Scattering of hadrons with spin and the Roper resonance

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Outline

Simulating scattering of two particles with spin on the lattice

- motivation
- the relation to extract the scattering matrix from energies is known [Collin Morningstar's talk]
- construction of operators (interpolators) by three different methods that give consistent results: reassuring
- example: Nucleon-pion scattering in the channel with $J^P=1/2^+$

lattice results and implications for the Roper resonance

Current status
of hadron-hadron scattering from lattice

- most detailed scattering results exist only for spin-less particles

  $\pi\pi$, $K\pi$, KK, DK, $D\pi$, ...

- $H^{(1)}$ $H^{(2)}$: where one or both H carry spin was explored mostly only for L=0

  many interesting channels still unexplored, particularly for L>0

  there are only few simulations for L>0 using Luscher-type method:

  example: Amy Nicholson's talk:

  Two-Nucleon Higher partial wave scattering from LQCD

  Berkowitz, Kurth, Nicholson, Joo, Rinaldi, Strother, Vranas, Walker-Loud

  1508.00886, Phys. Lett. B (2017)

  talks by Morningstar and Hortz

Motivation

- <u>in lattice QCD:</u>

  hadron-hadron scattering $H^{(1)}\ H^{(2)}$

  where H is one of P,V,N hadrons, which is (almost) stable with respect to strong decay:

P=psuedoscalar ($J^P=0^{\wedge -}$) = π , K, D, B, η_c , ...

V=vector ($J^P=1^{\wedge -}$) = D*, B*, J/ψ, Υ_b , B_c^*,... (but not ρ as is unstable...)

N=nucleon ($J^P=1/2^{\wedge +}$) = p, n, Λ, Λ_c, Σ, ... (but not $N^-(1535)$ as is unstable...)

I will consider interpolators for channels :

PV: meson resonances and <u>Q</u>Q-like exotics (for example Z_c in π J/ψ, D <u>D</u>* ..)

PN, VN: baryon resonances (e.g. in π N, K N ...) and pentaquarks (e.g. P_c in J/ψ N channel)

NN: nucleon-nucleon and deuterium, baryon-baryon

- <u>in any lattice field theory</u> (beyond SM)

  scattering channels with vector bosons and fermions

The need for interpolators

$$\langle O_i(t) | O_j^+(0) \rangle \rightarrow E_n \rightarrow \text{scattering matrix M}$$

O=HH needed to create/annihilate HH

Relation between scattering matrix M and energies E_n are known

- two spinless particles Luscher (1991):

- two particles with arbitrary spin

  Briceno, PRD89, 074507 (2014)

  (other authors: some specific cases)

scattering matrix: $S = I + i\,M = e^{2i\delta}$

$$\det_{oc}\left[\det_{lSJm_J}\left[\mathcal{M}^{-1} + \delta\mathcal{G}^V\right]\right] = 0$$

$$[\delta\mathcal{G}_j^V]_{Jm_J,lS;J'm_{J'},l'S'} = \frac{ik_j^*\delta_{SS'}}{8\pi E^*} n_j \left[\delta_{JJ'}\delta_{m_J m_{J'}}\delta_{ll'} + i\sum_{l'',m''}\frac{(4\pi)^{3/2}}{k_j^{*l''+1}} c^{\mathbf{d}}_{l''m''}(k_j^{*2};L)\right.$$

$$\left.\times \sum_{m_l,m_{l'},m_S}\langle lS, Jm_J | lm_l, Sm_S\rangle\langle l'm_{l'}, Sm_S | l'S, J'm_{J'}\rangle \int d\Omega\, Y^*_{l,m_l} Y^*_{l'',m''} Y_{l',m_{l'}}\right]$$

$$c^{\mathbf{d}}_{lm}(k_j^{*2};L) = \frac{\sqrt{4\pi}}{\gamma L^3}\left(\frac{2\pi}{L}\right)^{l-2} \mathcal{Z}^{\mathbf{d}}_{lm}[1;(k_j^* L/2\pi)^2]$$

related to eigen-energy E_n

Some previous related work on lattice HH operators for hadrons with spin and L≠0

Partial-wave method for HH:
Berkowitz, Kurth, Nicolson, Joo, Rinaldi, Strother, Walker-Loud, 1508.00886
Wallace, Phys. Rev. D92, 034520 (2015), [arXiv:1506.05492]

Projection method for HH:
M. Göckeler et al., Phys.Rev. D86, 094513 (2012), [arXiv:1206.4141].

Helicity operators for single-H:
Thomas, Edwards and Dudek, Phys. Rev. D85, 014507 (2012), [arXiv:1107.1930]

Some aspects of helicity operators for HH:
Wallace, Phys. Rev. D92, 034520 (2015), [arXiv:1506.05492].
Dudek, Edwards and Thomas, Phys. Rev. D86, 034031 (2012), [arXiv:1203.6041].

Which CG of H_1 and H_2 to H_1H_2 irreps are nonzero; values of CG not published:
Moore and Fleming, Phys. Rev. D 74, 054504 (2006), [arXiv:hep- lat/0607004].

Tetraqurak operators (appeared after our paper on operators)
Cheung, Thomas, Dudek, Edwards [1709.01417, JHEP 2017]

etc ...

However: for a lattice practitioner who was interested in a certain channel, for example
(PV scattering in L=2 or VN scattering with λ_V=1 and λ_N=1/2)
there were still lots of puzzles to beat before constructing a reliable interpolator ..

Constructing HH operators for scattering with spin: outline

based on S. P., U. Skerbis, C.B. Lang: arXiv:1607:06738, JHEP 2017

- three different methods to construct operators
- illuminate the proofs (given in the paper)
- verify they lead to consistent operators (that gives confidence in each one of them)
- they lead to complementary physics info
- present explicit ops for PV, PN, VN, NN for lowest two momentum shells.

We restrict to total momentum zero

$H^{(1)}(p)\, H^{(2)}(-p)$, $P_{tot}=0$

Advantage of $P_{tot}=0$:

- parity P is a good number
- channels with even and odd L do not mix in the same irrep

} not true for $P_{tot}\neq 0$

Building blocks H: required transformation properties of H

to prove correct transformation properties of HH

rotations R — Wigner D matrix — inversion I — $|p,s,m_s\rangle \equiv H^\dagger_{m_s}(p)|0\rangle$

$$R|p,s,m_s\rangle = \sum_{m'_s} \overbrace{D^s_{m'_s m_s}(R)}^{\text{Wigner D matrix}} |Rp,s,m'_s\rangle \,, \qquad I|p,s,m_s\rangle = (-1)^P|-p,s,m_s\rangle$$ state

$$RH^\dagger_{m_s}(p)R^{-1} = \sum_{m'_s} D^s_{m'_s m_s}(R) H^\dagger_{m'_s}(Rp) \,, \qquad IH^\dagger_{m_s}(p)I = (-1)^P H^\dagger_{m_s}(-p)\,.$$ creation field

note: D ➔ D*

$$RH_{m_s}(p)R^{-1} = \sum_{m'_s} \underbrace{D^s_{m'_s m_s}(R)^*}_{D^s_{m_s m'_s}(R^{-1})} H_{m'_s}(Rp) \,, \qquad IH_{m_s}(p)I = (-1)^P H_{m_s}(-p)$$ annihilation field

m_s is a good quantum number at p=0: $S_z H_{m_s}(0) S_z^{-1} = m_s H_{m_s}(0)$

m_s is not good quantum number in general for p≠0: in this case it denotes eigenvalue of S_z of corresponding $H_{ms}(p=0)$

Non-practical choice of $H_{ms}(p)$: canonical fields $H^{(c)}$

with correct transformation properties under R and I

$$H^{(c)}_{m_s}(p) \equiv L(p) H_{m_s}(0)$$

L(p) is boost from 0 to p; drawback: $H^{(c)}(p)$ depend on m, E,..

$$V_{m_s=1}(0) = \frac{1}{\sqrt{2}}[-V_x(0) + iV_y(0)] \;\rightarrow\; V^{(c)}_{m_s=1}(p_x) = \frac{1}{\sqrt{2}}[-\gamma V_x(p_x) + iV_y(p_x)]$$

$$\begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix} \xrightarrow{\Lambda^1(p_x)} \begin{pmatrix} \gamma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix} = \begin{pmatrix} -\gamma \\ i \\ 0 \end{pmatrix}$$

$$N_{m_s=1/2}(0) = \mathcal{N}_1(0) \;\rightarrow\; N^{(c)}_{m_s=1/2}(p_x) \propto \mathcal{N}_1(p_x) + \frac{p_x}{E+m}\mathcal{N}_4(p_x)$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\Lambda^{1/2}(p_x)} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \frac{p_x}{E+m} \end{pmatrix}$$

$\mathcal{N}_{\mu=1,..,4}$ are Dirac components in Dirac basis

Non-practical choice of H: canonical fields $H^{(c)}$

with correct transformation properties under R and I

$$H^{(c)}_{m_s}(p) \equiv L(p) H_{m_s}(0)$$

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$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\Lambda^{1/2}(p_x)} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \frac{p_x}{E+m} \end{pmatrix}$$

$\mathcal{N}_{\mu=1,..,4}$ are Dirac components in Dirac basis

Practical choice of $H_{ms}(p)$

with correct transformation properties under R and I

$$V_{m_s=\pm1}(p) = \frac{1}{\sqrt{2}}[\mp V_x(p) + iV_y(p)], \quad V_{m_s=0}(p) = V_z(p)$$

$$N_{m_s=1/2}(p) = \mathcal{N}_{\mu=1}(p)\ , \quad N_{m_s=-1/2}(p) = \mathcal{N}_{\mu=2}(p)$$

$$P(p) = \sum_x \bar{q}(x)\gamma_5 q(x) e^{ipx}$$

$$V_i(p) = \sum_x \bar{q}(x)\gamma_i q(x) e^{ipx}, \;\; i = x, y, z$$

$$\mathcal{N}_\mu(p) = \sum_x \epsilon_{abc}[q^{aT}(x) C\gamma_5 q^b(x)]\; q^c_\mu(x)\; e^{ipx}, \quad \mu = 1, .., 4$$

These H are employed as building block in our HH operators

simple examples

Required transformation properties of O=HH

$$RO^{J,m_J}(P_{tot}=0)R^{-1} = \sum_{m'_J} D^J_{m_J m'_J}(R^{-1})O^{J,m'_J}(0) \qquad IO^{J,m_J}(0)I = (-1)^P O^{J,m_J}(0)$$

continuum R

good parity since P_{tot}=0 !

relevant rotations: $R \in O^{(2)}$ O with 24 el. for J=integer ; O^2 with 48 elements for J=half-integer

The group including inversion I: O_h with 48 el. for J=integer ; O^2_h with 96 elements for J=half-integer

The representation O^J is irreducible under continuum R, but it is reducible under discrete R in $O^{(2)}$.

The operators should transform according to certain irreducible representation Γ and its row r.

$$R|\Gamma, r\rangle = \sum_{r'} T^{\Gamma}_{r',r}(R)|\Gamma, r'\rangle \quad R \in O^{(2)}, \qquad I|\Gamma, r\rangle = (-1)^P|\Gamma, r\rangle \ ,$$

discrete R

$$RO_{\Gamma,r}R^{-1} = \sum_{r'} \underbrace{T^{\Gamma}_{r,r'}(R^{-1})}O_{\Gamma,r'} \quad R \in O^{(2)}, \qquad IO_{\Gamma,r}I = (-1)^P O_{\Gamma,r}$$

T(R) given for all irreps in
Bernard, Lage, Meißner, Rusetsky,
JHEP 2008, 0806.4495
We use same conventions for rows.

J	Γ ($\dim_\Gamma$)
0	$A_1(1)$
$\frac{1}{2}$	$G_1(2)$
1	$T_1(3)$
$\frac{3}{2}$	$H(4)$
2	$E(2) \oplus T_2(3)$
$\frac{5}{2}$	$H(4) \oplus G_2(2)$
3	$A_2(1) \oplus T_1(3) \oplus T_2(3)$

Method I: Projection operators

$$O_{|p|,\Gamma,r,n} = \sum_{\tilde{R}\in O_h^{(2)}} T^{\Gamma}_{r,r}(\tilde{R})\ \tilde{R}\ H^{(1),a}(p)\ H^{(2),a}(-p)\ \tilde{R}^{-1}\ , \qquad n = 1,..,n_{max}$$

T(R) given for all irreps in
Bernard, Lage, Meißner, Rusetsky,
JHEP 2008, 0806.4495

"seed": Each H^a can have any polarization m_s and direction p with given |p|. Different choices lead to different linearly independent O_n

Some examples for |p|=1:

PV in T_1^+ , n_{max}=2:

$$O_{T_1^+,r=3,n=1} = \mathrm{P}(e_z)V_z(-e_z) + \mathrm{P}(-e_z)V_z(e_z)$$
$$O_{T_1^+,r=3,n=2} = \mathrm{P}(e_x)V_z(-e_x) + \mathrm{P}(-e_x)V_z(e_x) + \mathrm{P}(e_y)V_z(-e_y) + \mathrm{P}(-e_y)V_z(e_y)$$

PN in H^+ , n_{max}=1:

$$O_{H^+,r=1} = -iN_{\frac{1}{2}}(-e_x)\mathrm{P}(e_x) + iN_{\frac{1}{2}}(e_x)\mathrm{P}(-e_x) - N_{\frac{1}{2}}(-e_y)\mathrm{P}(e_y) + N_{\frac{1}{2}}(e_y)\mathrm{P}(-e_y)$$

VN in H^- , n_{max}=3:

$$O_{H^-,r=1,n=1} = iN_{\frac{1}{2}}(e_x)V_x(-e_x) + iN_{\frac{1}{2}}(-e_x)V_x(e_x) + N_{\frac{1}{2}}(e_y)V_y(-e_y) + N_{\frac{1}{2}}(-e_y)V_y(e_y)$$
$$O_{H^-,r=1,n=2} = \ldots$$
$$O_{H^-,r=1,n=3} = \ldots$$

Disadvantage:

not informative which continuum numbers (partial wave L or helicity) each O_n corresponds

This is remedied in next two methods

Method II: Partial-wave operators

Starting annihilation operator
(before sobduction to irreps)

Clebsch-Gordans

Spherical Harmonics

building blocks H
mentioned on slide 10 (bottom)

$$O^{|p|,J,m_J,S,L} = \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O} Y^*_{Lm_L}(\widehat{Rp})\; H^{(1)}_{m_{s1}}(Rp)\; H^{(2)}_{m_{s2}}(-Rp)$$

Proposed for NN in [Berkowitz, Kurth, Nicolson, Joo, Rinaldi, Strother, Walker-Loud, CALLAT, 1508.00886] There Y_{lm}^* appears where we have Y_{lm}

Proof (in our paper and next slide): the correct transformation properties

$$R_a O^{J,m_J,S,L} R_a^{-1} = \sum_{m'_J} D^J_{m_J m'_J}(R_a^{-1}) O^{J,m'_J,S,L}$$

follow from transformations of H (slide 8) and properties of C, Y_{lm} and D.

Example of PV operators

$$O^{|p|=1,J=1,m_J=0,L=0,S=1} = \sum_{p=\pm e_x,\pm e_y,\pm e_z} \mathrm{P}(p) V_z(-p)\ ,$$

$$O^{|p|=1,J=1,m_J=0,L=2,S=1} = \sum_{p=\pm e_x,\pm e_y} \mathrm{P}(p) V_z(-p) - 2\sum_{p=\pm e_z} \mathrm{P}(p) V_z(-p)$$

Subduction to irreps discussed later on.

Proof: partial-wave operators

$$O^{|p|,J,m_J,S,L} = \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O} Y^*_{Lm_L}(\widehat{Rp})\; H^{(1)}_{m_{s1}}(Rp)\; H^{(2)}_{m_{s2}}(-Rp)$$

Proof of correct transformation properties:

$$\begin{aligned} R_a O^{J,m_J,S,L} R_a^{-1} &= \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O^{(2)}} Y^*_{Lm_L}(\hat{R}p)\; \underbrace{R_a H_{m_{s1}}(Rp) H_{m_{s2}}(-Rp) R_a^{-1}} \\ &= \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O_h} Y^*_{Lm_L}(\hat{R}p) \\ &\quad \times \sum_{m'_{s1}} D^{s_1}_{m_{s1}m'_{s1}}(R_a^{-1}) H_{m'_{s1}}(R_aRp) \sum_{m'_{s2}} D^{s_2}_{m_{s2}m'_{s2}}(R_a^{-1}) H_{m'_{s2}}(-R_aRp)\ , \end{aligned}$$

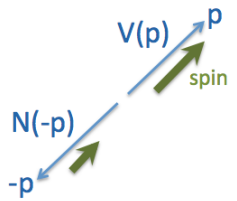
$$Y^*_{Lm_L}(Rp) = Y^*_{Lm_L}(R_a^{-1}(R'p)) = \sum_{m'_L} D^L_{m_Lm'_L}(R_a^{-1}) Y^*_{Lm'_L}(R'p) \qquad R' \equiv R_aR \qquad Y^*_{Lm_L}(R_1p) = \textstyle\sum_{m'_L} D^L_{m_Lm'_L}(R_1) Y^*_{Lm'_L}(p)$$

$$D^{s_1}_{m_{s1}m'_{s1}}(R_a^{-1}) D^{s_2}_{m_{s2}m'_{s2}}(R_a^{-1}) = \sum_{\tilde{S},\tilde{m}_S,m'_S} C^{\tilde{S},\tilde{m}_S}_{s_1m_{s1},s_2m_{s2}} C^{\tilde{S},m'_S}_{s_1m'_{s1},s_2m'_{s2}}\; D^{\tilde{S}}_{\tilde{m}_Sm'_S}(R_a^{-1}) \qquad \sum_{m_{s1},m_{s2}} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} C^{\tilde{S},\tilde{m}_S}_{s_1m_{s1},s_2m_{s2}} = \delta_{m_S,\tilde{m}_S}\delta_{S,\tilde{S}}$$

$$D^L_{m_Lm'_L}(R_a^{-1}) D^{\tilde{S}}_{\tilde{m}_Sm'_S}(R_a^{-1}) = \sum_{\tilde{J},\tilde{m}_J,m'_J} C^{\tilde{J},\tilde{m}_J}_{Lm_L,\tilde{S}\tilde{m}_S} C^{\tilde{J},m'_J}_{Lm'_L,\tilde{S}m'_S}\; D^{\tilde{J}}_{\tilde{m}_Jm'_J}(R_a^{-1}) \qquad \sum_{m_L,m_S} C^{Jm_J}_{Lm_L,Sm_S} C^{\tilde{J},\tilde{m}_J}_{Lm_L,Sm_S} = \delta_{m_J,\tilde{m}_J}\delta_{J,\tilde{J}}$$

$$\begin{aligned} &R_a O^{J,m_J,S,L} R_a^{-1} = \\ &= \sum_{m'_J} D^J_{m_Jm'_J}(R_a^{-1}) \sum_{m'_L,m'_S,m'_{s1},m'_{s2}} C^{Jm'_J}_{Lm'_L,Sm'_S} C^{Sm'_S}_{s_1m'_{s1},s_2m'_{s2}} \sum_{R'\in O^{(2)}} Y^*_{Lm'_L}(\hat{R'}p) H_{m'_{s1}}(R'p) H_{m'_{s2}}(-R'p) \\ &= \sum_{m'_J} D^J_{m_Jm'_J}(R_a^{-1}) O^{J,m'_J,S,L} \end{aligned}$$

Method III: helicity operators



[HH in continuum: Jacob, Wick (1959)]
[for single H on lattice: HSC, Thomas et al. (2012)]
[not widely used for HH on lattice yet]

- building blocks in partial-wave operators are $H_{ms}(p)$ and m_s is not good for $p \neq 0$:
- Helicity λ is projection of S to p. It is good also for particles in flight $h \equiv S \cdot p \,/\, |p|$
- Definition of single-hadron helicity operator

rotation from p_z to p

$$H^h_\lambda(p) \equiv R^p_0 \; H_{m_s=\lambda}(p_z) \; (R^p_0)^{-1}$$

denoted by superscript h

good m_s

- Helicity is not modified under R (p and S transform the same way) $R H^h_\lambda(p) R^{-1} = e^{i\varphi(R)} H^h_\lambda(Rp)$

p is arbitrary momentum in given shell |p|; R does not modify $\lambda_{1,2}$, so $H_{1,2}$ have chosen $\lambda_{1,2}$ in all terms

- Two-hadron O:

$$\boxed{O^{|p|,J,m_J,\lambda_1,\lambda_2,\lambda} = \sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R) \; R \; H^{(1),h}_{\lambda_1}(p) \; H^{(2),h}_{\lambda_2}(-p) \; R^{-1}}$$

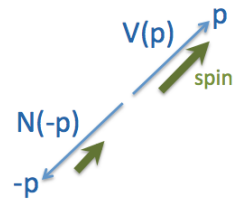
- Proof:

$$\begin{aligned}
R_a O^{J,m_J,\lambda_1,\lambda_2} R_a^{-1} &= \sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R) \; R_a R \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R^{-1} R_a^{-1} \\
&= \sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R_a^{-1} R') \; R' \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R'^{-1} \\
&= \sum_{R'\in O^{(2)}} \sum_{m'_J} D^J_{m_J,m'_J}(R_a^{-1}) D^J_{m'_J,\lambda}(R') \; R' \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R'^{-1} \\
&= \sum_{m'_J} D^J_{m_J,m'_J}(R_a^{-1}) \; O^{J,m'_J,\lambda_1,\lambda_2} \; ,
\end{aligned}$$

$R' = R_a R$

$D(R_1 R_2) = D(R_1) D(R_2)$

Method III: helicity operators (continued)



Using definitions of $H^h_\lambda(p) \equiv R^p_0 \; H_{m_s=\lambda}(p_z) \; (R^p_0)^{-1}$ and parity projection $\frac{1}{2}(\mathcal{O} + PI\mathcal{O}I)$

p is arbitrary momentum in given shell |p|

$$O^{|p|,J,m_J,P,\lambda_1,\lambda_2,\lambda} = \frac{1}{2}\sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R) \; RR^p_0 \; [H^{(1)}_{m_{s_1}=\lambda_1}(p_z) H^{(2)}_{m_{s_2}=-\lambda_2}(-p_z)$$

$$+ P \; I \; H^{(1)}_{m_{s_1}=\lambda_1}(p_z) H^{(2)}_{m_{s_2}=-\lambda_2}(-p_z) \; I] \; (R^p_0)^{-1} R^{-1}$$

$$\lambda = \lambda_1 - \lambda_2$$

- H are building blocks from slide 10 (bottom): actions of R and I on $H_{ms}(p)$ are given in slide 8
- p is arbitrary momentum in given shell |p|; there are several choices of R_0^p which rotate from p_z to p:
  - these lead to different phases in definition of H_λ^h : inconvenience
  - but they lead to the same O above (modulo irrelevant overall factor): so no problem for such construction
- Simple choice for momentum shell |p|=1 : $p=p_z$ and R_0^p=Identity
- paper provides details how to use functions from Mathematica for construction, also since Mathematica uses non-conventional defnition of D

$$D^j_{m,m'}[R^\omega_{\alpha\beta\gamma}] = F \cdot \texttt{WignerD}[\{j,m,m'\},-\alpha,-\beta,-\gamma], \qquad F = \begin{cases} 1 \; : \; j = \text{integer} \\ \pm 1 : \; j = \text{halfinteger}, \; \mathrm{F}(\omega + 2\pi) = -\mathrm{F}(\omega) \end{cases}$$

, choice of sign in our paper

$$\{\alpha,\beta,\gamma\} = \underbrace{\texttt{EulerAngles}}_{\text{MATHEMATICA}}[T] \quad T = \exp(-i\vec{n}\vec{J}\omega) \text{ and } (J_k)_{ij} = -i\epsilon_{ijk}$$

one last step before reaching the results ...

Subduction of $O^{J,mJ}$ to irreducible representations

subduction

continum R

discrete R in discrete group $O^{(2)}$

Partial-wave operators $O^{J, mJ, L, S}$

Helicity operators $O^{J, mJ, \lambda 1, \lambda 2}$

$$O^{[J,S,L]}_{|p|,\Gamma,r} = \sum_{m_J} \mathcal{S}^{J,m_J}_{\Gamma,r} O^{|p|,J,m_J,S,L}$$

$$O^{[J,P,\lambda_1,\lambda_2,\lambda]}_{|p|,\Gamma,r} = \sum_{m_J} \mathcal{S}^{J,m_J}_{\Gamma,r} O^{|p|,J,m_J,P,\lambda_1,\lambda_2,\lambda}$$

Subduction matrices S

[Dudek et al., PRD82, 034508 (2010)

Edwards et al, PRD84, 074508 (2011)]

The representation O^J is irreducible under continuum R.

But it is reducible under R in discrete group lattice $O^{(2)}$.

Operators that transform according to irrep Γ and row r obtained via subduction.

Single-hadron operators H: experience by Hadron Spectrum collaboration Phys. Rev. D 82, 034508 (2010)

- subduced operators $O^{[J]}_{\Gamma}$ carry memory of continuum spin and dominantly couple to states with this J

Expectation for partial-wave and helicity operators HH obtained by subduction :

- $O^{[J,S,L]}_{|p|,\Gamma,r}$ would dominantly couple to eigen-states with continuum (J,L,S)
- $O^{[J,P,\lambda_1,\lambda_2,\lambda]}_{|p|,\Gamma,r}$ would dominantly couple to eigen-states with continuum (J,λ1,λ2)

valuable for simulations

give physics intuition on quant. num.

Example: P(p)V(-p) operators

J	Γ ($\dim_\Gamma$)
0	$A_1(1)$
$\frac{1}{2}$	$G_1(2)$
1	$T_1(3)$
$\frac{3}{2}$	$H(4)$
2	$E(2) \oplus T_2(3)$
$\frac{5}{2}$	$H(4) \oplus G_2(2)$
3	$A_2(1) \oplus T_1(3) \oplus T_2(3)$

row=1 provided

Conventions for row
Bernard et al. , 0806.4495

rows of T1: (x,y,z)

rows of T2: (yz,xz,xy)

$|p| = 0$

T_1^+ :

$$O_{T_1^+,r=1} = \mathrm{P}(0)V_x(0)$$

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1}$$

other irreps: O=0

$|p| = 1$

A_1^- :

$$O_{A_1^-,r=1} = \mathrm{P}(e_x)V_x(-e_x) - \mathrm{P}(-e_x)V_x(e_x) + \mathrm{P}(e_y)V_y(-e_y) - \mathrm{P}(-e_y)V_y(e_y) + \mathrm{P}(e_z)V_z(-e_z) - \mathrm{P}(-e_z)V_z(e_z)$$

$$O^{[J=0,m_J=0,P=-,\lambda_V=0,\lambda_P=0]}_{A_1^-,r=1} = O^{[J=0,m_J=0,L=1,S=1]}_{A_1^-,r=1} = O_{A_1^-,r=1}$$

T_1^+ :

$$O_{T_1^+,r=1,n=1} = \mathrm{P}(e_x)V_x(-e_x) + \mathrm{P}(-e_x)V_x(e_x)$$
$$O_{T_1^+,r=1,n=2} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) + \mathrm{P}(e_z)V_x(-e_z) + \mathrm{P}(-e_z)V_x(e_z)$$

$$O^{[J=1,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=2}$$
$$O^{[J=1,P=+,\lambda_V=0,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1}$$

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$
$$O^{[J=1,L=2,S=1]}_{T_1^+,r=1} = -2\, O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$

T_1^- :

$$O_{T_1^-,r=1} = -\mathrm{P}(e_y)V_z(-e_y) + \mathrm{P}(-e_y)V_z(e_y) + \mathrm{P}(e_z)V_y(-e_z) - \mathrm{P}(-e_z)V_y(e_z)$$

$$O^{[J=1,P=-,\lambda_V=\pm1,\lambda_P=0]}_{T_1^-,r=1} = O^{[J=1,L=1,S=1]}_{T_1^-,r=1} = O_{T_1^-,r=1}$$

T_2^+ :

$$O_{T_2^+,r=1} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) - \mathrm{P}(e_z)V_x(-e_z) - \mathrm{P}(-e_z)V_x(e_z)$$

$$O^{[J=2,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_2^+,r=1} = O^{[J=2,L=2,S=1]}_{T_2^+,r=1} = O_{T_2^+,r=1}$$

T_2^- :

$$O_{T_2^-,r=1} = \mathrm{P}(e_y)V_z(-e_y) - \mathrm{P}(-e_y)V_z(e_y) + \mathrm{P}(e_z)V_y(-e_z) - \mathrm{P}(-e_z)V_y(e_z)$$

$$O^{[J=2,P=-,\lambda_V=\pm1,\lambda_P=0]}_{T_2^-,r=1} = O^{[J=2,L=1,S=1]}_{T_2^-,r=1} = O^{[J=2,L=3,S=1]}_{T_2^-,r=1} = O_{T_2^-,r=1}$$

E^- :

$$O_{E^-,r=1} = \mathrm{P}(e_x)V_x(-e_x) - \mathrm{P}(-e_x)V_x(e_x) + \mathrm{P}(e_y)V_y(-e_y) - \mathrm{P}(-e_y)V_y(e_y) - 2\mathrm{P}(e_z)V_z(-e_z) + 2\mathrm{P}(-e_z)V_z(e_z)$$

$$O^{[J=2,P=-,\lambda_V=0,\lambda_P=0]}_{E^-,r=1} = O^{[J=2,L=1,S=1]}_{E^-,r=1} = O^{[J=2,L=3,S=1]}_{E^-,r=1} = O_{E^-,r=1}$$

$$O_{A_1^+} = O_{A_2^+} = O_{A_2^-} = O_{E^+} = 0\,.$$

Example: P(p)V(-p) operators

$\boxed{|p| = 1}$

A_1^- :

$$O_{A_1^-,r=1} = \mathrm{P}(e_x)V_x(-e_x) - \mathrm{P}(-e_x)V_x(e_x) + \mathrm{P}(e_y)V_y(-e_y) - \mathrm{P}(-e_y)V_y(e_y) + \mathrm{P}(e_z)V_z(-e_z) - \mathrm{P}(-e_z)V_z(e_z)$$

$$O^{[J=0,m_J=0,P=-,\lambda_V=0,\lambda_P=0]}_{A_1^-,r=1} = O^{[J=0,m_J=0,L=1,S=1]}_{A_1^-,r=1} = O_{A_1^-,r=1}$$

T_1^+ :

projection operators

$$O_{T_1^+,r=1,n=1} = \mathrm{P}(e_x)V_x(-e_x) + \mathrm{P}(-e_x)V_x(e_x)$$

$$O_{T_1^+,r=1,n=2} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) + \mathrm{P}(e_z)V_x(-e_z) + \mathrm{P}(-e_z)V_x(e_z)$$

provides lin. combination of projection operators O_n that enhances the coupling to state with continuum (J^P, λ_V)

helicity operators

$$O^{[J=1,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, $\lambda_V=0$

$$O^{[J=1,P=+,\lambda_V=0,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1}$$ $J^P=1^+$, $\lambda_V=1$

provides lin. combination of projection operators O_n that enhances the coupling to state with continuum (J^P, S,L)

partial-wave operators

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, S=1,L=0

$$O^{[J=1,L=2,S=1]}_{T_1^+,r=1} = -2\, O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, S=1,L=2

T_1^- :

$$O_{T_1^-,r=1} = -\mathrm{P}(e_y)V_z(-e_y) + \mathrm{P}(-e_y)V_z(e_y) + \mathrm{P}(e_z)V_y(-e_z) - \mathrm{P}(-e_z)V_y(e_z)$$

$$O^{[J=1,P=-,\lambda_V=\pm1,\lambda_P=0]}_{T_1^-,r=1} = O^{[J=1,L=1,S=1]}_{T_1^-,r=1} = O_{T_1^-,r=1}$$

T_2^+ :

$$O_{T_2^+,r=1} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) - \mathrm{P}(e_z)V_x(-e_z) - \mathrm{P}(-e_z)V_x(e_z)$$

$$O^{[J=2,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_2^+,r=1} = O^{[J=2,L=2,S=1]}_{T_2^+,r=1} = O_{T_2^+,r=1}$$

T_2^- :

$$O_{T_2^-,r=1} = \mathrm{P}(e_y)V_z(-e_y) - \mathrm{P}(-e_y)V_z(e_y) + \mathrm{P}(e_z)V_y(-e_z) - \mathrm{P}(-e_z)V_y(e_z)$$

$$O^{[J=2,P=-,\lambda_V=\pm1,\lambda_P=0]}_{T_2^-,r=1} = O^{[J=2,L=1,S=1]}_{T_2^-,r=1} = O^{[J=2,L=3,S=1]}_{T_2^-,r=1} = O_{T_2^-,r=1}$$

E^- :

$$O_{E^-,r=1} = \mathrm{P}(e_x)V_x(-e_x) - \mathrm{P}(-e_x)V_x(e_x) + \mathrm{P}(e_y)V_y(-e_y) - \mathrm{P}(-e_y)V_y(e_y) - 2\mathrm{P}(e_z)V_z(-e_z) + 2\mathrm{P}(-e_z)V_z(e_z)$$

$$O^{[J=2,P=-,\lambda_V=0,\lambda_P=0]}_{E^-,r=1} = O^{[J=2,L=1,S=1]}_{E^-,r=1} = O^{[J=2,L=3,S=1]}_{E^-,r=1} = O_{E^-,r=1}$$

$$O_{A_1^+} = O_{A_2^+} = O_{A_2^-} = O_{E^+} = 0\,.$$

J	Γ ($\dim_\Gamma$)
0	$A_1(1)$
$\frac{1}{2}$	$G_1(2)$
1	$T_1(3)$
$\frac{3}{2}$	$H(4)$
2	$E(2) \oplus T_2(3)$
$\frac{5}{2}$	$H(4) \oplus G_2(2)$
3	$A_2(1) \oplus T_1(3) \oplus T_2(3)$

$\boxed{|p| = 0}$

T_1^+ :

$$O_{T_1^+,r=1} = \mathrm{P}(0)V_x(0)$$

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1}$$

other irreps: O=0

row=1 provided

Conventions for row
Bernard et al. , 0806.4495

rows of T1: (x,y,z)

rows of T2: (yz,xz,xy)

P(1)V(-1) operators, T_1^+, row=r=1

T_1^+ :

projection op.

$$O_{T_1^+,r=1,n=1} = \mathrm{P}(e_x)V_x(-e_x) + \mathrm{P}(-e_x)V_x(e_x)$$

$$O_{T_1^+,r=1,n=2} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) + \mathrm{P}(e_z)V_x(-e_z) + \mathrm{P}(-e_z)V_x(e_z)$$

partial-wave op.

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$

$J^P=1^+$, S=1,L=0

$$O^{[J=1,L=2,S=1]}_{T_1^+,r=1} = -2\, O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$

$J^P=1^+$, S=1,L=2

helicity op.

$$O^{[J=1,P=+,\lambda_V=0,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1}$$

$J^P=1^+$, $\lambda_V=0$

$$O^{[J=1,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=2}$$

$J^P=1^+$, $\lambda_V=1$

Partial-wave and helicity operators expressed in terms of projection operators throughout and consistency is found.

Results for operators

Explicit expressions all for $H^{(1)}(p)H^{(2)}(-p)$

- PV, PN, VN, NN

- in three methods

- all irreps, $|p|=0,1$

given in [S. P., U. Skerbis, C.B. Lang, arXiv:1607:06738, JHEP 2016]

operators from three methods are consistent (not equal) with each other

Relation between partial-wave and helicity operators is derived

$$O^{|p|,J,m_J,S,L} = \sqrt{\frac{2L+1}{4\pi}} \sum_{\lambda=-S}^{S} \sum_{\lambda_1,\lambda_2} \sum_{\lambda'} D^{J}_{\lambda',\lambda}(R_0^p) C^{J\lambda}_{L0,S\lambda} C^{S\lambda}_{s_1\lambda_1,s_2\,-\lambda_2}\; O^{|p|,J,m_J,\lambda',\lambda_1,\lambda_2}$$

Nπ scattering in ½$^+$ and the Roper resonance

C.B. Lang, L. Leskovec, M. Padmanath, S.P.

Phys. Rev. D 95 (2017) 014510; hep-lat:1610.01422

Brief intro to Roper resonance

uud

$N(1440)$ BREIT-WIGNER MASS	1410 to 1450 ($\approx$ 1430) MeV
$N(1440)$ BREIT-WIGNER WIDTH	250 to 450 ($\approx$ 350) MeV
$N\pi$	55 – 75%
$N\eta$	< 1%
$N\pi\pi$	25–50%
$\Delta(1232)\pi$	20–30%
$\Delta(1232)\pi$, $P-wave$	13 – 27%
$N\sigma$	11 – 23%

p
n
$N(1440)\ 1/2^+$
$N(1520)\ 3/2^-$
$N(1535)\ 1/2^-$
$N(1650)\ 1/2^-$
$N(1675)\ 5/2^-$
$N(1680)\ 5/2^+$
$N(1685)\ ?^?$
$N(1700)\ 3/2^-$
$N(1710)\ 1/2^+$
$N(1720)\ 3/2^+$
$N(1860)\ 5/2^+$
$N(1875)\ 3/2^-$
$N(1880)\ 1/2^+$
$N(1895)\ 1/2^-$
$N(1900)\ 3/2^+$
$N(1990)\ 7/2^+$
$N(2000)\ 5/2^+$
$N(2040)\ 3/2^+$
$N(2060)\ 5/2^-$
$N(2100)\ 1/2^+$

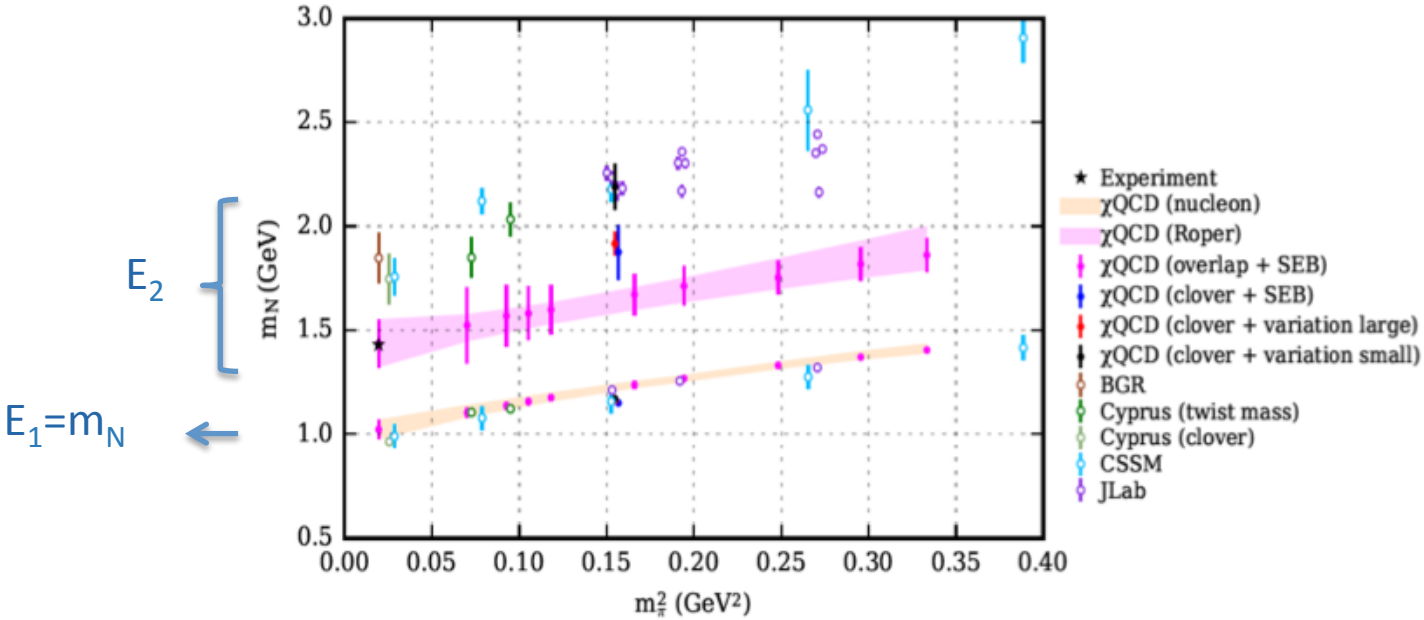
Puzzling since its discovery in 1964 by L.D. Roper.

In particular: why is it lighter than N(1535) with $\frac{1}{2}^-$?

Some previous simulations of the proton/Roper channel: $J^P=1/2^+$

- all used just O=qqq interpolators (with exception of Adelaide 1608.03051 which did not find two-hadron state in spite of that)
- ignored that Roper is strongly decaying resonance
- assumed that $E_1=m_N$ (correct)

  $E_2=m_R$ (not correct); E_2 could in principle be energy of Nπ eigenstate



χQCD : Liu *et al.*, arXiv:1403.6847[hep-ph]

BGR : Engel *et al.*, PRD, arXiv:1301.4318[hep-lat]

Cyprus : Alexandrou *et al.*, PRD, arXiv:1411.6765[hep-lat]

JLab : Edwards *et al.*, PRD, arXiv:1104.5152[hep-lat]

CSSM : Adelaide group, PLB, arXiv:1011.5724[hep-lat] Figure courtesy : K. F. Liu, arXiv:1609.02572

Lattice simulation

Lattice size	N_f	$N_{\rm cfgs}$	m_π [MeV]	a [fm]	L [fm]
$32^3 \times 64$	$2+1$	197(193)	156(7)(2)	0.0907(13)	2.9

PACS-CS lattices, Aoki *et al.*, PRD, arXiv:0807.1661.

- Wilson clover fermions

- Lowest non-interacting N(1)π(-1) states in p-wave expected at

$$E \approx \sqrt{(\tfrac{2\pi}{L})^2 + m_\pi^2} + \sqrt{(\tfrac{2\pi}{L})^2 + m_N^2} \approx 1.5\,\mathrm{GeV}$$

This is in the Roper resonance region: favorable

Implementing meson-nucleon interpolators in $J^P=1/2^+$ channel
(for the first time in this channel)

- only total momentum P=0 is simulated
- P≠0 not used (since p-wave mixes with s-wave)

momenta of hadrons in units of 2π/L

$$O^{N\pi}_{1,2} = -\sqrt{\tfrac{1}{3}}\,\Big[p^{1,2}_{-\frac{1}{2}}(-e_x)\pi^0(e_x) - p^{1,2}_{-\frac{1}{2}}(e_x)\pi^0(-e_x) - ip^{1,2}_{-\frac{1}{2}}(-e_y)\pi^0(e_y) + ip^{1,2}_{-\frac{1}{2}}(e_y)\pi^0(-e_y) + p^{1,2}_{\frac{1}{2}}(-e_z)\pi^0(e_z) - p^{1,2}_{\frac{1}{2}}(e_z)\pi^0(-e_z)\Big] + \sqrt{\tfrac{2}{3}}\,\big[\{p\to n, \pi^0\to\pi^+\}\big] \quad [narrower]$$

$$O^{N_w}_{3,4,5} = p^{1,2,3}_{\frac{1}{2}}(0) \quad [wider]$$

$$O^{N_n}_{6,7,8} = p^{1,2,3}_{\frac{1}{2}}(0) \quad [narrower]$$

$$O^{N\sigma}_{9,10} = p^{1,2}_{\frac{1}{2}}(0)\sigma(0) \quad [narrower]$$

Nπ in p-wave: irrep G_1^+

all three methods give one and the same O

"Lattice operators for scattering of particles with Spin", JHEP 2017,

S. P., U. Skerbis, C.B. Lang

Nσ in s-wave

$$N^i_{m_s=1/2}(\mathbf{n}) = \mathcal{N}^i_{\mu=1}(\mathbf{n})\ ,\ N^i_{m_s=-1/2}(\mathbf{n}) = \mathcal{N}^i_{\mu=2}(\mathbf{n})$$

$$\mathcal{N}^i_\mu(\mathbf{n}) = \sum_{\mathbf{x}} \epsilon_{abc}[u^{aT}(\mathbf{x},t)\Gamma^i_2 d^b(\mathbf{x},t)]\ [\Gamma^i_1 q^c(\mathbf{x},t)]_\mu\ e^{i\mathbf{x}\cdot\mathbf{n}\frac{2\pi}{L}}$$

$$i=1,2,3: \quad (\Gamma^i_1,\Gamma^i_2) = (\mathbf{1}, C\gamma_5),\ (\gamma_5, C),\ (i\mathbf{1}, C\gamma_t\gamma_4)$$

$$\pi^+(\mathbf{n}) = \sum_{\mathbf{x}} \bar{d}(\mathbf{x},t)\gamma_5 u(\mathbf{x},t) e^{i\mathbf{x}\cdot\mathbf{n}\frac{2\pi}{L}}$$

$$\sigma(0) = \frac{1}{\sqrt{2}}\sum_{\mathbf{x}}[\bar{u}(\mathbf{x},t)u(\mathbf{x},t) + \bar{d}(\mathbf{x},t)d(\mathbf{x},t)]\ .$$

Computing 10x10 matrix C: Wick contractions

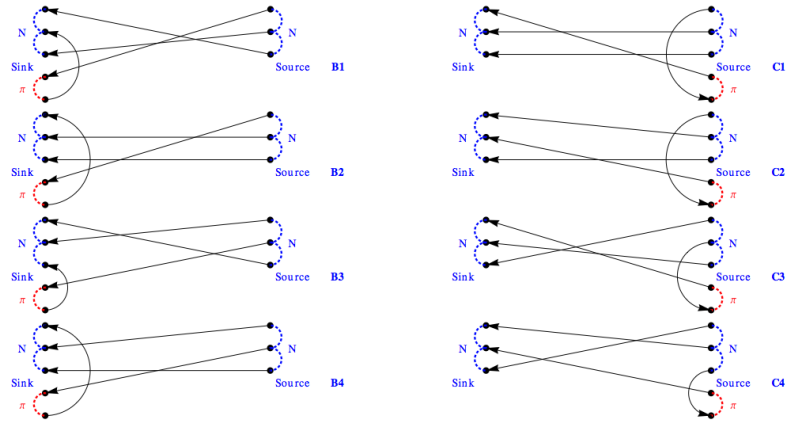
part of them are similar as in Nπ in s-wave

[Verduci, Lang, PRD 2013] plots taken from there

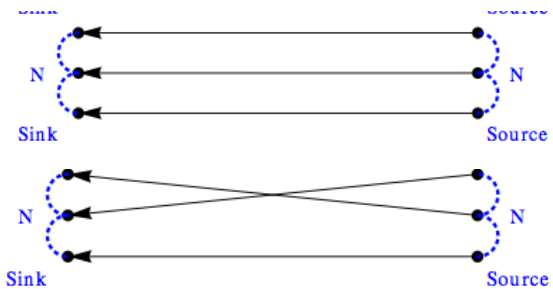
$$C_{ij}(t) = \langle \Omega | O_i(t + t_{\rm src}) \bar{O}_j(t_{\rm src}) | \Omega \rangle$$

TABLE III. Number of Wick contractions involved in computing correlation functions between interpolators in Eq. (7).

$O_i \backslash O_j$	O^N	$O^{N\pi}$	$O^{N\sigma}$
O^N	2	4	7
$O^{N\pi}$	4	19	19
$O^{N\sigma}$	7	19	33



- just part of Wick contractions plotted
- computational challenge:
- all-to-all quark propagators needed;
- full distillation employed [Peardon et al, 2009]

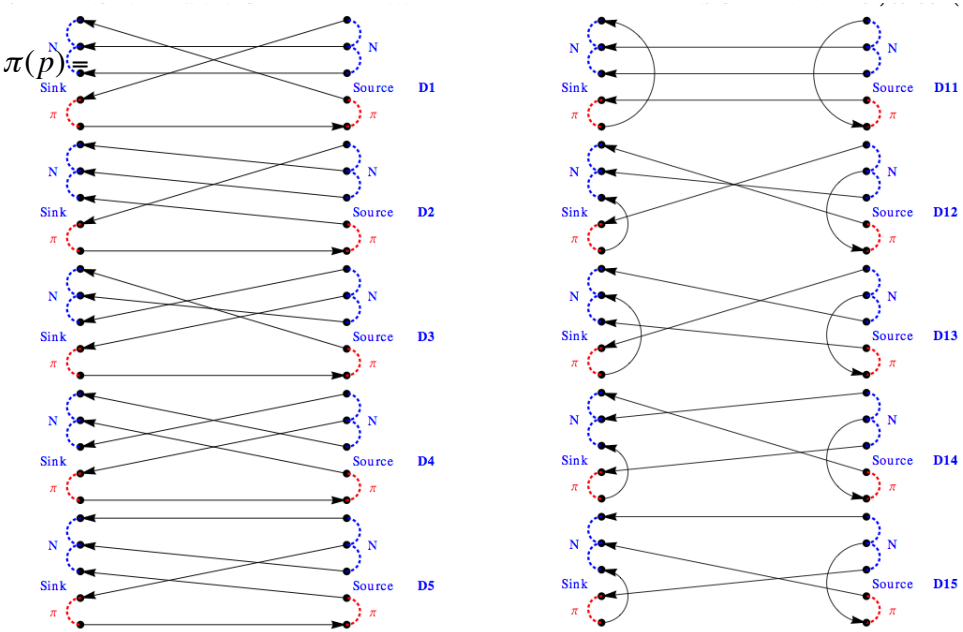


$$C_{ij}(t) = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

$$C(t) v^{(n)}(t) = C(t_0) \lambda(t) v^{(n)}(t)$$

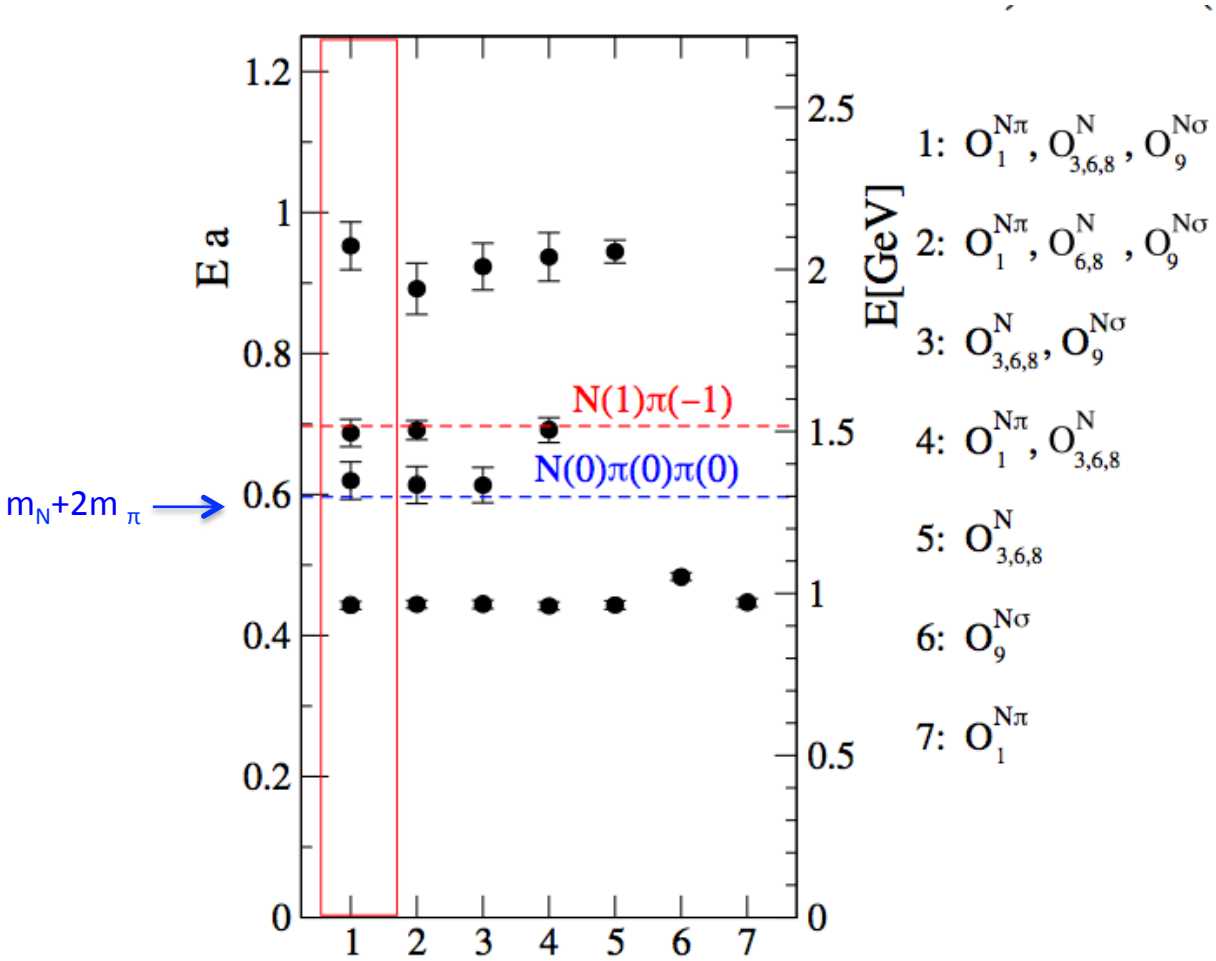
[Luscher & Wolf 1991, Blossier et al 2009]

$\pi(p) =$



$$\pi(p) = \sum_x e^{ipx} \bar{d}(x) \gamma_5 u(x)$$

E_n: dependence on the interpolators used



note again:

total momentum P=0

N(0)π(0) not possible

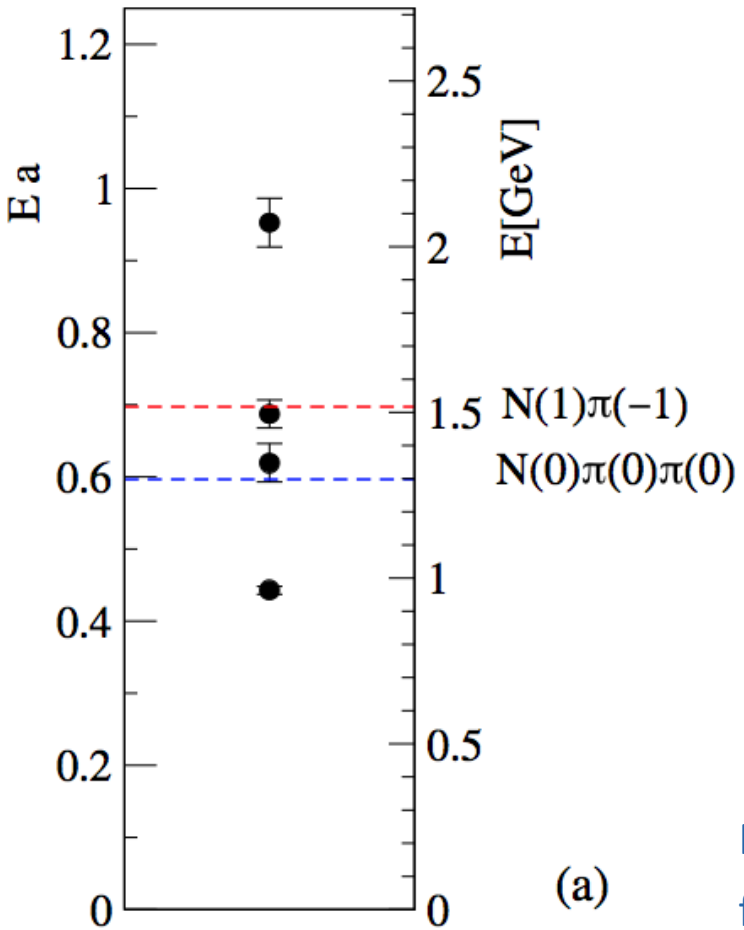
for p-wave

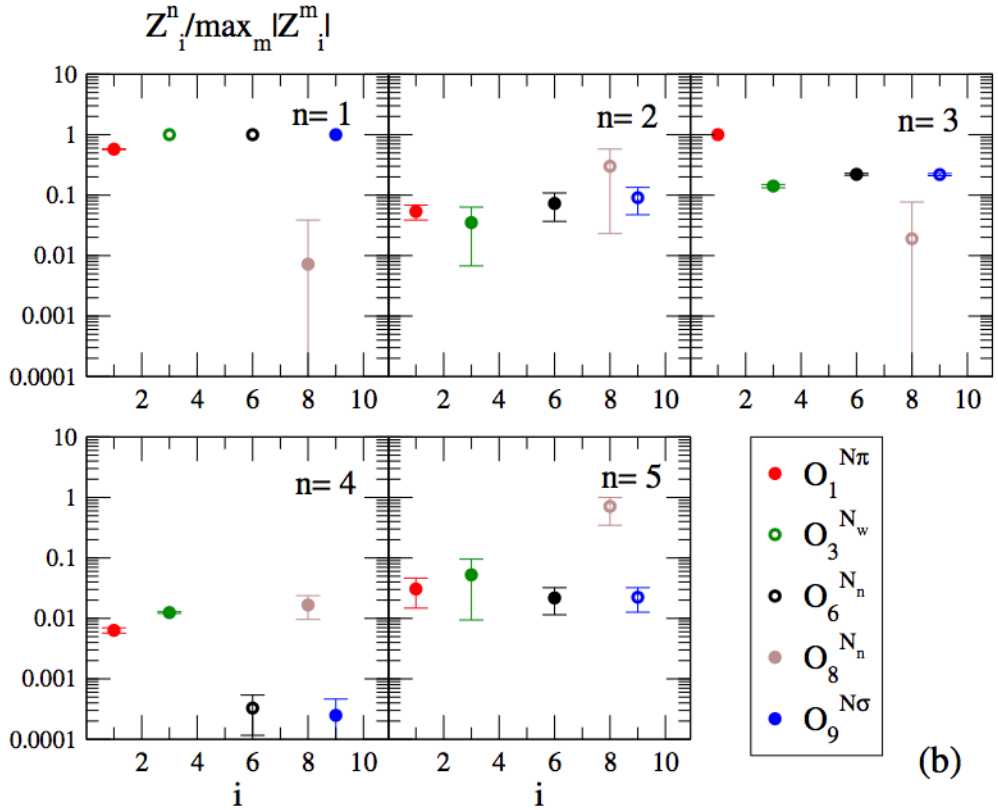
lowest non-interacting

Nπ state is N(1)π(-1)

p= 1 * 2π/L

Final E_n and overlaps $Z_i^n = \langle O_i | n \rangle$



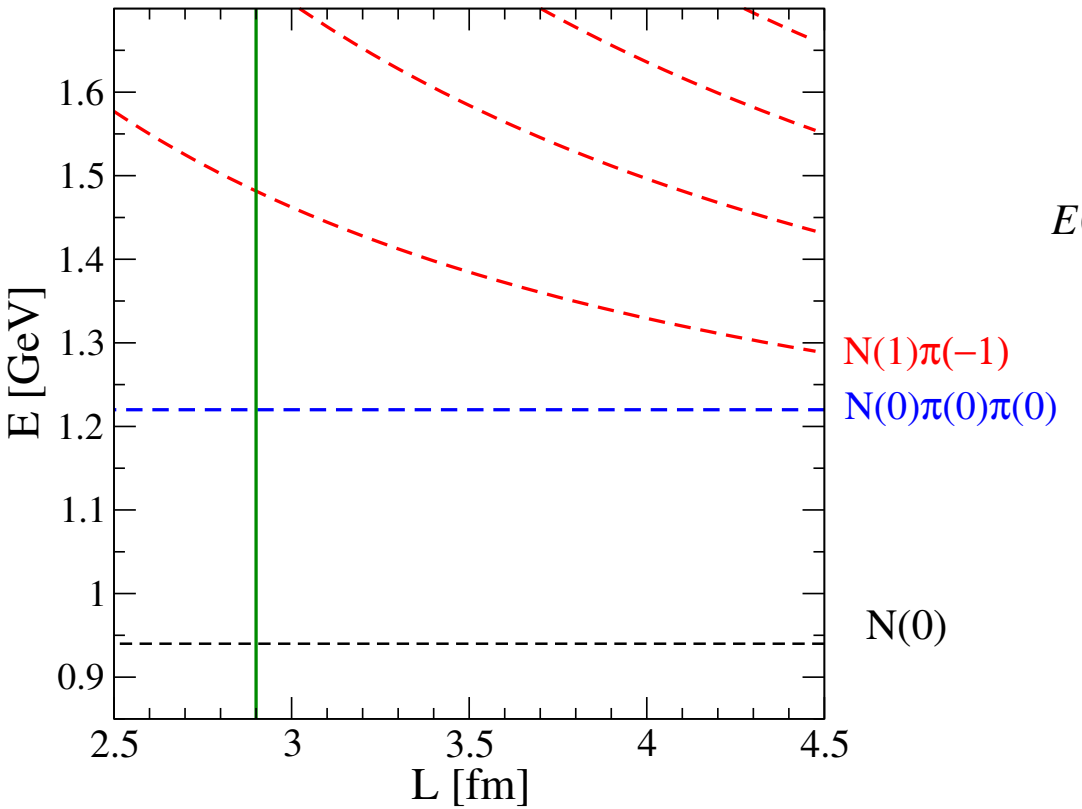


N(1)π(-1) lattice eigenstate established in ½⁺ channel for the first time: related to problems of "excited-state pollution" of nucleon observables by Nπ [work by Baer]

E not precise enough to reliably determine ΔE and δ: not unexpected for $m_\pi \approx 156$ MeV !!

Alternative path to reach physics conclusions from the results.

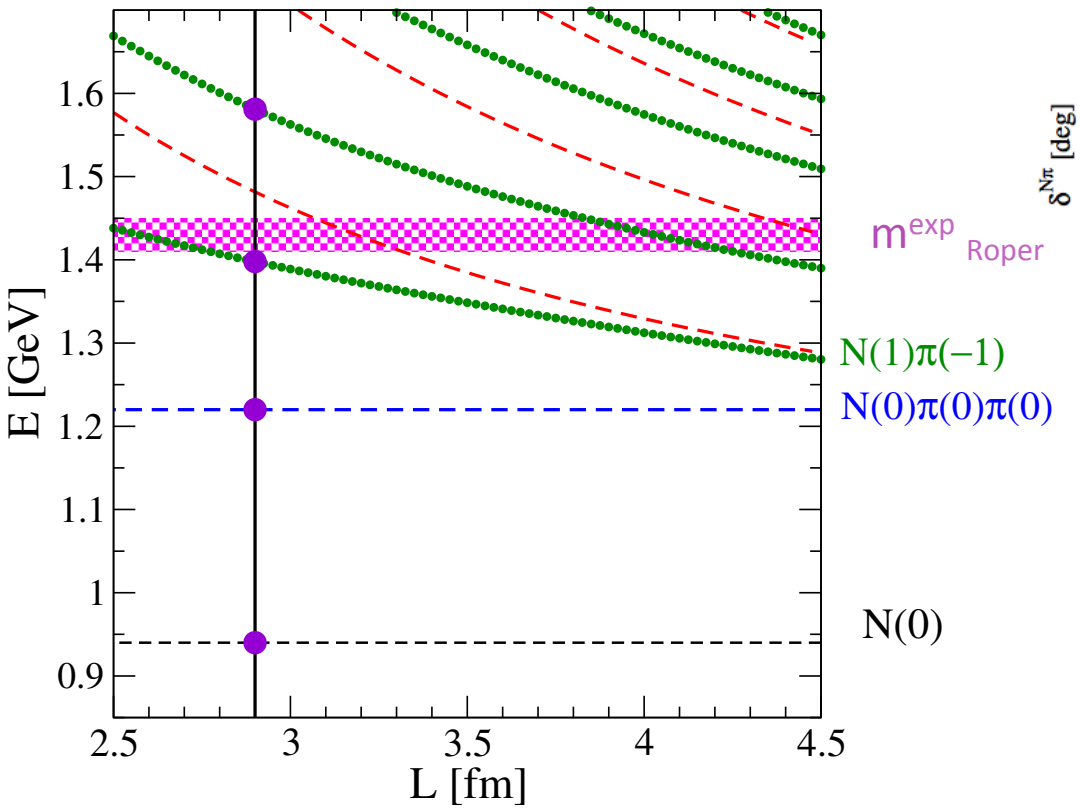
(A) Expectation from elastic Nπ scattering based on low-lying Roper (from experimental $\delta_{N\pi}$)

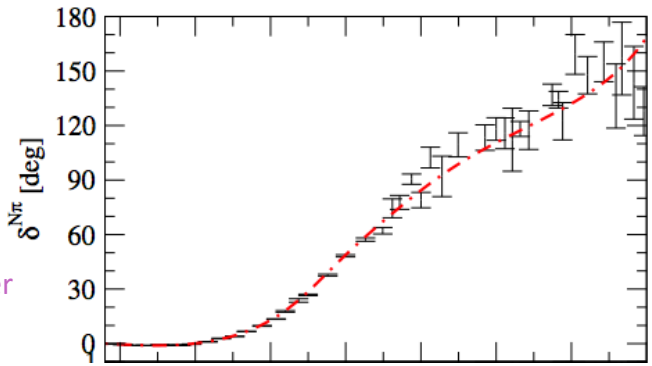


non-interacting case

$$E(L) = \sqrt{m_N^2 + \left(\frac{2\pi}{L}n\right)^2} + \sqrt{m_\pi^2 + \left(\frac{2\pi}{L}n\right)^2}$$

(A) Expectation from elastic Nπ scattering based on low-lying Roper (from experimental $\delta_{N\pi}$)





Luscher's relation

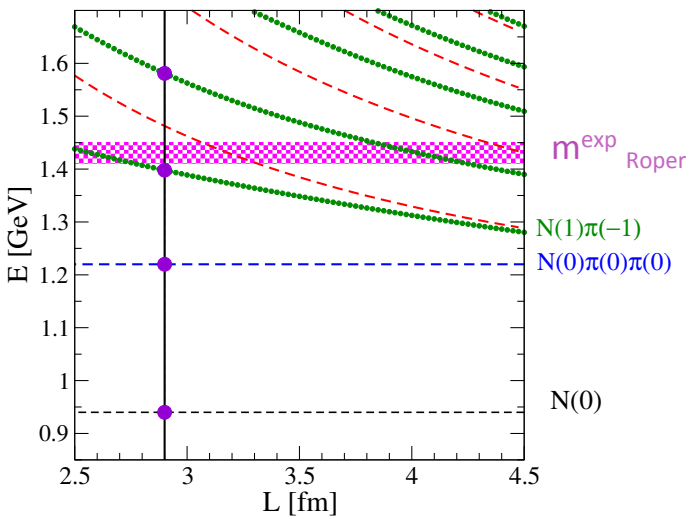
$$\cot\delta(p) = \frac{1}{pL\pi}\sum_{\vec{n}\in N^3}\frac{1}{\vec{n}^2-(\frac{pL}{2\pi})^2}$$

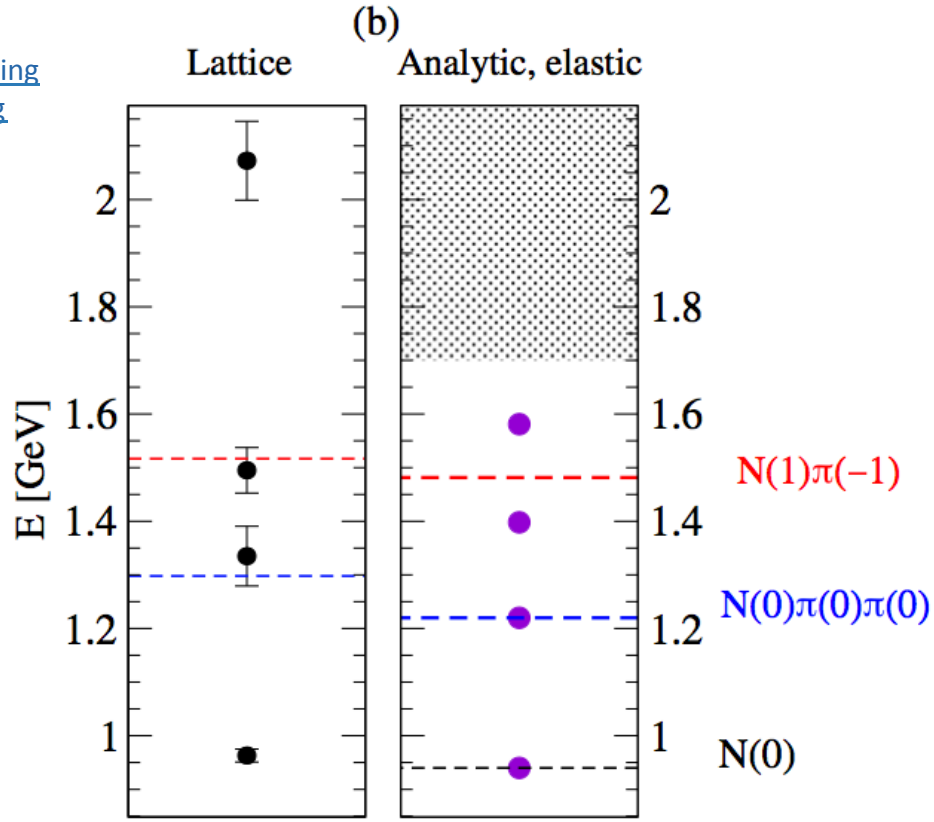
$$E(p) = \sqrt{p^2+m_1^2}+\sqrt{p^2+m_2^2}$$

(A) Expectation from elastic Nπ scattering
based on low-lying Roper (from experimental $\delta_{N\pi}$)

- our lattice data is qualitatively different from the prediction of the resonating Nπ phase shift for the low-lying Roper resonance, assuming it is decoupled from other channels
- the scenario of mainly elastic low-lying Roper is not supported by our lattice data
- this calls for other possibilities: one possibility is that the coupling of Nπ with other channels (Nσ or Nππ) is essential for low-lying Roper resonance in experiment:

  this is dubbed dynamically generated Roper resonance
  [Krehl, Hanhart, Krewald, Speth, PRC 62 025207 (2000), many other follow up-works]





(B) 3 scenarios with coupled Nπ – Nσ – Δπ scattering

Hamiltonian EFT study of Roper
Adelaide group, Leinneweber et al,
PRD 2017, 1607.04536

$$H = H_0 + H_I.$$

bare baryon

$$H_0 = \sum_{B_0} |B_0\rangle m_B^0 \langle B_0|$$

α= Nπ , Nσ, Δπ

$$+ \sum_{\alpha} \int d^3\vec{k} |\alpha(\vec{k})\rangle$$

$$\times \left[\sqrt{m_{\alpha_1}^2 + \vec{k}^2} + \sqrt{m_{\alpha_2}^2 + \vec{k}^2}\right] \langle \alpha(\vec{k})|,$$

$$H_I = g + v,$$

$$g = \sum_{\alpha B_0} \int d^3\vec{k} \{ |\alpha(\vec{k})\rangle G^{\dagger}_{\alpha,B_0}(k) \langle B_0| + |B_0\rangle G_{\alpha,B_0}(k) \langle \alpha(\vec{k})| \},$$

$$v = \sum_{\alpha,\beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^S_{\alpha,\beta}(k,k') \langle \beta(\vec{k}')|.$$

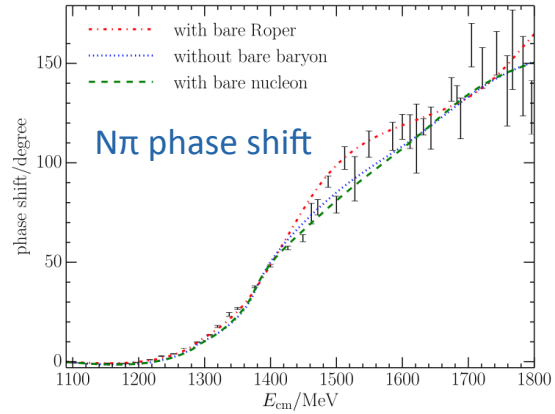
caveat: σ treated as stable

3 scenarios, which all fit experimental Nπ scattering well

I : with bare Roper B0
with bare nucleon
no coupling between Nπ – Nσ

II : without bare Roper B0
with bare nucleon N;
with strong Nπ – Nσ coupling

III: without bare Roper B0
with bare nucleon N;
with strong Nπ – Nσ coupling



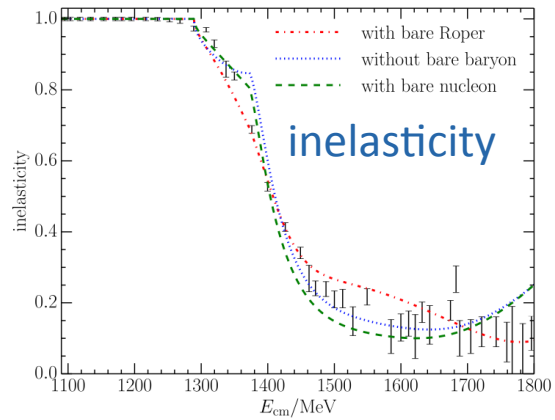


TABLE I. Best-fit parameters and resultant pole positions in the three scenarios: I, the system with the bare Roper; II, the system without a bare state; and III, the system with a bare nucleon. Underlined parameters were fixed in the fitting of that scenario. The experimental pole position for the Roper resonance is $(1365 \pm 15) - (95 \pm 15)i$ MeV [4].

Parameter	I	II	III
$g^S_{\pi N}$	0.161	0.489	0.213
$g^S_{\pi\Delta}$	−0.046	−1.183	−1.633
$g^S_{\pi N,\pi\Delta}$	0.006	−1.008	−0.640
$g^S_{\pi N,\sigma N}$	0	2.176	2.401
$g^S_{\sigma N}$	0	9.898	9.343
$g_{B_0\pi N}$	0.640	0	−0.586
$g_{B_0\pi\Delta}$	1.044	0	1.012
$g_{B_0\sigma N}$	2.172	0	2.739
m_B^0/GeV	2.033	∞	1.170
$\Lambda_{\pi N}$/GeV	0.700	0.562	0.562
$\Lambda_{\pi\Delta}$/GeV	0.700	0.654	0.654
$\Lambda_{\sigma N}$/GeV	0.700	1.353	1.353
Pole (MeV)	$1380 - 87i$	$1361 - 39i$	$1357 - 36i$

(B) 3 scenarios with coupled Nπ – Nσ – Δπ scattering

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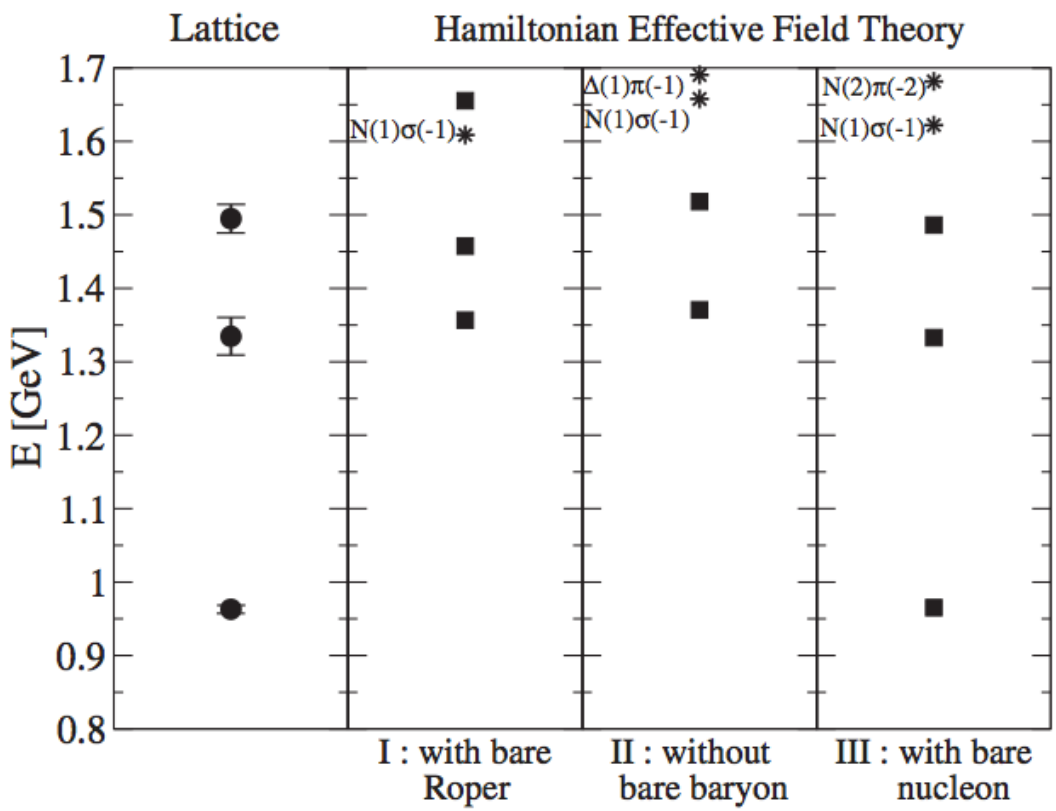
our lattice results

predicted E_n for PACS-CS lattice with L=2 fm

in 3 scenarios on previous page



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comparing analytic predictions and lattice data:

- scenario I disfavoured
- scenarios II, III favoured
- Roper as dynamically generated resonance favoured

(B) Scenarios with coupled Nπ – Nσ – Δπ scattering

Structure of the Roper resonance from Lattice QCD constraints

Adelaide group, Leineweber et al. 1703.10715

- analysis of our lattice data within Hamiltonian EFT

- Two different descriptions of the existing pion-nucleon scattering data in the region of the Roper resonance are constructed.
- Both descriptions fit the experimental data very well.
- Consideration of the finite volume spectra enable a discrimination of these two different descriptions.
- Lattice data supports the scenario where
  - the Roper resonance is the result of strong rescattering between coupled meson-baryon channels
  - the quark-model like state (first radial excitation of the nucleon) is heavy, approx. 2 GeV

If this is the case, the prospects of rigorous lattice treatment will be challenging:

- coupled channel scattering
- three-body Nππ decay: relation of E and scattering matrix under development

  [Rusetsky, Sharpe, Hansen, Briceno..]

  scattering matrix has never been extracted within QCD

Conclusions

(1) $H_1(p)H_2(-p)$ operators constructed for scattering of particles with spin

- Consistent results found in three methods: PV, PN, VN, NN

✧ Projection operators O_n: gives little guidance on underlying quantum numbers

✧ Partial-wave operators: provides linear combinations O_n to enhance coupling to (J, S, L)

✧ Helicity operators: provides linear combinations O_n to enhance coupling to (J, P, λ1, λ2)

- Operators will lead to E_n of HH. These are related to scattering matrix by a known relation.

(2) lattice simulation of Nπ scattering in p-wave , $J^P=1/2^+$

- meson-baryon eigenstates (Nπ and Nππ) are identified for the first time in this channel
- the scenario of the low-lying Roper that is mainly elastic in Nπ is not supported by our data
- coupling of Nπ with other channels (Nσ or Nππ) seems important to render low-lying Roper in exp
- this step was only the first on in more to follow ....

Thanks

- construction of operators: C.B. Lang, U. Skerbis
  R. Briceno

- Nπ scattering: C.B. Lang, M. Padmanath, L. Leskovec