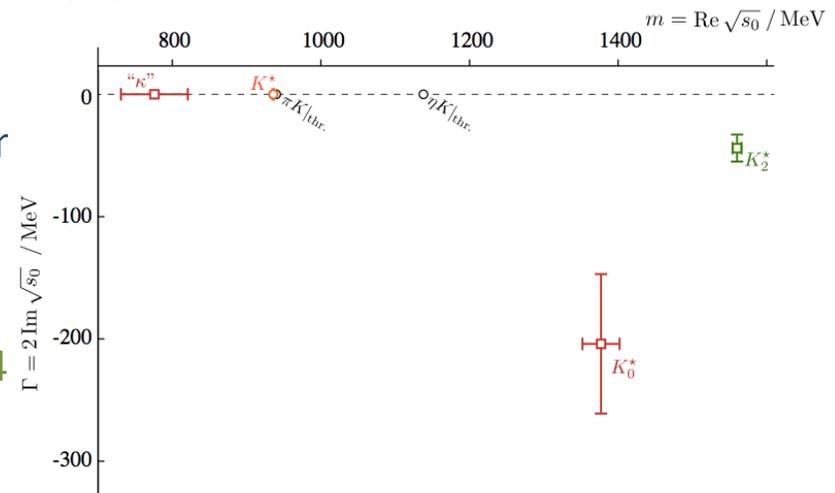
discussed by Yamazaki, Briceno, Wilson at Lat 14

- qq , $K\pi$, $K\eta$ interpolators
- a number of different $0 < P \leq 2$
- for each E_n : one determinant equation for many unknowns
- T-matrix parametrized to get around this problem
- the location of poles of T-matrix in complex given below
- $K^*(892)$ and κ are below threshold for this n
- K_0^* , K_2^* are resonances
- $m_\pi = 391$ MeV, $N_L = 16, 20, 24$

[Dudek, Edwards, Thomas, Wilson, HSC, 1406.4



$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, \quad t_{ij} = \frac{\sqrt{1-\eta^2} e^{i(\delta_i + \delta_j)}}{2\sqrt{\rho_i \rho_j}}$$



$$\det \left[\delta_{ij} \delta_{JJ'} + i\rho_i t_{ij}^{(J)}(E_{\text{cm}}) \left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L) \right) \right] = 0,$$

location of poles in T matrix in complex plane

Z_c^+ channel : $|^G=1^+, J^{PC}=1^{+-}$

Interpolating fields

18 two-meson (MM)

Aiming at 9 two-meson states listed in previous slide

$$\begin{aligned} \mathcal{O}_1^{\psi(0)\pi(0)} &= \bar{c}\gamma_i c(0) \bar{d}\gamma_5 u(0), \\ \mathcal{O}^{\psi(1)\pi(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k) \bar{d}\gamma_5 u(-e_k), \\ \mathcal{O}^{\eta_c(0)\rho(0)} &= \bar{c}\gamma_5 c(0) \bar{d}\gamma_i u(0), \\ \mathcal{O}_1^{D(0)D^*(0)} &= \bar{c}\gamma_5 u(0) \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\ \mathcal{O}^{D^*(0)D^*(0)} &= \epsilon_{ijk} \bar{c}\gamma_j u(0) \bar{d}\gamma_k c(0), \end{aligned}$$

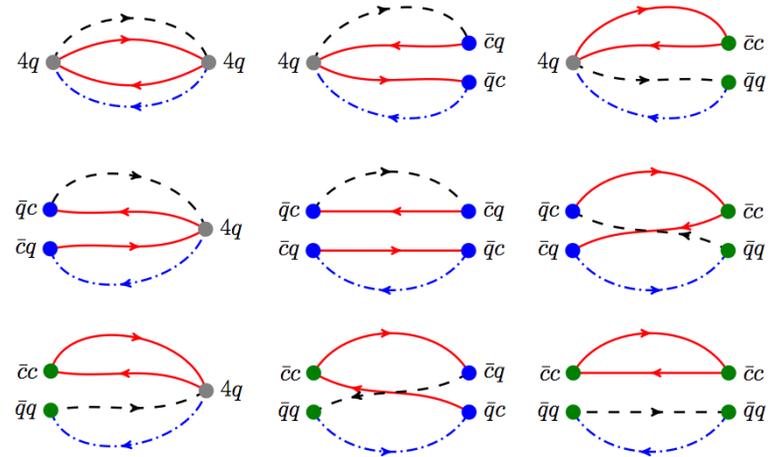
and 13 others ..

4 diquark-antidiquark (4Q)

Aiming to find additional state related to exotic Z_c^+

$$\begin{aligned} \mathcal{O}_1^{4q} &\approx [\bar{c} C \gamma_5 \bar{d}]_{3_c} [c \gamma_i C u]_{\bar{3}_c} \\ \mathcal{O}_2^{4q} &\approx [\bar{c} C \bar{d}]_{3_c} [c \gamma_i \gamma_5 C u]_{\bar{3}_c} \end{aligned}$$

and 2 others ..



Wick contractions

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^+(0) | 0 \rangle$$