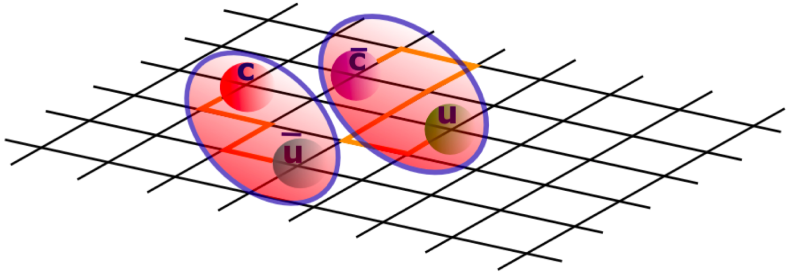
Exotic and conventional hadrons from lattice QCD



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HICforFAIR colloquium , JL University Giessen , 28th January 2016

in collaboration with

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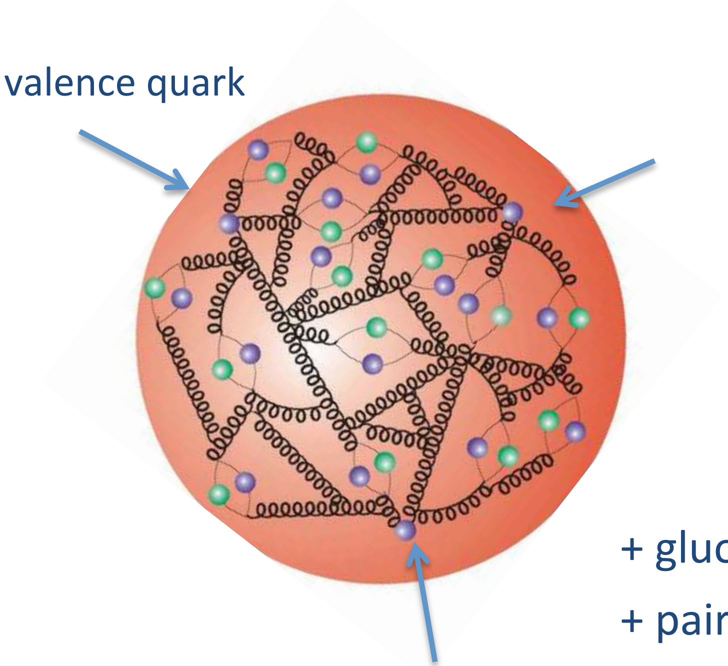
Outline

- Experimental appetizer concerning exotic hadrons:
  - pentaquarks & tetraquarks

- Theoretical approach to study
  conventional and exotic hadrons
  based directly on QCD – lattice QCD

Hadrons

Conventional



+ gluons

+ pairs of sea $\underline{q}q$

Exotic

minimal valence content

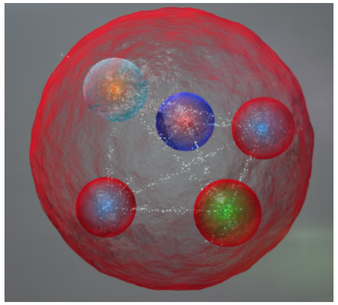
$\bar{q}\ \bar{q}\ q\ q$	tetraquarks
$\bar{q}\ q\ q\ q\ q$	pentaquarks
....	hybrids
	glueballs
	....

Terminology in this talk: tetra(penta) quarks indicate just the number of valence quarks in the state; it is not meant to say anything on how quarks are clustered in them

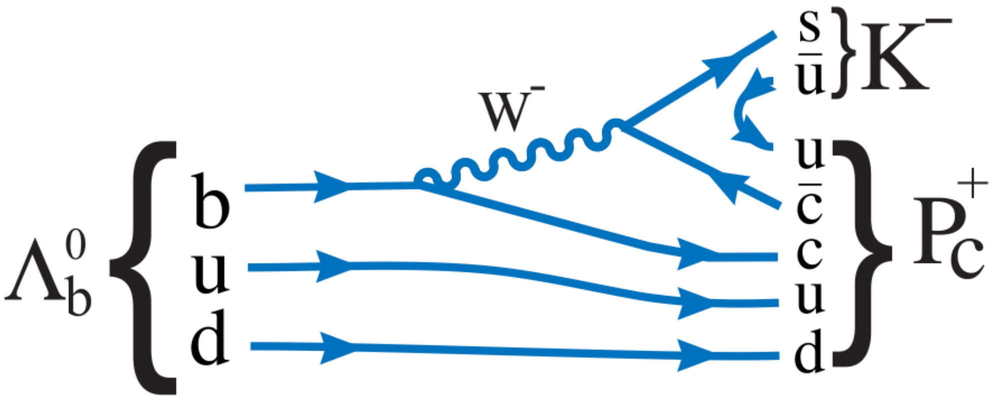
Pentaquarks

LHCb: 1507.03414

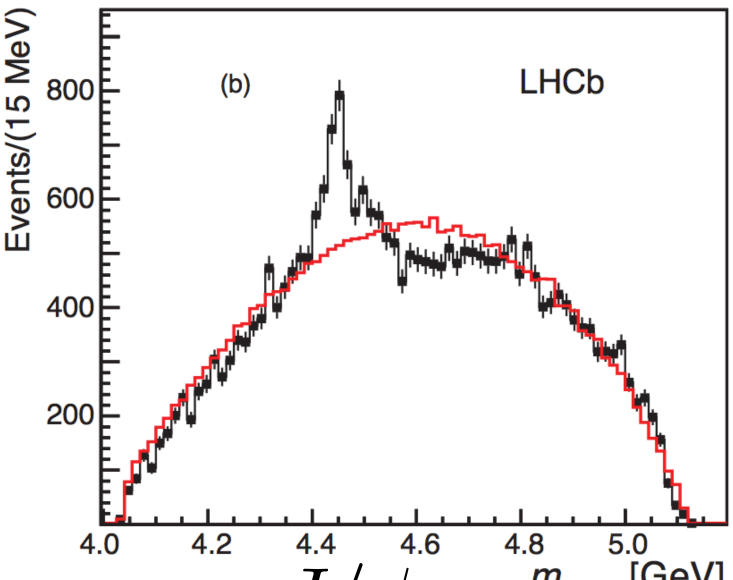
July 2015

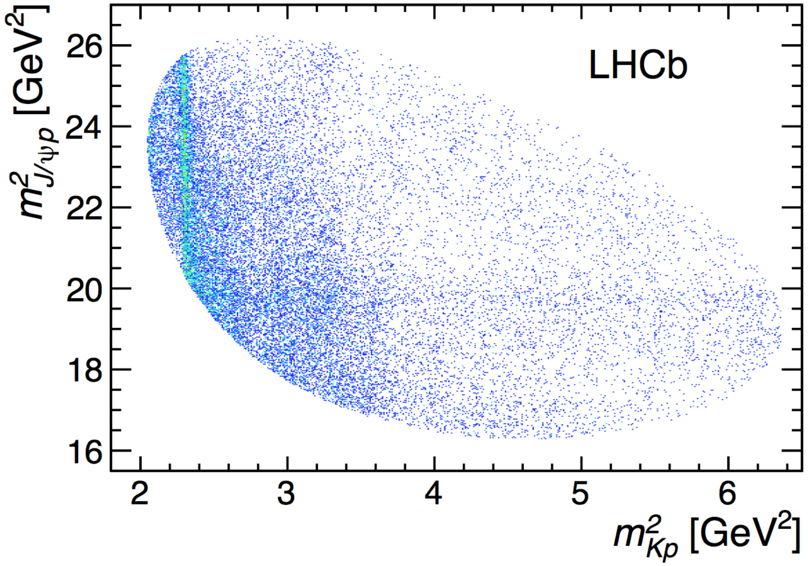


$$\Lambda_b^- \to K^- J/\psi\ p$$



$$P_c^+ \to J/\psi \quad p$$
$$\bar{c}c \quad uud$$



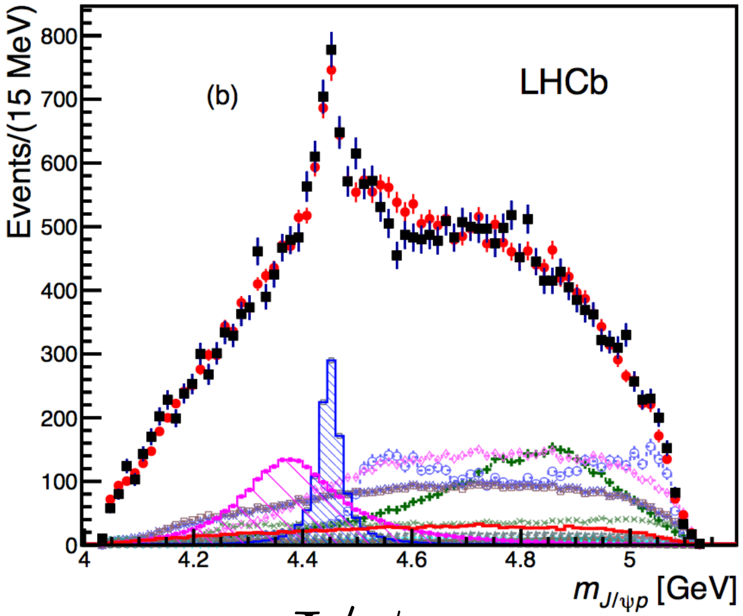


$J/\psi\ p$

$J/\psi\ p$

Two pentaquark candidates from LHCb

LHCb: 1507.03414

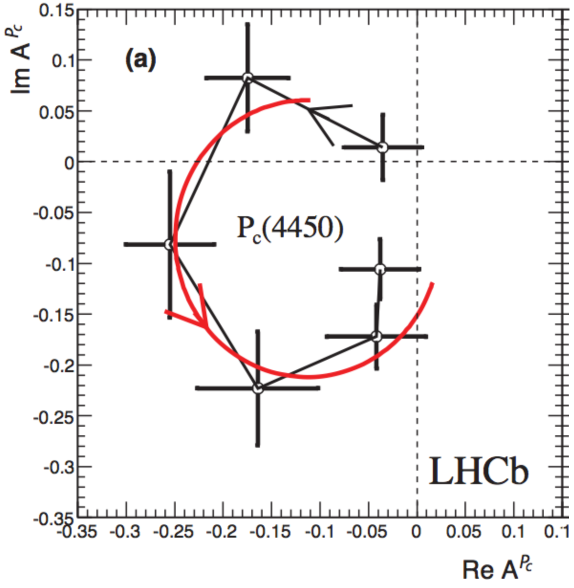


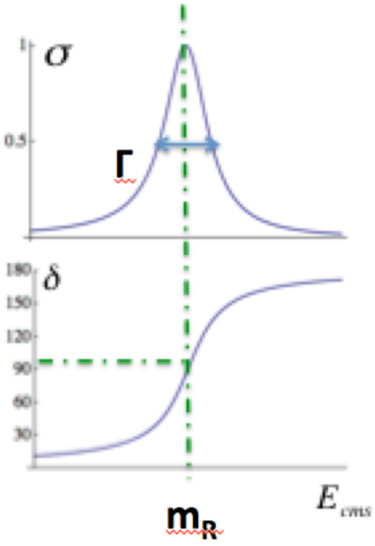
$$J/\psi\ p$$

scattering matrix T for elastic scattering

$$A \simeq T = \frac{1}{2i}(e^{2i\delta(E)} - 1)$$

$$|T + \frac{1}{2i}| = |\frac{1}{2i}e^{2i\delta(E)}| = \frac{1}{2}$$





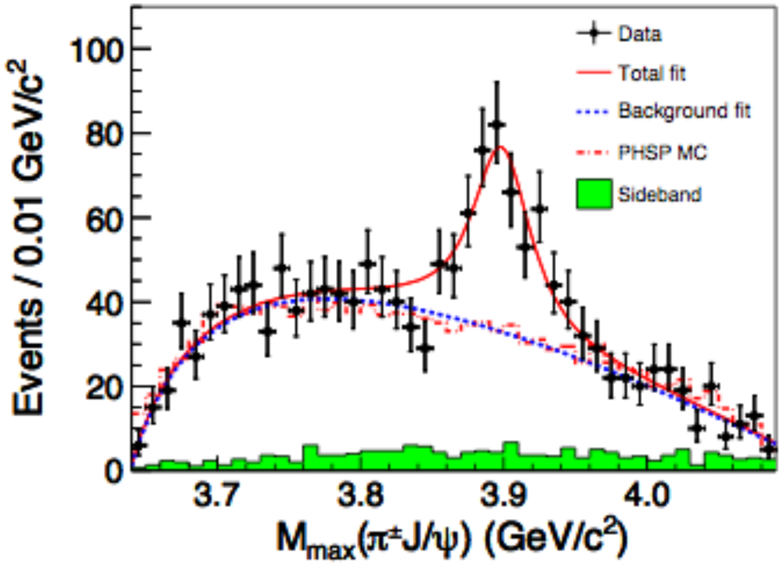
$P_c(4380)$: $M_1 = 4380 \pm 8 \pm 29$ MeV $\Gamma_1 = 205 \pm 18 \pm 86$ MeV $J^P = 3/2^+$ or $5/2^+$

$P_c(4450)$: $M_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV $\Gamma_2 = 39 \pm 5 \pm 19$ MeV $J^P = 5/2^-$ or $3/2^-$

Tetraquarks Z_c

BESIII, Belle, Cleo-c 2013

Example: $Z_c^+(3900)$



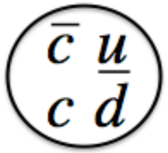
DD* thr.

[BESIII, 2013, 1303.5949, PRL]

state confirmed by Belle, Cleo-c

$$Z_c^+(3900) \to \; J/\psi\ \pi^+$$

flavor content of the hadron: $\boxed{\bar{c}c \;\; \bar{d}u}$

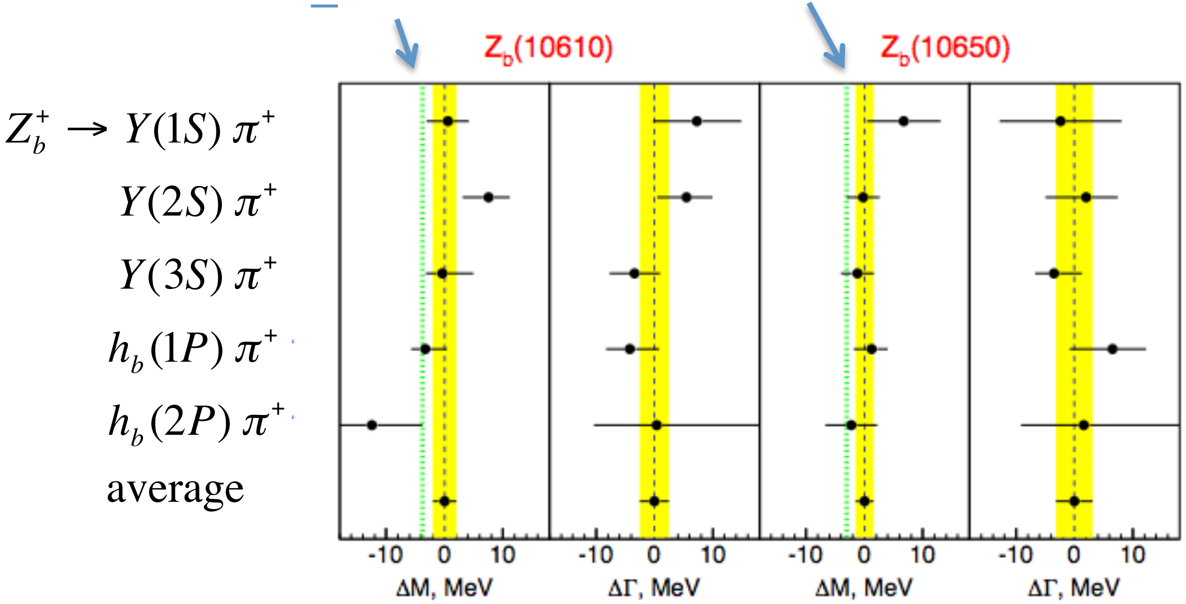


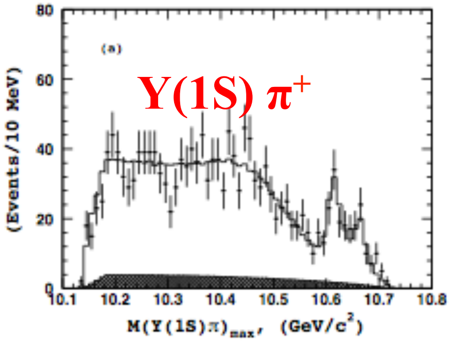
M = 3888 ± 3 MeV , Γ = 35 ± 7 MeV

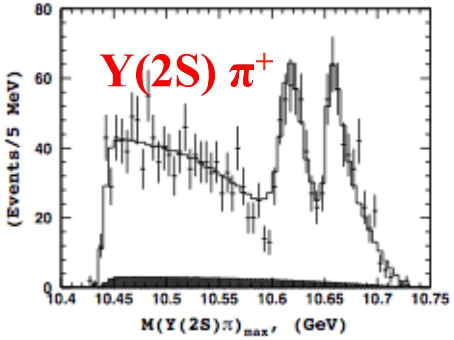
Tetraquarks Z_b

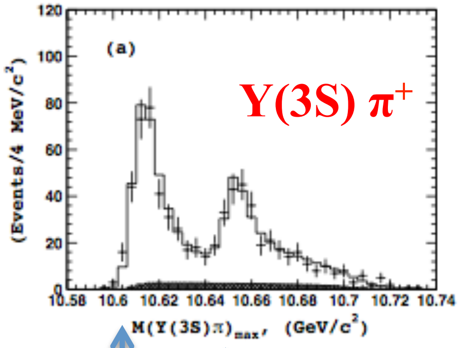
Belle 2011

BB*+ th. B*B*+ th.









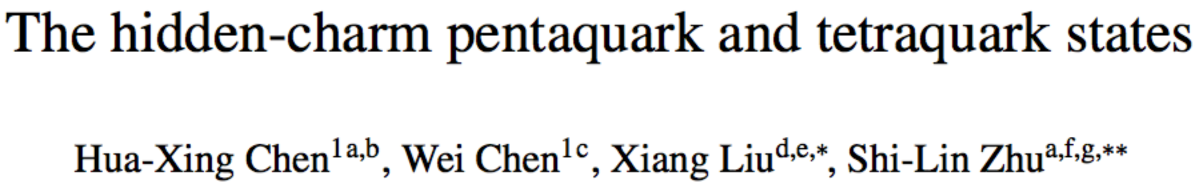
BB*+ B*B*+ th.

- both: $I^G(J^P)=1^+(1^+)$
- both: observed only by Belle 2011 [1105.4583]

$$Z_b^+ \to \Upsilon(nS)\ \pi^+$$

flavor content of the hadron: $\boxed{\bar{b}b\ \ \bar{d}u}$

A very recent review



The hidden-charm pentaquark and tetraquark states

Hua-Xing Chen[1a,b], Wei Chen[1c], Xiang Liu[d,e,*], Shi-Lin Zhu[a,f,g,**]

Review submitted to **Physics Reports**, 150 pages

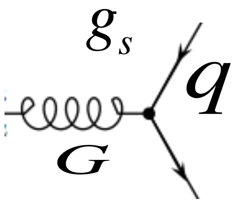
arXiv:1601.02092

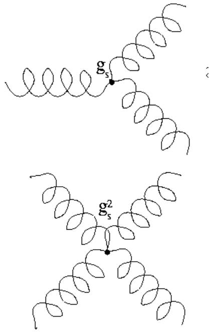
exp + theory

Theoretical problem

- Fundamental theory of strong interaction between **quarks** and **gluons** is known and reliable: chromodynamics (QCD)

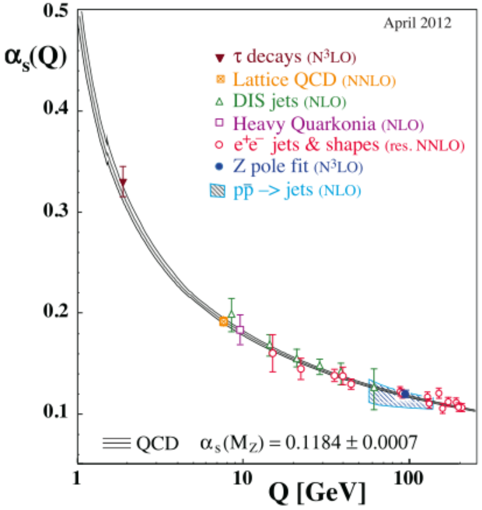
$$L_{QCD} = -\tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s,c,b,t} \bar{q} i\gamma_\mu (\partial^\mu + i g_s G^\mu_a T^a) q - m_q \bar{q} q$$





$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^\mu_b G^\nu_c$$

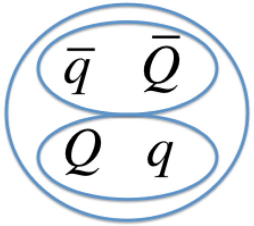
- But: $\alpha_s \equiv \dfrac{g_s^2}{4\pi} \approx O(1) \quad r \approx 1\,\mathrm{fm}$



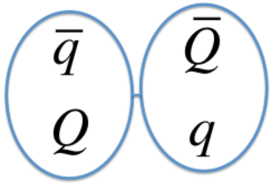
- Calculation of hadron properties (mass, ...) :
  - perturbative expansion in α_s not possible
  - non-perturbative method needed !

Phenomenological approaches

Approaches that support new exotic hadrons



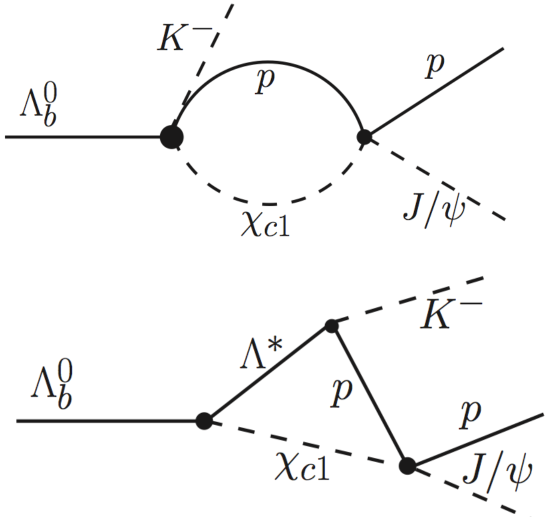
- compact tetraquark: diquark antidiquark
- meson molecules



- ...

Approaches that do not contain new exotic hadrons, and explain bumps in experimental cross sections via

- kinematical effects due to nearby threshold
- coupled channel effects
- kinematics interplay in triangle diagrams
- ...



[Guo, Meissner, Wang, Yang 1507.04950]

I will try to address the question whether observed exotic and conventional hadrons arise directly from QCD.

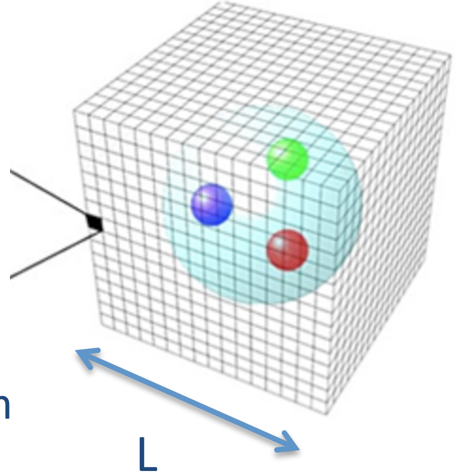
Non-perturbative method: QCD on lattice

$$L_{QCD} = -\tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G^\mu_a T^a) q - m_q \bar{q} q$$

input: g_s , m_q

output: hadron properties

hadron interactions



precision cal.

a~0.05-0.15 fm

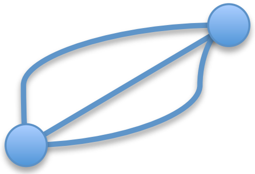
L~ 2-8 fm

Evaluation of Feynman path integrals in discretized space-time

quantum mechanics

$$\int Dx\, e^{i S/\hbar}$$

$$S = \int dt\, L[x(t)]$$



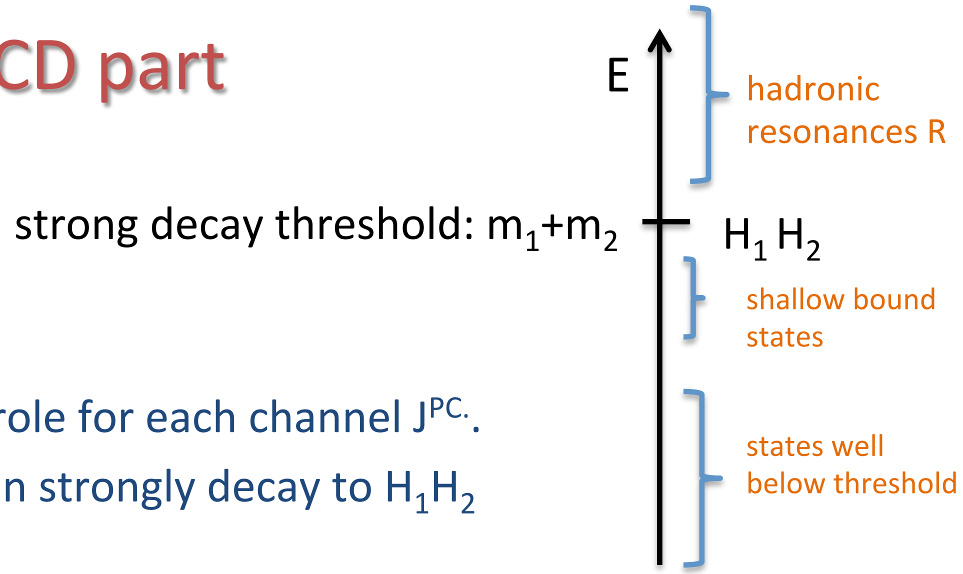
quantum field theory in Euclidian space-time

$$\int DG\, Dq\, D\bar{q}\, e^{-S_{QCD}/\hbar}$$

$$S_{QCD} = \int d^4x\, L_{QCD}[G(x), q(x), \bar{q}(x)]$$

x,t (Minkovsky) → x, i t (Euclidean)

Outline for lattice QCD part



E

hadronic resonances R

strong decay threshold: m_1+m_2

$H_1 H_2$

shallow bound states

states well below threshold

The location of the threshold pays a major role for each channel $J^{PC.}$.
If the hadron mass is above threshold, it can strongly decay to H_1H_2
as long as quantum numbers allow decay.

- Hadrons well bellow threshold: *"easy"*
  Unfortunately none of exotic experimental candidates is found well below threshold.

- Hadron resonances above threshold and states slightly below threshold : *challenging*
  - until recently: so-called single-hadron approximation
  - now: rigorous treatment by determining scattering matrix for two hadrons $H_1 H_2$

- lattice searches for manifestly exotic hadrons (usually above several thresholds): *very challenging*

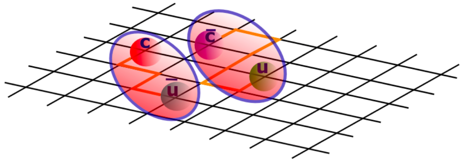
Discrete energies of eigenstates: E_n

$$J^{PC} \qquad \mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)_{\vec{p}_1}(\bar{q}\Gamma_2 q)_{\vec{p}_2}, \quad [\bar{q}\Gamma_3\bar{q}][q\Gamma_4 q],\ldots$$

mesons

$$\rho(770),\ 1^{--}: \qquad \bar{d}u, \quad (\bar{d}d)(\bar{d}u) = \pi\pi$$

$$X(3872),\ 1^{++}: \quad \bar{c}c\,, \quad (\bar{c}u)(\bar{u}c) = D\bar{D}^*\,, \qquad [\overline{cu}][cu]$$



baryon

$$p \qquad 1/2^+: \quad uud, \quad (\mathrm{uud})(\bar{u}c) = p\pi$$

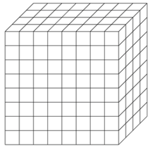
$$C_{ij}(t) = \left\langle 0\right|\mathcal{O}_i(t)\ \mathcal{O}_j^+(0)\left|0\right\rangle = \sum_n \left\langle 0\right|\mathcal{O}_i\left|n\right\rangle\ e^{-E_n t}\left\langle n\right|\mathcal{O}_j^+\left|0\right\rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$

All eigenstates with given J^{PC} appear in principle (example: proton channel ½⁺)

- single hadron states p=proton $m_p = E_1$ for P=0 (after extrapolation)
- two-hadron states p π E_n rigorously render two-hadron scattering matrix (for example pπ scattering matrix)

States well below threshold: “easy” precision spectrum

- $m=E_n$ for P=0 a→0, L→∞, $m_q \to m_q^{phy}$



- Available from a number of lattice QCD collaborations for a number of years
- Only few examples shown

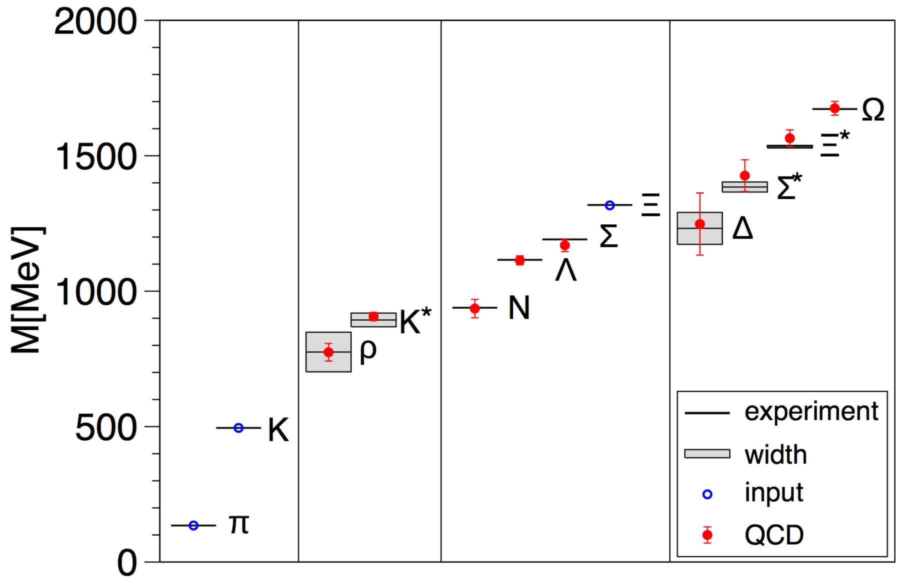
Proton and neutron

constitute more than 99%
of the bright side of universe

$$m_u c^2 \approx 2\ \mathrm{MeV} \qquad m_d c^2 \approx 5\ \mathrm{MeV}$$

$$m_p c^2 \approx 938.3\ \mathrm{MeV} \qquad m_n c^2 \approx 939.5\ \mathrm{MeV}$$

Higgs mechanism provides tiny contribution, the rest of visible mass in universe is due to the strong interaction in hadrons

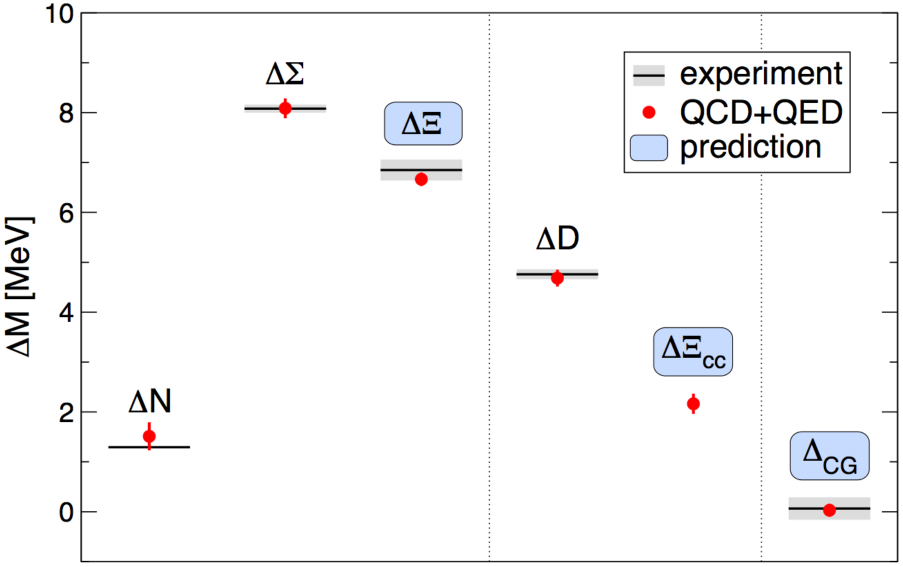


BMW collaboration
Science 322,2008

QCD

$m_u = m_d$

m_n , m_p



m_n - m_p

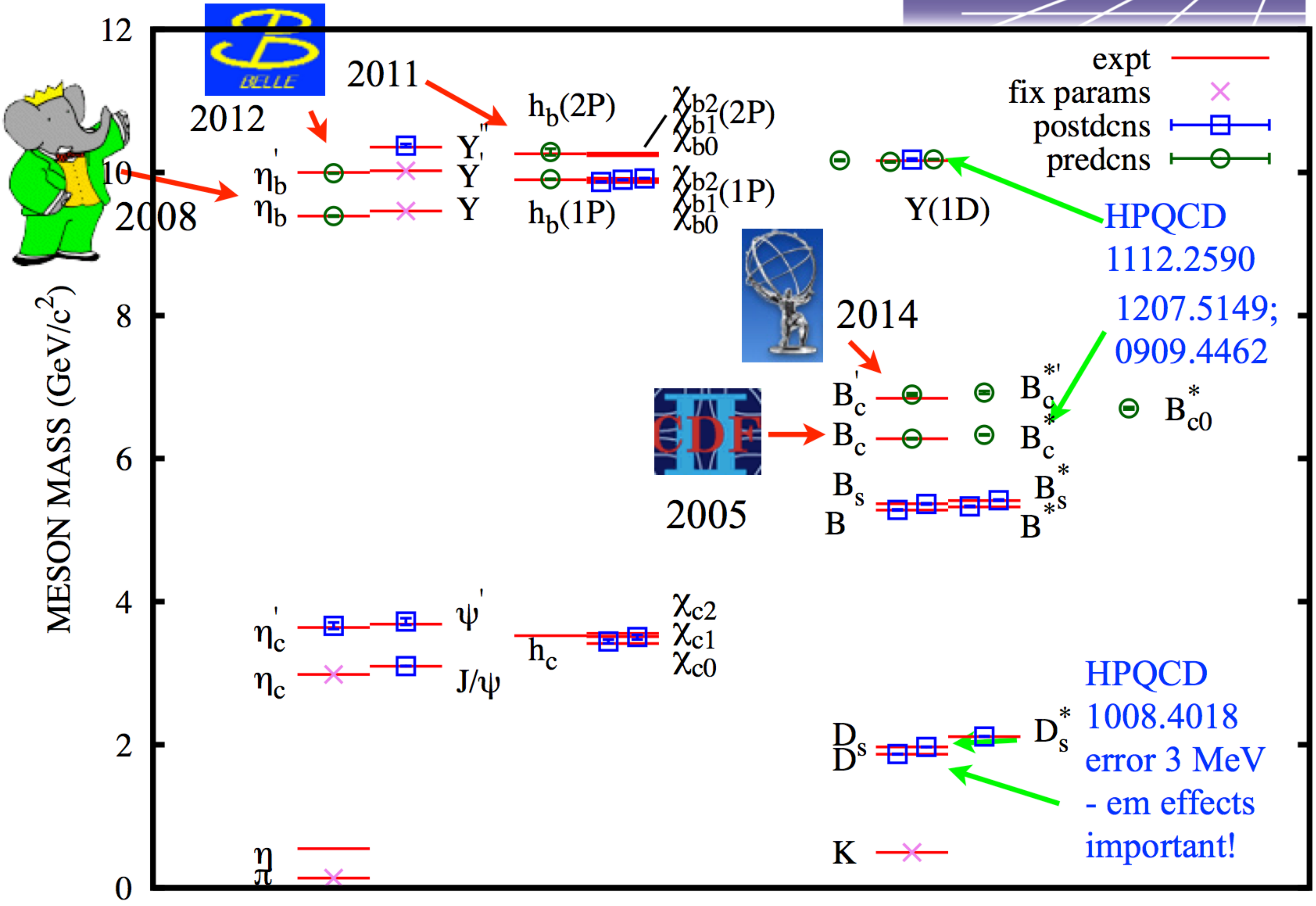
BMW collaboration
Science 347,2015

QCD + QED

$m_u \neq m_d$

High Precision QCD

$m=E_n$ The gold-plated meson spectrum



HPQCD
1112.2590

1207.5149;
0909.4462

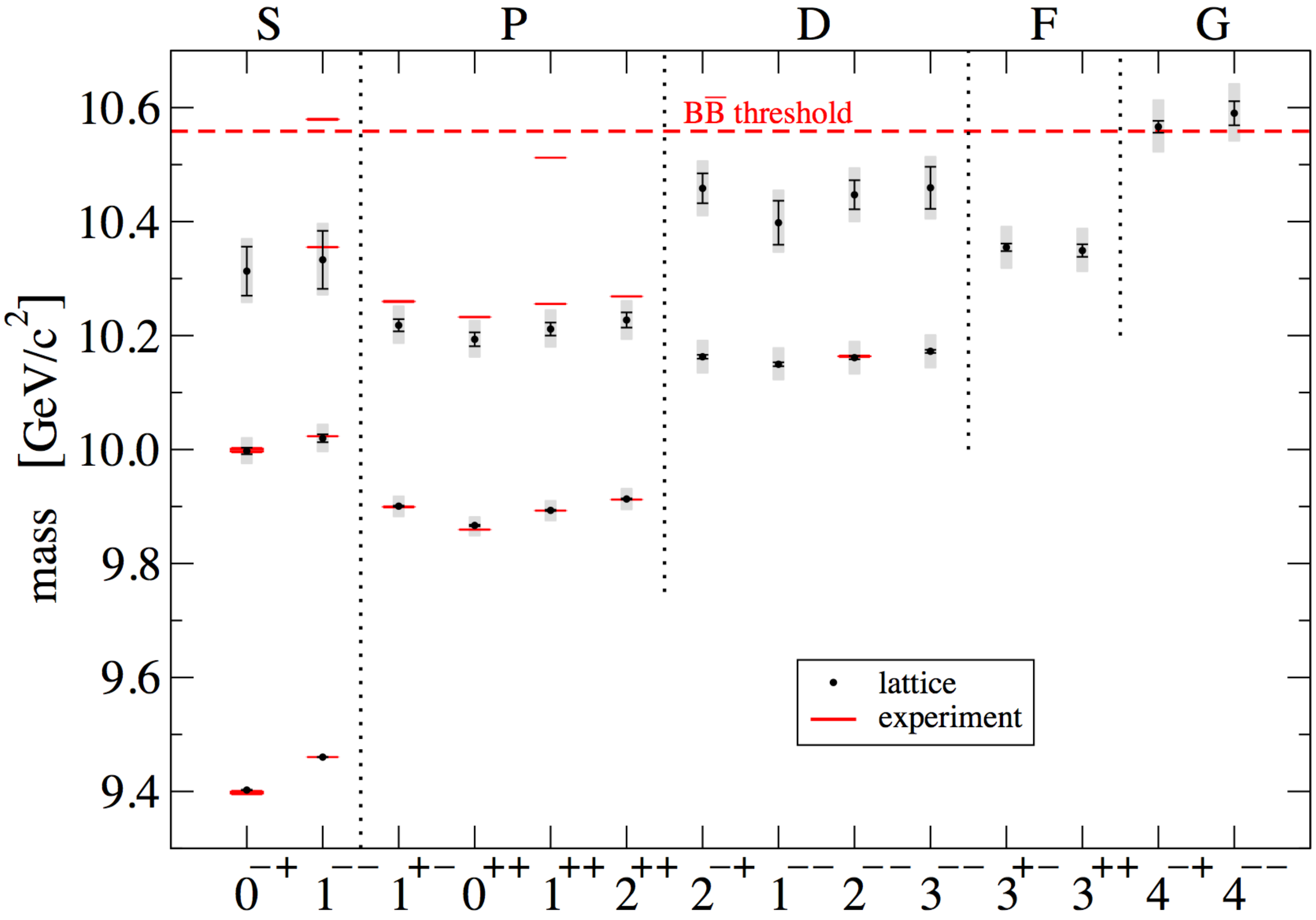
HPQCD
1008.4018
error 3 MeV
- em effects
important!

Bottomonium spectrum below threshold

... is rich

The most complete spectrum has been recently published:

[Wurtz, Lewis, Woloshyn, 0505.04410, PRD 2015] (see talk of Lewis at KEK 2014 meeting)



Valuable and reliable predictions for 1D and 2D states that have not been observed in exp yet !!

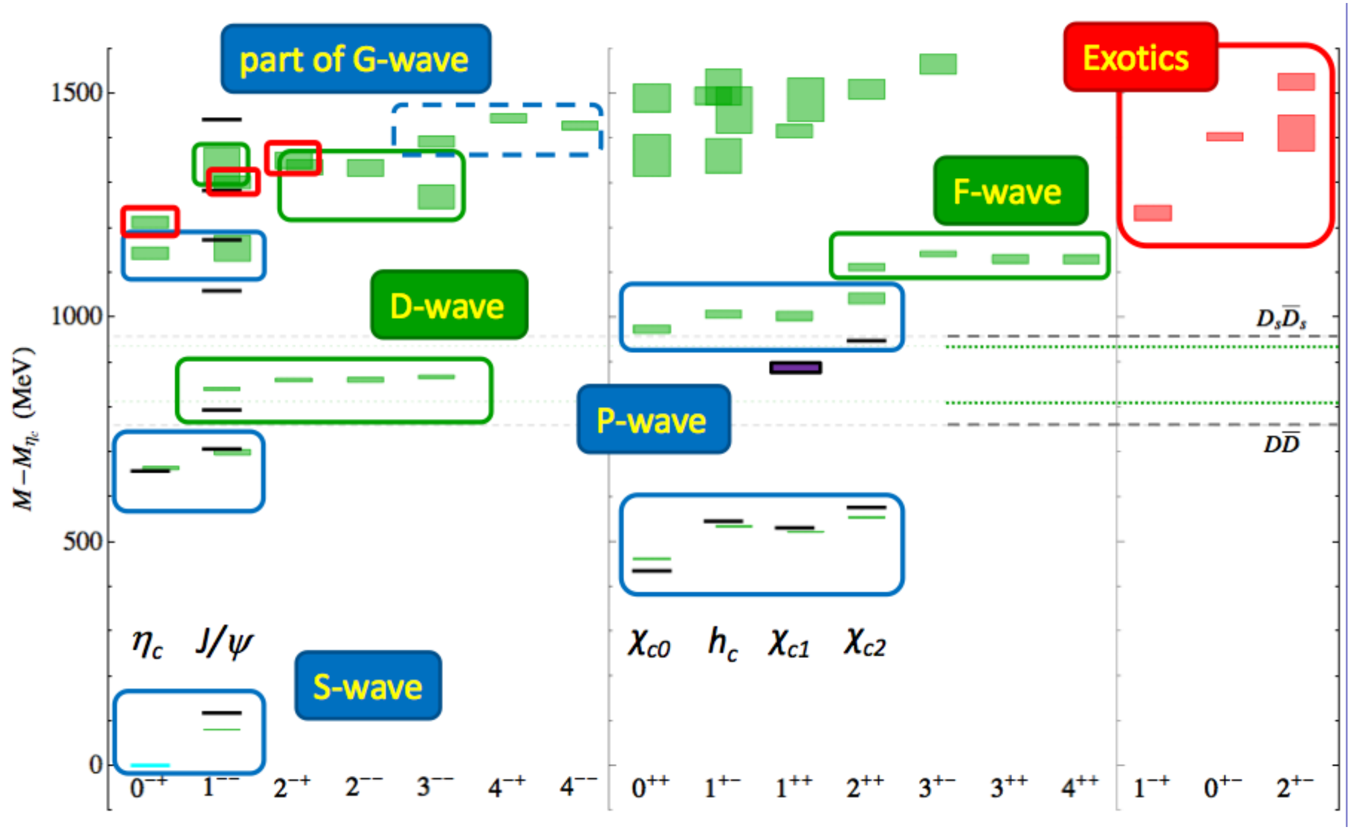
Hadrons near or above strong decay threshold
challenging

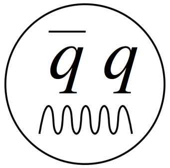
- Single–hadron approximation used until few years ago: ignores strong decays of resonances and effects of thresholds

- I will concentrate on “Rigorous approach” which addresses scattering of two hadrons ..... in experiment or theory

$\underline{c}c$ spectrum: single-hadron approximation

ignores strong decays of resonances and effects of thresholds





[HSC , L. Liu et al: 1204.5425, JHEP]

- $m_\pi \approx 400$ MeV, L≈2.9 fm, Nf=2+1
- identification with n $^{2S+1}L_J$ multiplets using <O|n>
- green: lat, black: exp

Hybrids:

some of them have exotic J^{PC}

large overlap with $O=\underline{q}\,F_{ij}\,q$

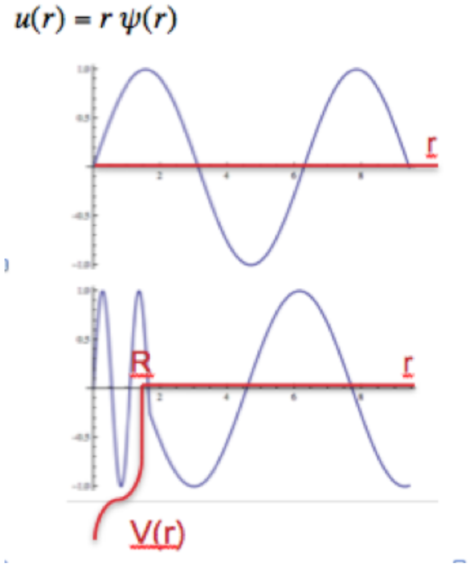
“Rigorous approach” to hadrons near or above threshold

which addresses scattering of two hadrons ..... in experiment or theory

challenging

Two-hadron scattering

Scattering phase shift δ



QM interpretation:

$$\psi(r) \propto \frac{\sin(pr)}{r}$$

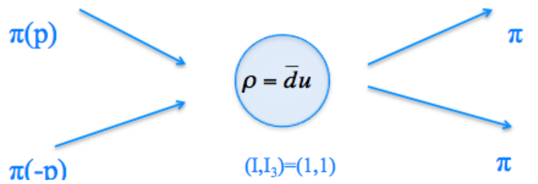
$$\psi(r) \propto \begin{cases} \dfrac{\sin(pr+\delta)}{r} & r > R \\ \text{unknown} & r < R \end{cases}$$

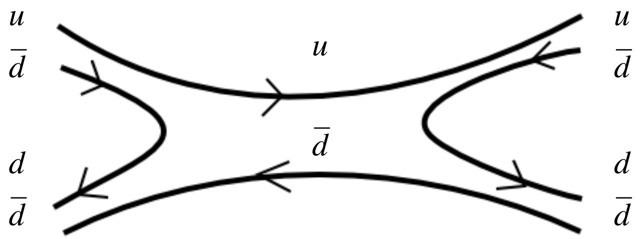
scattering matrix T(E) from δ(E) for elastic scat:

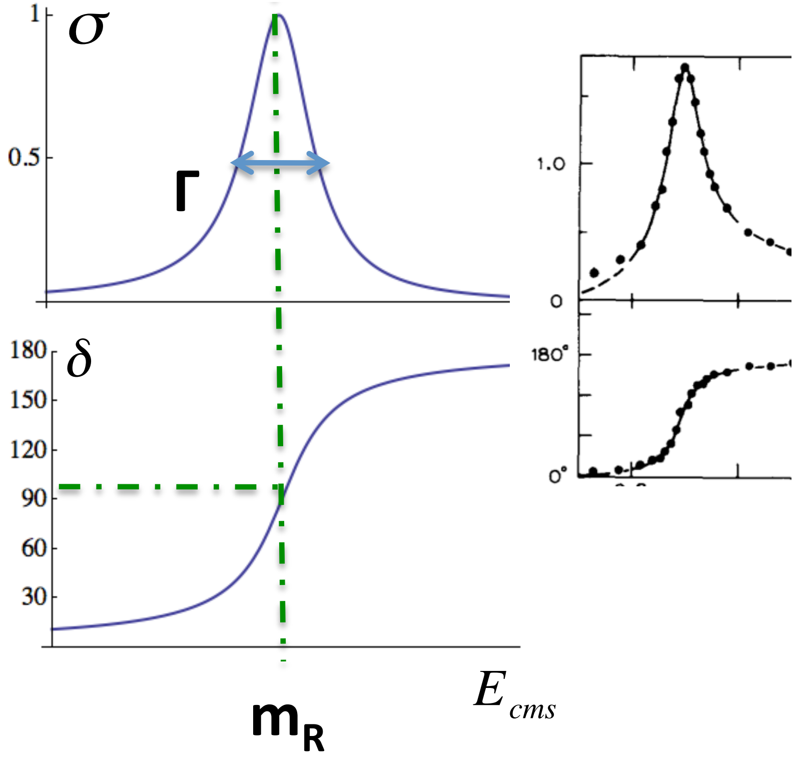
$S(E) = 1 + 2\,i\,T(E) = e^{2\,i\,\delta(E)}$

$$T = \frac{e^{2i\delta}-1}{2i} = \sin\delta\; e^{i\delta} = \frac{1}{\cot\delta - i}$$

$$\sigma \propto |T|^2 = \sin^2\delta$$

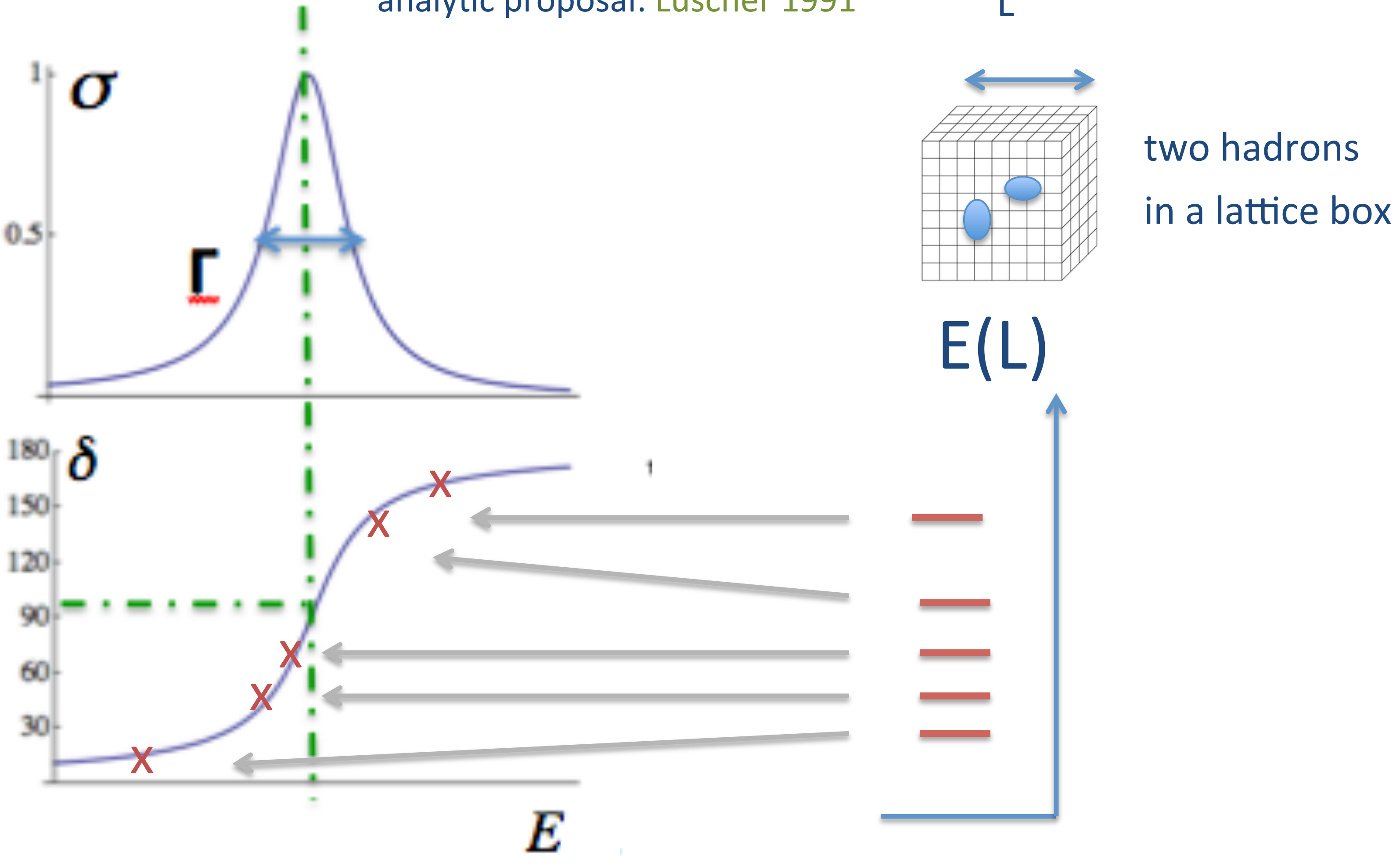






Rigorous treatment: scattering info from spectrum

analytic proposal: Luscher 1991



two hadrons

in a lattice box

scattering phase shifts at infinite volume δ(E) ← E(L) energies from lattice with spatial extent L

Relation between E and δ(E)

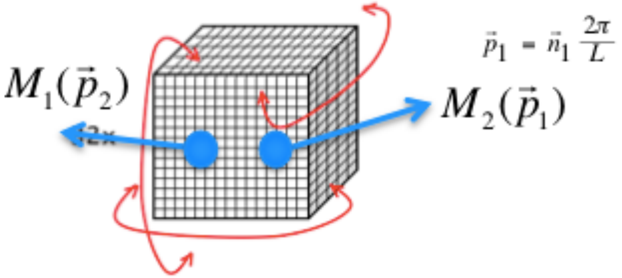
analytic proposal: Luscher 1991

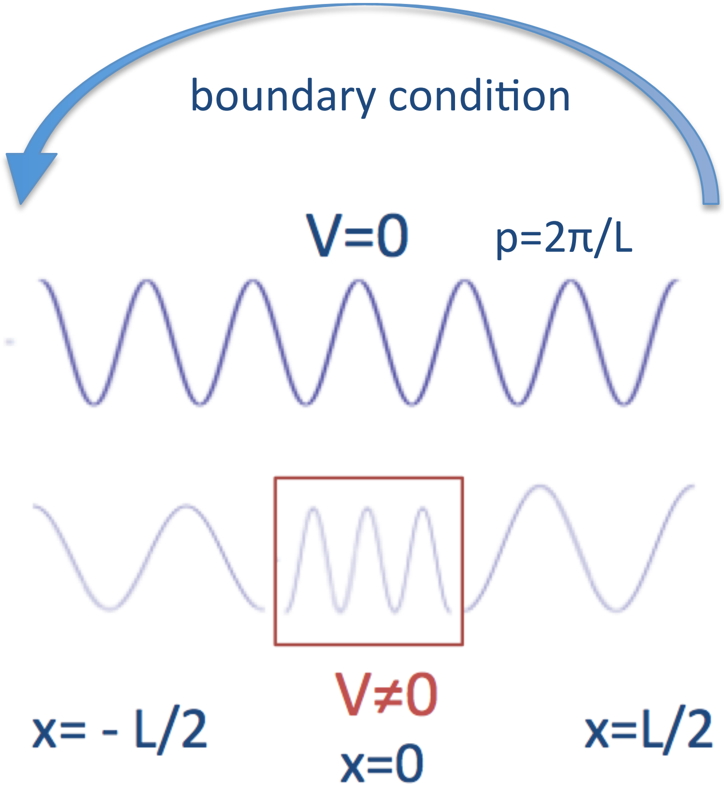
Energies of two hadrons in a box:

$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E$$

- due to strong interaction
- gives rigorous info on δ

$$\vec{p}_{1,2} = \frac{2\pi}{L}\vec{n}$$





boundary condition

V=0 p=2π/L

V≠0

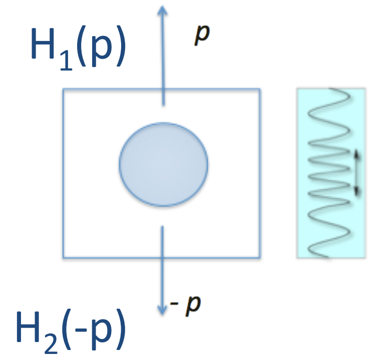
x= - L/2

x=0

x=L/2

Scattering of two hadrons

elastic scattering with total momentum P=0: E=E_{cm}



$$E_n(L) \xrightarrow{\text{Luscher's eq.}} \delta(E)$$

Scattering matrix for partial wave l:

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot\delta(E) - i}$$

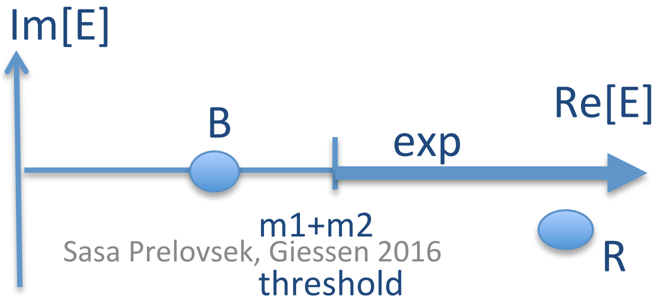
Bound state (B):

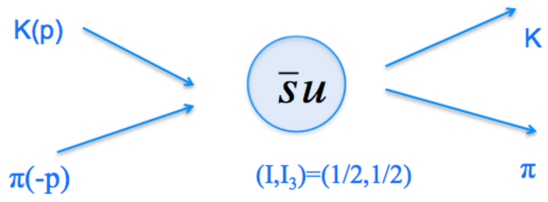
cot[δ(E_B)] = i , $E_B < m_1 + m_2$

Resonance (R) (of Breit-Wigner type):

$$T(E) = \frac{-E\ \Gamma}{E^2 - m_R^2 + i\ E\ \Gamma}, \quad \Gamma(E) = g^2\ \frac{p^{2l+1}}{E^2}$$

Locations of poles of T(E)
for res. and bound st.



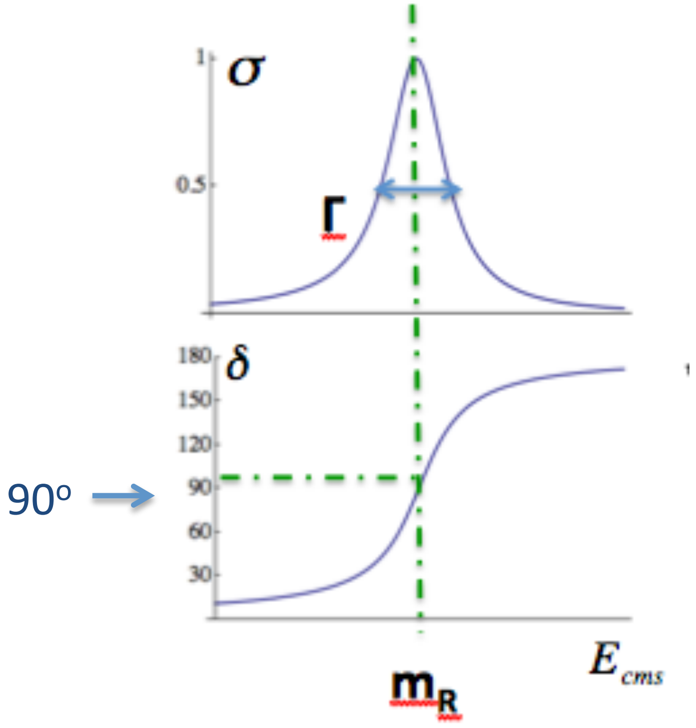


Resonances above threshold

Decay quickly through strong interaction

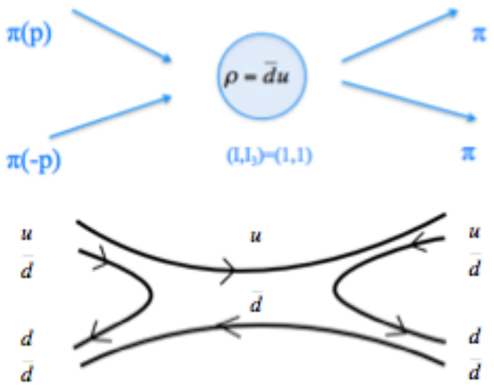
$$\tau = \hbar / \Gamma$$

Γ=10−300 MeV



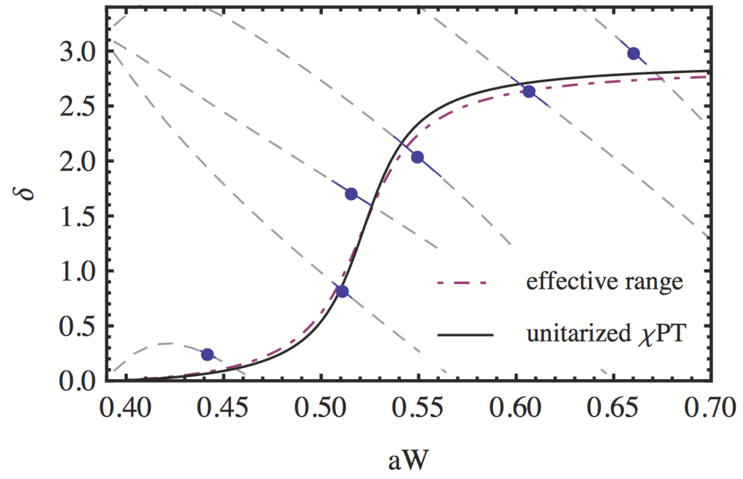
ρ resonance

(the only "mature" hadronic resonance from lattice)

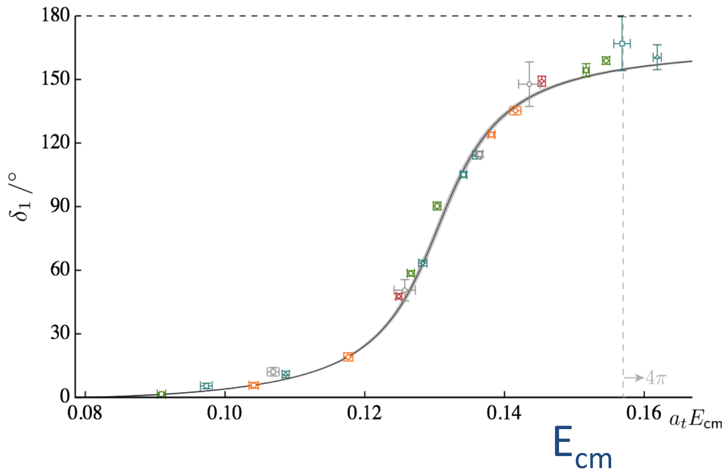


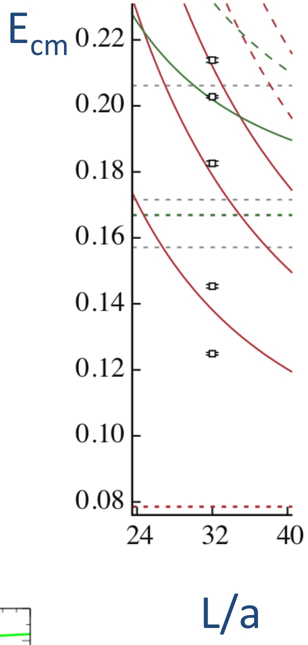
$$\mathcal{O}: \quad \bar{u}d, \quad \pi\pi$$

$$\boxed{E_{cm} \rightarrow \delta(E_{cm})}$$



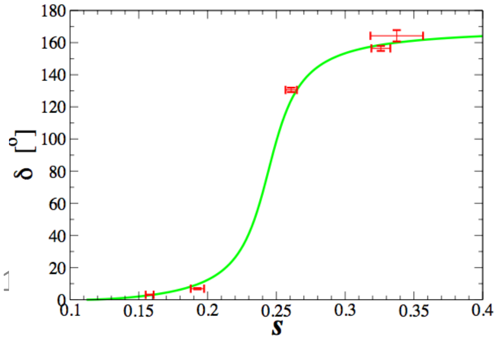
[Pelissier, Alexandru,PRD 2013], m_π≈300 MeV





[HSC, PRD 1507.02599] m_π≈236 MeV

$$\boxed{\mathrm{BW:}\quad \delta = \mathrm{acot}\frac{m_R^2 - E_{cms}^2}{m_R\,\Gamma} \quad \rightarrow \quad m_R, \quad \Gamma \text{ or } g_{\rho\pi\pi}}$$



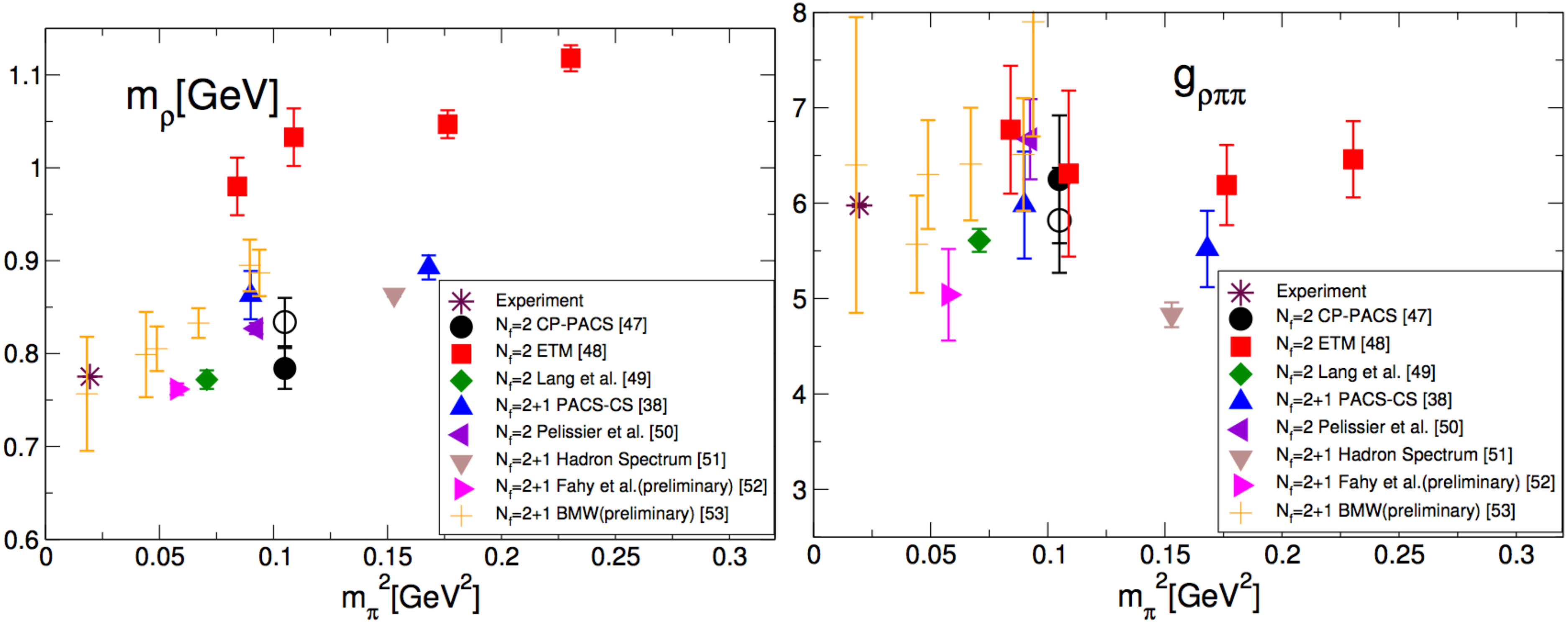
[Lang, Mohler, S.P. ,Vidmar, PRD 2011]

m_π≈266 MeV

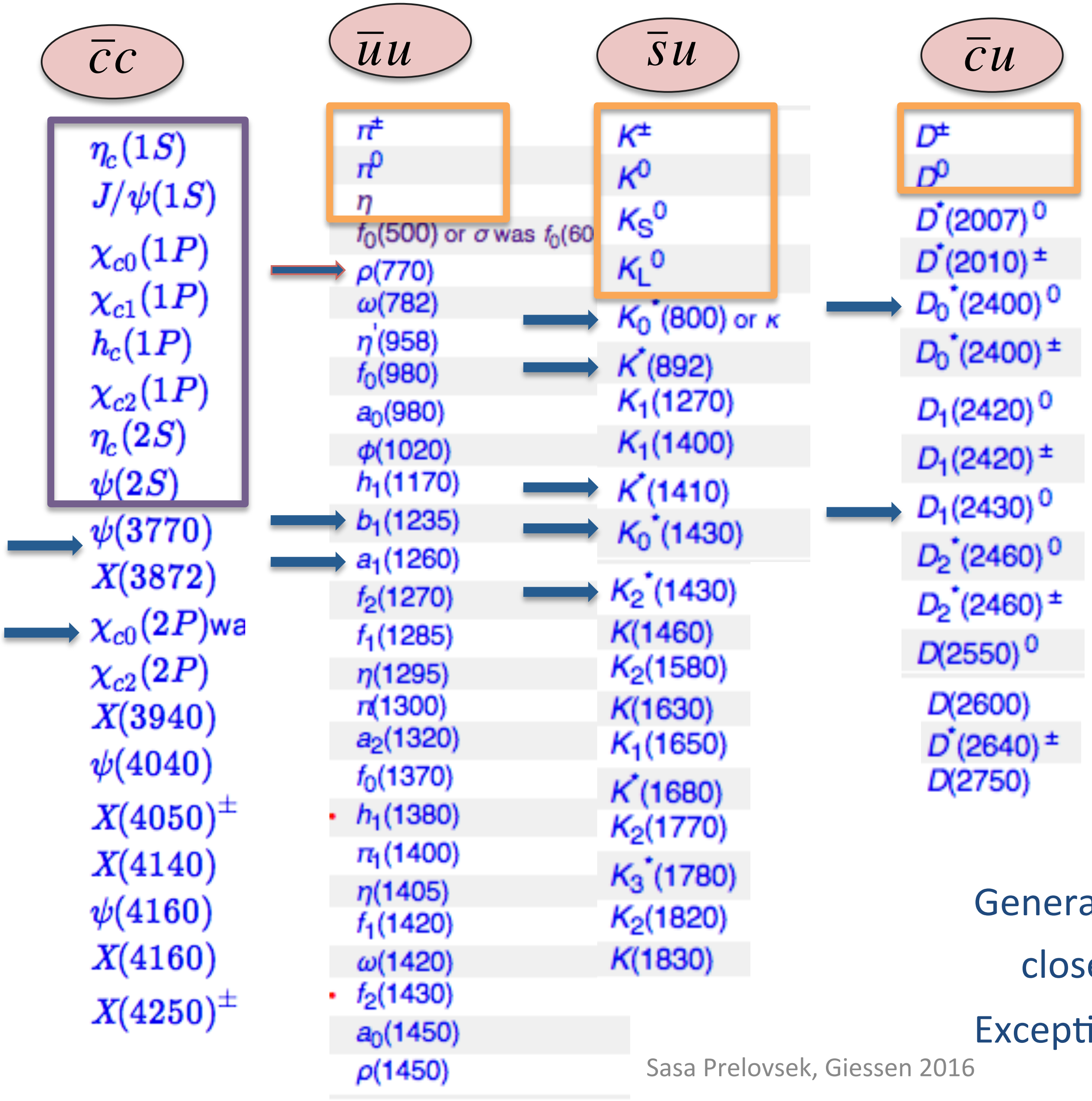
ρ resonance: lattice results

from T. Yamazaki [lat14 plenary, 1503.0867]

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E^2}$$



Other resonances simulated on the lattice



$\bar{c}c$

$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$h_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$
$\psi(3770)$
$X(3872)$
$\chi_{c0}(2P)$wa
$\chi_{c2}(2P)$
$X(3940)$
$\psi(4040)$
$X(4050)^{\pm}$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^{\pm}$

$\bar{u}u$

$\pi^{\pm}$
π^0
η
$f_0(500)$ or σ was $f_0(60$
$\rho(770)$
$\omega(782)$
$\eta'(958)$
$f_0(980)$
$a_0(980)$
$\phi(1020)$
$h_1(1170)$
$b_1(1235)$
$a_1(1260)$
$f_2(1270)$
$f_1(1285)$
$\eta(1295)$
$\pi(1300)$
$a_2(1320)$
$f_0(1370)$
$h_1(1380)$
$\pi_1(1400)$
$\eta(1405)$
$f_1(1420)$
$\omega(1420)$
$f_2(1430)$
$a_0(1450)$
$\rho(1450)$

$\bar{s}u$

$K^{\pm}$
K^0
$K_S{}^0$
$K_L{}^0$
$K_0^*(800)$ or κ
$K^*(892)$
$K_1(1270)$
$K_1(1400)$
$K^*(1410)$
$K_0^*(1430)$
$K_2^*(1430)$
$K(1460)$
$K_2(1580)$
$K(1630)$
$K_1(1650)$
$K^*(1680)$
$K_2(1770)$
$K_3^*(1780)$
$K_2(1820)$
$K(1830)$

$\bar{c}u$

$D^{\pm}$
D^0
$D^*(2007)^0$
$D^*(2010)^{\pm}$
$D_0^*(2400)^0$
$D_0^*(2400)^{\pm}$
$D_1(2420)^0$
$D_1(2420)^{\pm}$
$D_1(2430)^0$
$D_2^*(2460)^0$
$D_2^*(2460)^{\pm}$
$D(2550)^0$
$D(2600)$
$D^*(2640)^{\pm}$
$D(2750)$

well below open charm decay th.

well below strong decay th.

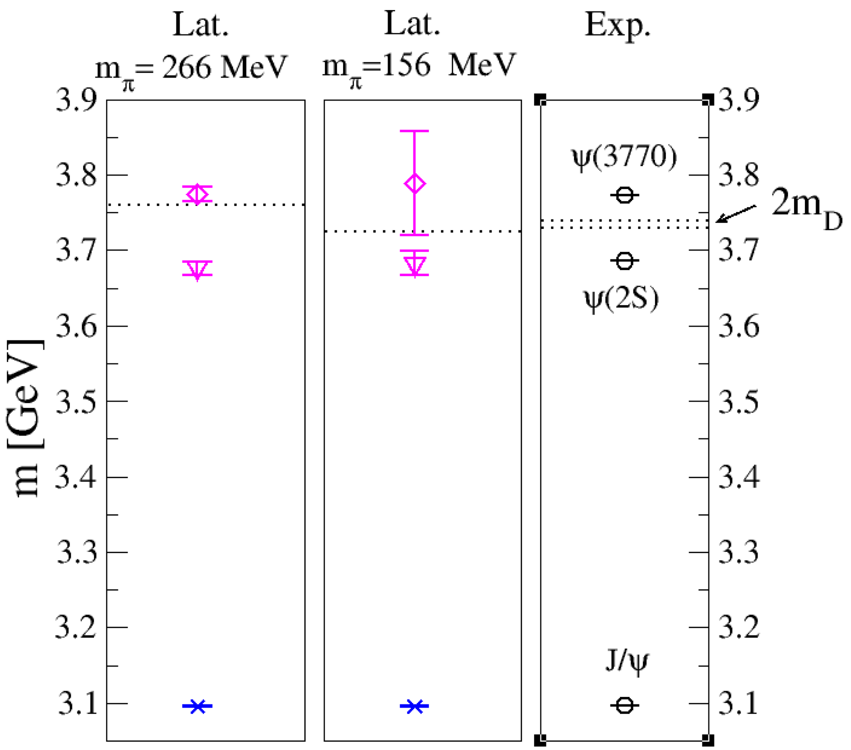
→ pioneering simulations of resonances with rigorous treatment 2012-2015

$\bar{c}u$ [Mohler, S.P. , Woloshyn: PRD 2013] $\bar{s}u$ [S.P., Lang, Leskovec, Mohler, PRD 2013; Wilson, Dudek, Edwards, Thomas, PRL 20114, PRD 2015] $\bar{c}c$ [Lang, Leskovec, Mohler, S.P., 1503.05363] a1,b1 [Lang, Leskovec, Mohler, S.P., JHEP 2014]

General result: m_R and Γ (or coupling) close to experimental values.

Exception: $\chi_{c0}(2P)$ - puzzling state

Resonance ψ(3770) and bound st. ψ(2S) from $D\underline{D}$ scattering in p-wave



$\mathcal{O}: \bar{c}\,c,\ D\bar{D},\quad J^{PC}=1^{--}$

$D\underline{D}$ scat. in p-wave is simulated

T-matrix is determined from E_n

Fit of T-matrix gives:

BW resonance ψ(3770):

m_R (magenta diamonds)

Γ (given below)

Bound state ψ(2S) from pole in T:

m_B (magetna triangles)

$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$h_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$
$2m_D$
$\psi(3770)$
$X(3872)$
$\chi_{c0}(2P)$wa
$\chi_{c2}(2P)$
$X(3940)$
$\psi(4040)$
$X(4050)^{\pm}$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^{\pm}$

ψ(3770)	Mass [MeV]	g (no unit)
Lat (m_π=266 MeV)	3774 ±6±10	19.7 ±1.4
Lat (m_π=156 MeV)	3789 ±68±10	28 ± 21
Exp.	3773.15± 0.33	18.7 ± 1.4

$$\Gamma = \frac{g^2}{6\pi}\,\frac{p^3}{s}$$

Lang, Leskovec, Mohler, S.P.,
1503.05363

Other resonances simulated on the lattice

- scalar charmonium resonances from DD scattering in s-wave:
  - It is still not commonly accepted which exp state corresponds to first exc. scalar charmonium
  - lattice data on DD indicate bound state χ_{c0}(1P) [mass agrees with experiment]
  - yet unobserved narrow scalar resonance with m_R=3.9-4.0 GeV Γ<100 MeV

  Lang, Leskovec, Mohler, S.P., 1503.05363, JHEP 2015

channel	resonance	
$D\bar{D}$:	$\psi(3770),\ \chi'_{c0}$	Lang, Leskovec, Mohler, S.P., 1503.05363, JHEP 2015]
$K\pi$:	$K^*(892),\ K_2$	[S.P. ,Lang, Leskovec, Mohler, 1307.0736, PRD 2013]
$D\pi$:	$D_0^*(2400)$	[D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD]
$D^*\pi$:	$D_1(2430)$	
$\rho\pi$:	$a_1(1230)$	[Lang, Leskovec, Mohler, S.P. , 1401.2088, 2014 JHEP]
$\omega\pi$:	b_1	

States slightly below threshold

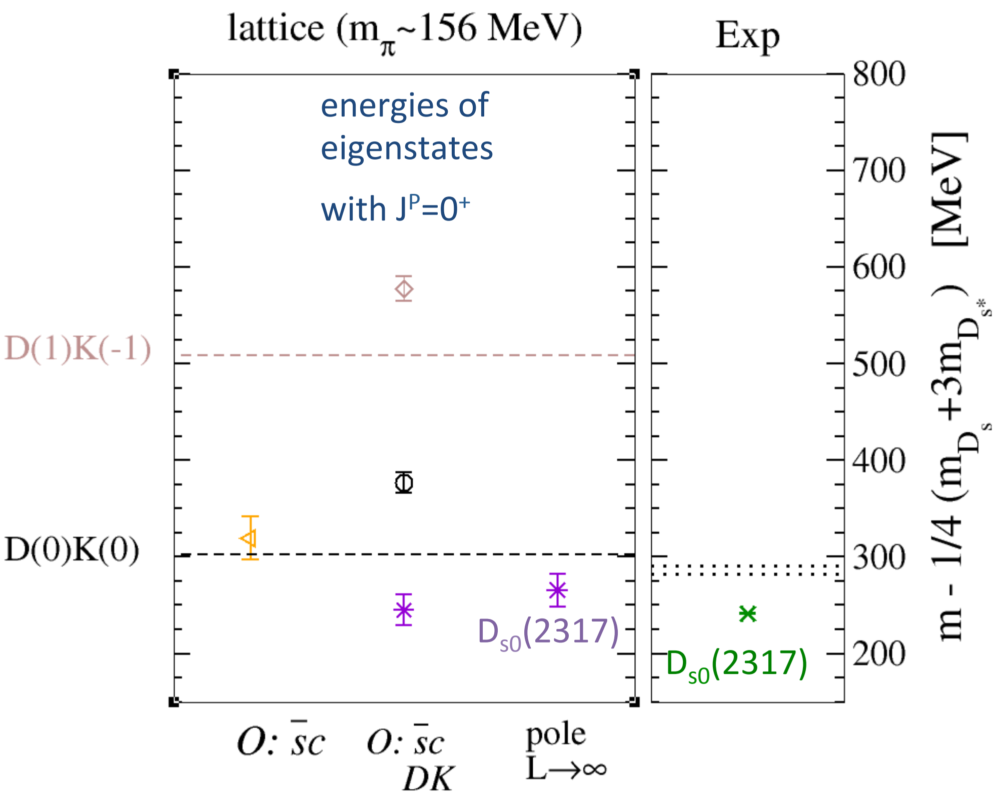
$p\eta_c$ pentaquark

D_{s0}^*, D_{s1} , B_{s0}^* , B_{s1} , X(3872)

deuterium-like systems

Scalar meson D_{s0} below DK threshold

$$\mathcal{O} = \bar{s}c,\ \ DK$$



[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn, 1308.3175, Phys. Rev. Lett.]

- phase shift for DK scattering
  E → δ(E)
- interpolation of δ(E) near th. using effective range expansion

$$p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2}\, r_0\ p^2$$

$$a_0 = -1.33(20)\ \text{fm} \qquad r_0 = 0.27(17)\ \text{fm}$$

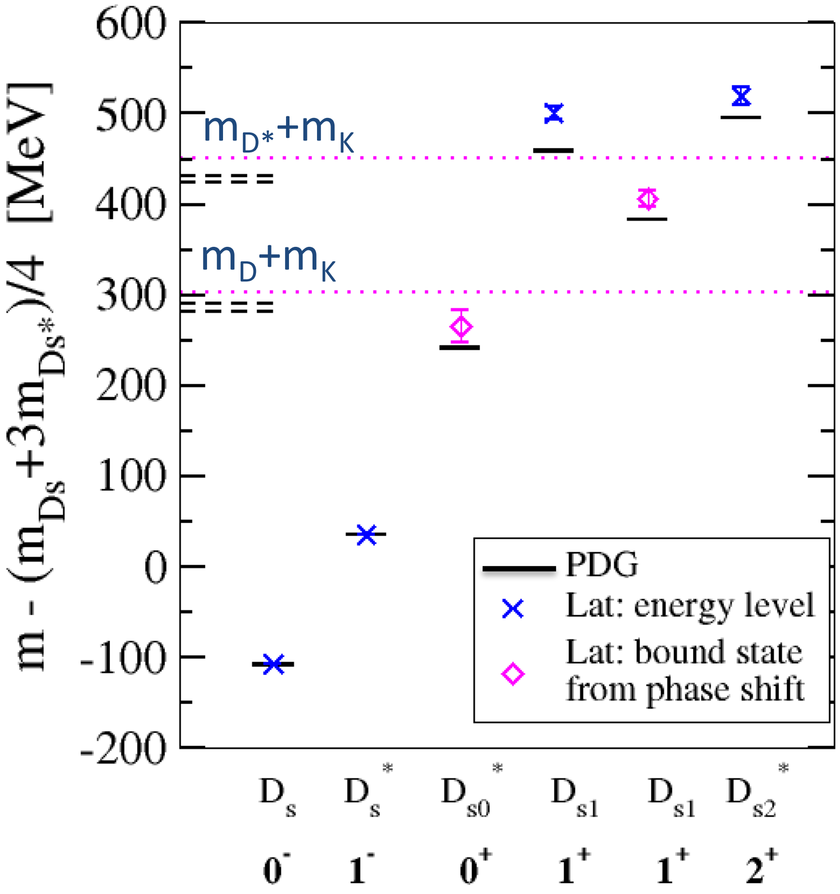
- This renders scattering matrix in all region near threshold
- pole in T(E) found slightly below th.

$$T \propto \frac{1}{\cot\delta - i} = \infty$$

- attributed to $D_{s0}(2317)$

D_s scalar meson from DK scattering

D_s meson spectrum



$$\mathcal{O} = \bar{s}c\ ,\ \ DK$$

Taking into account the effect of

DK threshold for the first time:

Simulation of DK scattering;

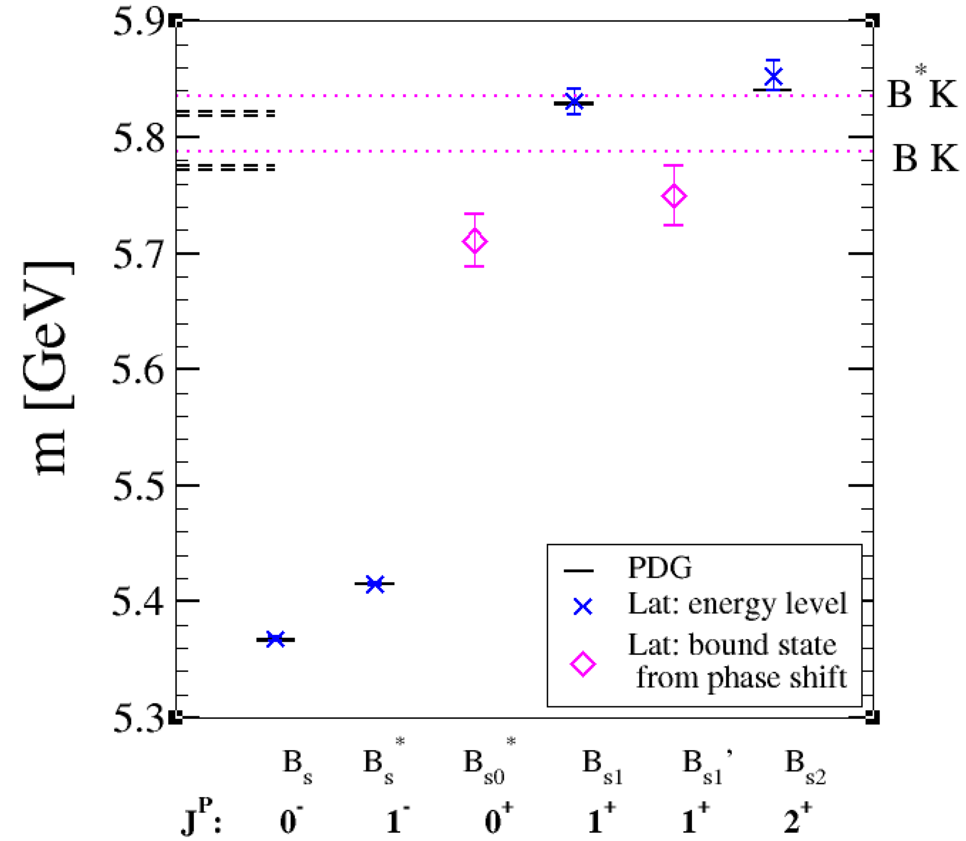
pole in T matrix found slightly below DK threshold

[C. Lang, L. Leskovec, D. Mohler, S.P. ,
R. Woloshyn: PRL 2013, PRD 2014]

Mass prediction for missing B_{s0} and B_{s1}

$$\mathcal{O} = \bar{s}b, \;\; BK$$

m_π=156 MeV



Quantities shown:

for two bound states :

$$m_B = (m_B - E_{th})^{lat} + E_{th}^{exp}$$

for other states :

$$m = (m - \bar{m})^{lat} + \bar{m}^{lat}$$

for dotted lattice thresholds :

$$E_{th} = (E_{th} - \bar{m})^{lat} + \bar{m}^{lat}$$

$$\bar{m} \equiv \tfrac{1}{4}(m_{Bs} + 3m_{Bs^*})$$

- B_{s1}' and B_{s2} agree well with exp

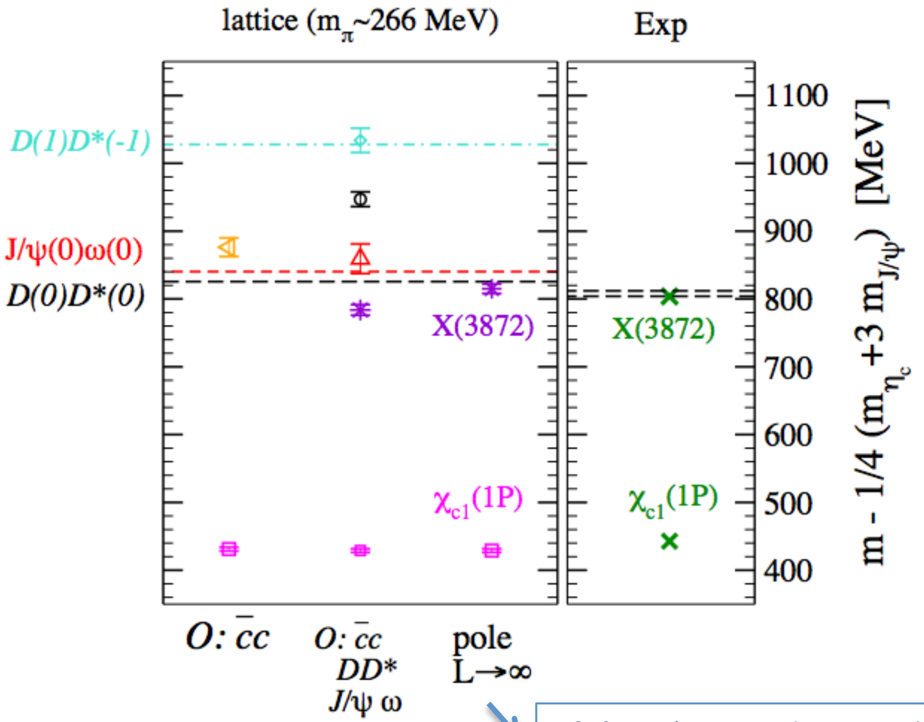
Predictions:

- B_{s0} bound state below BK th.
- B_{s1} bound state below B*K th.

[C. Lang, D. Mohler, S.P. ,
R. Woloshyn: 1501.0164]

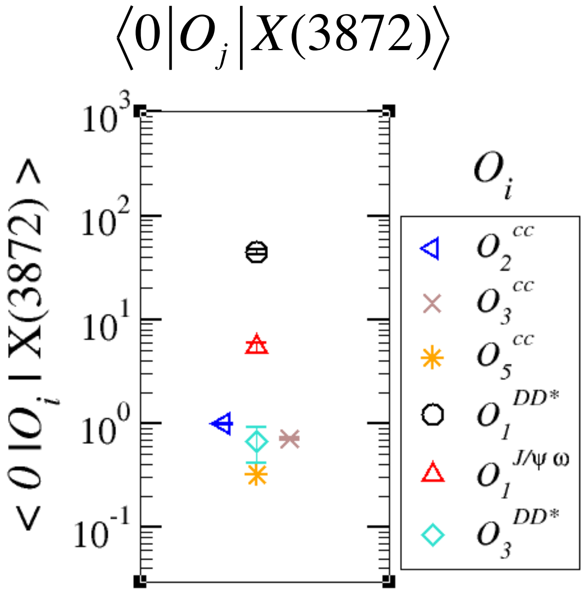
Evidence for X(3872) : $J^{PC}=1^{++}$, I=0

$\mathcal{O}: \bar{c}\,c, \quad DD^*, \quad J/\psi\omega$



- δ_0 for DD* extracted using Luscher's rel. and interpolated near threshold
- pole in T-matrix $T \propto [\cot\delta - i]^{-1} = \infty$ found just below DD* threshold.

$\langle 0 | O_j | X(3872) \rangle$



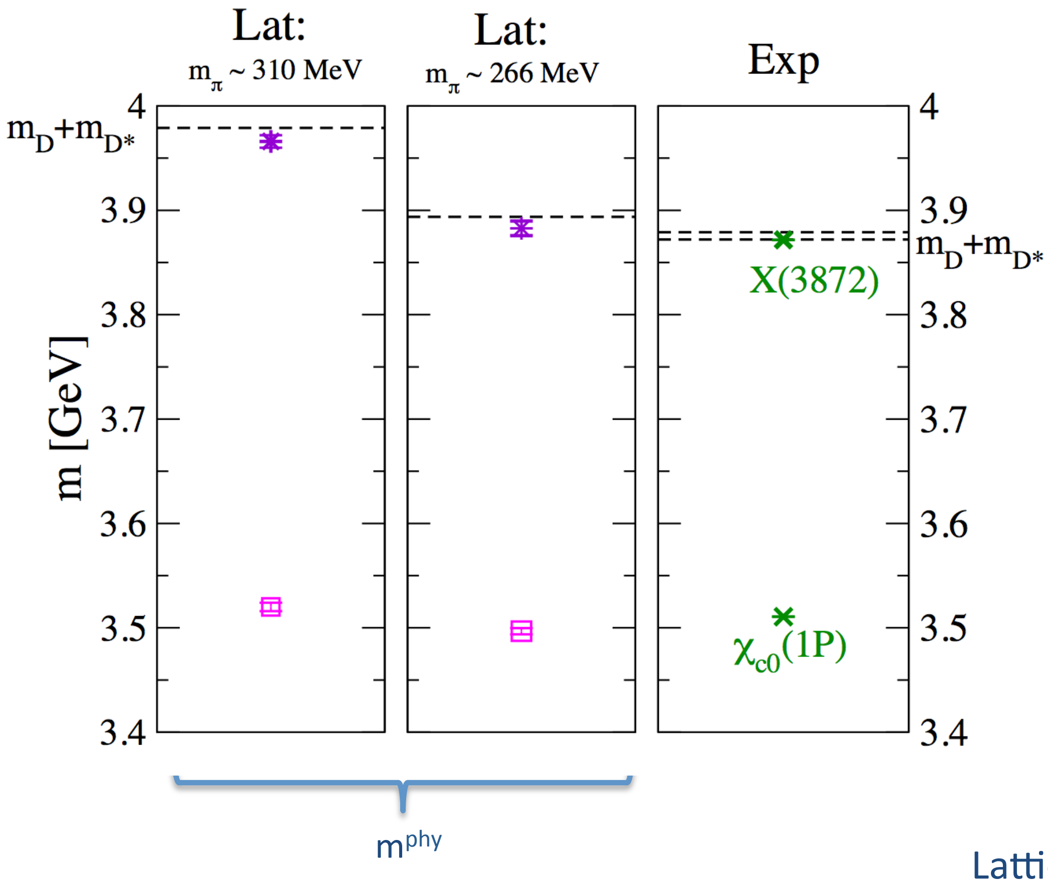
Overlaps normalized to $<0|O_1^{cc}|X(3872)>$

X(3872)	m - $(m_{D0}+m_{D0*})$
lat	- 11 ± 7 MeV
exp	- 0.14 ± 0.22 MeV

[S.P. and L. Leskovec : 1307.5172, Phys. Rev. Lett. 2013]

$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

X(3872) as bound state from $D\underline{D}^*$ scattering, $J^{PC}=1^{++}$, I=0



$\mathcal{O}: \bar{c}\, c,\ D\bar{D}^*$

- ground state: $\chi_{c1}(1P)$
- $D\underline{D}^*$ scattering matrix near th. determined
- A pole of $T \propto \frac{1}{\cot\delta - i} = \infty$ found just below th. (violet star)
- The pole attributed to X(3872), which is a shallow bound state in both simulations
- Position of DD* threshold depends on $m_{u/d,}$ and may be affected by discretization effects related to charm quark

X(3872)	m - $(m_{D0}+m_{D0^*})$
lat (m_π=310 MeV)	- 13 ± 6 MeV
lat (m_π=266 MeV)	- 11 ± 7 MeV
exp	- 0.14 ± 0.22 MeV

Lattice evidence for X(3872):

- m_π≈266 MeV, a=0.124 fm, L= 2 fm

[S.P. and Leskovec: 1307.5172, PRL 2013]

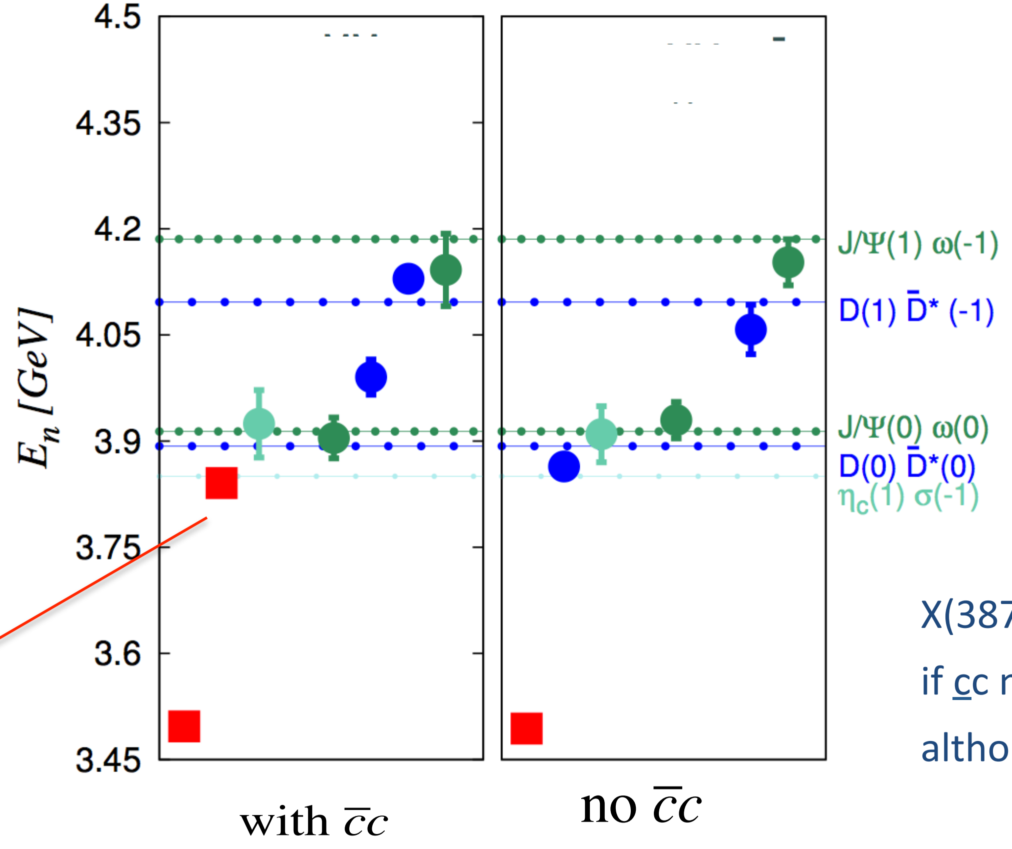
- m_π≈310 MeV, a=0.15 fm, L=2.4 fm , HISQ

[Lee, DeTar, Na, Mohler , update of proc 1411.1389]

Which Fock components are essential for X(3872) with I=0 ?

$J^{PC}=1^{++}$ $\mathcal{O}$: $\underbrace{\bar{c}\,c,\quad D\bar{D}^*}_{\text{essential}}$, $J/\psi\omega$, $\chi_{c1}\eta$, $\eta_c\sigma$, $\underbrace{[\overline{cu}]_{3c}[cu]_{3c},\ [\overline{cu}]_{6c}[cu]_{6c}}_{\text{do no seem not essential}}$

energies of eigenstates



X(3872) found

only if $\underline{c}c$ in the basis

X(3872) not found

if $\underline{c}c$ not in the basis,

although [$\underline{cu}$][cu] in the basis

[M. Padmanath, C.B. Lang, S.P., 1503.03257]

Bound state of a η_c and p from lattice

[NPLQCD, 1410.7069, PRD, m_π~800 MeV]

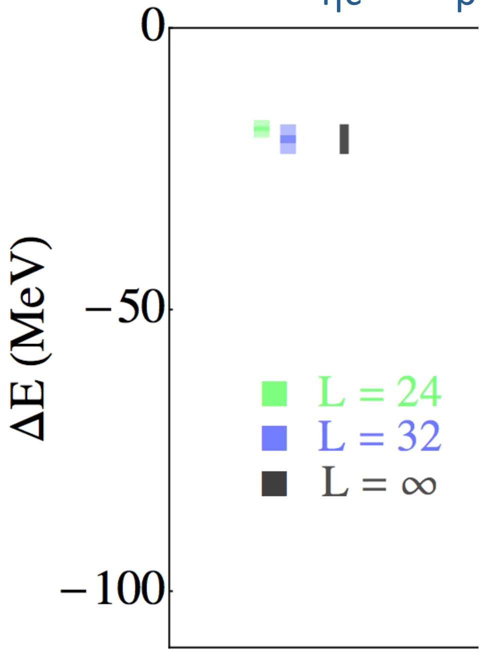
$$\eta_c \; p$$

$$\bar{c}c \; uud$$

~ 20 MeV below

th. $m_{\eta c} + m_p$

$m - (m_{\eta c} + m_p)$



Two pentaquark resonances in J/ψ and p from exp

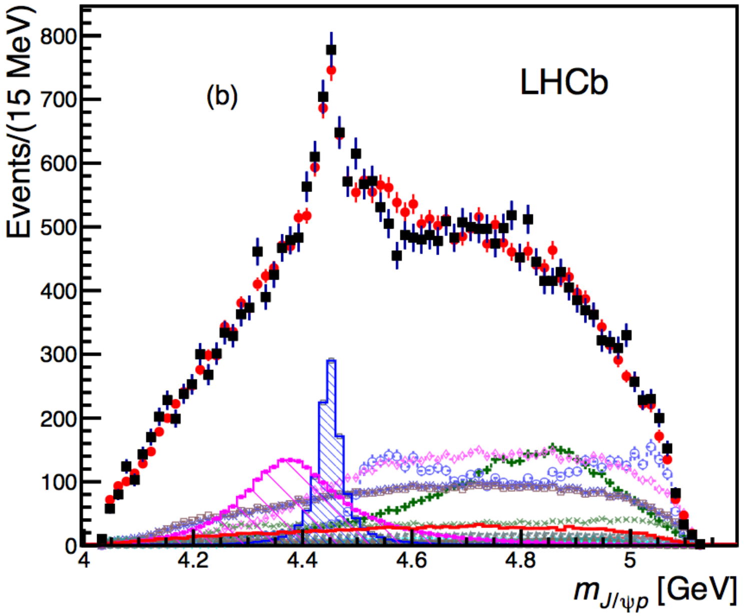
LHCb: 1507.03414

$$J/\psi \; p$$

$$\bar{c}c \; uud$$

~ 400 MeV above

th. $m_{J/\psi} + m_p$



Searches for hadrons with exotic flavor

very challenging

$$Z_c = \bar{c}c\bar{d}u, \quad Z_b = \bar{b}b\bar{d}u, \quad P_c = \bar{c}cuud$$

few searches challenging challenging and recent

Even more challenging since most of experimental exotic states

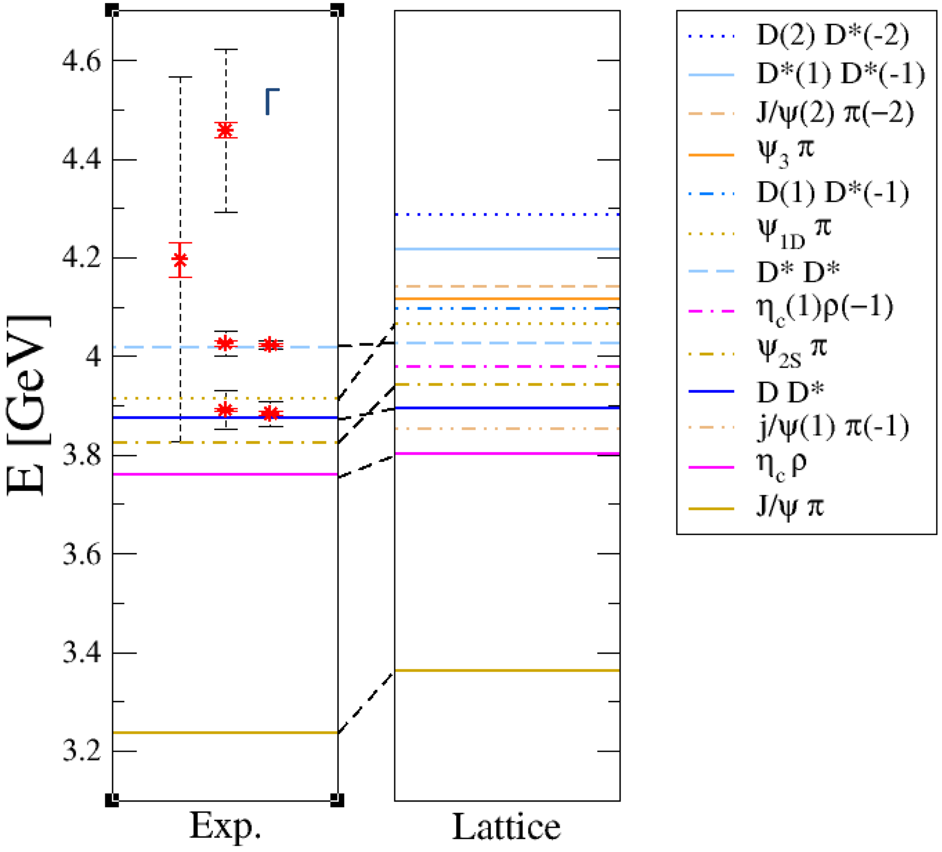
- are above several thresholds and decay to several two-meson final states
- require simulation of coupled-channels

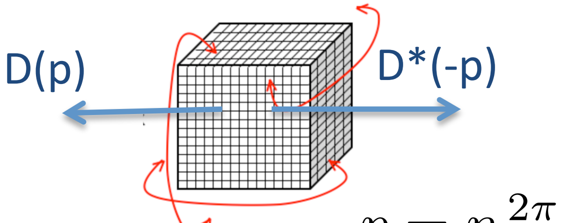
Coupled-channel scattering matrix extracted on lattice so far only in

- a-la Luscher: Kπ, Kη system [Willson, Dudek, Edward, Thomas, HSC, PRL 2014]

Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$

$\begin{matrix}\bar{c} & u \\ c & \bar{d}\end{matrix}$





$$p = n\frac{2\pi}{L}$$

Lattice:

Horizontal lines represent energies of 13 two-meson states in non-interacting case

E = E[$M_1(p_1)$] + E[$M_2(p_2)$]

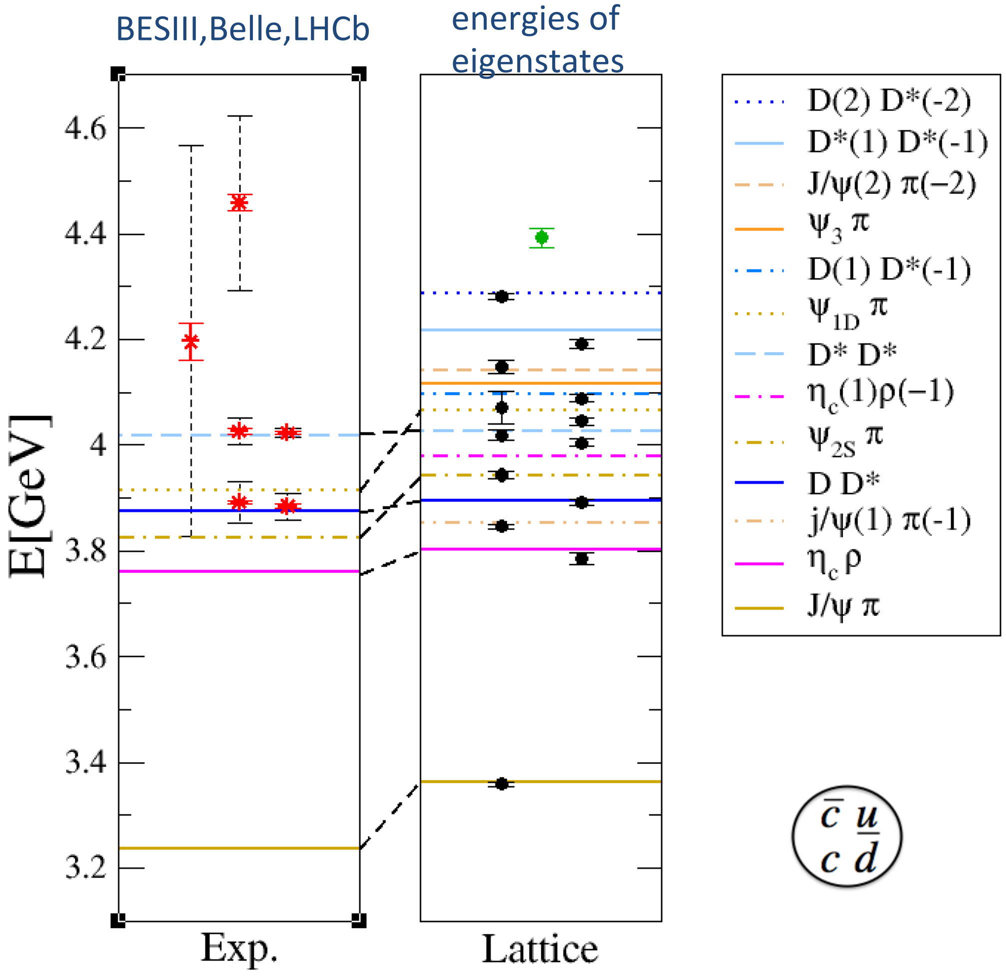
Extracting 13 two-meson states is a challenge

$$\mathcal{O} : (\bar{c}u)(\bar{d}c),\ (\bar{c}c)(\bar{d}u),\ [\bar{c}\bar{d}][cu]$$

[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

Ensemble (2), m_π≈266 MeV, L≈2 fm, Nf=2

Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$



$$\mathcal{O}:(\bar{c}u)(\bar{d}c),\ (\bar{c}c)(\bar{d}u),\ [\bar{c}\bar{d}][cu]$$

Lattice results:

- 13 expected two-meson eigenstates found as expected (black circles)
- no additional eigenstate below 4.2 GeV
- no candidate for Z_c^+ below 4.2 GeV

Exp:

- Z_c^+(3900) confirmed by three exp

Puzzle:

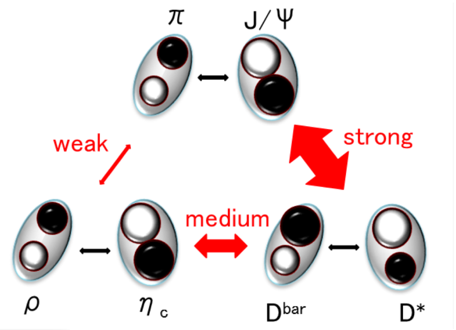
- why no eigenstate for Zc(3900) ?
- it would be naively expected if Zc(3900) related to a resonance pole
- is Zc(3900) of a different origin?
- perhaps coupled channel effect?

[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

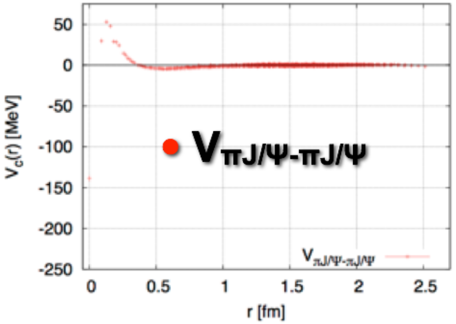
similar conclusion [S.-H. Lee, C. DeTar, H. Na, 1411.1389]

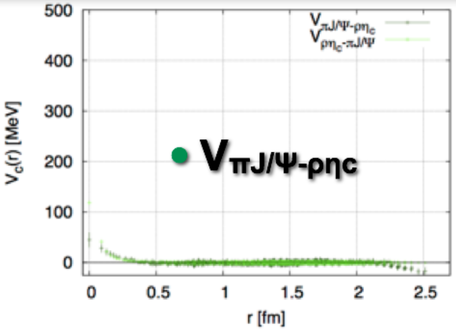
Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$

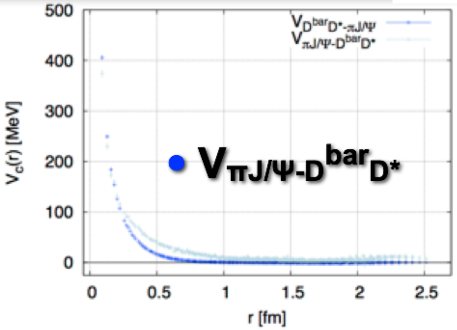
- HALQCD method [application on H-dibaryon: HALQCD, 1504.01717]



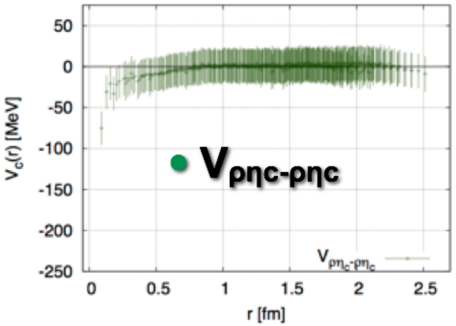
indication for coupled channel eff.

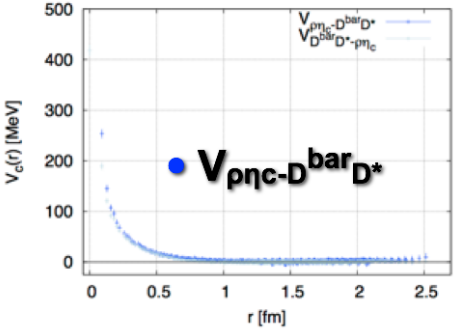


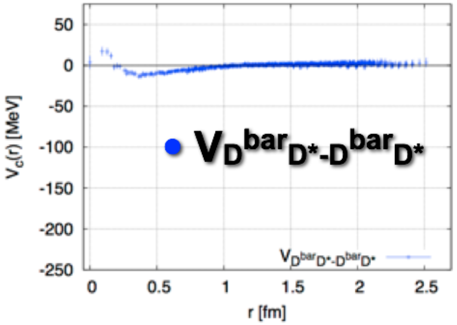




r [fm]







HALQCD coll, Y. Ikeda et al $m_\pi \approx 410$ MeV, Nf=2+1+1

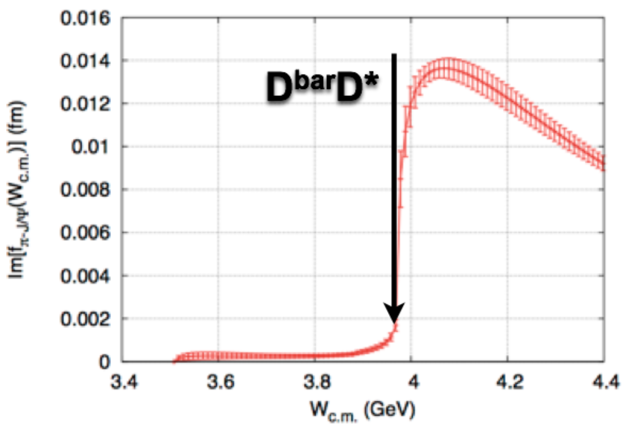
[private communication with Y. Ikeda]

Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$

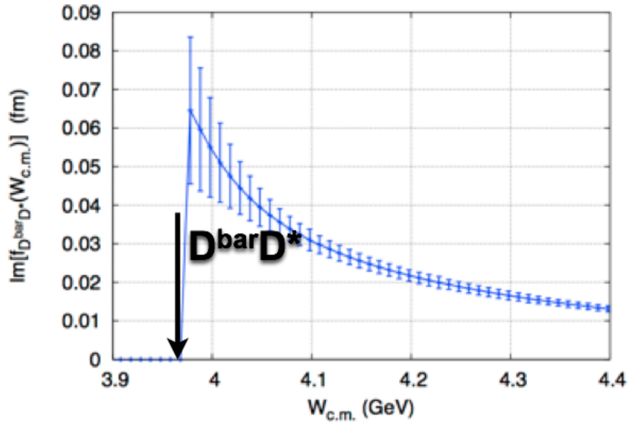
Lattice quantity related to cross-section

Experimental cross-section related to $Z_c^+(3900)$

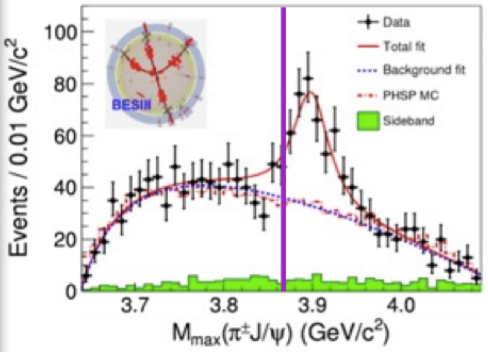
- **$\pi J/\Psi$ invariant mass (m_π=410MeV)**



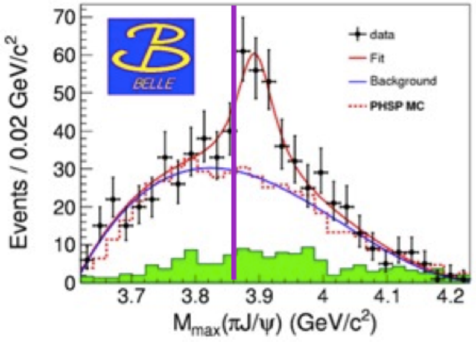
- **$D^{bar}D^*$ invariant mass (m_π=410MeV)**



- **e^+e^- --> $\pi(\pi J/\Psi)$ @ 4.26GeV**

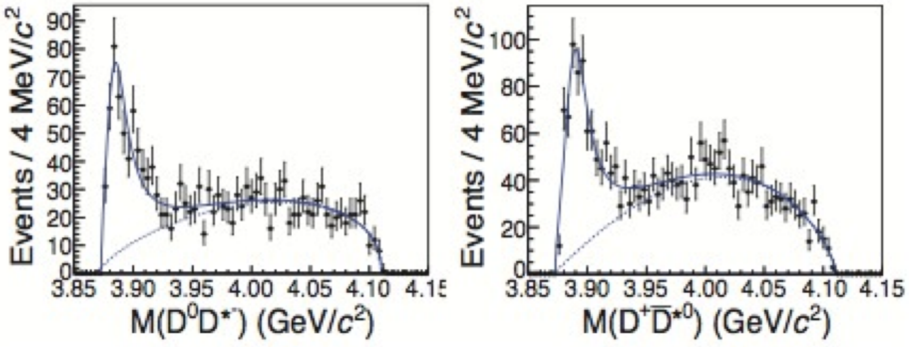


BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).

- **e^+e^- --> $\pi^{+/-}$ $(D^{bar}D^*)^{-/+}$**



BESIII Coll., PRL112 (2014).

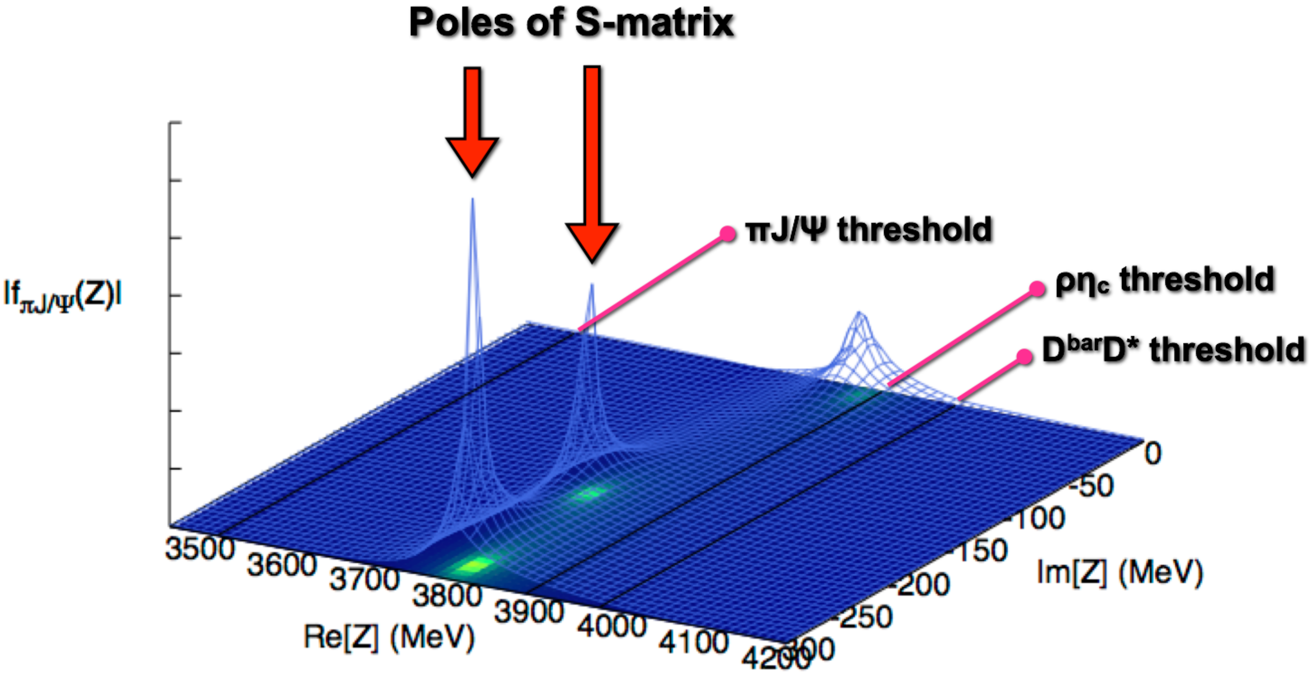
HALQCD coll, Ikeda et al m_π≈410 MeV, Nf=2+1+1

[private communication with Y. Ikeda]

Lineshapes resemble

experimental Zc(3900).

Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$



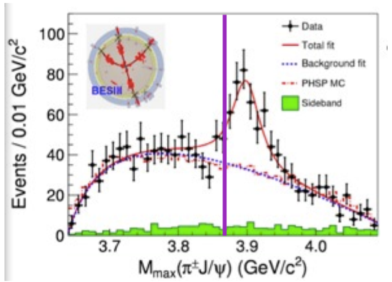
Poles of S-matrix

$\pi J/\Psi$ threshold

$\rho\eta_c$ threshold

$D^{bar}D^*$ threshold

DD*



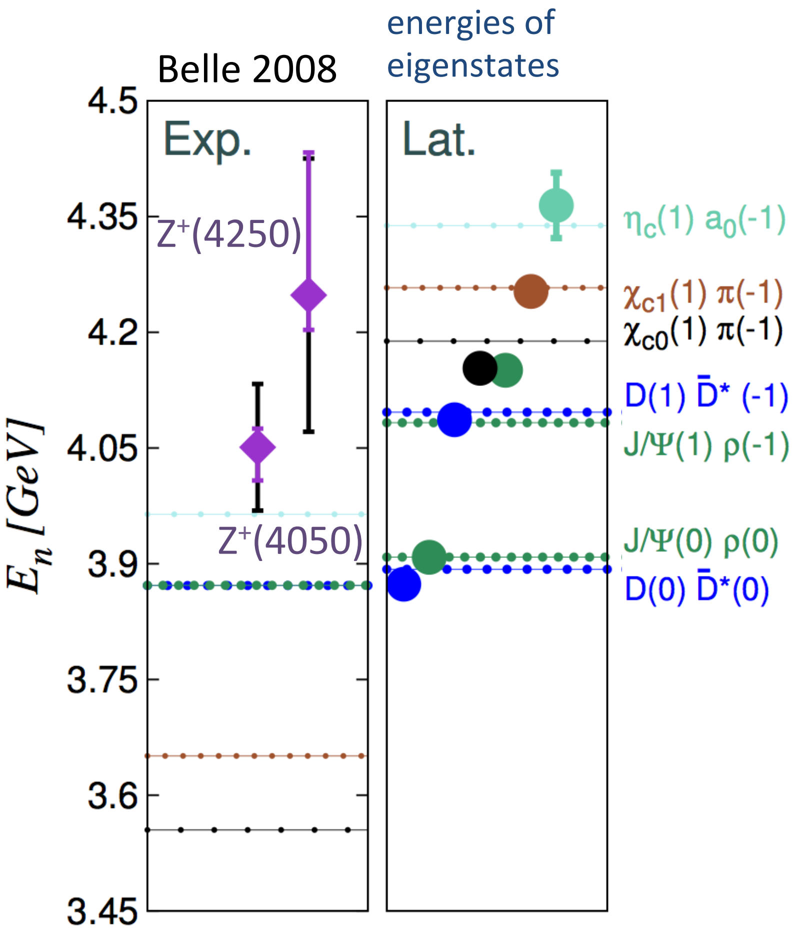
- exp: $Z_c^+(3900)$ peak appears above DD* th.
- HALQCD: poles found BELOW DD* th.

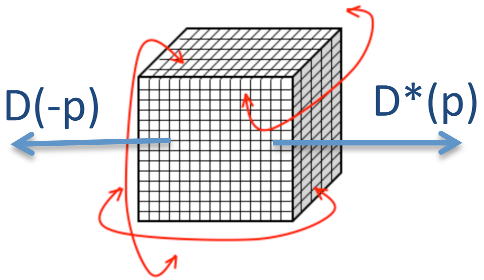
  pole NOT interpreted as a resonance (such a pole would be expected above DD*)
- Remains to be seen if HALQCD result is consistent with absence of Zc eigenstate in S.P. et al, PRD 2015
- Scattering matrix has not yet been extracted from Luscher approach

HALQCD coll, Ikeda et al $m_\pi \approx 410$ MeV, Nf=2+1+1

[private communication with Y. Ikeda]

charged partner of X(3872); channel $I^G=1^-$, $J^{PC}=1^{++}$, $\underline{c}c\underline{d}u$





$$p = n\frac{2\pi}{L}$$

$$\mathcal{O} : (\bar{c}u)(\bar{d}c),\ \ (\bar{c}c)(\bar{d}u),\ \ [\bar{c}\bar{d}][cu]$$

- Horizontal lines: energies of expected two-meson states in limit of no interaction: $E = E[\ M_1(p_1)\ ] + E[\ M_2(p_2)\ ]$

- Circles: energies of eigenstates from latt

- Only expected two-meson states observed.

- No lattice candidate for charged X(3872). In agreement with absence of such state in exp.

- No lattice candidate for other charged state. Two Belle 2008 states are exp. unconfirmed.

[M. Padmanath, C.B. Lang, S.P., 1503.03257]

Y(4140), $J^{PC}=?^{?+}$, $\underline{c}c\underline{s}s$

$$Y(4140) \rightarrow \; J/\psi \; \phi$$
$$\bar{c}c \quad \bar{s}s$$

Experiment:

peak in J/ψ Φ just above J/ψ Φ threshold

found: CDF 2009, CMS 2012, D0 2013, Babar 2015

not found: Belle 2010, LHCb 2012

Lattice:

- S. Ozaki and S. Sasaki, 1211.5512, PRD

  strange quark annihilation neglected

  no resonant structure found for

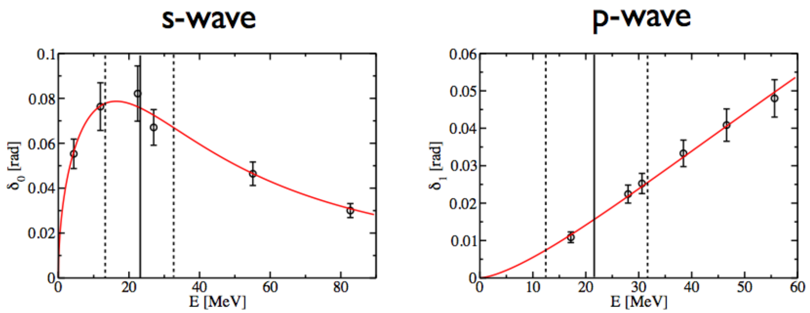
- M. Padmanath, C.B. Lang, S.P., 1503.03257

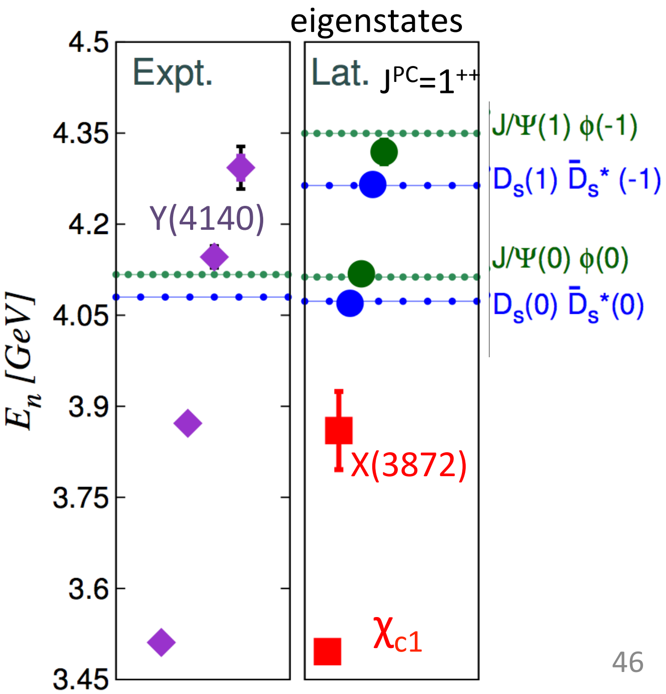
  $$\mathcal{O}: \;\; \bar{c}c, \;\; (\bar{c}s)(\bar{s}c), \;\; (\bar{c}c)(\bar{s}s), \;\; [\bar{c}\bar{s}][cs]$$

  channel $J^P=1^+$ considered only: expected two-particle

  eigenstates found and χ_{c1} , X(3872) but not Y(4140)

J/Ψ Φ scattering phase shift [rad]





Conclusions

- **Experiment** (in brief): exciting tetraquark and pentaquark candidates

- **Phenomenology:** many many interpretations of exotic experimental “bumps”, some of them do not need to introduce exotic hadrons to explain experimental data

- **Lattice QCD** (in brief):

☐ Evidence found for many hadrons with **non-exotic** flavor:

- states well below th. : charmonium , D, π, K ... and all the others
- shallow bound states : D_{s0}, D_{s1}, B_{s0}, B_{s1} , X(3872) with I=0
- resonances via BW : ρ, K*, $K_0^*(1430)$, K_2, D_0^*, D_1, a_1, b_1 , Ψ(3770)

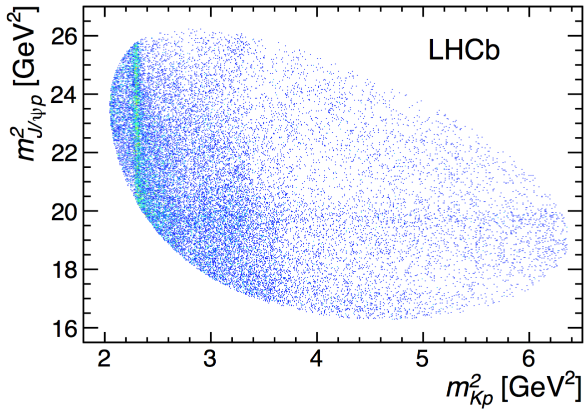
☐ Hadrons with **exotic** flavor

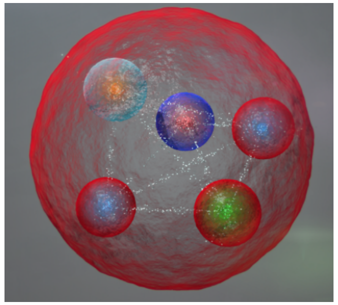
- if exotic hadrons were below strong decay threshold, they would be easy to (dis)prove in LQCD
- unfortunately, most of exotic hadrons can decay into several channels via strong interaction
- therefore lattice has not given yet a final answer which (if any) exotic hadrons arise from QCD
- I have (hopefully) given you some flavor in which direction progress is going
- this is an exciting topic at present and I am looking forward to face further challenges it poses ...

Backup slides

Pentaquarks

$$\Lambda_b^- \to K^- J/\psi\ p$$





LHCb: 1507.03414

14th July 2015

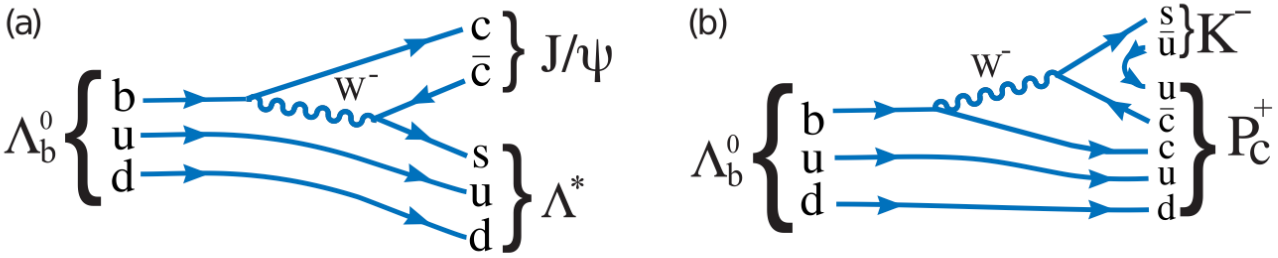
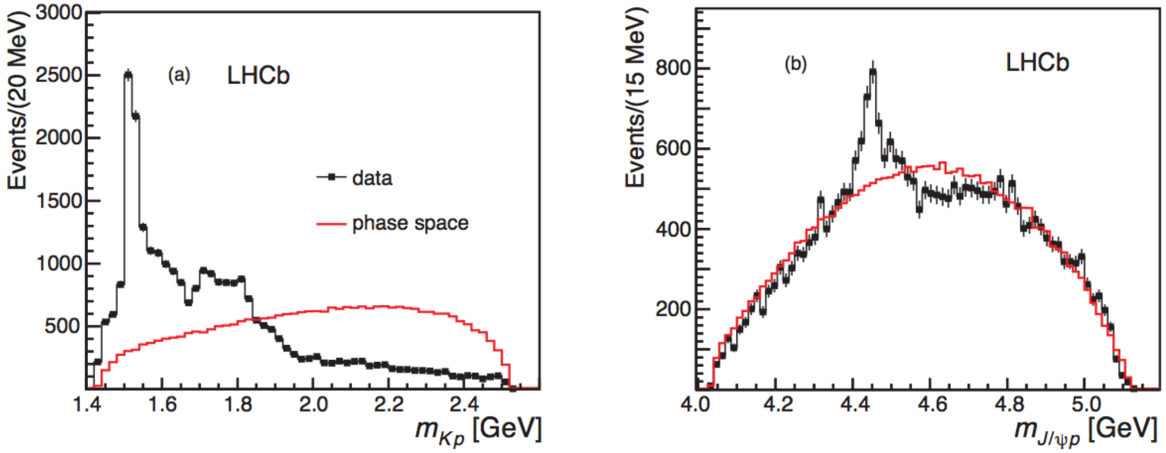


Figure 1: Feynman diagrams for (a) $\Lambda_b^0 \to J/\psi\,\Lambda^*$ and (b) $\Lambda_b^0 \to P_c^+ K^-$ decay.

$$P_c^+ \to J/\psi\ \ p$$

$$\bar{c}c\ \ uud$$

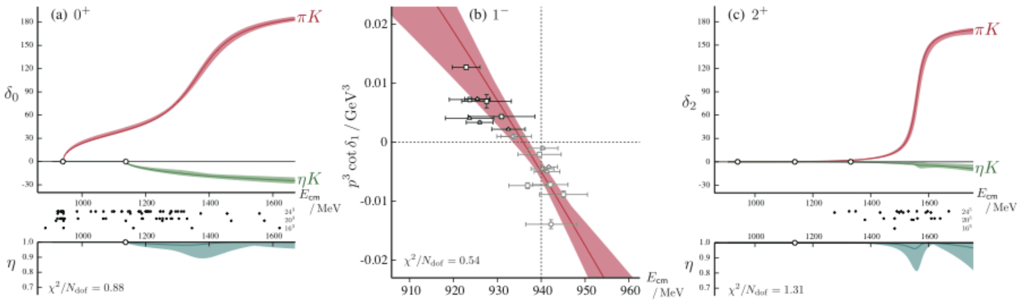


State	J^P	M_0 (MeV)	Γ_0 (MeV)
$\Lambda(1405)$	$1/2^-$	$1405.1^{+1.3}_{-1.0}$	50.5 ± 2.0
$\Lambda(1520)$	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0
$\Lambda(1600)$	$1/2^+$	1600	150
$\Lambda(1670)$	$1/2^-$	1670	35
$\Lambda(1690)$	$3/2^-$	1690	60
$\Lambda(1800)$	$1/2^-$	1800	300
$\Lambda(1810)$	$1/2^+$	1810	150
$\Lambda(1820)$	$5/2^+$	1820	80
$\Lambda(1830)$	$5/2^-$	1830	95
$\Lambda(1890)$	$3/2^+$	1890	100
$\Lambda(2100)$	$7/2^-$	2100	200
$\Lambda(2110)$	$5/2^+$	2110	200
$\Lambda(2350)$	$9/2^+$	2350	150
$\Lambda(2585)$	?	≈2585	200

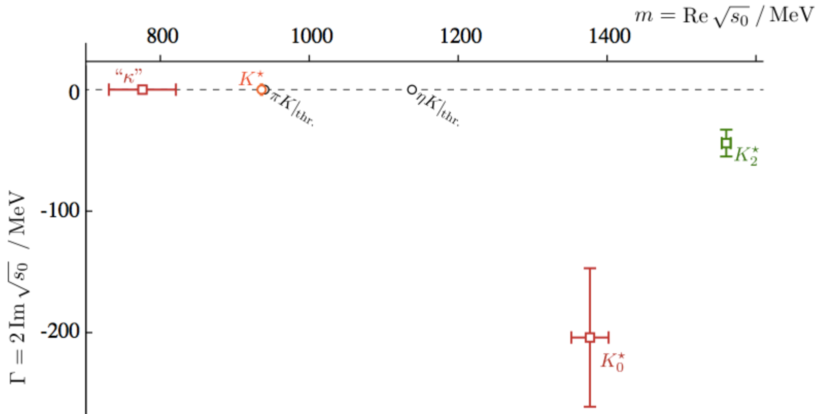
Resonances in Kπ, Kη coupled channels

- $\underline{q}q$, Kπ, Kη interpolators
- a number of different 0<P≤2
- for each E_n: one determinant equation for many unknowns
- T-matrix parametrized to get around this problem
- the location of poles of T-matrix in complex given below
- K*(892) and κ are below threshold for this m
- K_0^* , K_2^* are resonances
- m_π=391 MeV, N_L=16, 20, 24

[Dudek, Edwards, Thomas, Wilson, HSC, 1406.4158, PRL 2014]



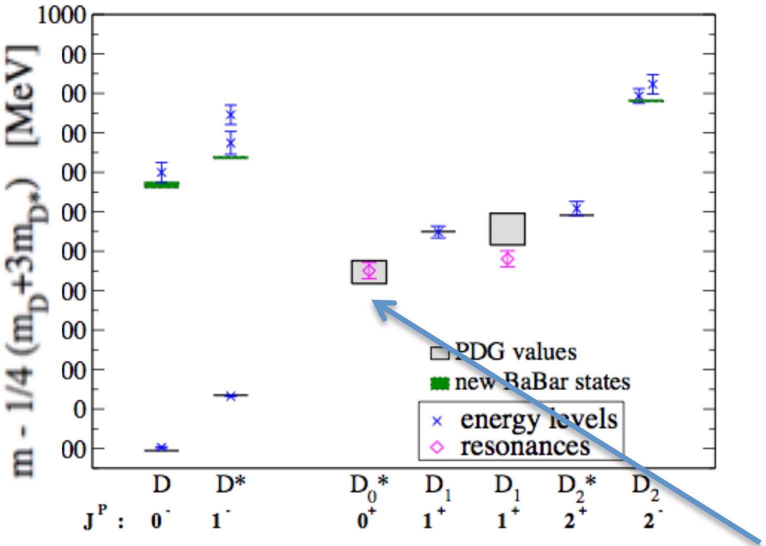
$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, \; t_{ij} = \frac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}}$$



$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i\, t_{ij}^{(J)}(E_{\mathsf{cm}})\left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L)\right)\right] = 0,$$

location of poles in T matrix in complex plane

D_s and D scalar meson puzzle



D mesons (resonances)

scalar D resonance in Dπ

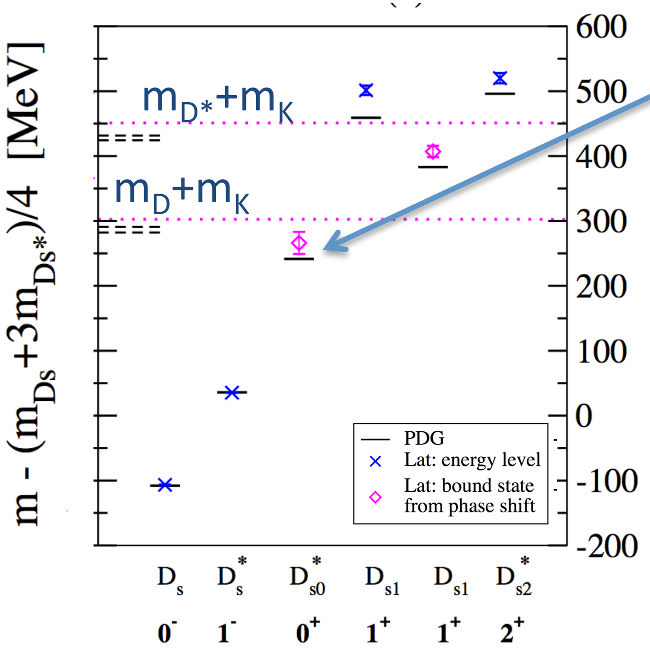
[D. Mohler, S.P. , R. Woloshyn: PRD 2013]

- why do these scalar partners have mass so close ?

$D_0^*(2400):\quad M\approx 2318\ \text{MeV}\quad \Gamma\approx 267\ \text{MeV}\quad \bar{c}u\ \text{ or }\ \bar{c}u\bar{s}s\,?$

$D_{s0}(2317):\quad M\approx 2318\ \text{MeV}\quad \Gamma\approx 0\ \text{MeV}\quad \bar{c}s\ \text{ or }\ \bar{c}s[\bar{u}u+\bar{d}d]\ ?$

1) is D_0^* mass pushed up : valence ss pair ?? ✖
2) is D_{s0}^* mass pushed down : effect of DK threshold ?? ✔



D_s mesons (near-threshold)

[C. Lang, L. Leskovec, D. Mohler, S.P. , R. Woloshyn: PRL 2013, PRD 2014]