Doubly charm tetraquark and charmonium-like resonances

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Karl-Franzens Universität Graz, 5th October, 2022

Lattice QCD study of

Doubly charm tetraquark $cc\bar{d}\bar{u}$

Charmonium(like) states $\bar{c}c$, $\bar{c}q\bar{q}c$

in collaboration with

M. Padmanath

2202.101101 Phys. Rev. Lett. 2022

S. Collins, D. Mohler,
M. Padmanath, S. Piemonte
(Regensburg)
2011.02541 JHEP 2021
1905.03506 Phys. Rev. D
2111.02934 (proceedins)

Motivation

$\bar{c}c$, $\bar{c}q\bar{q}c$

majority of exotic hadrons contain $c\bar{c}$

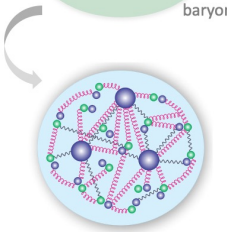
$cc\bar{d}\bar{u}$

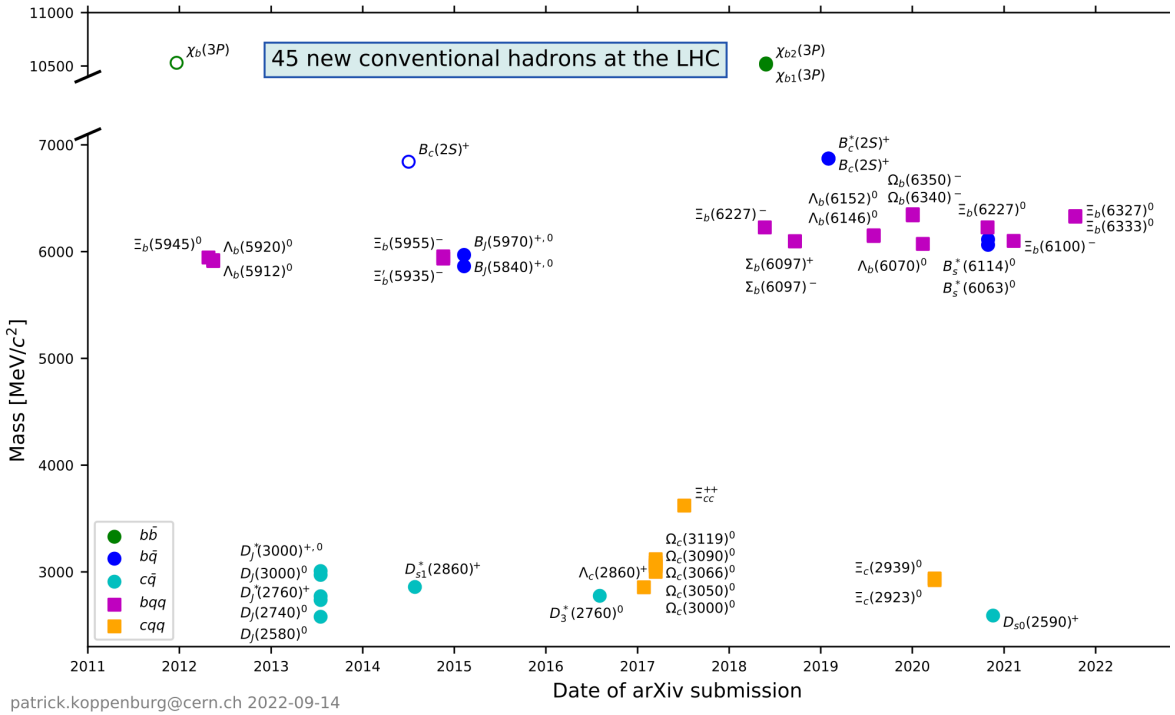
the longest lived exotic hadron discovered to date

conventional hadrons

$\bar{q}\ q$ meson

$q\ q\ q$ baryon



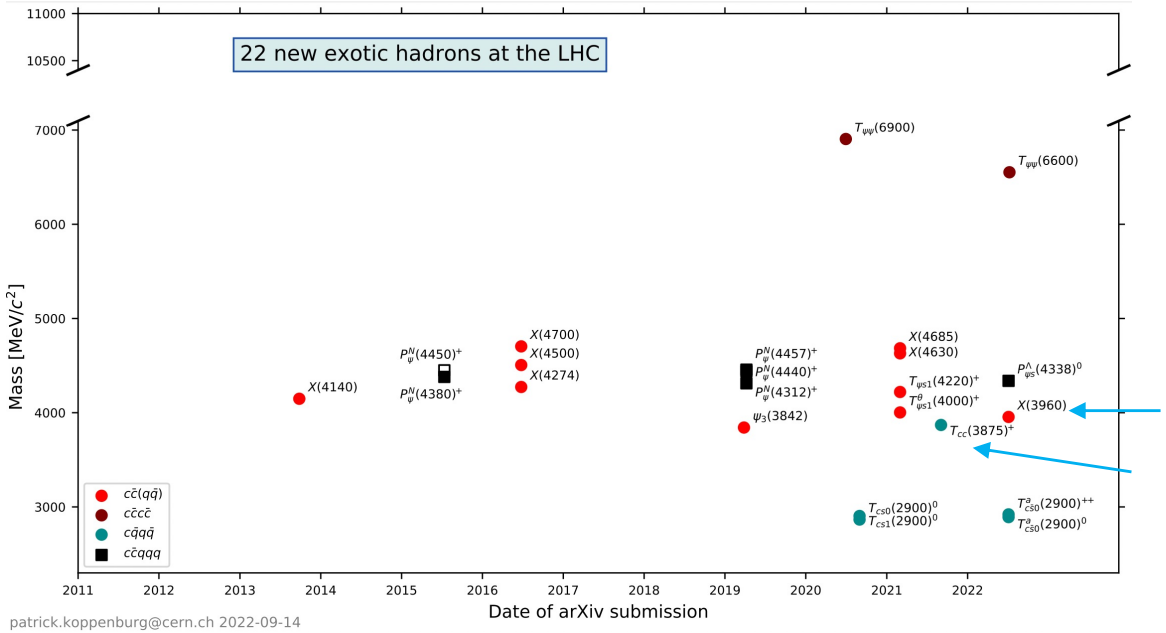


exotic hadrons

$q\ q$ $\bar{q}\ \bar{q}$ tetraquark

$q\ q\ q$ $\bar{q}\ q$ pentaquark

$\bar{q}\ G\ q$ hybrid



https://www.nikhef.nl/~pkoppenb/particles.html

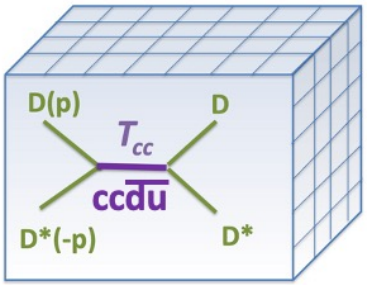
Outline and main lattice QCD results

Doubly charm tetraquark (T_{cc})

$$cc\bar{d}\bar{u}$$

I=0, $J^P=1^+$

DD^* scattering



- T_{cc} found as a virtual bound state ≈10 MeV below DD* threshold

- likely related to T_{cc} discovered by LHCb

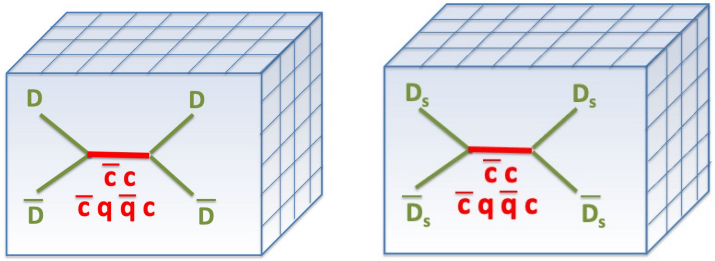
Charmonium(like) states

q=u,d,s

$$\bar{c}c\ ,\ \bar{c}q\bar{q}c$$

I=0, $J^{PC}=0^{++},1^{--},2^{++},3^{-}$

$D\bar{D} - D_s\bar{D}_s$ scattering



- masses and decay widths of conventional charmonia roughly confirmed
- two additional exotic charmonium-like states with $J^{PC}=0^{++}$ found just below thresholds

$\bar{D}D$ seen in dispersive re-analysis of exp. [Danilkin et al 2111.15033]

$\bar{D}_s D_s$ likely related to X(3915) / χ_{c0}(3930) / X(3960)
LHCb2020 LHCb2022

Lattice details

CLS ensembles with u/d, s dynamical quarks

a≃0.086 fm, **m_π =280(3) MeV**

L = 2.1 fm, 2.7 fm

relativistic charm quarks

lat exp

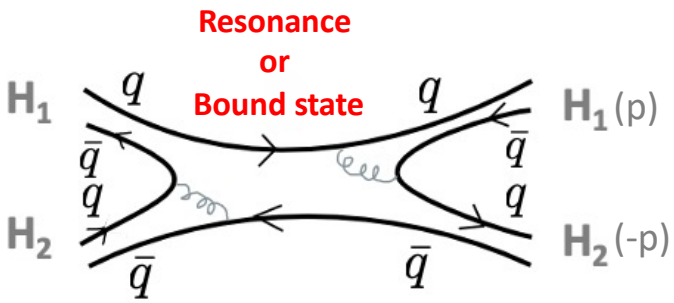
$m_{u/d} > m_{u/d}^{exp}$

$m_s < m_s^{exp}$

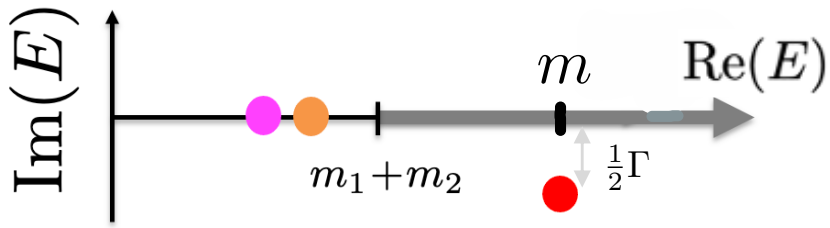
$m_u+m_d+m_s = m_u^{exp} + m_d^{exp} + m_s^{exp}$

only qualitative comparison to exp !!

Extract resonances and (virtual) bound states from H_1 H_2 scattering



scattering matrix T(E)



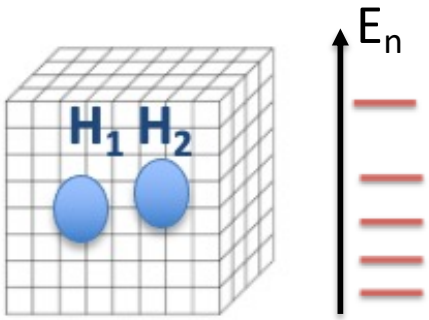
Virtual bound st. | Bound st. | Resonance

p = - i |p| | p = i |p|

$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$

Scattering matrix T(E) from lattice QCD



periodic b.c.

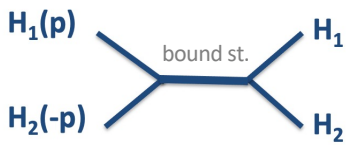
real E | for real E | for complex E

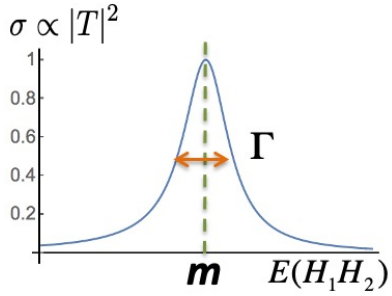
$$E \rightarrow T(E) \rightarrow T(E^c)$$

analytic relation: Luscher 1991

analytic contin. to complex E

simple argument: next slide





one-channel scattering

$$S = 1 + i\frac{4p}{E}T = e^{2i\delta} \qquad T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

Relation between E and δ(E), T(E):
1D nonrelativistic quantum mechanics

V= 0: outside the region of potential

$$\psi(x) = A\cos(p|x|+\gamma) = \begin{cases} A\cos(px+\gamma) & x>R \\ A\cos(-px+\gamma) & x<-\frac{R}{2} \end{cases}$$

- this form already ensures $\psi(L/2) = \psi(-L/2)$
- the other BC: $\psi'(L/2) = \psi'(-L/2)$

this requires

$$Ap\sin\left(p\left(\tfrac{L}{2}\right)+\gamma\right) = -Ap\sin\left(-p\left(-\tfrac{L}{2}\right)+\gamma\right)$$

$$\rightarrow \psi'(L/2)=0, \quad \sin\left(p\tfrac{L}{2}+\gamma\right)=0$$

$$p\tfrac{L}{2}+\delta = n\pi \qquad \boxed{p = n\frac{2\pi}{L} - \frac{2}{L}\gamma}$$

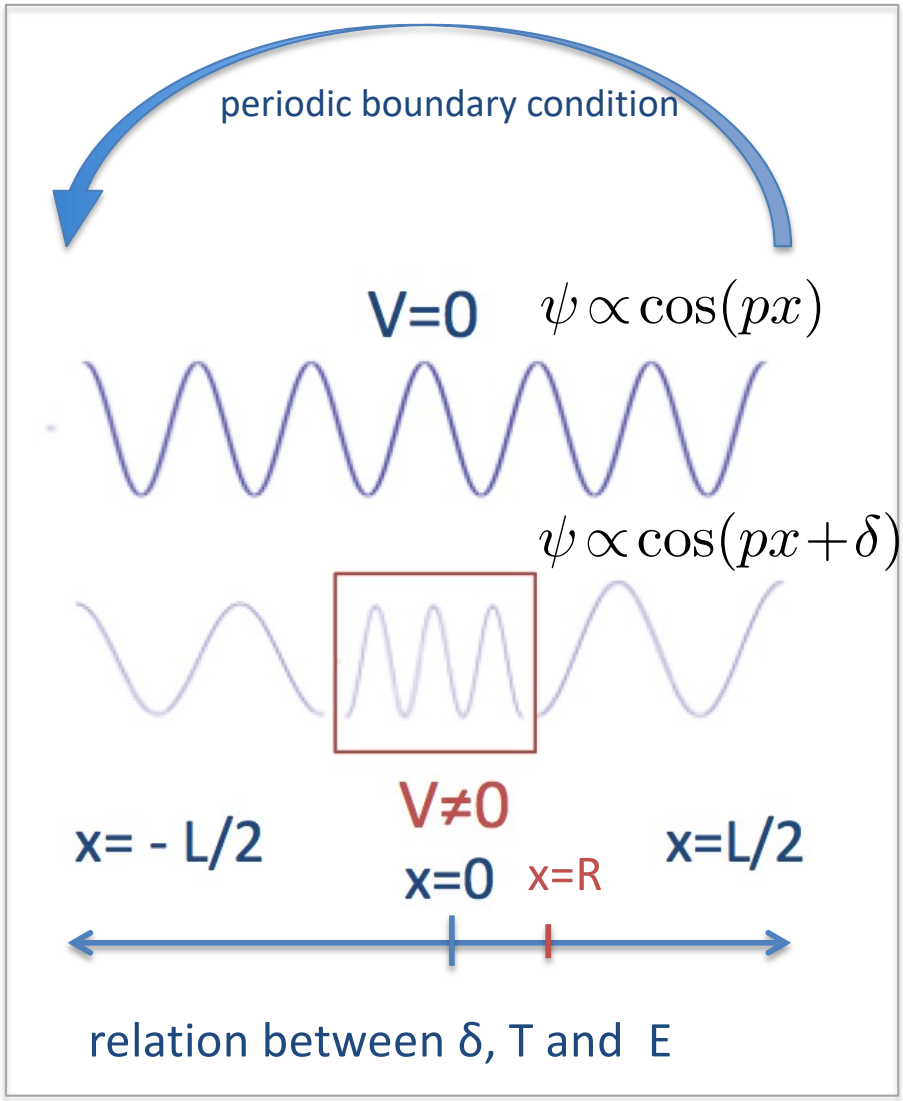
relation between δ, L

$$p = \frac{2\pi}{L}n$$

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta$$

$E=p^2/2m$

in both cases



Doubly heavy tetraquarks

$QQ'\bar{q}\bar{q}'$

$$J^P = 1^+$$

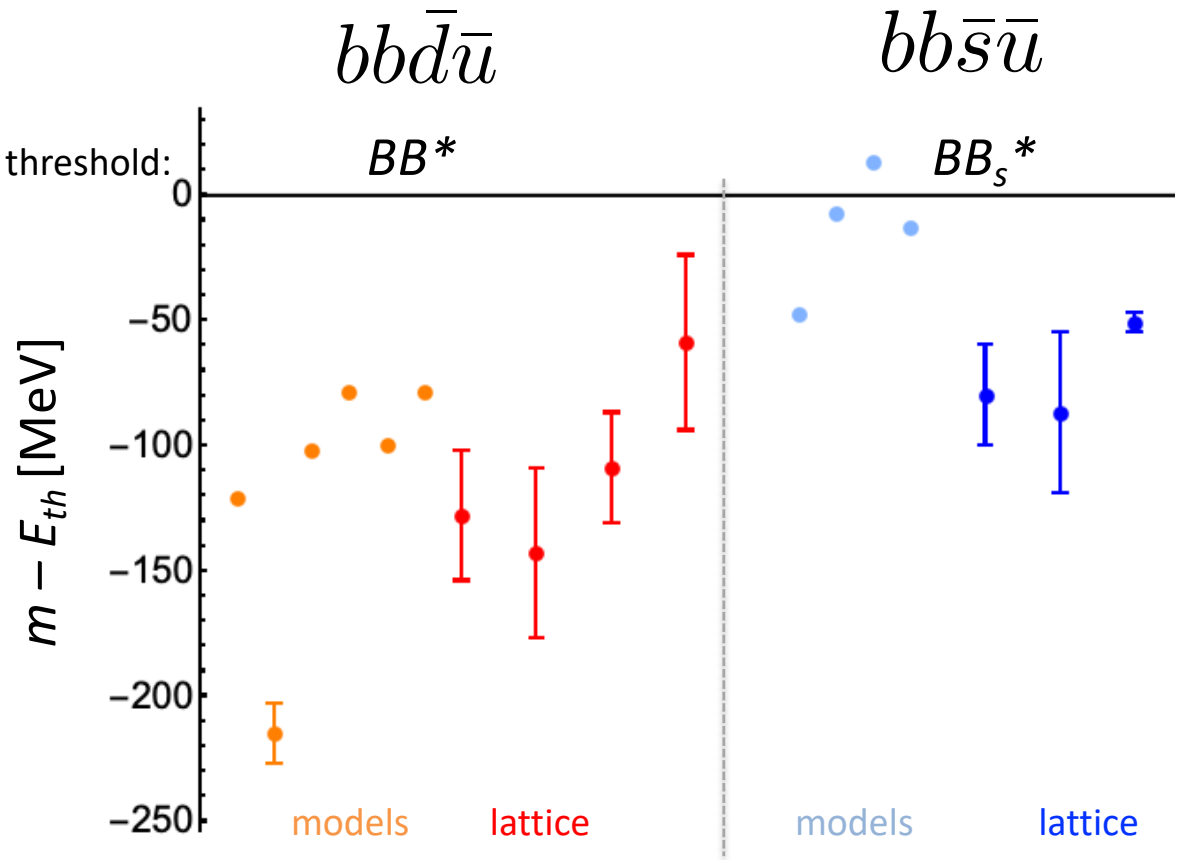
Q=c,b q=u,d,s

Two strongly stable tetraquarks

not found in exp, difficult to find

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $J^P = 1^+$

I=0



references from left to right

models (many more references):
Eichten and Quigg (2017) 1707.09575 PRL
Karliner and Rosner (2017) 1707.07666 PRL
Ebert et al. (2007) 0706.3853
Silvestre-Brac and Semay (1993)
Janc and Rosina (2004) hep-ph/0405208

lattice: most updated results
Leskovec, Meinel, Pflaumer, Wagner (2019) 1904.04197
Junnarkar, Mathur, Padmanth (2018) 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021) preliminary
Bicudo, Wagner et al. 1612.02758 static potentials

models (many more references)
Eichten and Quigg (2017) 1707.09575 PRL
Parket al. (2018) 1809.05257
Ebert et al. (2007) 0706.3853
Silvestre-Brac and Semay (1993)

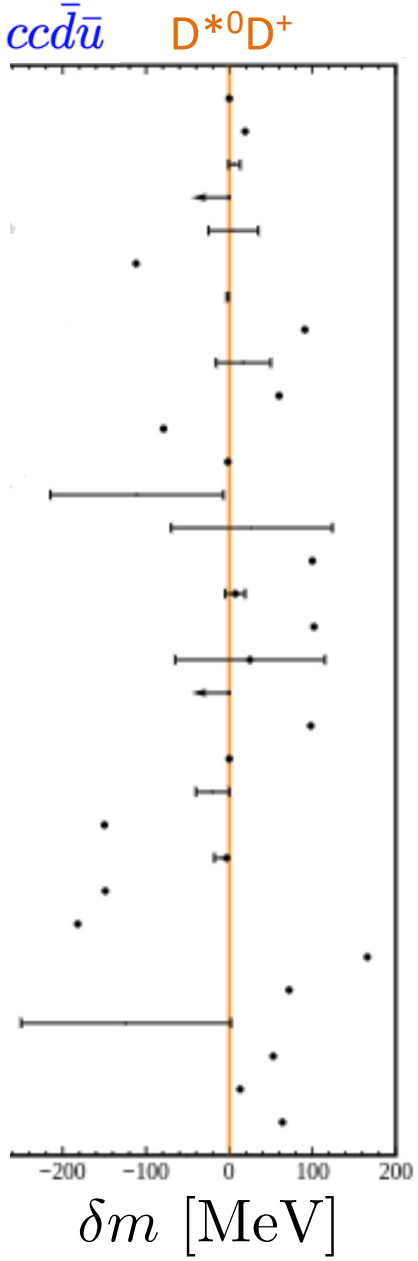
lattice: most updated results
Pflaumer, Leskovec,Meinel,Wagner (2021) 2108.10704
Junnarkar, Mathur, Padmanth (2018) 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

Other $QQ'\bar{q}\bar{q}'$ and J^P

$bc\bar{d}\bar{u}$ $cc\bar{d}\bar{u}$

Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering T(E)



J. Carlson *et al.*	1987
B. Silvestre-Brac and C. Semay	1993
C. Semay and B. Silvestre-Brac	1994
S. Pepin *et al.*	1996
B. A. Gelman and S. Nussinov	2003
J. Vijande *et al.*	2003
D. Janc and M. Rosina	2004
F. Navarra *et al.*	2007
J. Vijande *et al.*	2007
D. Ebert *et al.*	2007
S. H. Lee and S. Yasui	2009
Y. Yang *et al.*	2009
G.-Q. Feng *et al.*	2013
Y. Ikeda *et al.*	2013
S.-Q. Luo *et al.*	2017
M. Karliner and J. Rosner	2017
E. J. Eichten and C. Quigg	2017
Z. G. Wang	2017
G. K. C. Cheung *et al.*	2017
W. Park *et al.*	2018
A. Francis *et al.*	2018
P. Junnarkar *et al.*	2018
C. Deng *et al.*	2018
M.-Z. Liu *et al.*	2019
G. Yang *et al.*	2019
Y. Tan *et al.*	2020
Q.-F. Lü *et al.*	2020
E. Braaten *et al.*	2020
D. Gao *et al.*	2020
J.-B. Cheng *et al.*	2020
S. Noh *et al.*	2021
R. N. Faustov *et al.*	2021

Theoretical PREdictions before 2021

for T_{cc} mass (I=0, $J^P=1^+$)

[compilation by Polyakov]

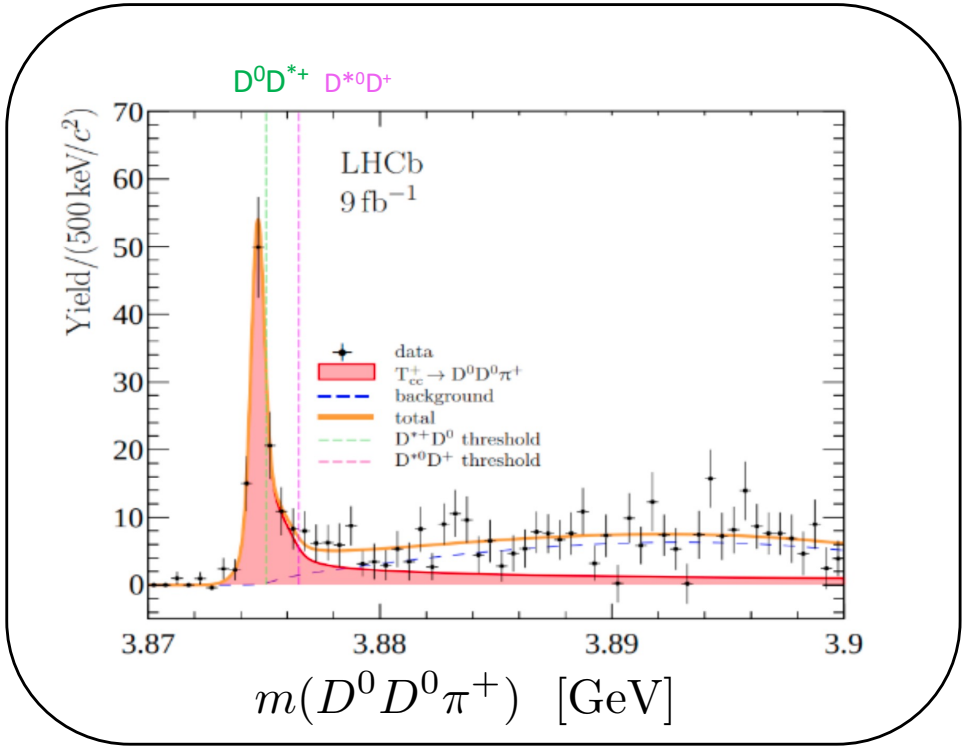
LHCb discovery of T_{cc}

$c c \bar{u} \bar{d}$

The longest lived exotic hadron ever discovered

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

I=0, $J^P=1^+$

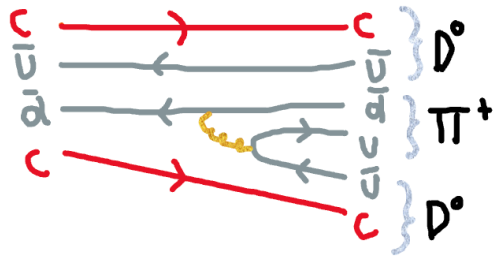


LHCb July 2021, 2109.01038, 2109.01056, Nature Physics

The doubly charmed tetraquark T_{cc}^+, $I = 0$ and favours $J^P = 1^+$.

No states observed in $D^0 D^+ \pi^+$: eliminates possibility of $I = 1$.

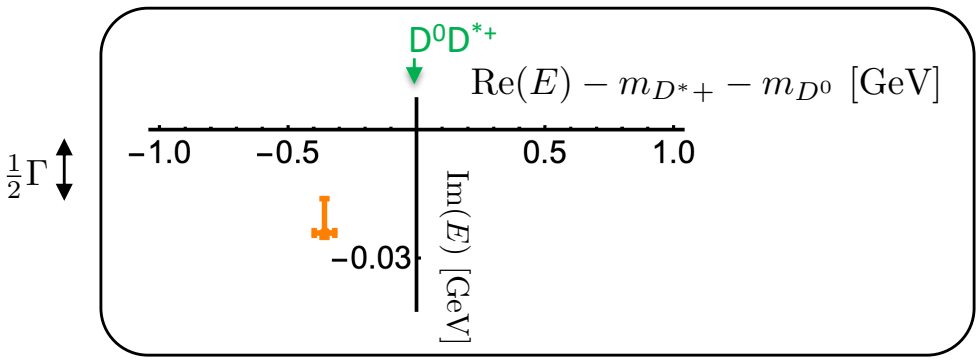
Near-threshold state: Demands pole identification to confirm existence.



Omitting $D^* \to D\pi$, $T_{cc} \to DD\pi$

T_{cc} would be a bound state

Pole in T(E) $\delta m = -0.36$ MeV



$$\delta m_{\rm pole} = -360 \pm 40^{+4}_{-0}\ {\rm keV}/c^2$$
$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14}\ {\rm keV}\,,$$

Lattice study of T_{cc}

$c c \bar{d} \bar{u}$ = T_{cc}

Padmanath, S.P.: 2202.101101,
Phys.Rev.Lett. 129 (2022) 3, 032002
&
subsequent studies with S. Collins

This is the first lattice extraction of the scattering amplitude T(E):

Previous lattice studies: Had. Spec. *JHEP*11(2017)033, Junnarkar, Matur, Padmanath (2019) *PRD*.99.034507

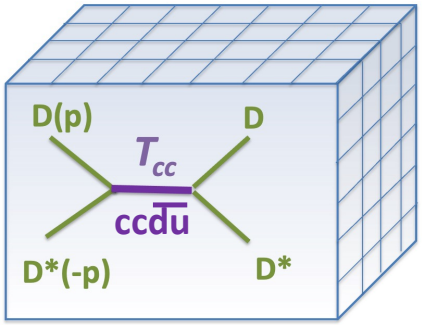
Subsequent study: Shi et al, Physics Letters B 833 (2022) 137391 (previous talk)

Lattice study

$c c \bar{d} \bar{u}$

I=0

$J^P=1^+$



$m_\pi \simeq 280$ MeV :

$D^* \not\to D\pi,\ T_{cc} \not\to DD\pi$

$DD\pi$ above analyzed region

$$C \to E \to T(E)$$

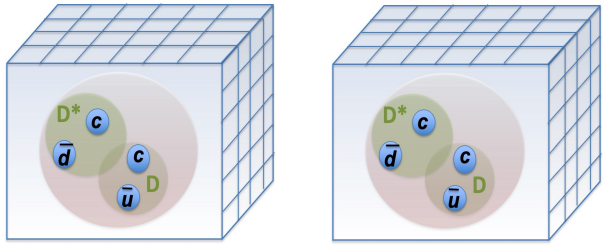
$\sum_n |n\rangle\langle n|$

Eucledian time

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\, e^{-E_n t}\, \langle n | \mathcal{O}_j^+ | 0 \rangle \qquad \langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$

t

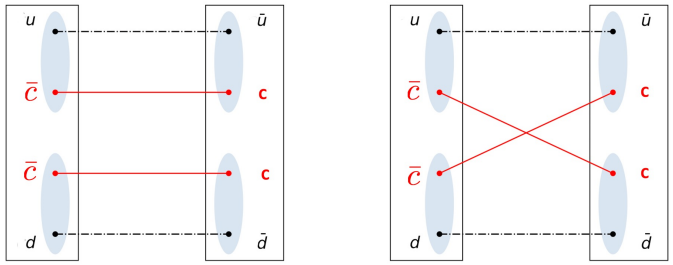
t=0

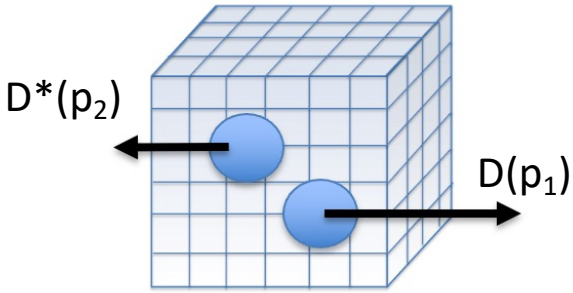


D D*

$$\mathcal{O} = (\bar{u}\gamma_5 c)_{\vec{p}_1}\ (\bar{d}\gamma_i c)_{\vec{p}_2} - (\vec{p}_1 \leftrightarrow \vec{p}_2) \qquad \vec{p}_{1,2} = \vec{n}_{1,2}\ \frac{2\pi}{L}$$

$$(\bar{u}\gamma_5\gamma_t c)_{\vec{p}_1}\ (\bar{d}\gamma_i\gamma_t c)_{\vec{p}_2}$$



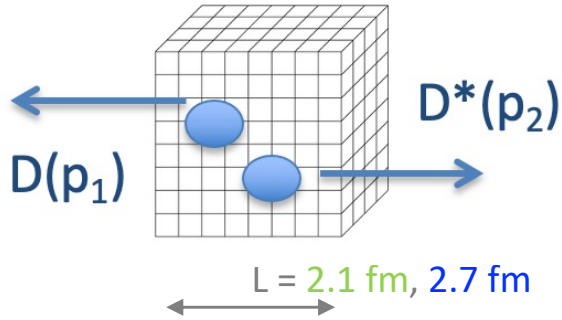


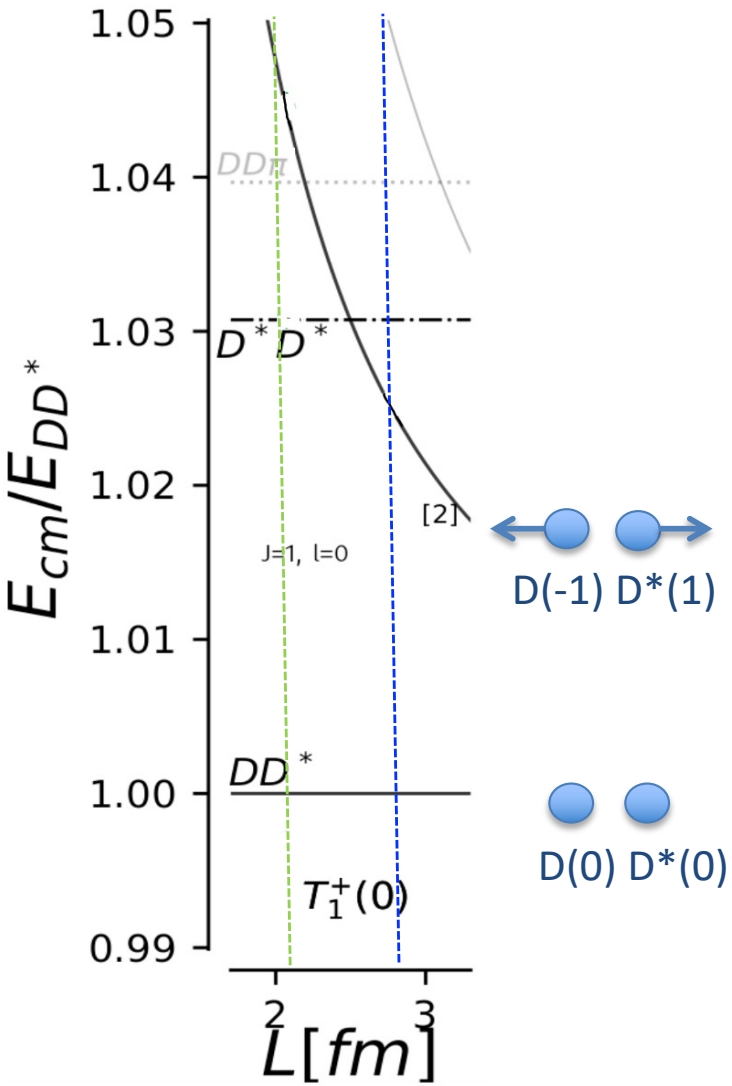
Non-interacting energies and interpolators

Example: $\vec{P} = \vec{0}$ $J^P=1^+$ -> T_1^+

$$E = \sqrt{m_D^2 + \vec{p}_1^{\,2}} + \sqrt{m_{D^*}^2 + \vec{p}_2^{\,2}}$$

$$\vec{p}_{1,2} = \vec{n}_{1,2}\ \frac{2\pi}{L}$$





$$O^{l=0} = V_{1x}[0,0,0]V_{2_y}[0,0,0] - V_{1y}[0,0,0]V_{2x}[0,0,0]$$

$$\begin{aligned} O^{l=2} =& P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\ &+P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\ &-2[P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \end{aligned}$$

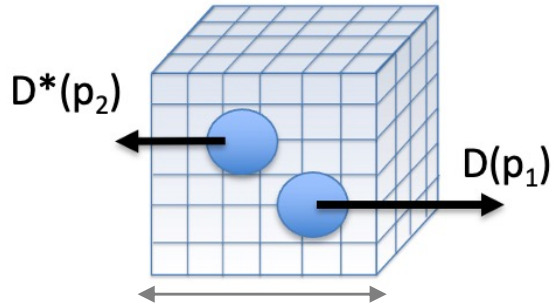
$$\begin{aligned} O^{l=0} =& P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\ &+P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\ &+P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \end{aligned}$$

$$O^{l=0} = P(\{0,0,0\})V_z(\{0,0,0\})$$

$$E_{DD^*} \equiv m_D + m_{D^*}$$

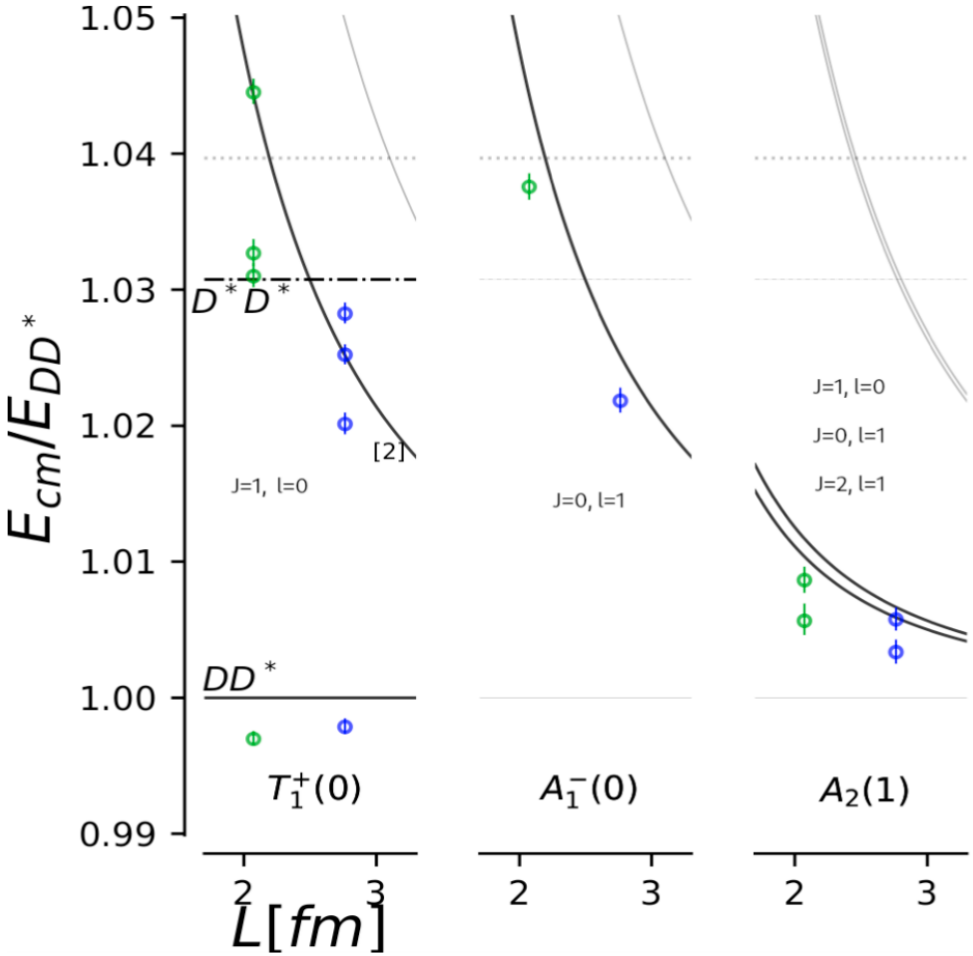
Eigen-energies on the lattice

at $m_\pi \approx 280\ MeV$



L = 2.1 fm, 2.7 fm

$\vec{P} = \vec{0}$ $\quad$ $\vec{P} = \frac{2\pi}{L}\ (0,0,1)$ $\quad$ $\vec{P} = \vec{p}_1 + \vec{p}_2$



lines

non-interacting energies

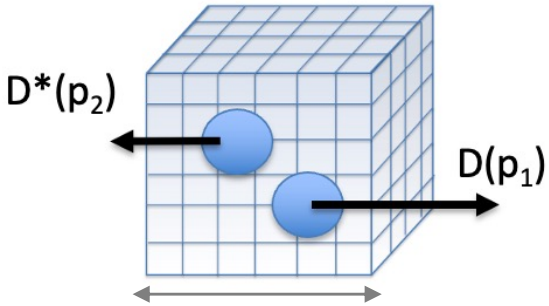
$$E^{n.i.} = \sqrt{m_D^2 + \vec{p}_1^{\,2}} + \sqrt{m_{D^*}^2 + \vec{p}_2^{\,2}}$$

$$\vec{p}_i = \vec{n}_i \frac{2\pi}{L}$$

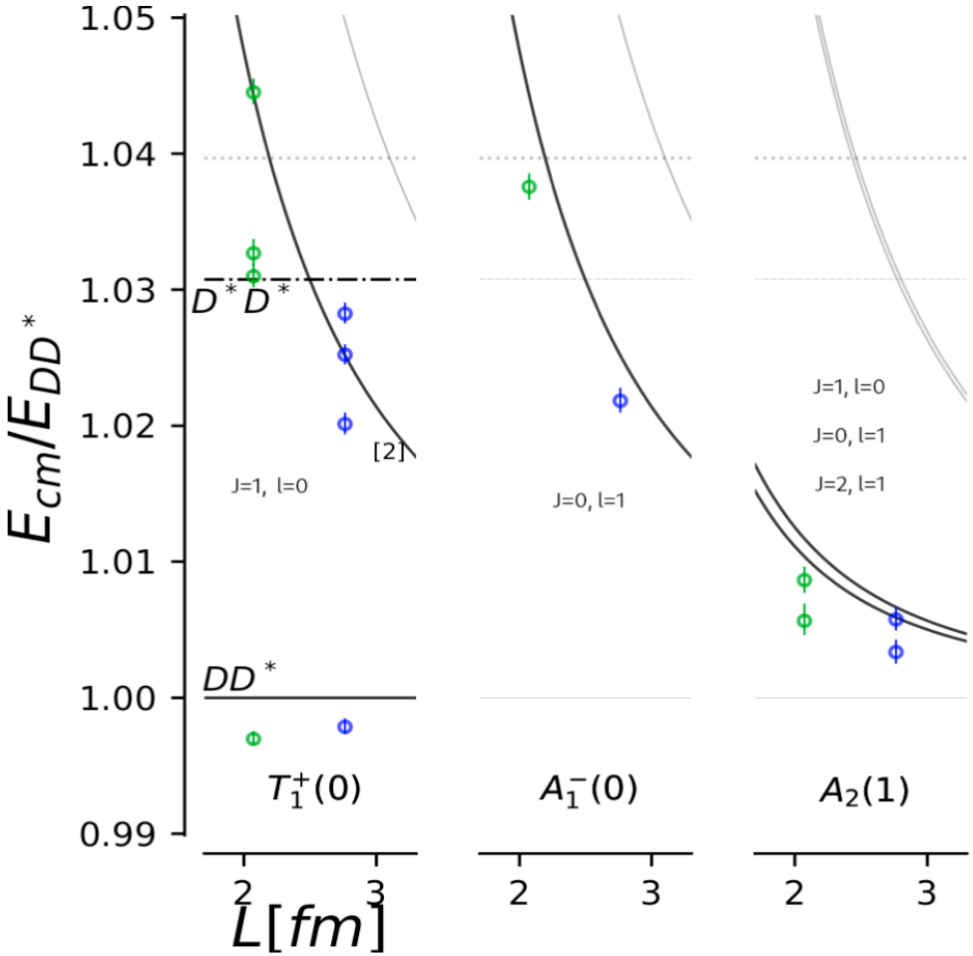
$$E_{DD^*} \equiv m_D + m_{D^*}$$

Eigen-energies and scattering amplitude

at $m_\pi \approx 280\ MeV$



L = 2.1 fm, 2.7 fm



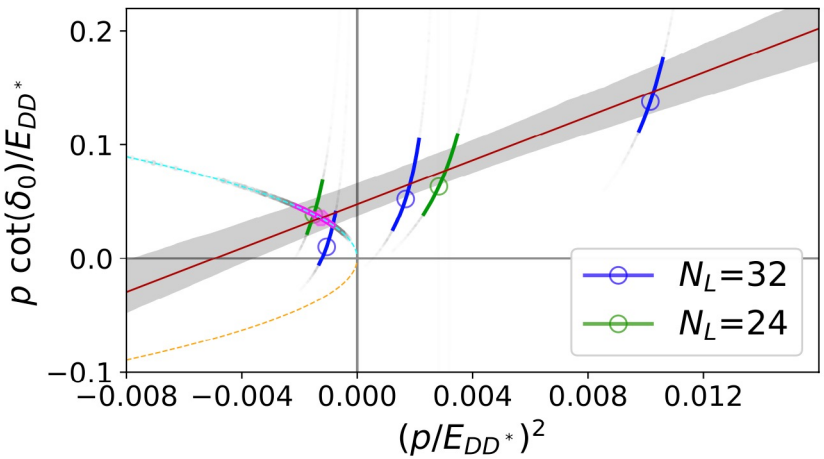
$E_{DD^*} \equiv m_D + m_{D^*}$

Luscher's relation

E -> T(E), $\delta(E)$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

$p\cot\delta$ (for partial wave l=0)



$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$

parametrizing l=0,1

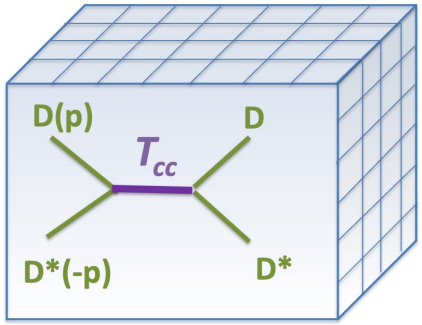
with effective range expansion

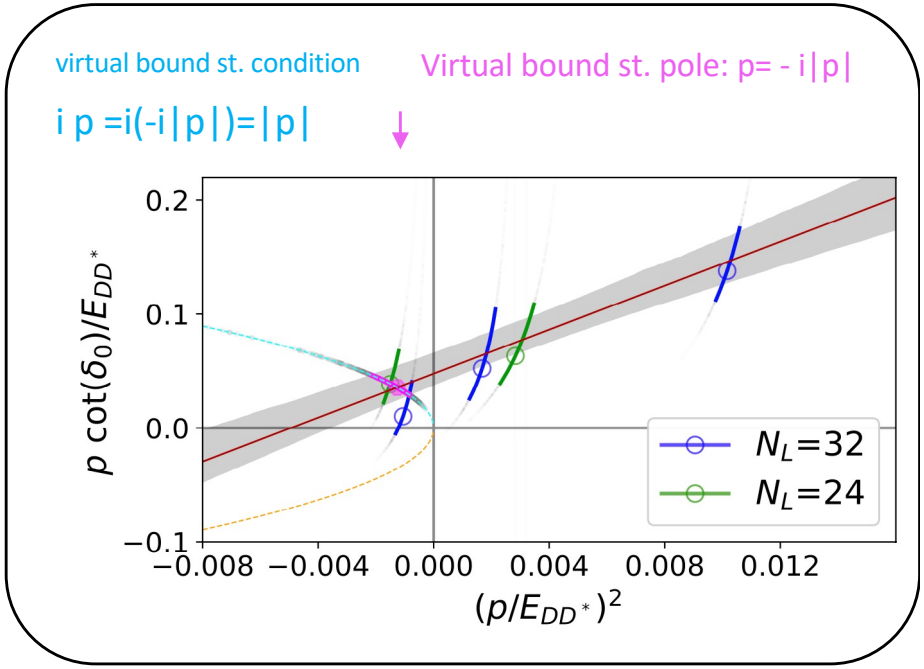
$a_0 = 1.04(0.29)$ fm & $r_0 = 0.96(^{+0.18}_{-0.20})$ fm

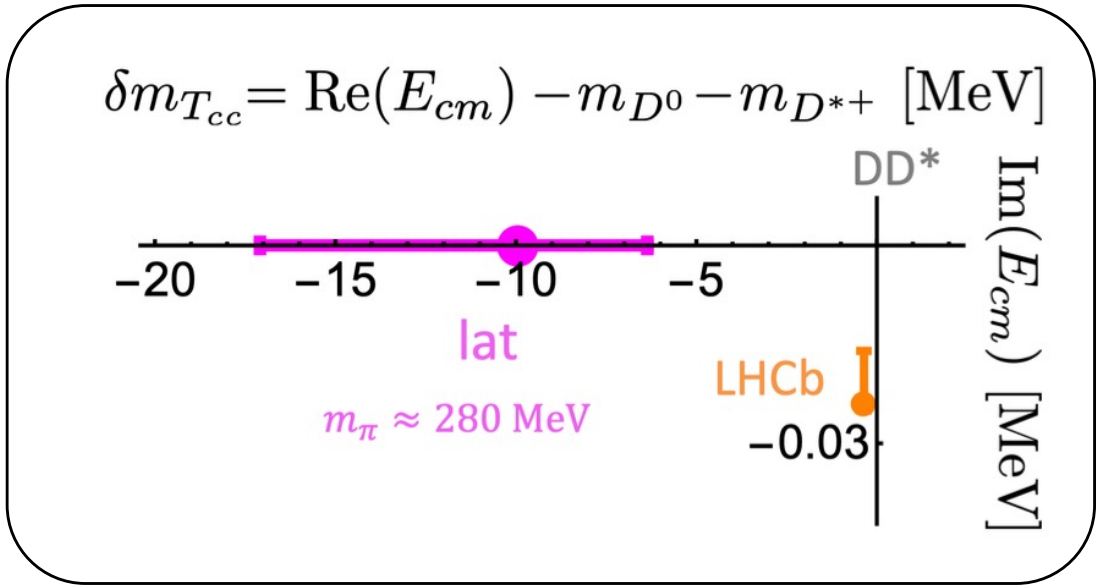
Scattering amplitude for l=0

at $m_\pi \approx 280\ MeV$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$







Lattice: virtual bound st. pole

Binding energy:

$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV

LHCb: bound st. pole

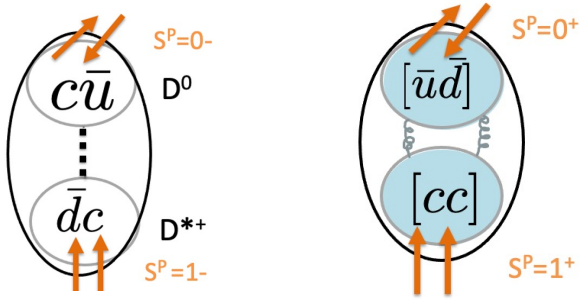
omitting $D^* \to D\pi,\ T_{cc} \to DD\pi$

Possible binding mechanisms of T_{cc}

molecular

likely dominant

[e.g. Janc, Rosina 2003]



"molecular"

Molecular component in simplest toy model: dependence on $m_{u/d}$

exchanged particles:

light mesons $\pi, \rho, ..$

increasing $m_{u/d}$

increasing m_{ex}

decreasing R or

decreasing attraction |V|

Yukava-like potential

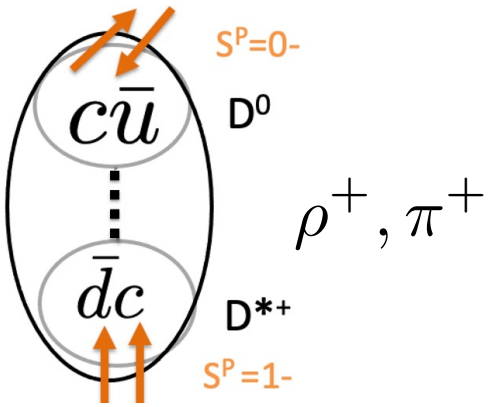
$$V(r) \propto -\frac{e^{-m_{ex} r}}{r}$$

analogous conclusion for any

fully attractive

$$V(r) = -V_0 \; f(r/R)$$

$$f = e^{-r/R}, e^{-r^2/R^2}, \theta(R-r), ...$$



subsequent lattice study:

CLQCD, Chen et al. 2206.06185

comparison of I=0,1 :

attraction in I=0 channel arises

mainly from ϱ exchange

Simplest Example: scattering in square-well potential in QM

$\delta = \arctan[\tan(qR)\frac{p}{q}] - pR$

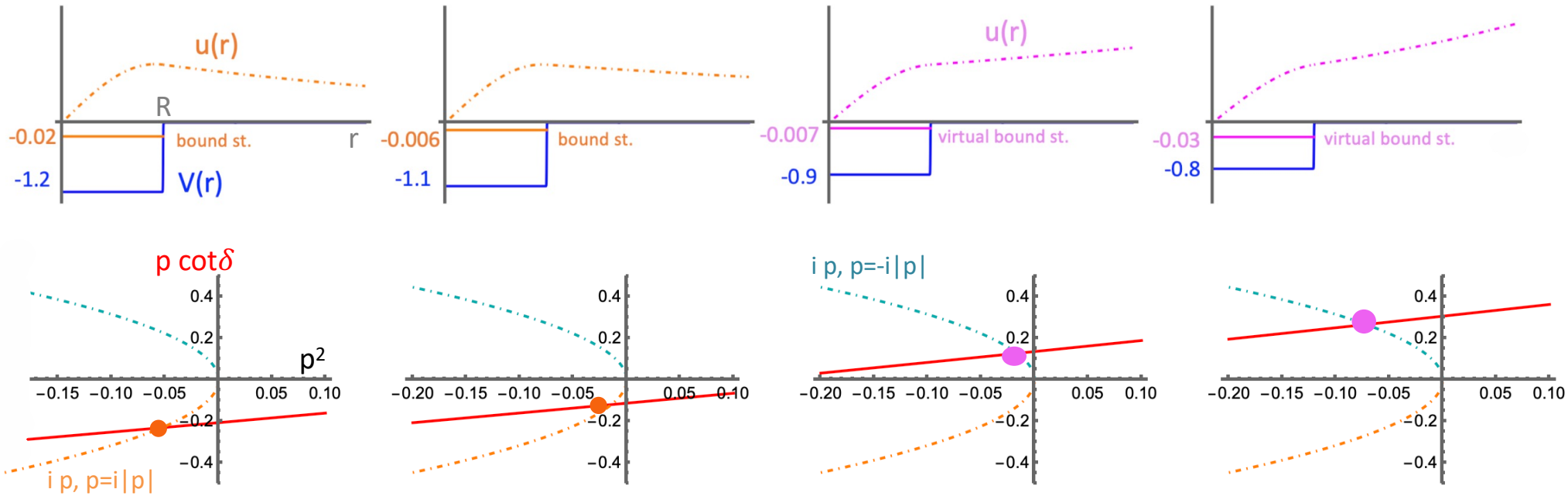
partial wave l=0

$t \propto (p\cot\delta - ip)^{-1}$

$u(r) = A\sin(qr)$ $\quad u(r) = B\sin(pr+\delta)$

p=i|p| $\quad e^{ipr} = e^{-|p|r}$

p=-i|p| $\quad e^{ipr} = e^{|p|r}$



increasing $m_{u/d}$, decreasing attraction V_0 (or decreasing R)

Simplest Example: scattering in square-well potential in QM

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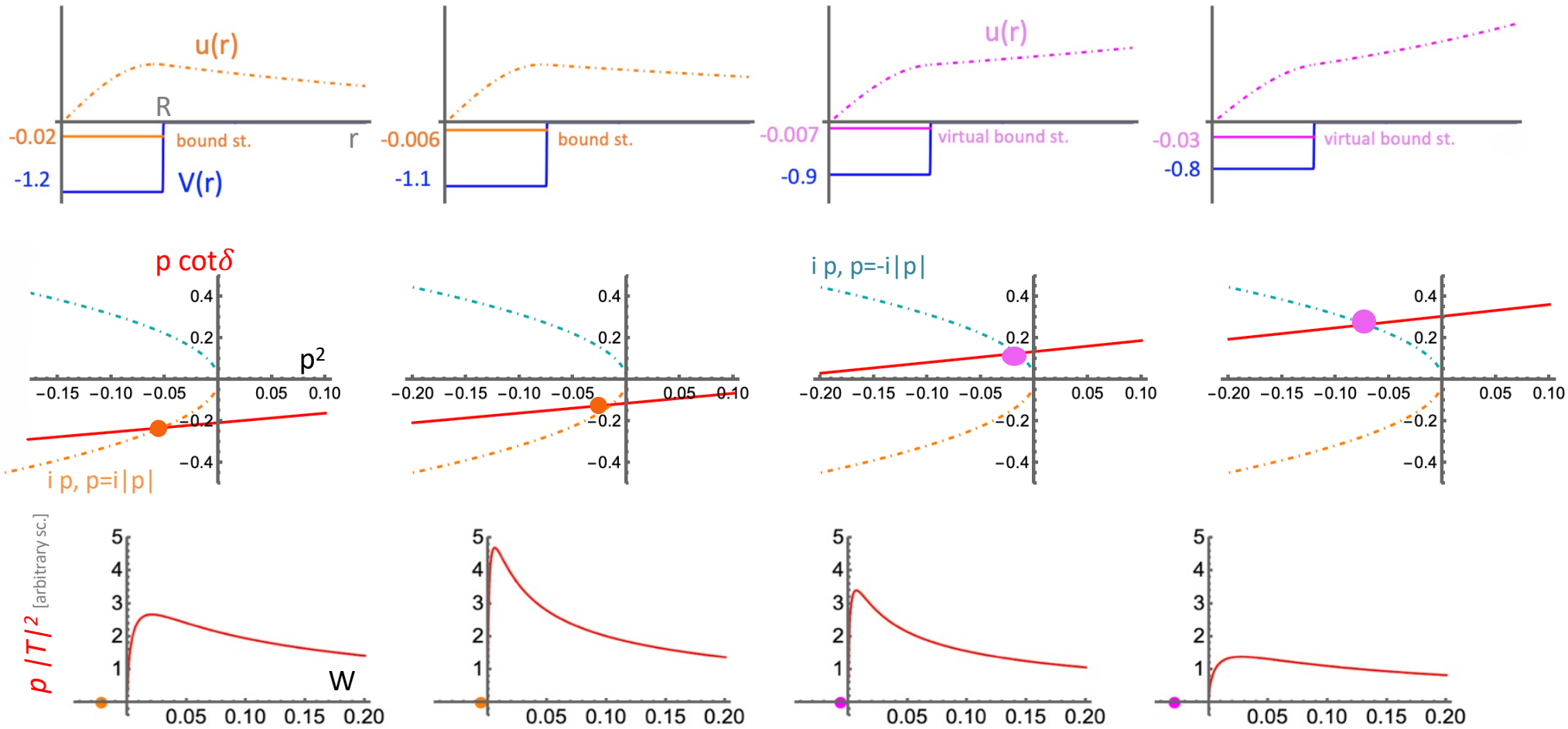
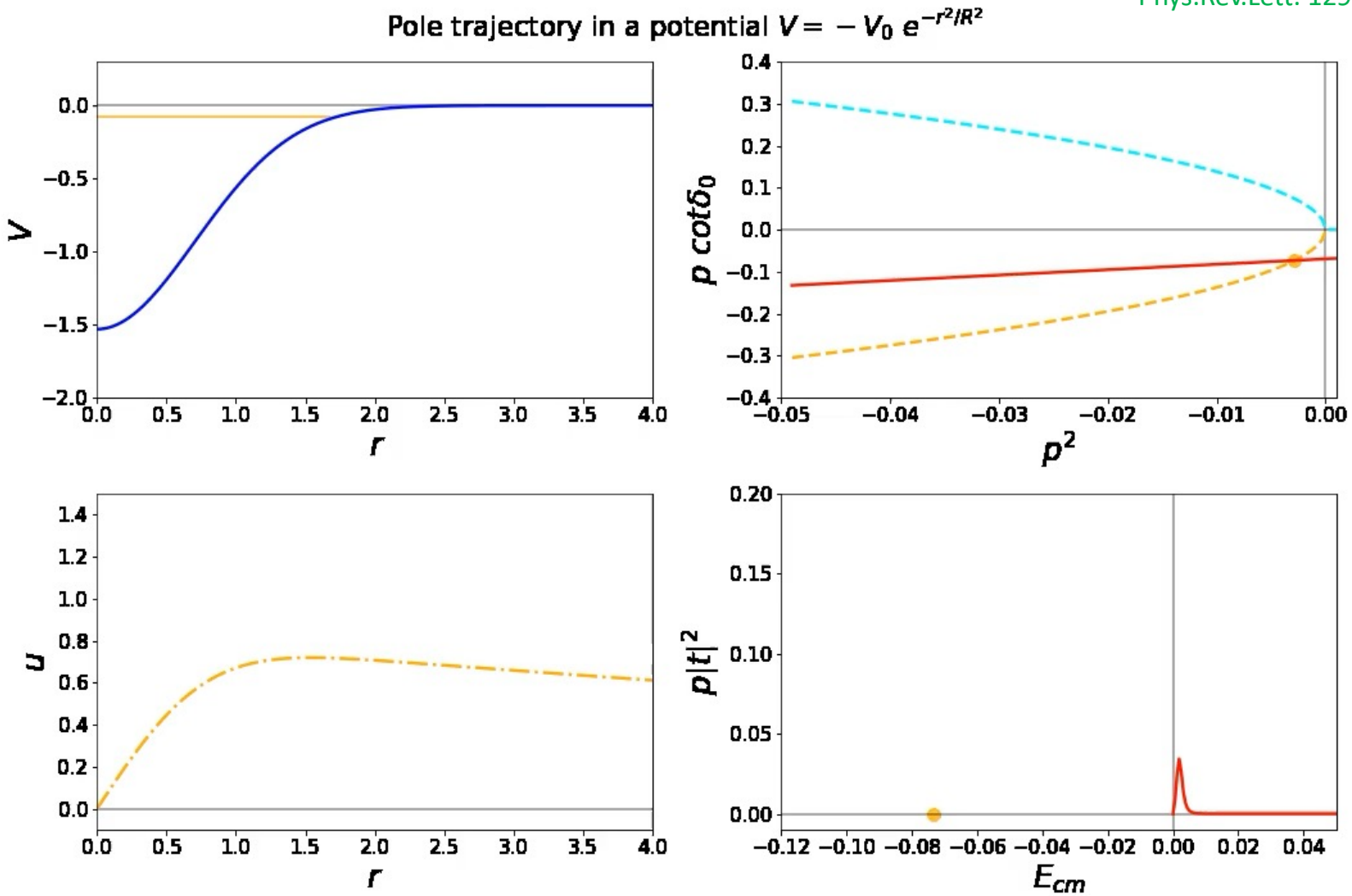


increasing $m_{u/d}$, decreasing attraction V_0 (or decreasing R)

All fully attractive potentials lead to analogous conclusions

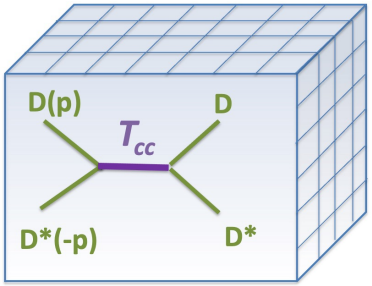
video: courtesy M. Padmanath
supplemental material of
Phys.Rev.Lett. 129 (2022) 3, 032002



Conclusions on T_{cc}

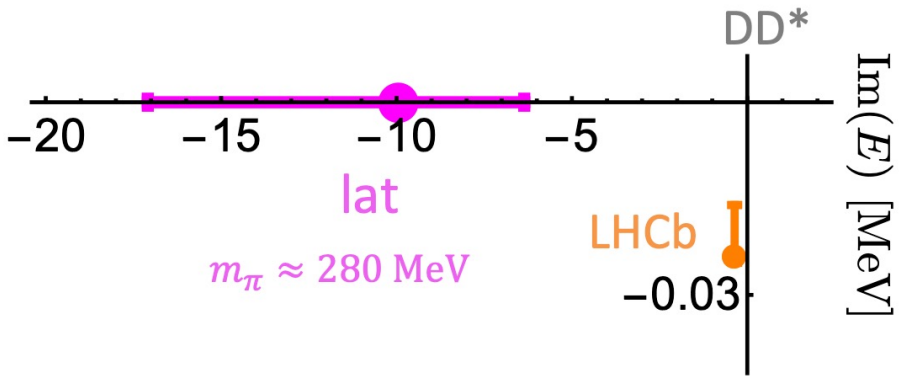
$cc\bar{d}\bar{u}$

The longest lived exotic hadron discovered to date



Pole of T(E) at $m_c^{(h)}$

$$\delta m_{T_{cc}} = \mathrm{Re}(E) - m_{D^0} - m_{D^{*+}} \ [\mathrm{MeV}]$$



	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	$-0.36(4)$	bound st.

closer-to physical m_c

$T_{cc} \to DD\pi$
$D^* \to D\pi$ omitting

Simple arguments within molecular picture:

$m_{u/d}$ increases :
$m_{u/d}^{phy} \to m_{u/d}^{lat}$

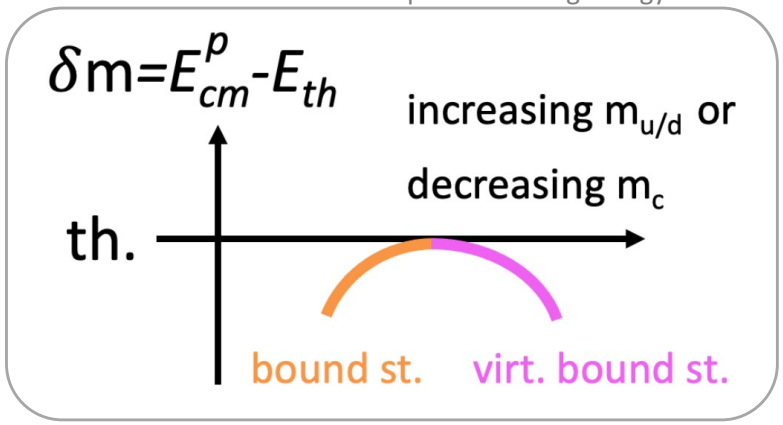
(LHCb) would-be bound st. → virtual bound st.

m_c decreases
$|\delta m_{T_{cc}}|$ increases for virtual bound st.

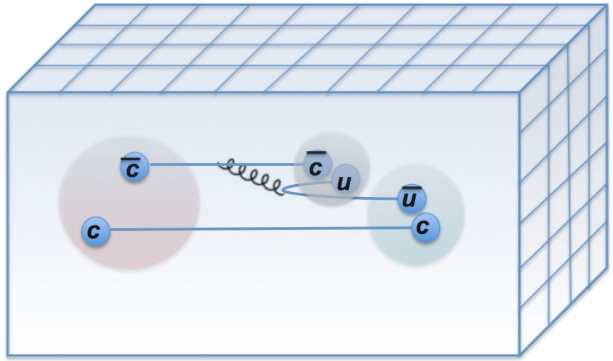
(see backup slides)

Both in agreement with the lattice result

sketch of expected binding energy



Hypothesis to be verified by future simulations



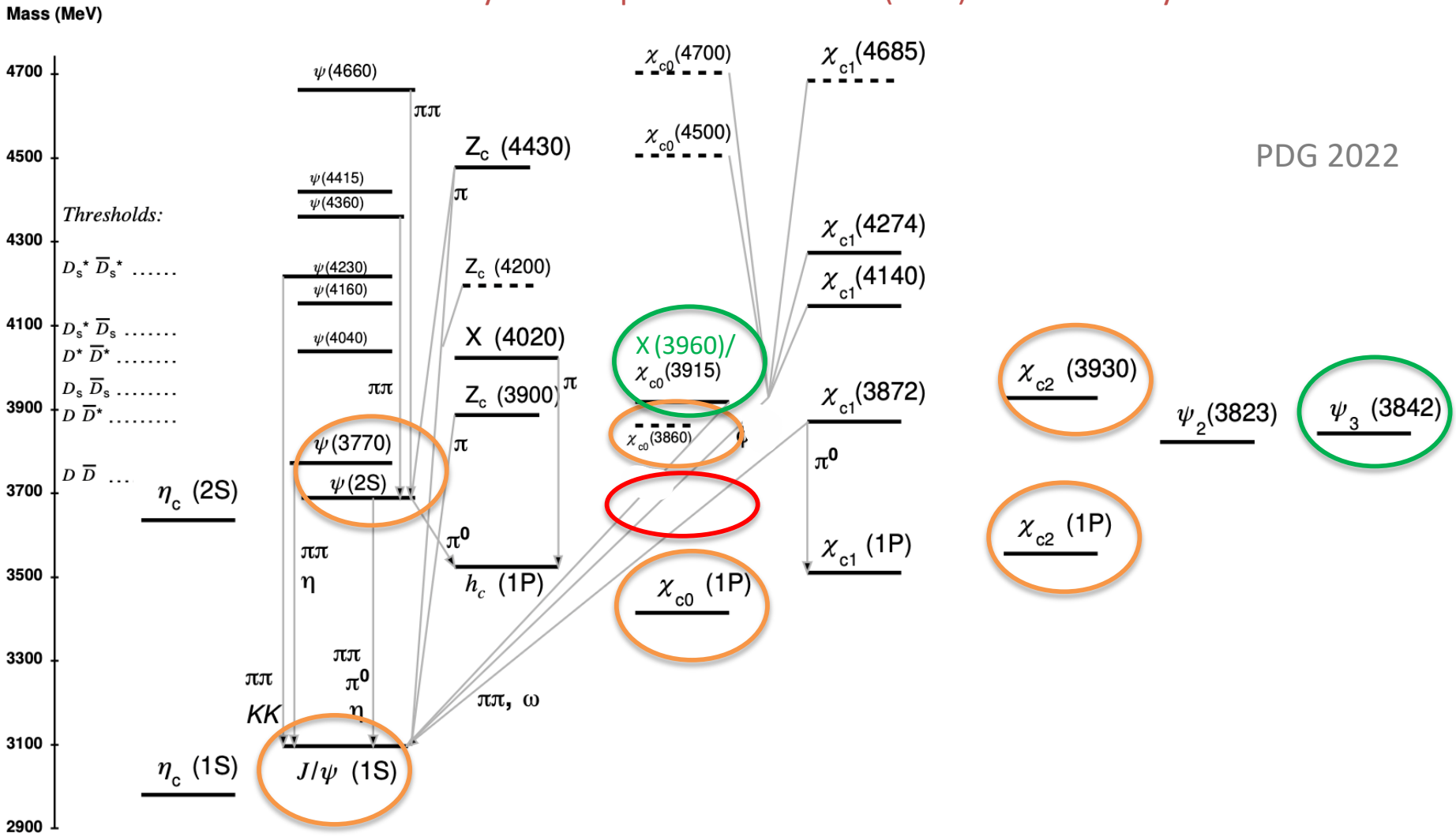
$\bar{c}c$, $\bar{c}q\bar{q}c$ I=0

S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP, 1905.03506 PRD, 2111.02934

This is the first coupled-channel extraction of T(E) in the charmonium system with I=0.

The only earlier scattering lattice study: Lang, Leskovec, Mohler, SP, JHEP(2015)

Charmonium system: experimental status (PDG) and summary of our lattice results

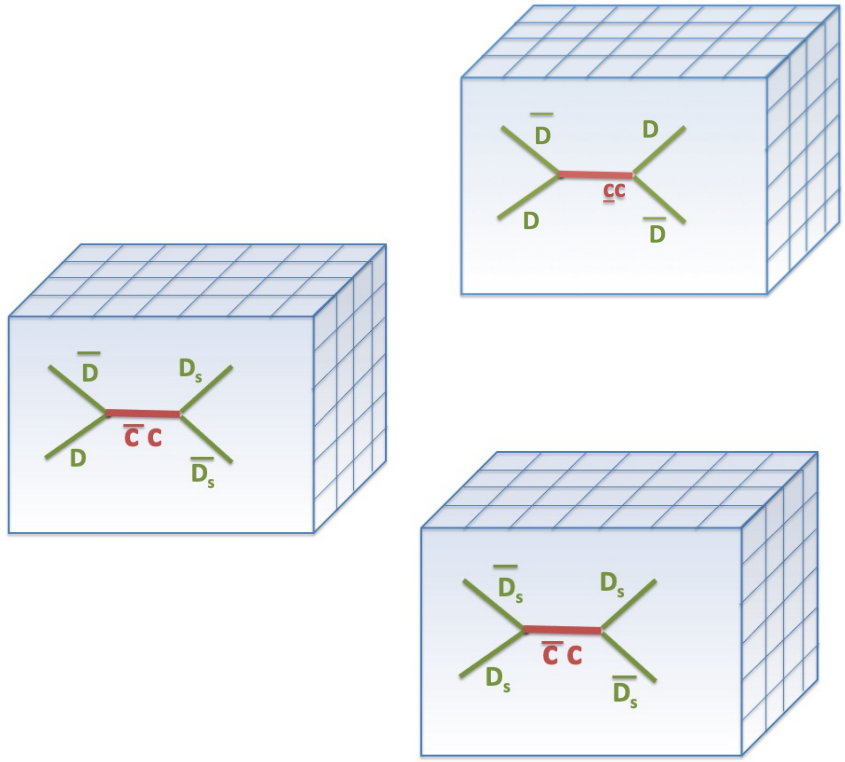


PDG 2022

$J^{PC} =$	0^{-+}	1^{--}	1^{+-}	0^{++}	1^{++}	2^{++}	2^{--}	3^{--}
our lattice study:		postdicted		predicted, exp discovered		predicted		

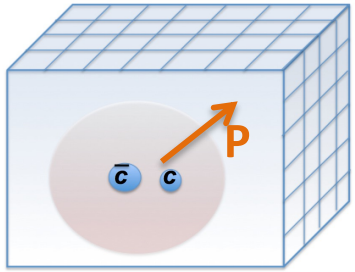
Coupled-channel scattering $D\bar{D} - D_s\bar{D}_s$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^{+}(0) | 0 \rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$

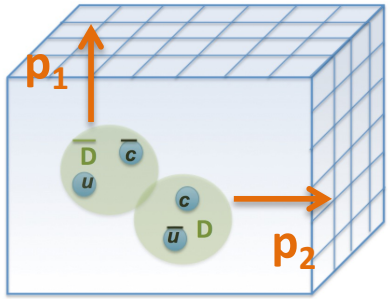


Operators

$$\mathcal{O}^{\bar{c}c} = (\bar{c}\Gamma c)_{\vec{P}}$$



$$\mathcal{O}^{\bar{D}D} = (\bar{c}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 c)_{\vec{p}_2}$$
$$= \bar{D}(\vec{p}_1) D(\vec{p}_2)$$

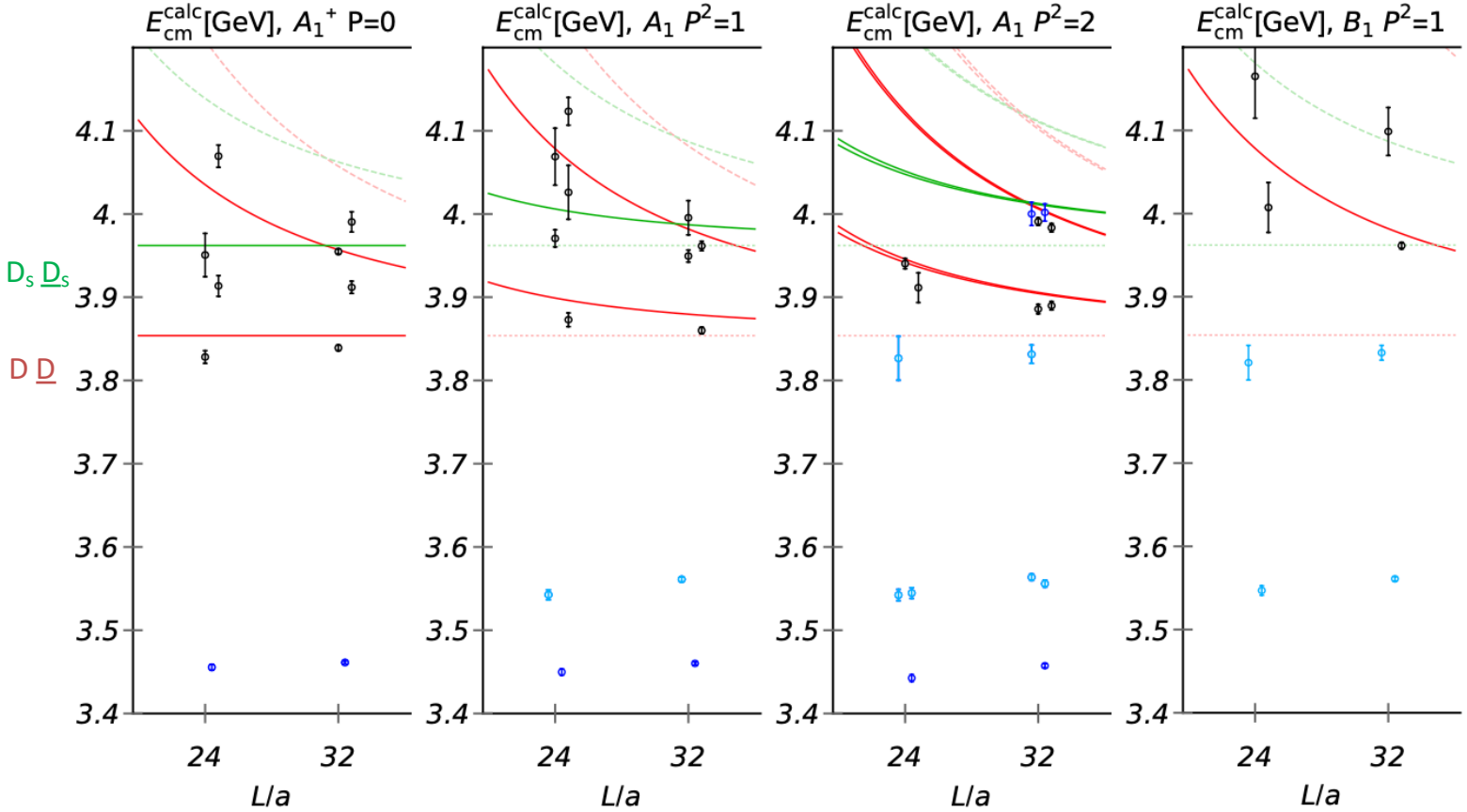


$$\mathcal{O}^{\bar{D}_s D_s} = (\bar{c}\Gamma_1 s)_{\vec{p}_1} (\bar{s}\Gamma_2 c)_{\vec{p}_2}$$
$$= \bar{D}_s(\vec{p}_1) D_s(\vec{p}_2)$$

$\vec{P} = \vec{p}_1 + \vec{p}_2$ P: 0
(0,0,1) $2\pi/N_L$
(1,1,0) $2\pi/N_L$

NL=24,32

Energies of eigen-states E_n in irreps that contain $J^{PC}=0^{++}, 2^{++}$



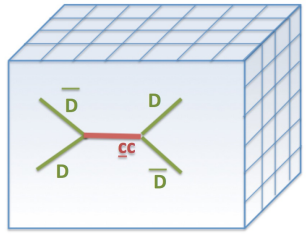
non-interacting energies denoted

D $\underline{D}$

D_s $\underline{D}_s$

$J^{PC}=0^{++}$

$J^{PC}=2^{++}, 2^{-+}$



$$S_{ij}(E_{cm}) = 1 + 2i\ \rho\ t_{ij}(E_{cm})$$

$$t(E_{cm}) = \begin{vmatrix} t_{11}(E_{cm}) & t_{12}(E_{cm}) \\ t_{12}(E_{cm}) & t_{22}(E_{cm}) \end{vmatrix}$$

1: $\underline{D}$D, 2: $\underline{D}$sDs

Extraction of matrix t(E) : NOT straightforward !

one equation, three unknowns (at each E_{cm})

$$det[1 + i\ t(E_{cm})\ F(E_{cm})] = 0$$

known 2x2 matrix

$$(t^{-1})_{ij} = \frac{2}{E_{cm}\ p_i^l p_j^l}\ (\tilde{K}^{-1})_{ij} - i\ \rho_i\ \delta_{ij}$$

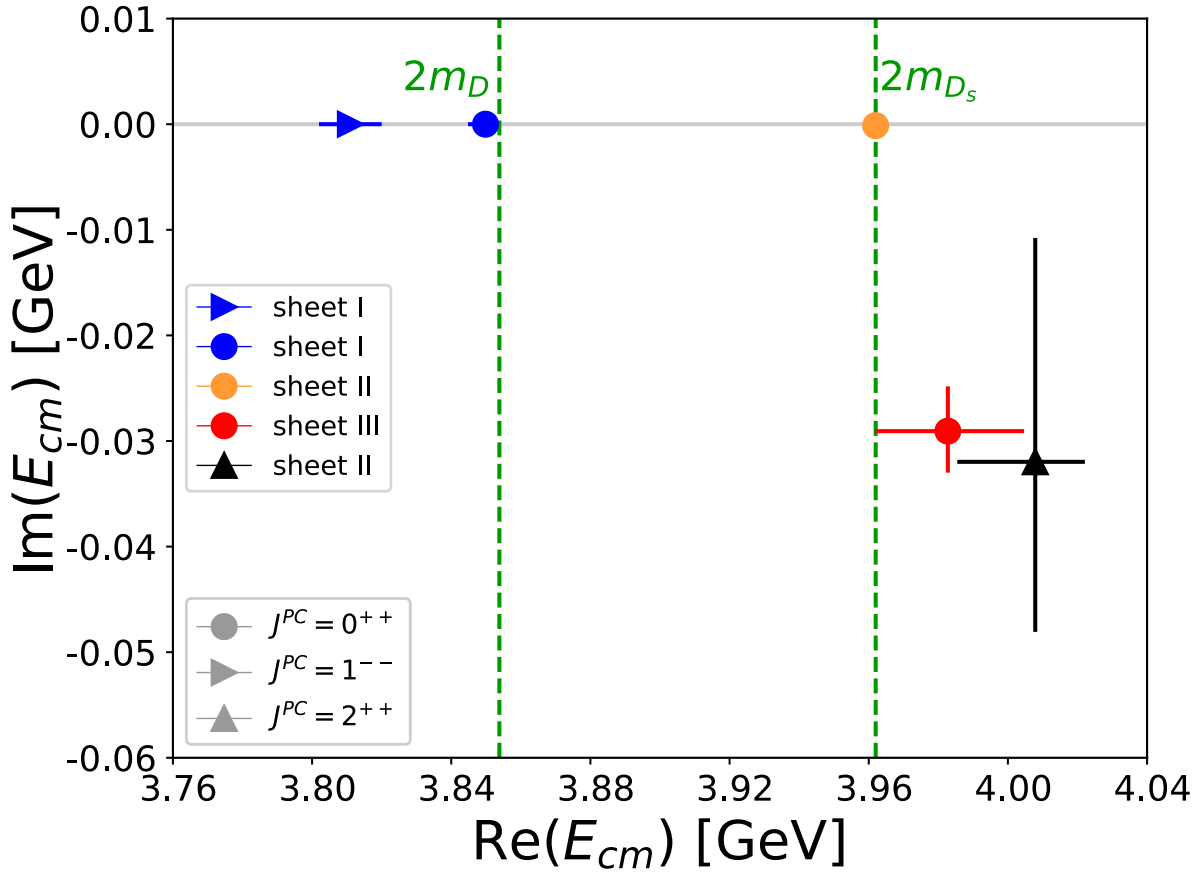
$$\frac{\tilde{K}_{ij}^{-1}(s)}{\sqrt{s}} = a_{ij} + b_{ij} s$$

$$s = E_{cm}^2$$

$$\rho_i \equiv 2p_i/E_{cm}$$

Poles of the scattering amplitude $D\bar{D} - D_s\bar{D}_s$

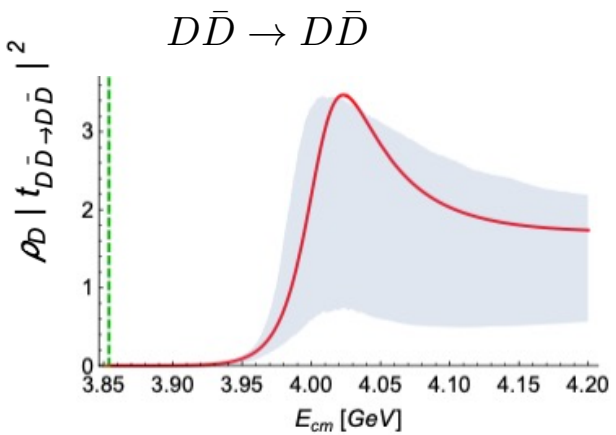
$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$

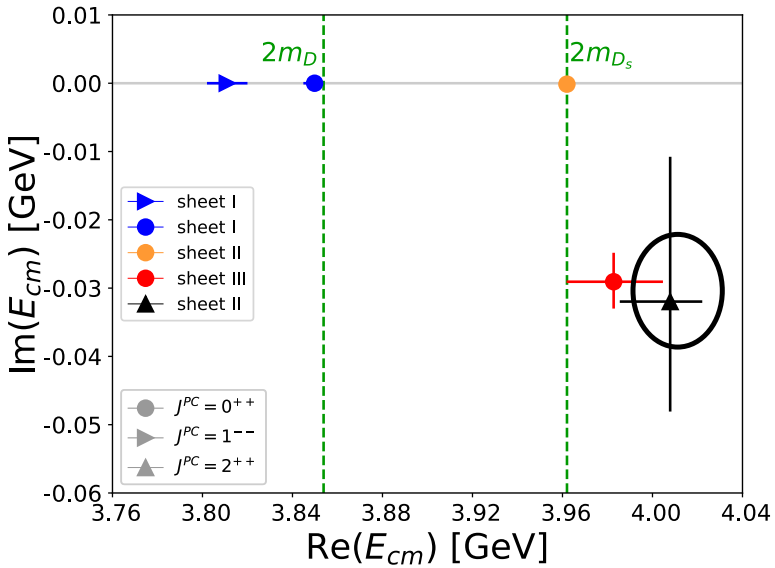


$$\text{sheet I}: \quad \mathrm{Im}(\rho_D) > 0, \mathrm{Im}(\rho_{D_s}) > 0\,, \quad \text{sheet II}: \quad \mathrm{Im}(\rho_D) < 0, \mathrm{Im}(\rho_{D_s}) > 0\,, \quad (\rho_i = 2p_i/E_{\mathrm{cm}})$$
$$\text{sheet III}: \quad \mathrm{Im}(\rho_D) < 0, \mathrm{Im}(\rho_{D_s}) < 0\,, \quad \text{sheet IV}: \quad \mathrm{Im}(\rho_D) > 0, \mathrm{Im}(\rho_{D_s}) < 0\,.$$

$J^{PC}=2^{++}$: conventional resonance

D-wave (L=2, $J^{PC}=2^{++}$)





- 2++ resonance

$$\Gamma \equiv g^2 p_D^{2l+1}/m^2$$

$$lat : m = 3973^{+14}_{-22} \text{ MeV} \quad g = 4.5^{+0.7}_{-1.5} \text{ GeV}^{-1}$$

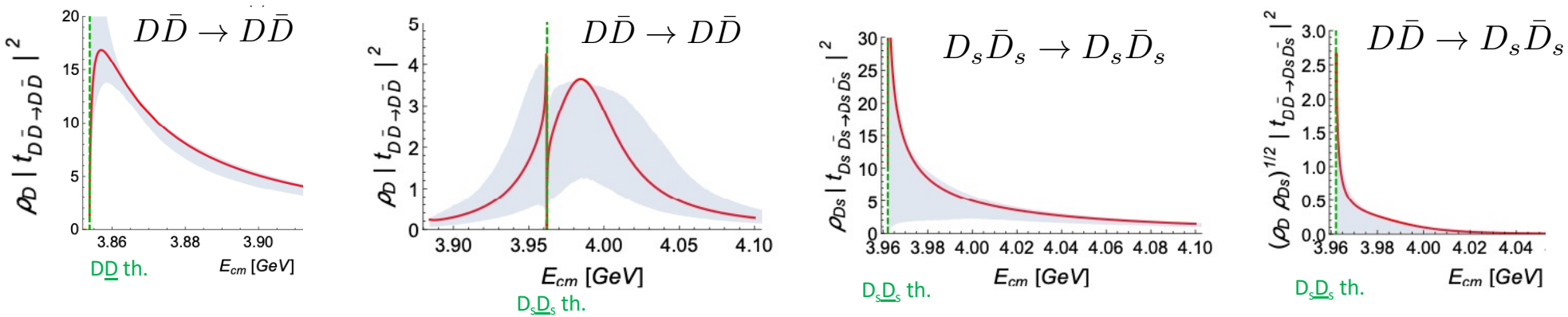
$$\chi_{c2}(3930) : m = 3923 \pm 1 \text{ MeV} \quad g = 2.65 \pm 0.12 \text{ GeV}^{-1}$$

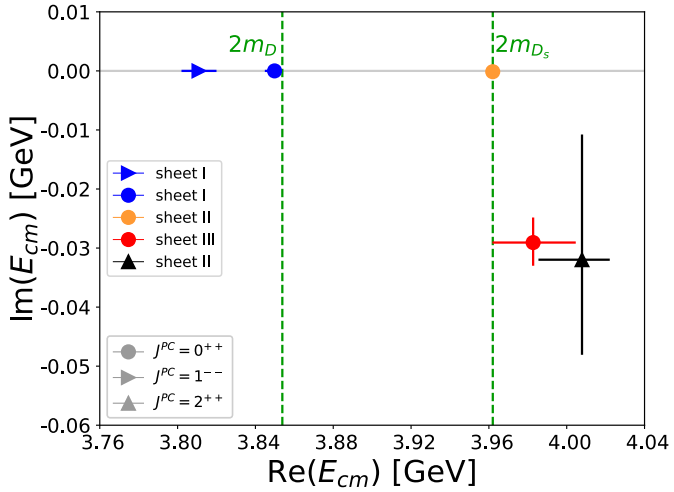
PDG

$J^{PC}=0^{++}$: some expected and unexpected states found

$$D\bar{D} - D_s\bar{D}_s$$

S-wave (L=0, $J^{PC}=0^{++}$)





- broad resonance coupling mostly to DD

$$lat : m = 3949^{+28}_{-20}\ MeV \quad g = 1.35^{+0.04}_{-0.08}\ GeV$$

$$X(3860) : m = 3862^{+48}_{-35}\ MeV \quad g = 2.5^{+1.2}_{-0.9}\ GeV \qquad \Gamma \equiv g^2 p_D^{2l+1}/m^2$$

Belle 2017

- state near $D_s\bar{D}_s$ threshold coupling mostly to $D_s\bar{D}_s$

$$\frac{|c^2_{D\bar{D}}|}{|c^2_{D_s\bar{D}_s}|} = 0.02\ ^{+0.02}_{-0.01}$$

lat : $m - 2m_{D_s} = -0.2\ ^{+0.16}_{-4.9}$ MeV , $g = 0.10\ ^{+0.21}_{-0.03}$ GeV

$\chi_{c0}(3930)$: $m - 2m_{D_s} = -12.9 \pm 1.6$ MeV , $\Gamma = 17 \pm 5$ MeV , $g = 0.67 \pm 0.10$ GeV

LHCb

- state near DD threshold

near pole $$t_{ij} \sim \frac{c_i\ c_j}{(E^p_{cm})^2 - E^2_{cm}}$$

Charmonium(like) resonances and bound states

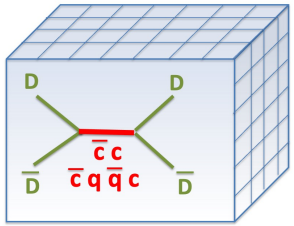
$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$

$$t_{ij} \sim \frac{c_i\, c_j}{(E^p_{cm})^2 - E^2_{cm}}$$

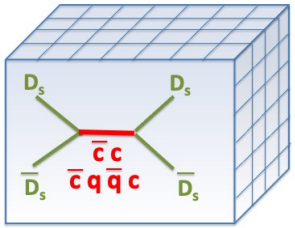
near the pole

lat: $\frac{|c^2_{D\bar{D}}|}{|c^2_{D_s\bar{D}_s}|} = 0.02\ ^{+0.02}_{-0.01}$

$\bar{c}s\bar{s}c$



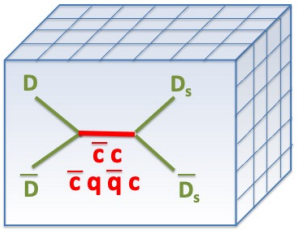
large coupling to $D_s\bar{D}_s$ and small to $D\bar{D}$
likely related to X(3915) / χ_{c0}(3930) / X(3960)
[BaBar, LHCb 2009.00026, LHCb 2022] explaining why it has narrow width to $D\bar{D}$. Supported by some pheno studies: Lebed, Polosa 1602.08421, Oset et al . 2207.08490, Guo et al, 2101.01021, ...



q=u,d $\bar{c}q\bar{q}c$

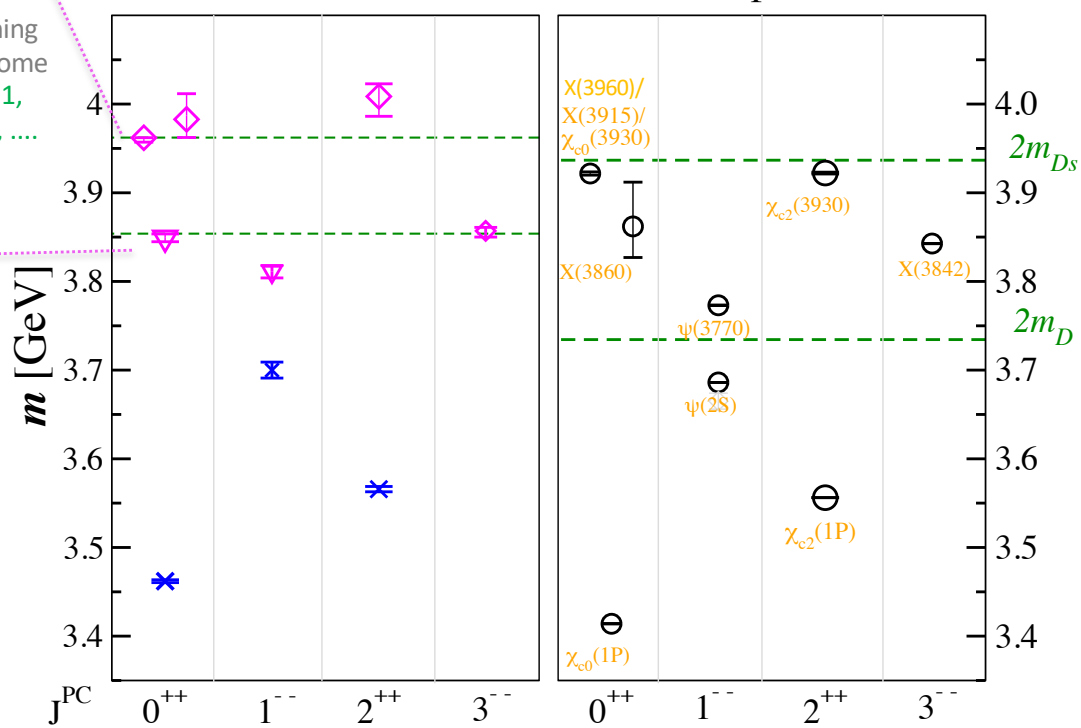
predicted in models [Oset et al, 0612179 PRD, Hildago Duque et al 1305.4487, Baru et al 1605.09649 PLB]

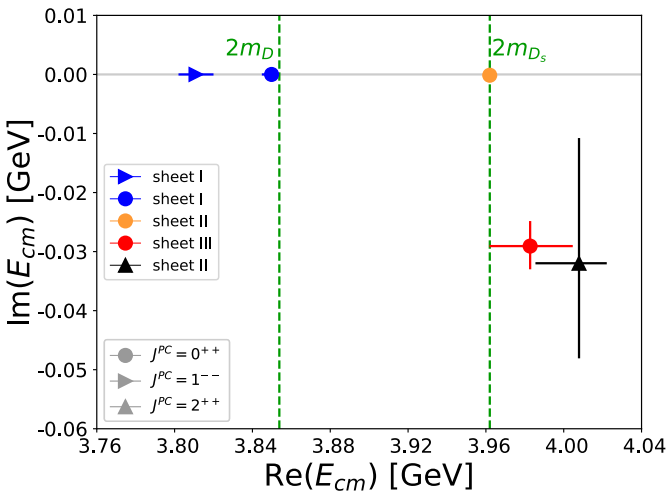
seen in dispersive re-analysis of exp. [Danilkin et al 2111.15033]



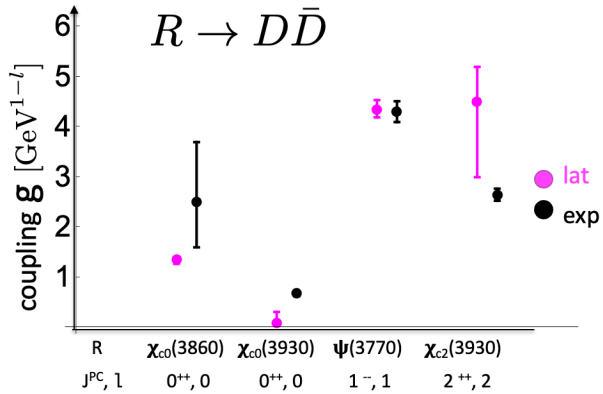
+ expected conventional charmonia

$m_\pi \simeq 280$ MeV





$$\Gamma \equiv g^2\, \frac{p_D^{2l+1}}{m^2}$$



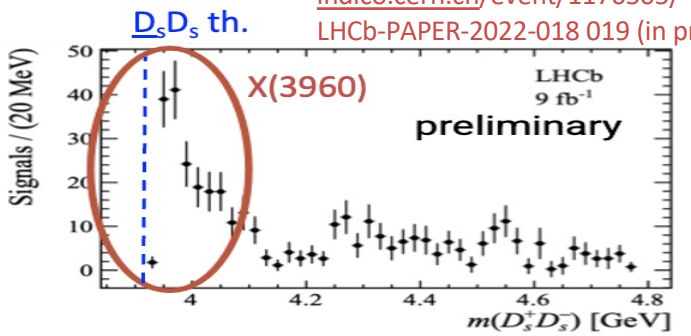
$J^{PC}=0^{++}$ $\bar{c}s\bar{s}c$ likely related to X(3915) / χ_{c0}(3930) / X(3960)

lat: $\frac{|c^2_{D\bar{D}}|}{|c^2_{D_s\bar{D}_s}|} = 0.02\ ^{+0.02}_{-0.01}$

all three likely the same state
currently named χ_{c0}(3914) in PDG

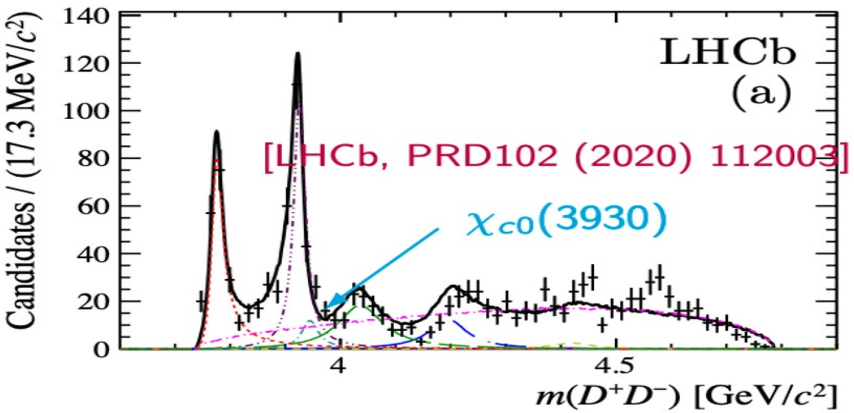
talk by
Chen Chen
today

indico.cern.ch/event/1176505/ july 2022
LHCb-PAPER-2022-018 019 (in preparation)

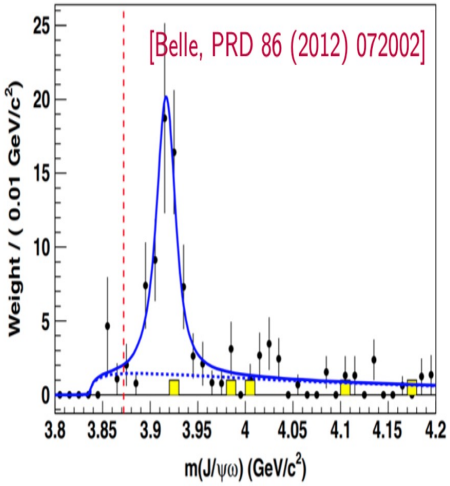


$$X(3960) \to D_s\bar{D}_s$$

exp: $\frac{Br(D\bar{D})}{Br(D_s\bar{D}_s)} \simeq 0.3$



$$\chi_{c0}(3930) \to D\bar{D}$$



$$X(3915) \to J/\psi\ \omega$$

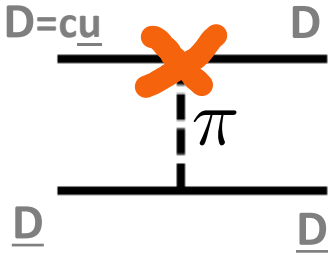
Possible interpretation of some near-threshold states:

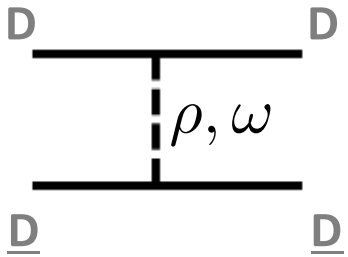
a number of pheno studies
Oset et al, 0612179 PRD,
Wu, Molina, Oset, Zou, 1007.0573, PRL
Guo et al, 2101.01021,...

"molecules" attracted by V exchange

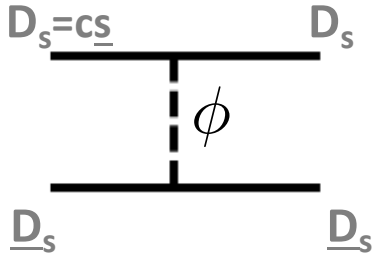
$\bar{c}q\bar{q}c$ $I=0$ $J^P=0^+$

$\bar{D}D$





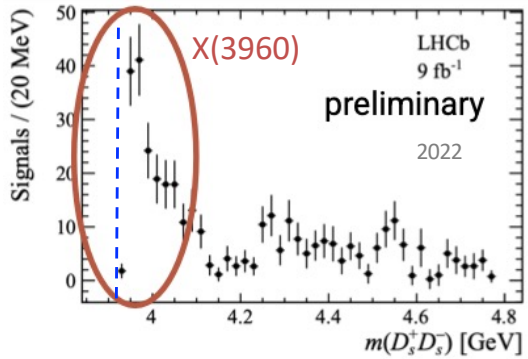
$\bar{D}_s D_s$



now support also from lattice

indico.cern.ch/event/1176505/, july 2022

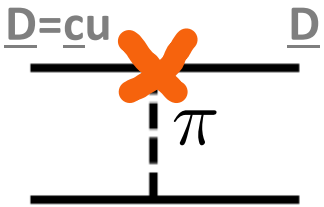
D_sD_s th.



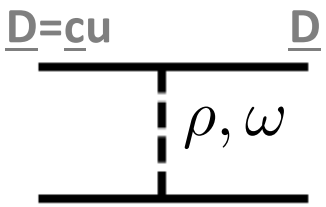
LHCb discoveries:
talk by Mikhasenko

$c\bar{c}uud$

$\bar{D}\Sigma_c$



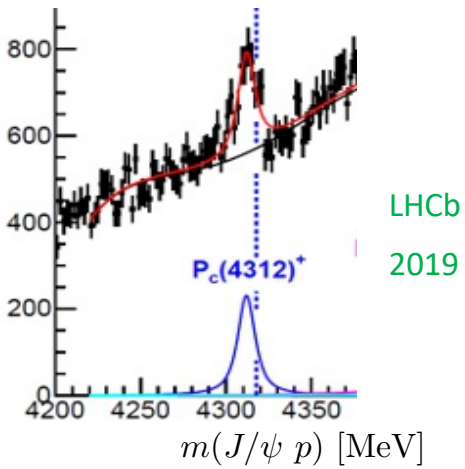
$\Sigma_c = \text{udc}$ Σ_c



$\Sigma_c = \text{udc}$ Σ_c

currently to challenging for lattice

$\Sigma_c^+\bar{D}^0$ th



LHCb
2019

Summary

Doubly charm tetraquark (T_{cc}) $\quad c c \bar{d} \bar{u}$ $\quad$ I=0, $J^P=1^+$

DD^* scattering

- T_{cc} found as a virtual bound state ≈10 MeV below DD* threshold

- likely related to T_{cc} discovered by LHCb

Charmonium(like) states $\quad \bar{c}c$, $\bar{c}q\bar{q}c$ $\quad$ I=0, $J^{PC}=0^{++},1^{--},2^{++},3^{--}$, q=u,d,s

$D\bar{D} - D_s\bar{D}_s$ scattering

- masses and decay widths of conventional charmonia confirmed : ground states (bound states)
  first excitations (resonances)

- two additional exotic charmonium-like states with $J^{PC}=0^{++}$ found just below thresholds

$\bar{D}D$ seen in dispersive re-analysis of exp. [Danilkin et al 2111.15033]

$\bar{D}_s D_s$ likely related to X(3915) / χ_{c0}(3930) / X(3960)
LHCb2020 LHCb2022

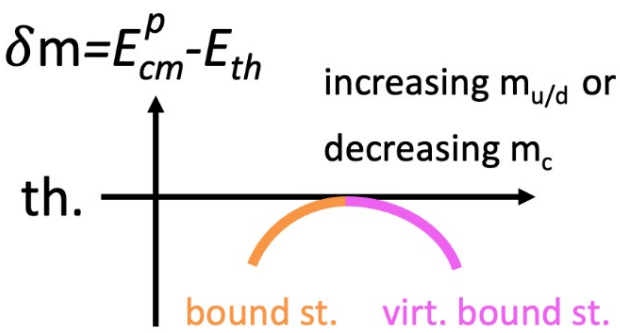
Backup

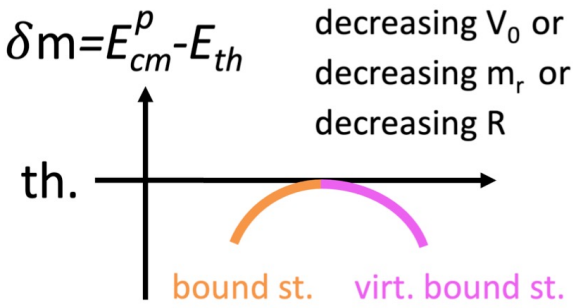
Details on Tcc

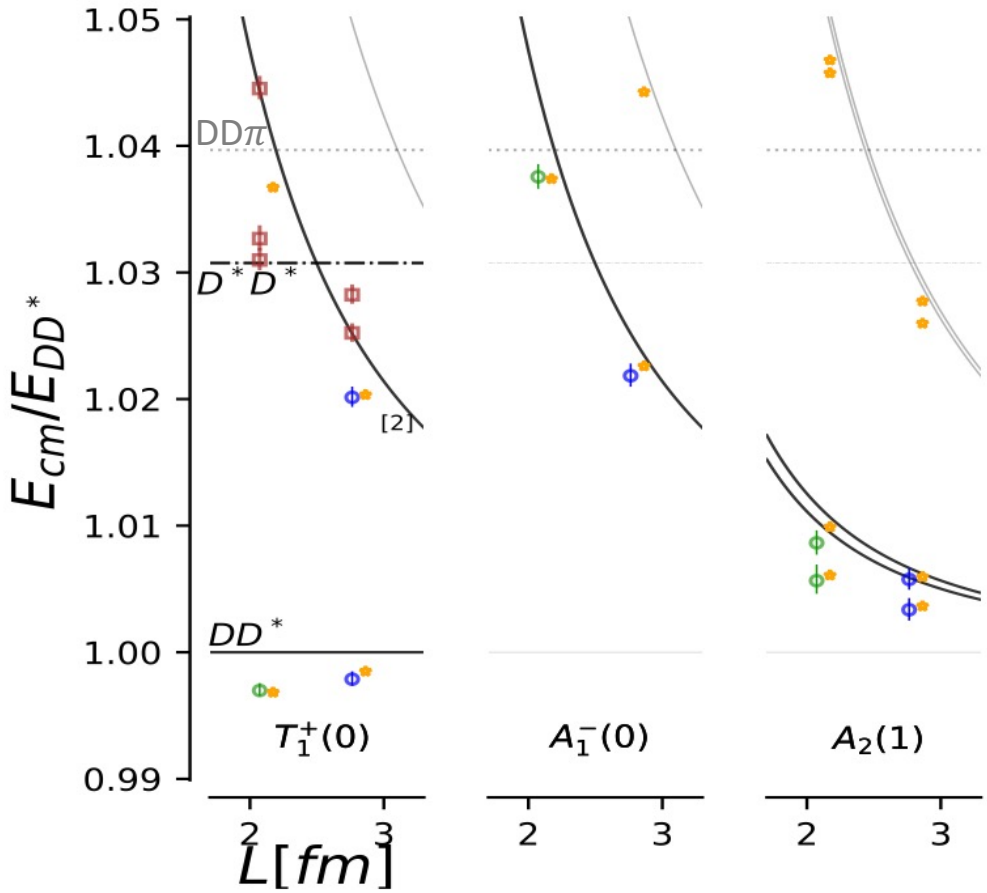
$\vec{P}$	LG	Λ^P	J^P	l	interpolators: $M_1(\vec{p}_1{}^2)M_2(\vec{p}_2{}^2)$
$(0,0,0)$	O_h	T_1^+	1^+	$0,2$	$D(0)D^*(0),\ D(1)D^*(1)$ [2], $D^*(0)D^*(0)$
$(0,0,0)$	O_h	A_1^-	0^-	1	$D(1)D^*(1)$
$(0,0,1)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(0)D^*(1),\ D(1)D^*(0)$
$(1,1,0)\frac{2\pi}{L}$	Dic_2	A_2	$0^-,1^+,2^-,2^+$	$0,1,2$	$D(0)D^*(2),\ D(1)D^*(1)$ [2], $D(2)D^*(1)$
$(0,0,2)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(1)D^*(1)$

	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$r_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp. [2, 37]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	[-11.9(16.9),0]	$-0.36(4)$	bound st.

$$V(r) = -V_0\ f(r/R)$$







$$\chi^2(\{a\}) = \sum_L \sum_{\vec{P}\Lambda n} \sum_{\vec{P}'\Lambda' n'} dE_{cm}(L, \vec{P}\Lambda n; \{a\}) \qquad (1)$$
$$\mathcal{C}^{-1}(L; \vec{P}\Lambda n; \vec{P}'\Lambda' n') dE_{cm}(L, \vec{P}'\Lambda' n'; \{a\}) \ .$$

Here

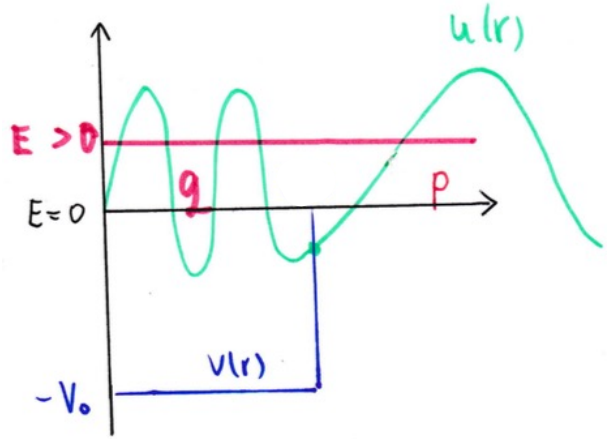
$$dE_{cm}(L, \vec{P}\Lambda n; \{a\}) = E_{cm}(L, \vec{P}\Lambda n) - E_{cm}^{an.}(L, \vec{P}\Lambda n; \{a\})$$

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm} p^{2l}} - i\frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1} \cot \delta_l^{(J)} \qquad (5)$$

We parametrize it with the effective range expansion

$$\tilde{K}^{-1} = \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)} p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)} p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix} . \qquad (6)$$

s-wave scattering on spherical potential well



$$A \sin qr \qquad B \sin(pr + \delta_0)$$

$$\left.\begin{aligned} u(R) &= A \sin qR = B \sin(pR + \delta) \\ u'(R) &= q\, A \cos qR = p\, B \cos(pR + \delta) \end{aligned}\right\}$$

dividing both eqs $\quad \frac{1}{q}\tan qR = \frac{1}{p}\tan(pR + \delta)$

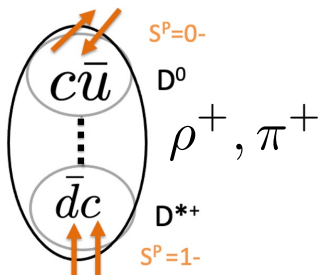
$$\boxed{\delta_0(p) = \arctan\left(\frac{p}{q}\tan(qR)\right) - pR + n\pi}$$

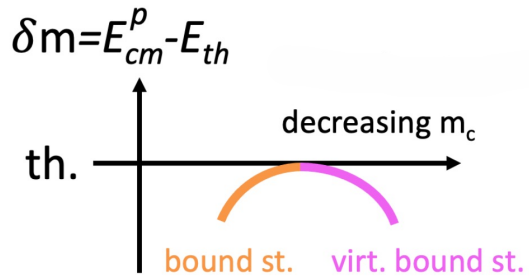
$$q = \sqrt{2\mu(V_0 + E)} = \sqrt{2\mu V_0 + p^2}$$

Molecular component: dependence on m_c

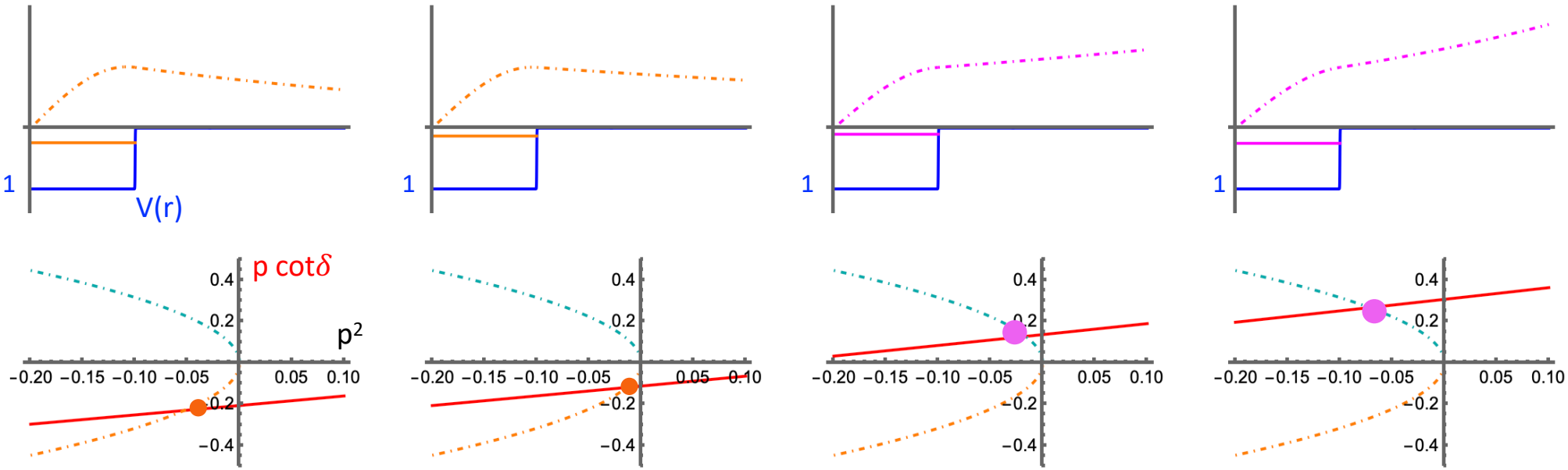
V(r) independent on m_c ,

m_c decreases : reduced mass m_r of D,D* system decreases



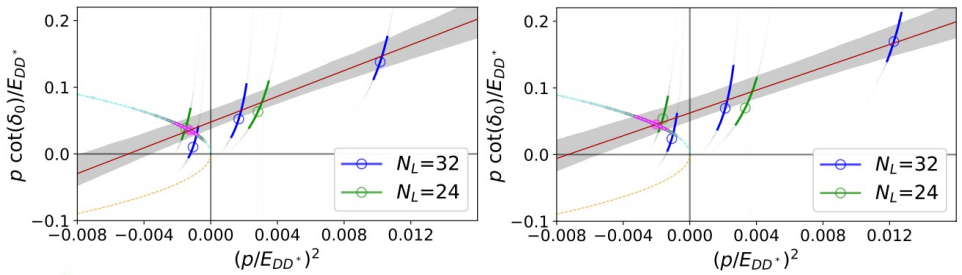


Square well potential (analogous conclusion for other shapes)



decreasing m_c and m_r

	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(h)}$	1927(1)	1.04(29)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
$m_c^{(l)}$	1762(1)	0.86(0.22)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.

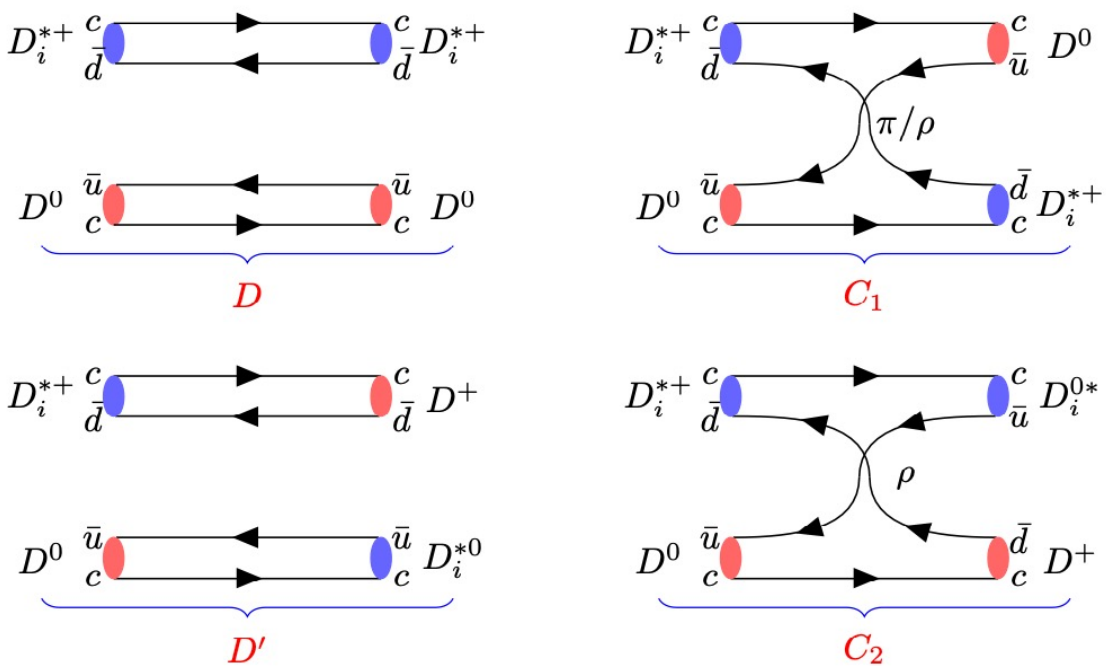


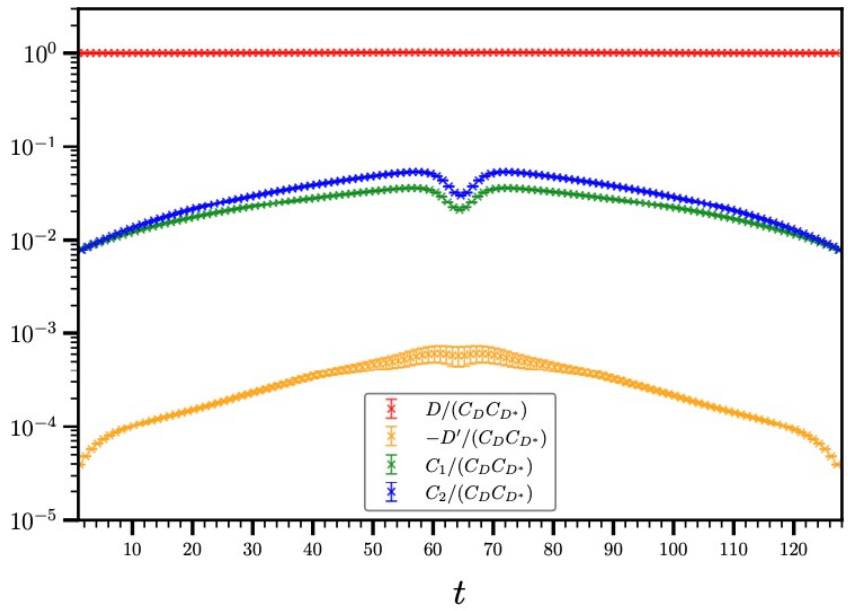
lattice results

Subsequent lattice QCD study of T_{cc} channel

CLQCD, Chen et al. 2206.06185

comparison of I=0,1 :
attraction in I=0 channel arises
mainly from ϱ exchange





$$C^{(I)}(p,t) = D - C_1(\pi/\rho) + (-)^{I+1}\left(D' - C_2(\rho)\right)$$