Conventional and exotic charmonia \( \text{from lattice QCD} \)

and bottomonia

Sasa Prelovsek

University of Ljubljana & Jozef Stefan Institute, Slovenia

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KEK meeting \& B2TiP

in collaboration with: D. Mohler, L. Leskovec, C. Lang, M. Padmanath
Outline

• charmonia and bottomonia below DD/BB thresholds

• First lattice QCD study of “conventional” charmonium resonances above open-charm threshold taking into account strong decay

\[
J^{PC} = 1^{--} : \, \psi(3770) \rightarrow D\bar{D} \\
J^{PC} = 0^{++} : \, \chi_{c0}(2P) \rightarrow D\bar{D}
\]

[Lang, Leskovec, Mohler, S.P., 1503.05363, PRD 2015]

• Lattice QCD study of three channels with \( J^{PC}=1^{++} \)

\[
\overline{c}c(\bar{u}u + \bar{d}d), \, I = 0, \, X(3872) \\
\overline{c}c\bar{u}\bar{d}, \, I = 1, \, \text{charged} \, X(3872) ? \\
\overline{c}c\bar{s}\bar{s}, \, I = 0, \, Y(4140) ?
\]

[M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]

• Pentaquarks with charm form lattice QCD

\( \eta p \) bound state

[NPLQCD, 1410.7069, PRD 2015]

• Conclusions & Outlook: what is feasible and what is challenging for lattice

• not covered: search for Zc  [see my talk at KEK 2014, Sasaki’s talk at this meeting]
Evaluation of Feynman path integrals in discretized space-time

Non-perturbative method: QCD on lattice

\[ L_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G^a_\mu T^a) q - m_q \bar{q} q \]

input: \( g_s, m_q \)

output: hadron properties

hadron interactions (if we are lucky)

precision cal.

\( a \sim 0.05 \text{ fm} \)

\( L \sim 4 \text{ fm} \)

**Evaluation of Feynman path integrals in discretized space-time**

quantum mechanics

\[ \int Dx \ e^{i S/\hbar} \]

\[ S = \int dt \ L[x(t)] \]

quantum field theory in Euclidean space-time

\[ \int DG \ Dq \ D\bar{q} \ e^{-S_{QCD}/\hbar} \]

\[ S_{QCD} = \int d^4x \ L_{QCD}[G(x), q(x), \bar{q}(x)] \]

\( x,t \ (\text{Minkovsky}) \rightarrow \ x, i \ t \ (\text{Euclidean}) \)

S. Prelovsek, KEK 2015
### Lattice setup

<table>
<thead>
<tr>
<th>PACS-CS</th>
<th>Ensemble (1)</th>
<th>Ensemble (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_L^3 \times N_T$</td>
<td>$16^3 \times 32$</td>
<td>$32^3 \times 64$</td>
</tr>
<tr>
<td>$N_f$</td>
<td>2</td>
<td>2+1</td>
</tr>
<tr>
<td>$a$ [fm]</td>
<td>0.1239(13)</td>
<td>0.0907(13)</td>
</tr>
<tr>
<td>$L$ [fm]</td>
<td>1.98(2)</td>
<td>2.90(4)</td>
</tr>
<tr>
<td>$m_\pi$ [MeV]</td>
<td>266(3)(3)</td>
<td>156(7)(2)</td>
</tr>
</tbody>
</table>

- **Wilon-clover quarks**

- **Fermilab method for $c$ and $b$**: [El Khadra, Kronfeld et al, 1997]
  
  Rest hadron energies have sizable discretization errors but these largely cancel in splittings. Only splittings with respect to a chosen reference mass are compared to experiment.

- **evaluating Wick contractions** to simulate scattering on the lattice is challenging and computationally intensive – that is part of the reason why a small number of studies have been made. We apply two methods
  
  - distillation (Ensemble 1) [Peardon et. al., HSC, 2009]
  - stochastic distillation (Ensemble 2) [Morningstar et al., 2011]
Meson(like) system with given $J^{PC}$ is created by a number of interpolating fields

$$J^{PC} \quad \mathcal{O} = \bar{q} \Gamma q, \quad (\bar{q} \Gamma_1 q) \mathcal{p}_1 (\bar{q} \Gamma_2 q) \mathcal{p}_2, \quad [\bar{q} \Gamma_3 q][q \Gamma_4 q],...$$

$X(3872), 1^{++}$: $\bar{c}c$, $(\bar{c}u)(\bar{u}c) = DD^*$, $[\bar{c}u][cu]$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^+(0) | 0 \rangle = \sum_{n} Z_i^n Z_j^{n*} e^{-E_n t}, \quad Z_i^n = \langle 0 | \mathcal{O}_i | n \rangle$$

All physical states with given $J^{PC}$ appear as $E_n$ in principle (example: charmonium with $1^{++}$)

- "single-meson" states $\chi_{c1}$
  $$m_{\chi_{c1}} = E_1 \quad \text{for } P=0 \text{ (after extrapolations)}$$
  $$X(3872)$$

- "two-meson" states $DD^*$, ...
  $$E_n \text{ rigorously render two-hadron scattering matrix}$$
  (for example $DD^*$ scattering matrix)
Charmonia well below $D\bar{D}$: recent precision results

$m=E\ (P=0)\ : \ a\to0, \ L\to\infty, \ m_q\to m_q^{\text{phy}}$

HPQCD, 1411.1318

chiQCD, update of 1410.3343

The omission of charm annihilation is the main remaining uncertainty
Bottomonium spectrum below threshold

... is richer than the charmonium one

The most complete spectrum has been recently published:
[Wurtz, Lewis, Woloshyn, 0505.04410, PRD 2015] (see talk of Lewis at KEK 2014 meeting)

Valuable and reliable predictions for 1D and 2D states that have not been observed in exp yet!!
Charmonia near or above DD threshold: single-meson approximation

- only interpolating fields \( \mathcal{O} \approx \bar{c} c \)
- assumptions:
  - strong decays of resonances above threshold ignored
  - effects of thresholds on near-threshold states ignored

\( m = E \) (for \( P=0 \))

these are strong assumptions ...

but results still present valuable reference point
Charmonia: single-meson approximation

$\mathcal{O} : \bar{c}c$

- Lattice
- Experiment

Part of G-wave

D-wave

F-wave

Exotics

P-wave

S-wave

Hybrids:
- some of them have exotic $J^{PC}$
- large overlap with $O = q F_{ij} q$

$[\text{HSC}, \text{L. Liu et al: 1204.5425, JHEP}]$

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

S. Prelovsek, KEK 2015
Rigorous treatment of hadrons near or above threshold:

\[ D\bar{D} \rightarrow \psi(3770) \rightarrow D\bar{D} \]

Example

scattering of two mesons

analytic proposal: Luscher 1991

two mesons in a lattice box

\[ E(L) \]

\[ \delta(E) \]

scattering phase shifts at infinite volume

energies from lattice with spatial extent \( L \)

S. Prelovsek, KEK 2015
One-channel (elastic) scattering with total momentum $P=0$: $E=E_{\text{cm}}$

Scattering of two mesons

$L_n(L) \xrightarrow{\text{Luscher's eq.}} \delta(E)$

Scattering matrix for partial wave $l$

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot \delta(E) - i}$$

**Bound state (B):**

$$\cot[\delta(E_\text{B})] = i, \quad E_\text{B} < m_1 + m_2$$

**Resonance (R) (of Breit-Wigner type):**

$$T(E) = \frac{-E \Gamma}{E^2 - m_R^2 + i E \Gamma}, \quad \Gamma(E) = g^2 \frac{p^{2l+1}}{E^2}$$

Locations of poles of $T(E)$ for res. and bound st.

Two types of plots will be shown:
- $E(L)$ energies of lat. eigenstates
- $m_{\text{phys}} = m_R, m_B$ extracted from $E(L)$
Resonance $\psi(3770)$ and bound st. $\psi(2S)$ from $DD$ scattering in p-wave

$\Theta$: $\bar{c}c, D\bar{D}$, $J^{PC} = 1^{--}$

<table>
<thead>
<tr>
<th></th>
<th>$\bar{c}c$</th>
<th>$D\bar{D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{c}c$</td>
<td>$-1$</td>
<td>$+2$</td>
</tr>
<tr>
<td>$D\bar{D}$</td>
<td>$+2$</td>
<td>$+2$      $-4$</td>
</tr>
</tbody>
</table>

$D\bar{D} \rightarrow \psi(3770) \rightarrow D\bar{D}$

Charm annihilation Wick contractions omitted
(like in all charmonium-like lattice simulations)

First lattice simulation of a charmonium resonance above open-charm threshold
taking into account its strong decay

$\psi(3770) \rightarrow D\bar{D}$

$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$h_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$
$\psi(3770)$
$X(3872)$
$\chi_{c0}(2P)^{we}$
$\chi_{c2}(2P)$
$X(3940)$
$\psi(4040)$
$X(4050)^{\pm}$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^{\pm}$

Lang, Leskovec, Mohler, S.P.,
1503.05363, JHEP 2015

S. Prelovsek, KEK 2015
Resonance $\psi(3770)$ and bound st. $\psi(2S)$ from DD scattering in p-wave

$E_n \rightarrow \delta(E_n) \rightarrow p^3 \cot \delta(p)/\sqrt{s}$

This quantity presented as it is approx. linear near simple BW resonance:

- BW fit (i):
  $$ T_i(s) = \frac{\sqrt{s} \Gamma(s)}{m_R^2 - s - i\sqrt{s} \Gamma(s)} = \frac{1}{\cot \delta_i(s) - i} $$
  $$ \Gamma(s) = \frac{g^2}{6\pi} \frac{p^3}{s} $$

- fit (ii): captures also bound st.:
  $R: m_R$: zero, $\Gamma_R$: slope near zero
  $B: \cot \delta = i$

$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$h_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$
$\psi(3770)$
$X(3872)$
$\chi_{c0}(2P)_{we}$
$\chi_{c2}(2P)$
$X(3940)$
$\psi(4040)$
$X(4050)^{\pm}$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^{\pm}$

$m_R = 266$ MeV
$m_R = 156$ MeV

$p = i|p| \rightarrow p^3 \cot \delta = (i|p|)^3 i = |p|^3 > 0$
Resonance $\psi(3770)$ and bound st. $\psi(2S)$ from DD scattering in p-wave

$\mathcal{O}$: $\bar{c}c$, $DD$, $J^{PC} = 1^{--}$

DD scat. in p-wave is simulated

T-matrix is determined from $E_n$

Fit of T-matrix gives:

BW resonance $\psi(3770)$:

$\Gamma$ (given below)

Bound state $\psi(2S)$ from pole in T:

$\psi(2S)$ (magenta diamonds)

$m_B$ (magenta triangles)

<table>
<thead>
<tr>
<th>$\psi(3770)$, fit (ii)</th>
<th>Mass [MeV]</th>
<th>$g$ (no unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lat ($m_\pi = 266$ MeV)</td>
<td>3774 ±6±10</td>
<td>19.7 ±1.4</td>
</tr>
<tr>
<td>Lat ($m_\pi = 156$ MeV)</td>
<td>3789 ±68±10</td>
<td>28 ±21</td>
</tr>
<tr>
<td>Exp.</td>
<td>3773.15±0.33</td>
<td>18.7 ±1.4</td>
</tr>
</tbody>
</table>

$\Gamma = \frac{g^2 p^3}{6\pi s}$

Lang, Leskovec, Mohler, S.P.,
1503.05363, JHEP 2015
Scalar charmonia from $\bar{D}D$ scattering in s-wave, $J^{PC}=0^{++}$

- $\chi_{c0}(1P)$: the only settled state; $\chi_{c0}(2P)$: PDG assigned it with $X(3915)$
- Meissner & Guo [1208.1134], Olsen [1410.6534]: arguments against this assignment
- Zhou et al [1501.00879, PRL 2015]: experimental $X(3915)$ could be compatible with $J^{PC}=2^{++}$
  talk at the meeting
- It is still not commonly accepted which exp state corresponds to $\chi_{c0}(2P)$
- $\bar{D}D$ and $J/\Psi$ $\omega$ scattering in s-wave simulated on lattice:

$\Theta$: $\bar{c}c$, $D\bar{D}$, $J/\psi\omega$

Assumption for extracting $D\bar{D}$ phase shifts: $J/\Psi$ $\omega$ channel approximately decoupled
Verified in lattice data (when $J/\Psi$ $\omega$ interpolator removed all other $E_n$ and $Z$ remain unaffected)
Scalar charmonia from DD scattering in s-wave, $J^{PC}=0^{++}$

**DD scattering phase shift**

$$E_n \rightarrow \delta(E_n) \rightarrow p \cot \delta/\sqrt{s}$$

real also below threshold

Near simple BW resonance:

$$\frac{p \cot \delta(s)}{\sqrt{s}} = \frac{4}{g^2}(p_R^2 - p^2), \quad \Gamma(s) = g^2\frac{p}{s}$$

At the bound state pole:

$$p = i|p| \rightarrow p \cot \delta = (i|p|)i = -|p| < 0$$

This curious shape seems to suggest narrow resonance and influence from the bound state pole at $\chi_{c0}(1P)$

Lat: $m_\pi = 266$ MeV

$E=3.4$ GeV

$\chi_{c0}(1P)$

$2m_D \quad E=3.96$ GeV

S. Prelovsek, KEK 2015
Scalar charmonia from DD scattering in s-wave, $J^{PC}=0^+$

DD scattering in s-wave simulated on lattice: comparison to several hypothesis made

Hypothesis:
- one narrow resonance & bound state pole at $\chi_{c0}(1P)$

m$_R$=3.966(20) GeV  $\Gamma^{\text{predict}}$=67(18) MeV
such narrow res. not (yet) found in exp DD inv. mass
does not describe our results near threshold

Hypothesis:
- one narrow resonance

supported by lat.
- narrow resonance in DD:
m$_R$=4.002(24) GeV
$\Gamma^{\text{predict}}$=32(48) MeV

Lang, Leskovec, Mohler, S.P., 1503.05363, JHEP 2015

S. Prelovsek, KEK 2015
Scalar charmonia from DD scattering in s-wave, $J^{PC}=0^{++}$

Hypothesis:

**one broad BW resonance**

This hypothesis leads to additional energy eigenstate (at the crossing of green and orange lines) which is not observed in simulation.

Hypothesis:

**two BW resonances**

not supported by lat. data near and above th.

more detailed DD and J/$\Psi$ $\omega$ lineshapes needed from lattice and exp
\( X(3872) \) as bound state from \( DD^* \) scattering, \( J^{PC}=1^{++} \), \( I=0 \)

**Diagram**

- **\( \Theta: \bar{c} \ c, DD^* \)**
  - ground state: \( \chi_{c1}(1P) \)
  - \( DD^* \) scattering matrix near th. determined
    \[ T \propto \frac{1}{\cot \delta - i} = \infty \]
  - A pole of \( X(3872) \) found just below th. (violet star)
  - The pole attributed to \( X(3872) \), which is a shallow bound state in both simulations
  - Position of \( DD^* \) threshold depends on \( m_{u/d} \), and may be affected by discretization effects related to charm quark

**Lattice evidence for \( X(3872) \):**
- \( m_\pi \approx 266 \text{ MeV}, \ a=0.124 \text{ fm, } L=2 \text{ fm} \)
  - [S.P. and Leskovec: 1307.5172, PRL 2013]
- \( m_\pi \approx 310 \text{ MeV}, \ a=0.15 \text{ fm, } L=2.4 \text{ fm} \), HISQ
  - [Lee, DeTar, Na, Mohler, update of proc 1411.1389]
Which Fock components are essential for X(3872) with I=0?

\[ J^{PC}=1^{++} \]

\[ \mathcal{O}: \quad \bar{c}c, \quad D\bar{D}^*, \quad J/\psi \omega, \quad \chi_{c1}\eta, \quad \eta_c\sigma, \quad [\bar{c}u]_{3c}[cu]_{3c}, \quad [\bar{c}u]_{6c}[cu]_{6c} \]

\[ (\bar{c}q)_{1c} (c\bar{q})_{1c}, \quad (\bar{c}c)_{1c} (\bar{q}q)_{1c} \]
Which Fock components are essential for $X(3872)$ with $I=0$?

$\mathcal{O}$: $\bar{c}c$, $D\bar{D}^*$, $J/\psi \omega$, $\chi_{c1}\eta$, $\eta\sigma$, $[cu]_{3c} [cu]_{3c}$, $[cu]_{6c} [cu]_{6c}$

$\theta$: interpolating fields

$O$: all

$O$: all without $cc$

$m_\pi \approx 266$ MeV

upper red square is candidate for $X(3872)$: it is found only if $cc$ in the basis

energies of eigenstates on lattice $E(L)$ (these are not $m^{phy}$)

X($3872$) not found if $cc$ not in the basis, although $[cu][cu]$ in the basis

[M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]
Illustration how eigenstate $D_0^*(2400)$ dominated by "qq" dissapears when qq interpolators omitted

Effective energies: $E_n(t)\rightarrow E_n$ in plateau region

[Mohler, S.P., Woloshyn, PRD 87 (2013) 034501]
Search for charged partner of X(3872); channel \( J^P = 1^+, J^{PC} = 1^{++}, ccd\)

\[ p = n \frac{2\pi}{L} \]

- Horizontal lines: energies of expected two-meson states in limit of no interaction:
  \[ E = E[M_1(p_1)] + E[M_2(p_2)] \]

- Circles: energies of eigenstates from lattice
  - Only expected two-meson states observed.
  - **No lattice candidate for charged X(3872).** In agreement with absence of such state in exp.

- **No lattice candidate for other charged state below 4.3 GeV.**

- Two Belle 2008 states are exp. unconfirmed.

---

\[ \mathcal{O} : (\bar{c}u)(\bar{d}c), (\bar{c}c)(\bar{d}u), [\bar{c}d][cu] \]

\[ m_\pi \approx 266 \text{ MeV} \]

[M. Padmanath, C.B. Lang, S.P., PRD 2015, 1503.03257]
**Y(4140), J^P_C=??^+, ccss**

Experiment:
- peak in \(J/\psi\ Φ\) just above \(J/\psi\ Φ\) threshold
- found: CDF 2009, CMS 2012, D0 2013, Babar 2015
- not found: Belle 2010, LHCb 2012

Lattice:
- S. Ozaki and S. Sasaki, 1211.5512, PRD
  - caveat: strange quark annihilation neglected
  - no resonant \(Y(4140)\) structure found
- M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD
  - channel \(J^p=1^+\) considered only: expected two-particle eigenstates found and \(\chi_{c1}\), \(X(3872)\) but not \(Y(4140)\)
  - \(m_\pi=266\) MeV

\[Y(4140) \rightarrow J/\psi \ Φ\]
\[\bar{c}c \quad \bar{s}s\]

\(J/Ψ\ Φ\) scattering phase shift [rad]

\[E(L)\]

Expt. \ Lat. \(J^P_C=1^{++}\)

- \(Y(4274)\)
- \(J/Ψ(1)\ Φ(-1)\)
- \(D_s(1)\ \bar{D}_s^*(-1)\)
- \(J/Ψ(0)\ Φ(0)\)
- \(D_s(0)\ \bar{D}_s^*(0)\)

- \(X(3872)\)
- \(\chi_{c1}\)
- \(X_{c1}\)
Bound state of a $\eta_c$ and $p$ from lattice

[NPLQCD, 1410.7069, PRD, $m_\pi \sim 800$ MeV]

$\eta_c ~ 20$ MeV below $m_{\eta_c} + m_p$

$\bar{c}c$ $uud$

no lattice results for these Pc yet: challenging as it can in principle decay to several decay ch.

LHCb: 1507.03414

Two pentaquark resonances in $J/\psi$ and $p$ from exp

$J/\psi ~ 400$ MeV above $m_{J/\psi} + m_p$

$LHCb$

S. Prelovsek, KEK 2015
Conclusions & Outlook

• states well below strong decay threshold: easy, under control
  First extensive results on bottomonium spectrum

• resonances and shallow bound states where one channel dominates:
  first rigorous results obtained during past two years:
  - first simulation of charmonium resonance
    \[ D\bar{D} \rightarrow \psi(3770) \rightarrow D\bar{D} \]
  - first evidence for shallow bound state
    \[ D\bar{D}^* \rightarrow \psi(3872) \rightarrow D\bar{D}^* \]

Improved lattice results for these and other channels expected

• States that can decay to two or three channels:
  - S for two-coupled channels from Luscher approach
    \[ K\pi, K\eta \text{ system } [\text{Willson, Dudek, Edward, Thomas, HSC, PRL 2014}] \]
    Very challenging! But feasible. There are efforts in this direction. More results expected.
  - S for three-coupled channels with HALQCD approach: Zc(3900)  [Sasaki, talk at this meeting]
Some of very interesting states can decay to more than three channels
- $Z_b^+$ by Belle in 5 decay modes $Y(1S, 2S, 3S)\pi^+$, $h_c(1S, 2S)\pi^+$
- $Z^+(4430)$

Rigorous results on 5-coupled channels nor realistic soon ...
Simplifying assumptions will be necessary.
Analytic efforts necessary to understand which simplifying assumptions could be relied on ...