# Charmonium-resonances from lattice QCD



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in collaboration with S. Collins, D. Mohler, M. Padmanath and S. Piemonte

[arxiv: 2011.02541 (accpeted to JHEP)] spins J=0 and 2

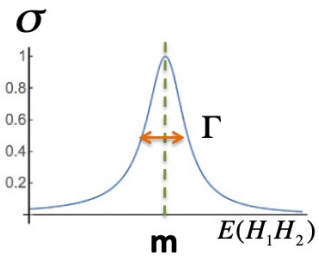
[arxiv: 074505, PRD 2019] spins J=1 and 3

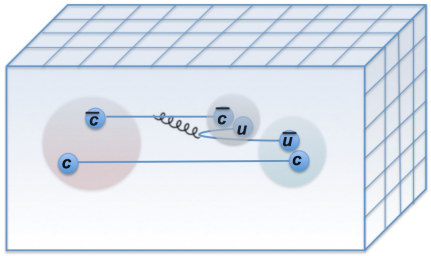
# Motivation to study charmonium resonances

Experimentally discovered exotic hadrons

- Most of them contain cc
- All of them are resonances (decay strongly)

$$Z_c : \bar{c}c\bar{d}u$$
$$P_c : \bar{c}cuud$$
$$X(3872) : \bar{c}c\bar{q}q$$



Current study: Charmonium(like) resonances with isospin=0 and J=0,1,2,3

q=u,d

$\bar{c}\ c$ $\quad$ $\bar{c}c\bar{q}q$ $\quad$ $\bar{c}c\bar{s}s$

conventional + exotic ?

$$D\bar{D} - D_s\bar{D}_s$$

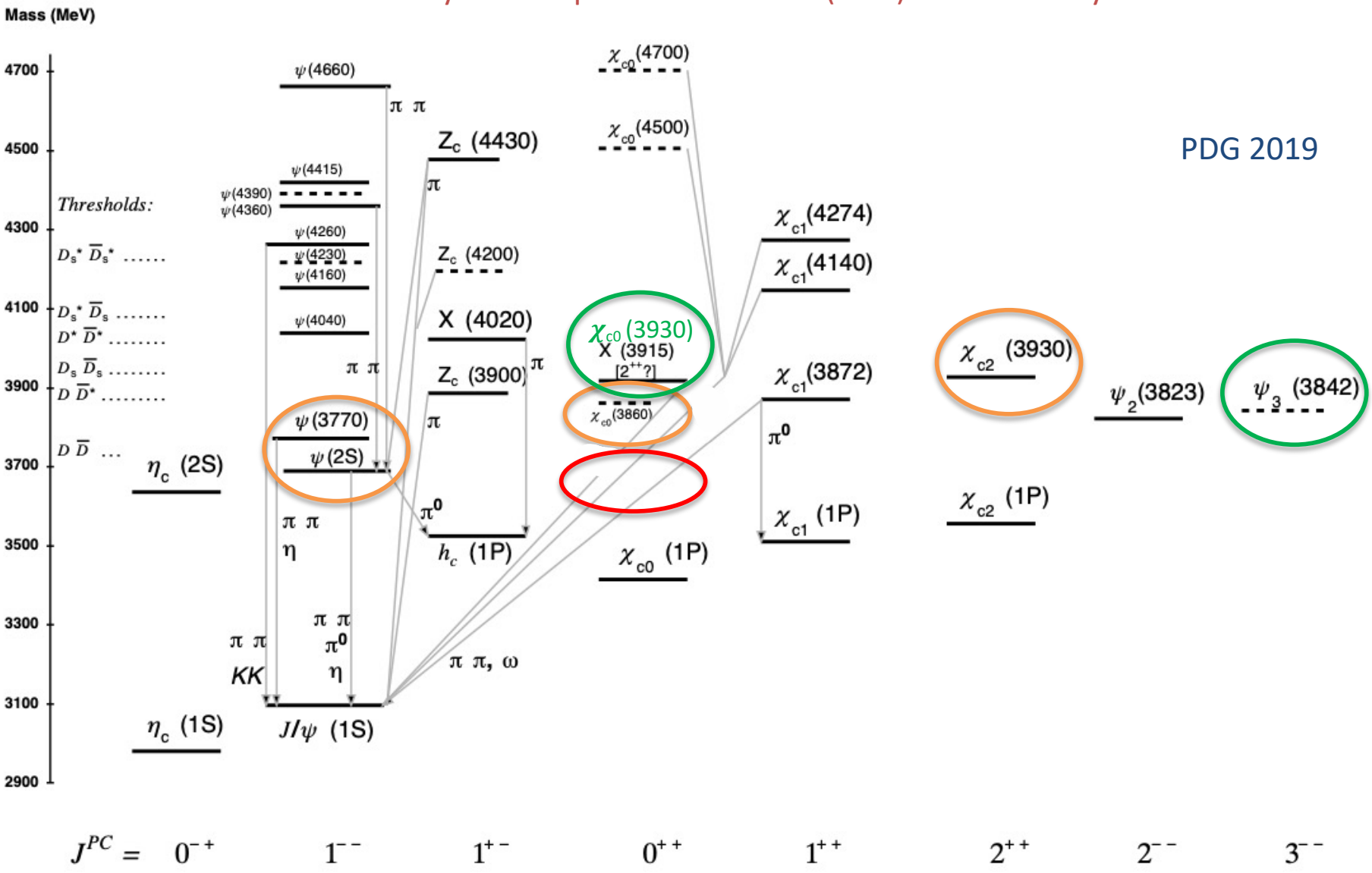
The first extraction of the scattering matrix for coupled channels

- for me
- besides the Had. Spec. Colll

Only one previous lattice study took into account decaying nature of charmonium resonances and determined width

Lang, Leskovec, Mohler, Prelovsek, JHEP (2015)

# Charmonium system: experimental status (PDG) and summary of our lattice results

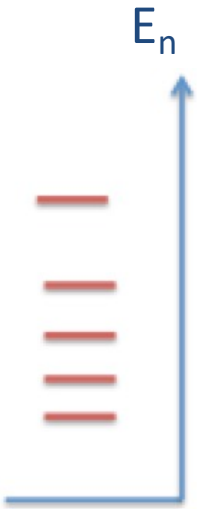


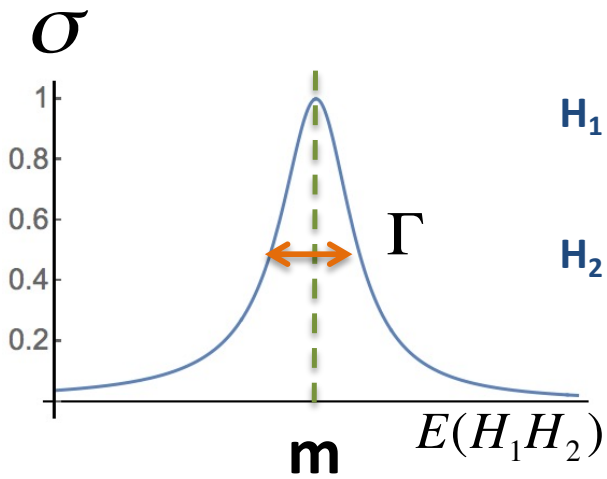

PDG 2019

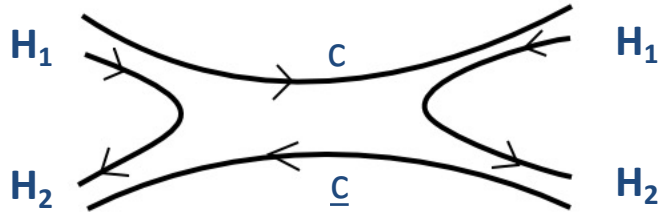
our lattice study: postdicted | predicted, exp discovered | predicted, exp not (yet) discovered

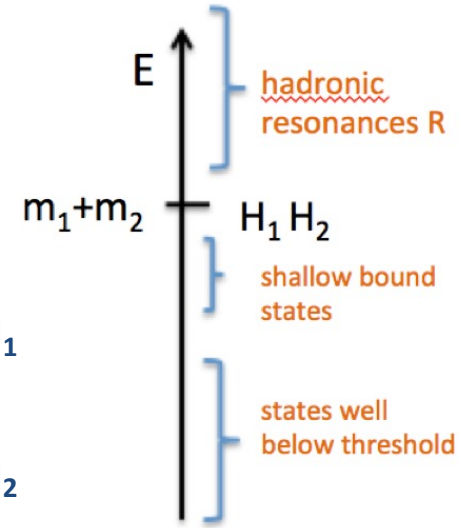
Most of hadrons and all experimentally discovered exotic hadrons are strongly decaying resonances !

# Hadronic resonances and shallow bound states from lattice (near/above one threshold)



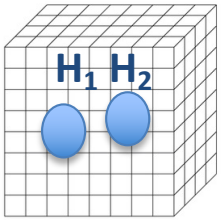




$$T_B(E) \propto \frac{1}{s - m_B^2} \qquad T_R(E) = \frac{-m_R\,\Gamma}{E^2 - m_R^2 + i\,m_R\,\Gamma}$$

$$T_B(E = m_B) = \infty$$

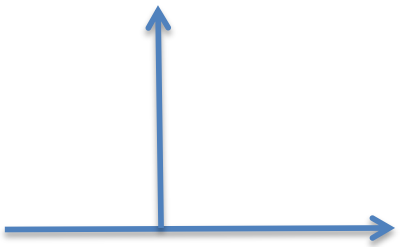



$$\sigma(E) \propto |T(E)|^2$$

energy of eigenstate

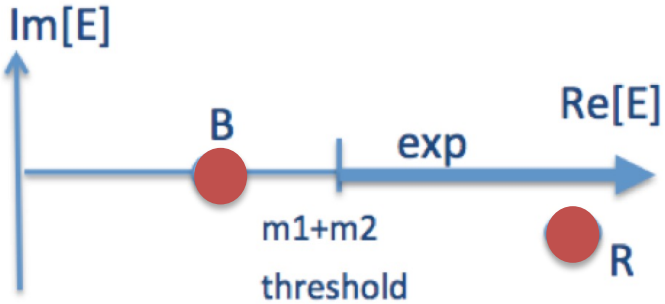
scattering matrix for real E

E → $T(E)$



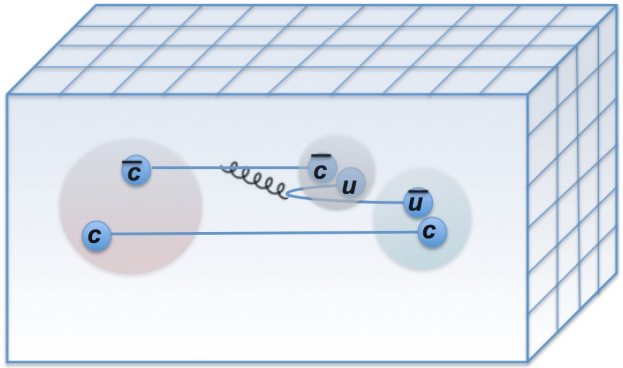
continuation to complex E

analytic relation:
Luscher 1991




location of poles in complex E plane

# Lattice details




CLS ensebmles with u/d, s dynamical quarks (Regesnburg)

$a \simeq 0.086$ fm

$N_L = 24, 32$

$m_\pi \simeq 280$ MeV $\quad m_{u/d} > m^{phy}$

$m_s < m_s^{phy}$

$m_u + m_d + m_s = m_u^{phy} + m_d^{phy} + m_s^{phy}$

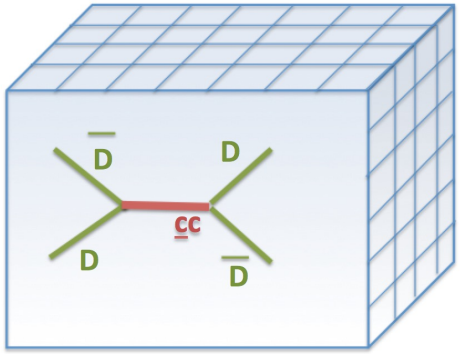
two charm quark masses analyzed

$m_D \simeq 1762$ MeV $\quad m_c < m_c^{phy}$

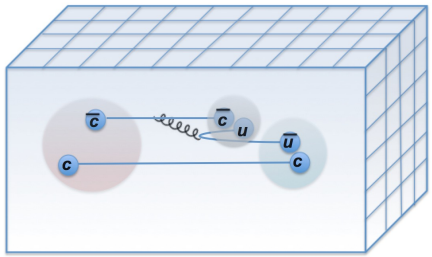
$m_D \simeq 1927$ MeV $\quad m_c > m_c^{phy}$

lattice

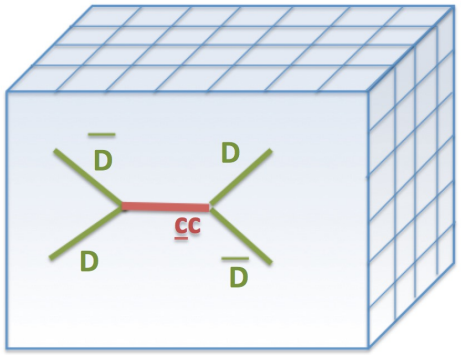
# Charmonium resonances with $J^{PC}=1^{--}, 3^{--}$ in one-channel $\bar{D}D$ scattering



# Resonances with $J^{PC}=1^{--}, 3^{--}$ in one-channel $\bar{D}D$ scattering

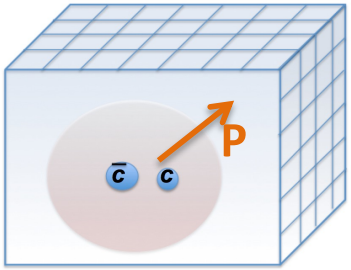


$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^{+}(0) | 0 \rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$

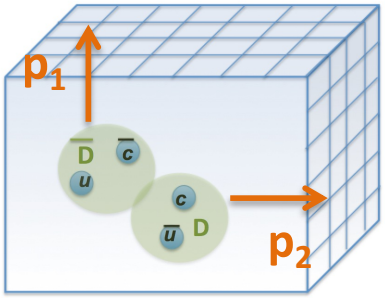



## Operators

$$\mathcal{O}^{\bar{c}c} = (\bar{c}\Gamma c)_{\vec{P}}$$




$$\mathcal{O}^{\bar{D}D} = (\bar{c}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 c)_{\vec{p}_2}$$
$$= \bar{D}(\vec{p}_1) D(\vec{p}_2)$$




$\vec{P} = \vec{p}_1 + \vec{p}_2$

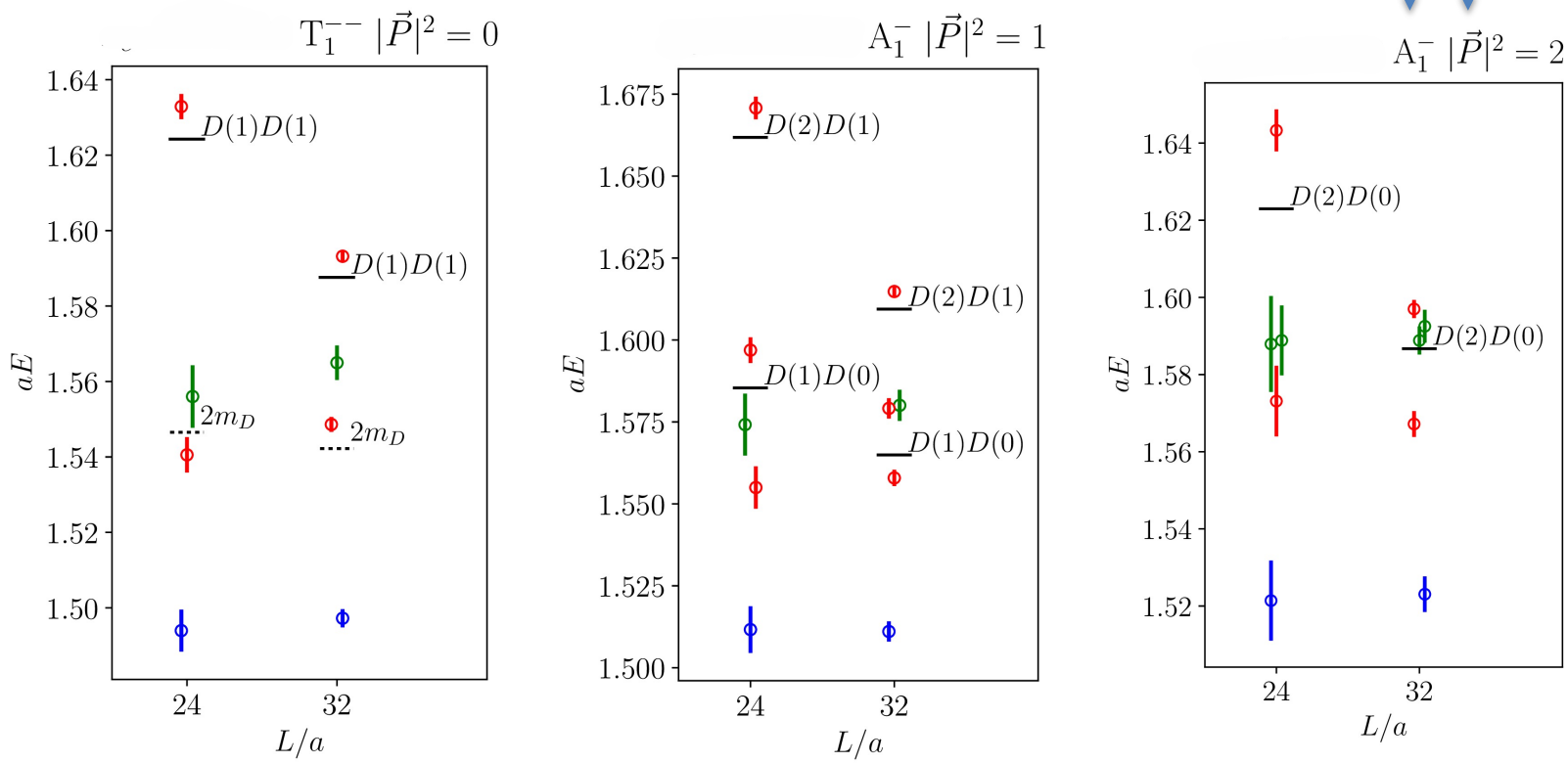
P: 0
(0,0,1) $2\pi/N_L$
(1,1,0) $2\pi/N_L$

NL=24,32

for $m_D$=1762 MeV

## Energies of eigen-states $E_n$ in irreps that contain $J^{PC}=1^{--}, 3^{--}$

irrep  total momentum



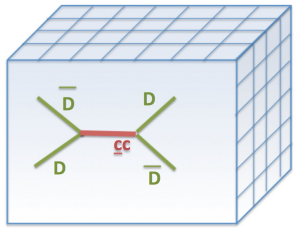

$J^{PC}=1^{--}$

$J^{PC}=3^{--}$

DD non-int

$$S(E_{cm}) = 1 + 2i\ \rho\ t(E_{cm}) = e^{2i\delta(E_{cm})}$$

$$\rho \equiv 2p/E_{cm}$$




Extraction of scattering amplitude t(E) for one-channel scattering: straightforward

Luscher's eq. $$det[1 + i\ t(E_{cm})\ F(E_{cm})] = 0 \longrightarrow t(E_{cm}) = -\frac{i}{F(E_{cm})}$$

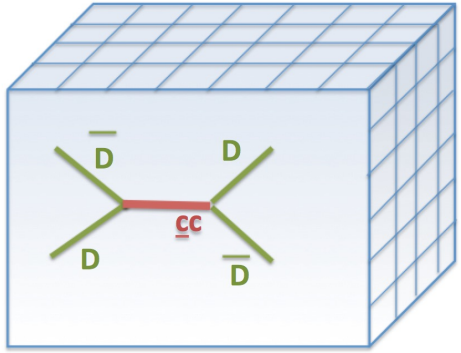
known function for each irrep and L

at $E_{cm}=E_{cm}^{lat}$

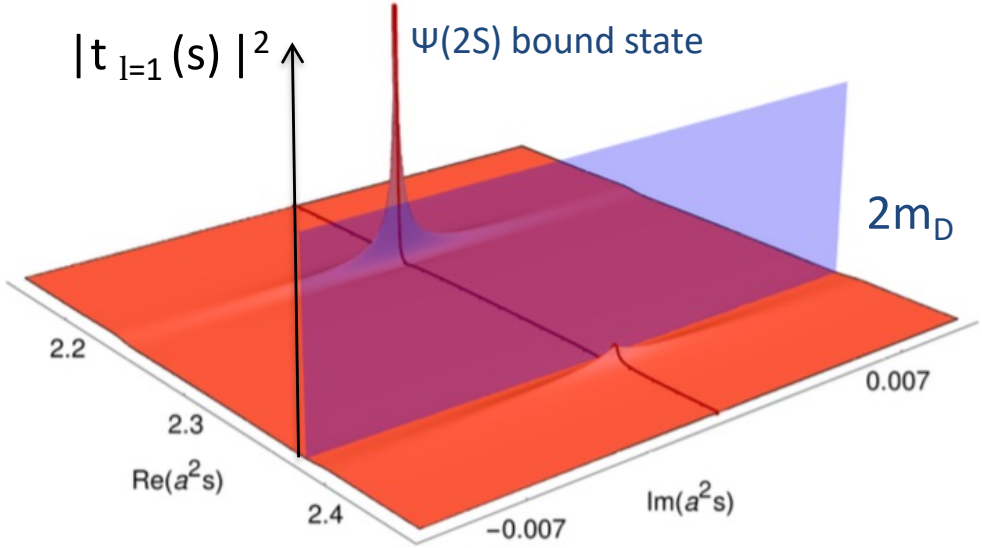
# Extracted scattering amplitude t(E) in complex energy planepartial wave l=1

$J^{PC}=1^{--}$

$$S_l(E_{cm}) = 1 + 2i\ \rho\ t_l(E_{cm}) = e^{2i\delta_l(E_{cm})}$$

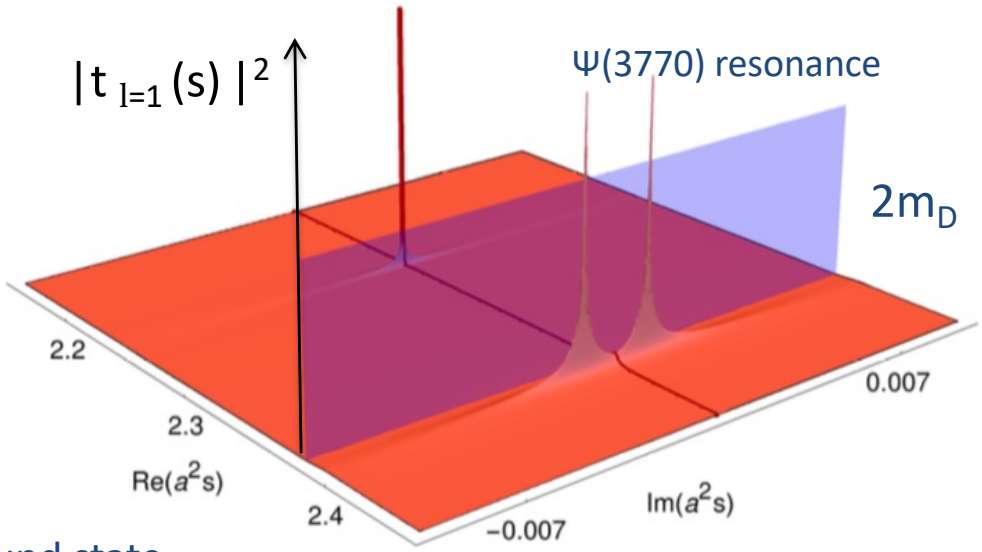



$s=E_{cm}^2$




(a) I Riemann sheet

virtual bound state




(b) II Riemann sheet

poles in t(s) related to
resonance and bound states

Fig for l=1 and $m_D \approx 1762$ MeV:
one resonance, one bound state, one virtual bound state

# Charmonium resonances with $J^{PC}=1^{--}, 3^{--}$: results for masses and widths

## Resonances

**Ψ(3770)** $J^{PC}=1^{--}$, l=1

**X(3842)** $J^{PC}=3^{--}$, l=3

**Mass:**

$\delta_1(E_{cm}=m_R)=90^o$

**Width** or coupling g:

Ψ(3770) → DD , l=1

$$\frac{p^3 \cot \delta_1}{\sqrt{s}}|_{s\simeq m^2} = \frac{6\pi}{g^2}(m^2 - s)$$

$$\Gamma = \frac{g^2\ p^3}{6\pi\ s}$$

| | g |
|---|---|
| lat | $16.0\ ^{+2.1}_{-0.2}$ |
| exp | 18.7 ± 0.9 |

X(3842): to narrow to resolve

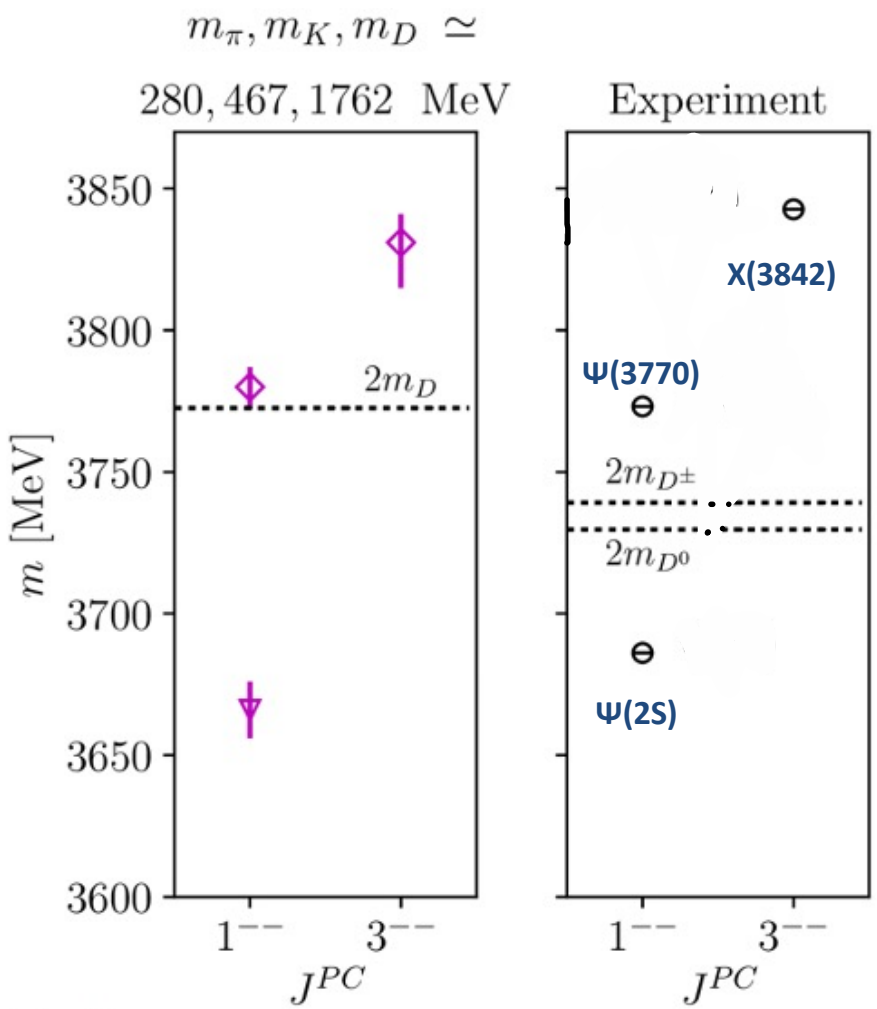
## Bound state

**Ψ(2S)** $J^{PC}=1^{--}$, l=1

**Mass:**

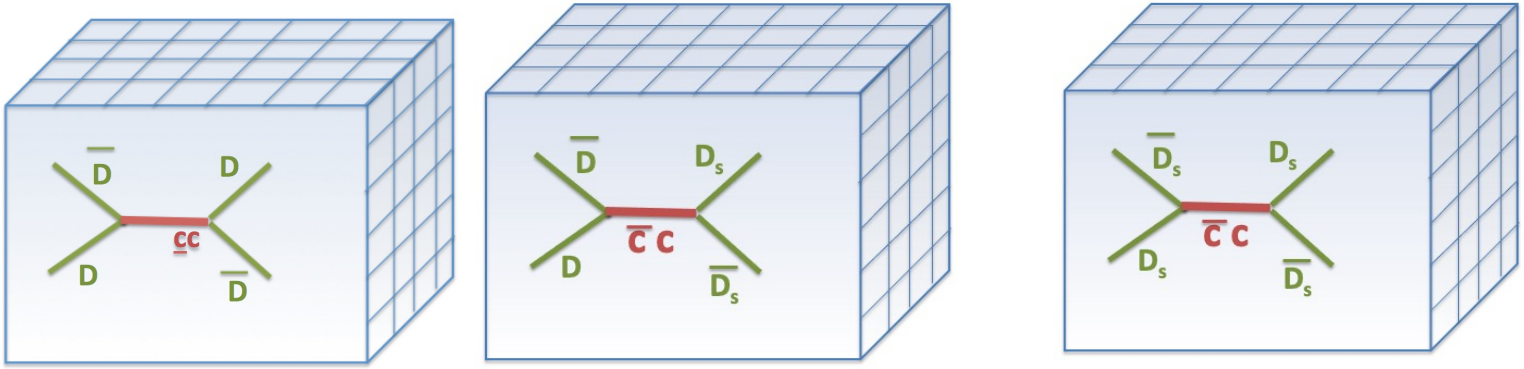
$|t(E_{cm}=m_B)| = \infty$

$m_B$ given by the pole on the first sheet



$$D\bar{D} - D_s\bar{D}_s$$

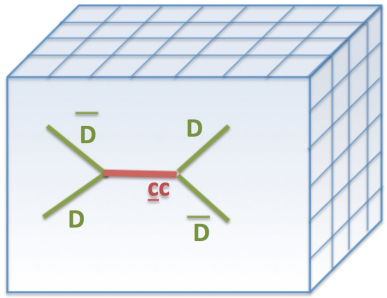
## Charmonium resonances with $J^{PC}=0^{++}$, $2^{++}$ in coupled $\underline{D}D - \underline{D}_s D_s$ scattering

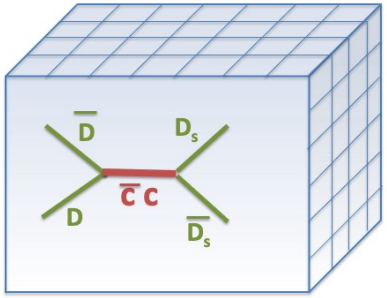


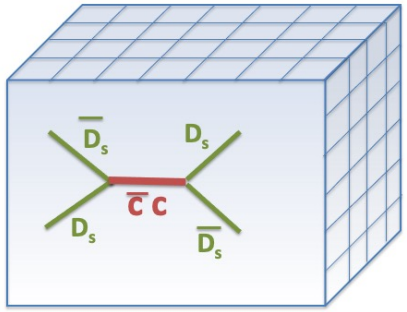

these resonances appear at higher energies and both decay channels need to be taken into account

# Resonances with $J^{PC}=0^{++}, 2^{++}$ in coupled-channel scattering

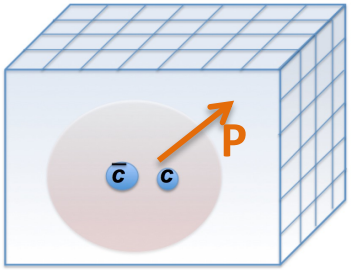
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^{+}(0) | 0 \rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$



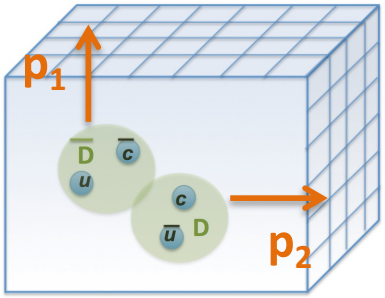




## Operators




$$\mathcal{O}^{\bar{c}c} = \left(\bar{c}\Gamma c\right)_{\vec{P}}$$

$$\mathcal{O}^{\bar{D}D} = \left(\bar{c}\Gamma_1 q\right)_{\vec{p}_1} \left(\bar{q}\Gamma_2 c\right)_{\vec{p}_2}$$
$$= \bar{D}(\vec{p}_1) D(\vec{p}_2)$$




$$\mathcal{O}^{\bar{D}_s D_s} = \left(\bar{c}\Gamma_1 s\right)_{\vec{p}_1} \left(\bar{s}\Gamma_2 c\right)_{\vec{p}_2}$$
$$= \bar{D}_s(\vec{p}_1) D_s(\vec{p}_2)$$

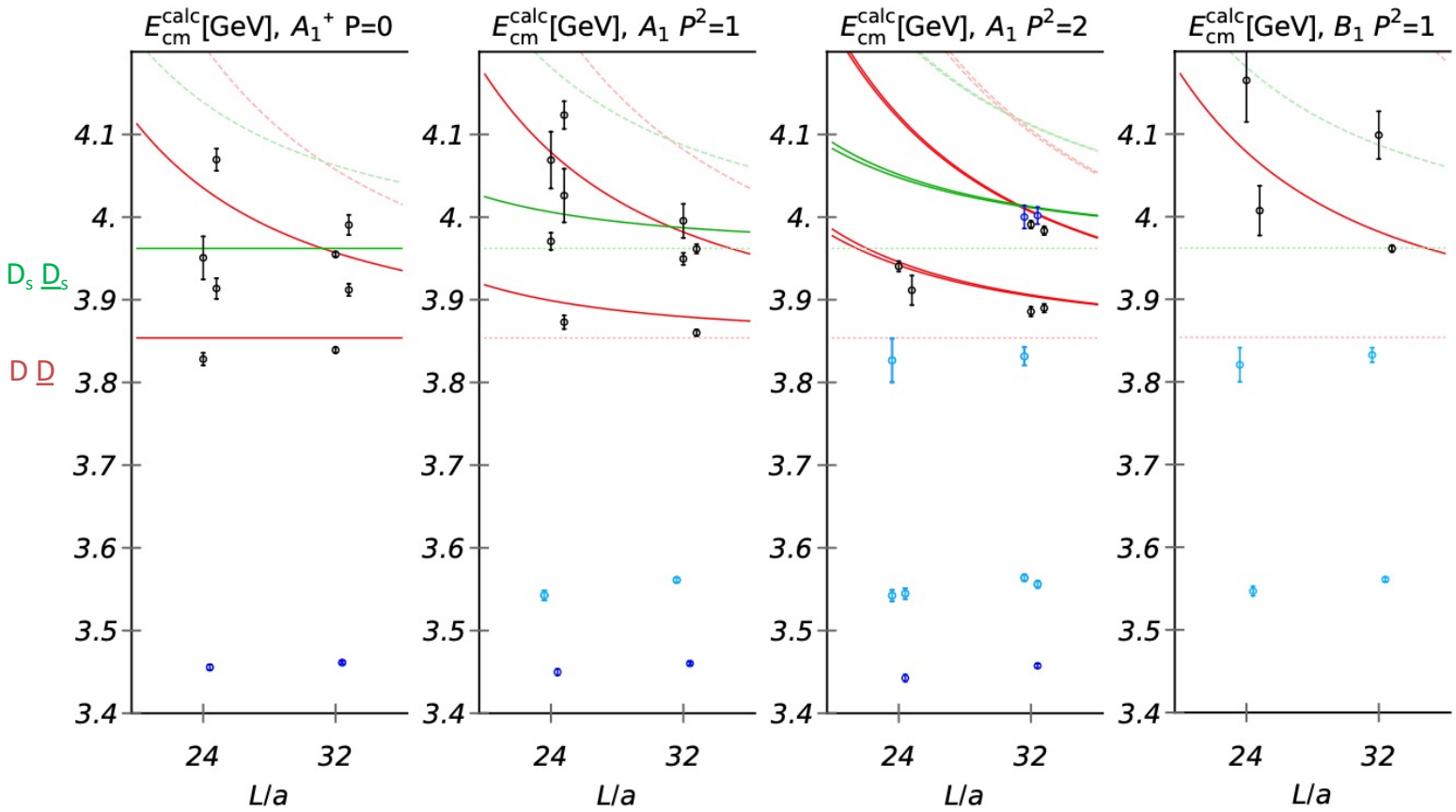
$\vec{P} = \vec{p}_1 + \vec{p}_2$

P: 0
(0,0,1) $2\pi/N_L$
(1,1,0) $2\pi/N_L$

NL=24,32

# Energies of eigen-states $E_n$ in irreps that contain $J^{PC}=0^{++}, 2^{++}$

for $m_D$=1927 MeV




non-interacting energies denoted

D $\underline{D}$

$D_s$ $\underline{D}_s$

$J^{PC}=0^{++}$

$J^{PC}=2^{++}, 2^{+-}$

$$S_{ij}(E_{cm}) = 1 + 2i\ \rho\ t_{ij}(E_{cm})$$

$$t(E_{cm}) = \begin{vmatrix} t_{11}(E_{cm}) & t_{12}(E_{cm}) \\ t_{12}(E_{cm}) & t_{22}(E_{cm}) \end{vmatrix}$$

1: $\underline{D}$D, 2: $\underline{D}$sDs

Extraction of matrix t(E) : NOT straightforward !

one equation, three unknowns (at each $E_{cm}$)

$$det[1 + i\ t(E_{cm})\ F(E_{cm})] = 0$$

known 2x2 matrix

$$(t^{-1})_{ij} = \frac{2}{E_{cm}\ p_i^l p_j^l}\ (\tilde{K}^{-1})_{ij} - i\ \rho_i\ \delta_{ij}$$

$$\frac{\tilde{K}_{ij}^{-1}(s)}{\sqrt{s}} = a_{ij} + b_{ij} s$$

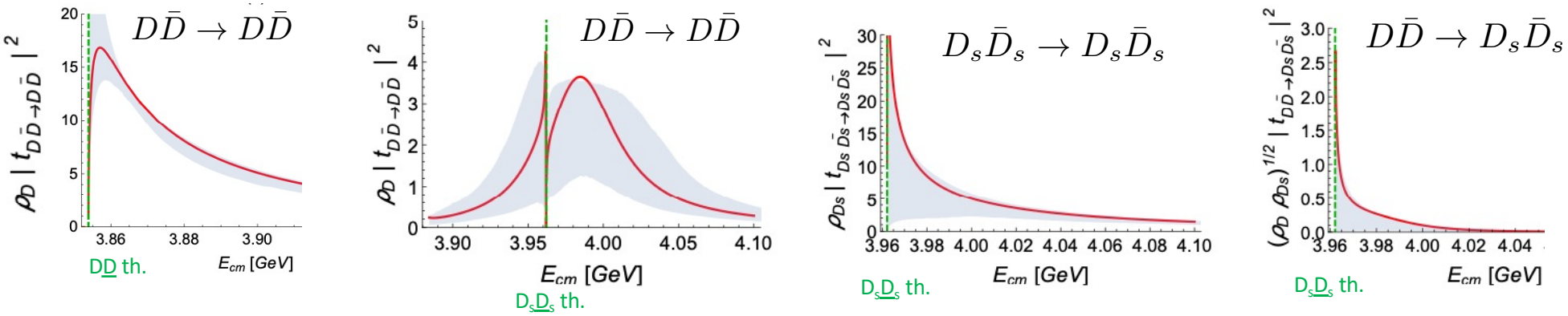
$$s = E_{cm}^2$$

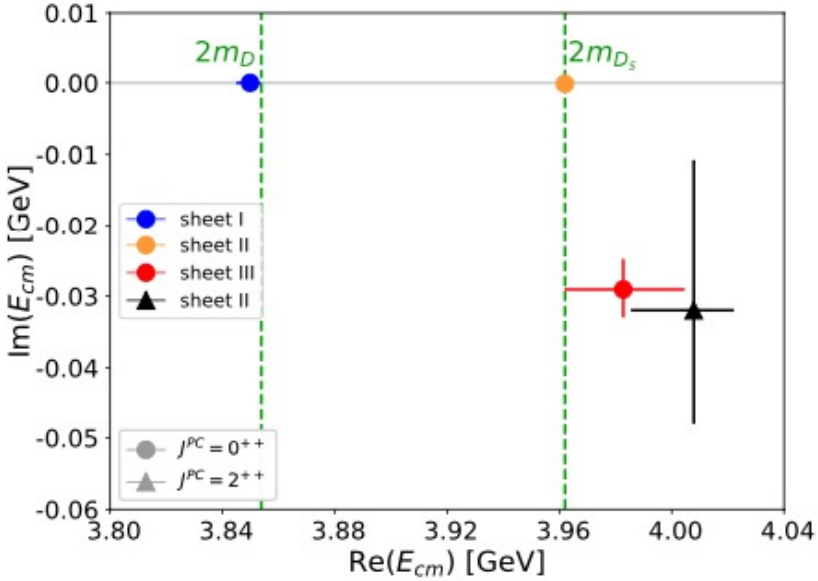
$$\rho_i \equiv 2p_i/E_{cm}$$

# $J^{PC}=0^{++}$ : some expected and unexpected states found

$$D\bar{D} - D_s\bar{D}_s$$

S-wave (L=0, $J^{PC}=0^{++}$ )




- broad resonance coupling mostly to DD

$$lat: m = 3949^{+28}_{-20}\ MeV \quad g = 1.35^{+0.04}_{-0.08}\ GeV$$

$$X(3860): m = 3862^{+48}_{-35}\ MeV \quad g = 2.5^{+1.2}_{-0.9}\ GeV \qquad \Gamma \equiv g^2 p_D^{2l+1}/m^2$$

Belle 2017

- state near $D_sD_s$ threshold coupling mostly to $D_sD_s$

lat : $m - 2m_{D_s} = -0.2\ ^{+0.16}_{-4.9}$ MeV , $g = 0.10\ ^{+0.21}_{-0.03}$ GeV

$\chi_{c0}(3930)$ : $m - 2m_{D_s} = -12.9 \pm 1.6$ MeV , $\Gamma = 17 \pm 5$ MeV , $g = 0.67 \pm 0.10$ GeV

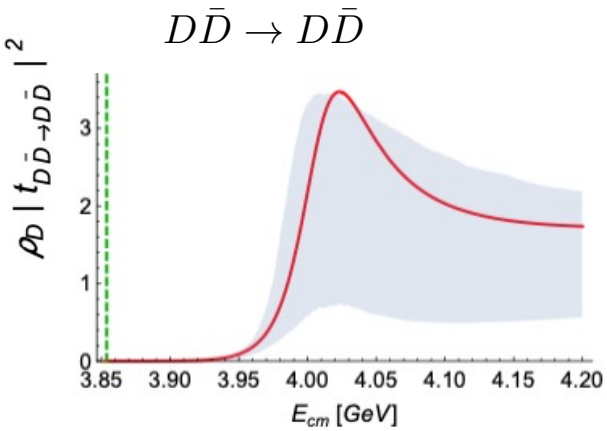
LHCb 2020

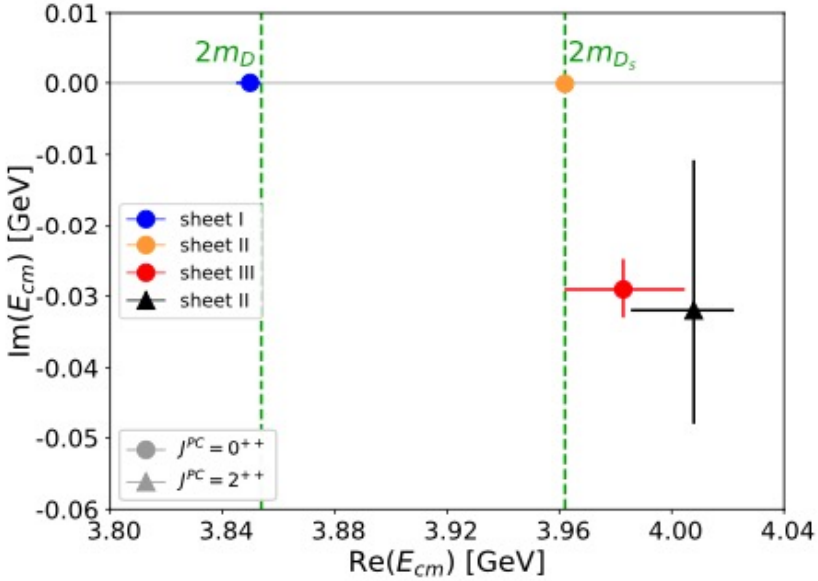
- state near DD threshold

near pole $$t_{ij} \sim \frac{c_i\ c_j}{(E^p_{cm})^2 - E^2_{cm}}$$

# $J^{PC}=2^{++}$ : conventional resonance found

D-wave (L=2, $J^{PC}=2^{++}$ )




- 2++ resonance

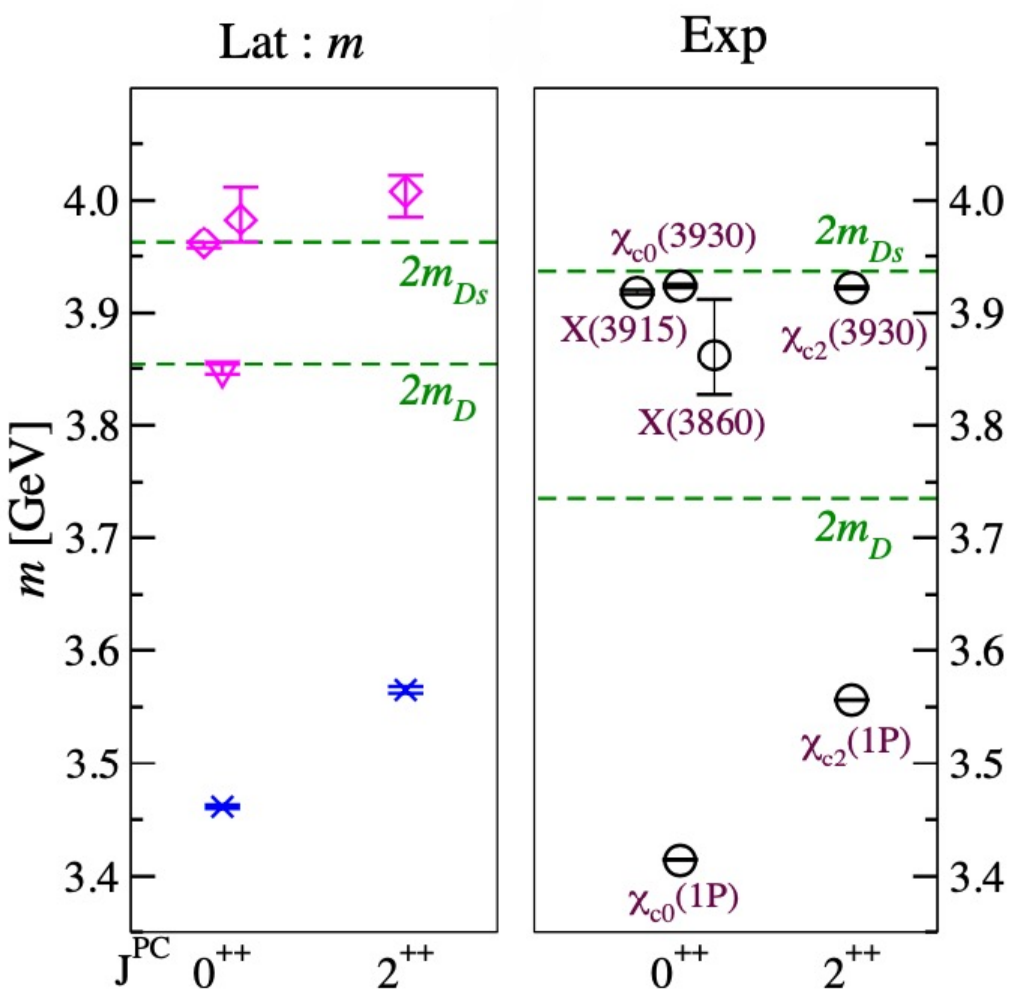
$$\Gamma \equiv g^2 p_D^{2l+1}/m^2$$

$$lat : m = 3973^{+14}_{-22}\ \mathrm{MeV} \quad g = 4.5^{+0.7}_{-1.5}\ \mathrm{GeV}^{-1}$$

$$\chi_{c2}(3930) : m = 3923 \pm 1\ \mathrm{MeV} \quad g = 2.65 \pm 0.12\ \mathrm{GeV}^{-1}$$

PDG

# Charmonium resonances with $J^{PC}=0^{++}, 2^{++}$ : results for masses



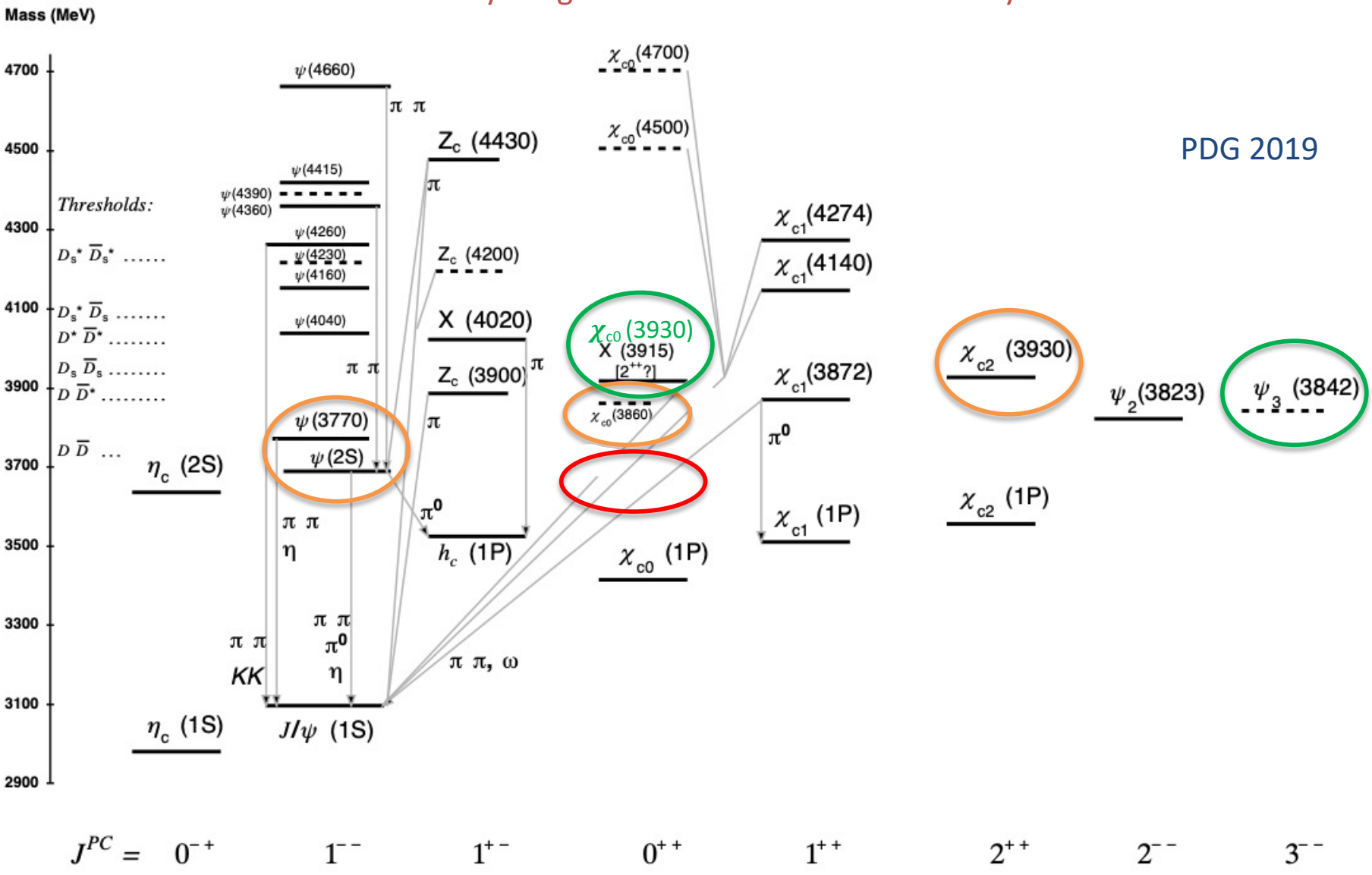
# Challenges

- accurate determination of highly excited $E_n$
- several $J^P$ contribute to each irrep
- extraction of scattering matrices for coupled-channel scattering

# Simplifications / assumptions

- $J^{PC}=0^{++}$ : assuming that channels $\boldsymbol{\eta_c\ \eta}$ and $\mathbf{J/\psi\ \omega}$ are decoupled from $D\bar{D}$, $D_s\bar{D}_s$
- further assumptions: see section 5 of 2011.02541, JHEP

# Kind of a summary ... again a look at the charmonium system




PDG 2019

$J^{PC} =$ $0^{-+}$ $1^{--}$ $1^{+-}$ $0^{++}$ $1^{++}$ $2^{++}$ $2^{--}$ $3^{--}$

our lattice study: postdicted | predicted, exp discovered | predicted, exp not (yet) discovered

# Backup

M. Padmanath, S. Collins, D. Mohler, S. Piemonte, S.P., A. Schafer, S. Weishaeupl : 1811.04116, PRD 2019

# The challenge to determine $J^P$

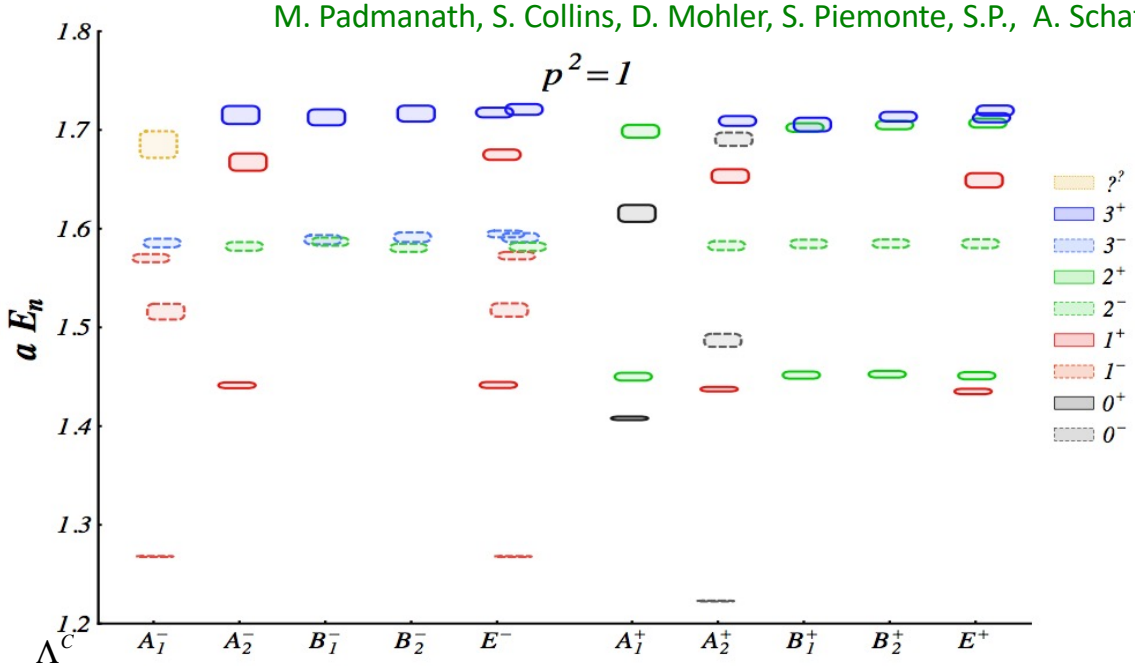
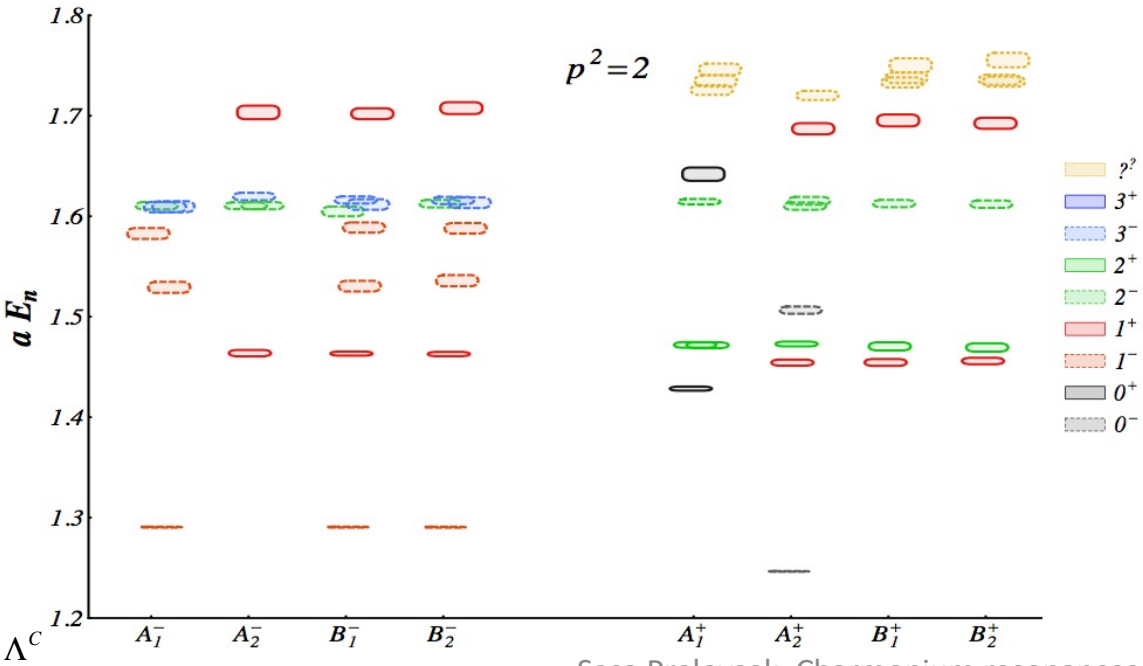



FIG. 11. $J^P$-identified charmonium spectrum in the moving frame with $\mathbf{p}=(0,0,1)$. Irreps $\Lambda^C$ of group $Dic_4$ are presented. The colors indicate $J^P$ of states according to the color-coding (21).

| $\mathbf{p}=(0,0,1)$, $Dic_4$ | | |
|---|---|---|
| $\Lambda$ $(dim)$ | $\lvert\lambda\rvert^{\tilde{\eta}}$ | $J^P$ (at rest) |
| $A_1$ (1) | $0^+$ | $0^+,\ 1^-,\ 2^+,\ 3^-$ |
| $A_2$ (1) | $0^-$ | $0^-,\ 1^+,\ 2^-,\ 3^+$ |
| $E$ (2) | 1 | $1^\pm,\ 2^\pm,\ 3^\pm$ |
| | 3 | $3^\pm$ |
| $B_1$ (1) | 2 | $2^\pm,\ 3^\pm$ |
| $B_2$ (1) | 2 | $2^\pm,\ 3^\pm$ |




| $\mathbf{p}=(1,1,0)$, $Dic_2$ | | |
|---|---|---|
| $\Lambda$ $(dim)$ | $\lvert\lambda\rvert^{\tilde{\eta}}$ | $J^P$ (at rest) |
| $A_1$ (1) | $0^+$ | $0^+,\ 1^-,\ 2^+,\ 3^-$ |
| | 2 | $2^\pm,\ 3^\pm$ |
| $A_2$ (1) | $0^-$ | $0^-,\ 1^+,\ 2^-,\ 3^+$ |
| | 2 | $2^\pm,\ 3^\pm$ |
| $B_1$ (1) | 1 | $1^\pm,\ 2^\pm,\ 3^\pm$ |
| | 3 | $3^\pm$ |
| $B_2$ (1) | 1 | $1^\pm,\ 2^\pm,\ 3^\pm$ |
| | 3 | $3^\pm$ |

1811.04116, PRD 2019

# T(E) near poles

$$t_{ij} \sim \frac{c_i\ c_j}{(E^p_{cm})^2 - E^2_{cm}} \quad \text{for} \quad E_{cm} \simeq E^p_{cm}\ , \qquad i = \ 1\ (D\bar{D}),\ 2\ (D_s\bar{D}_s)$$