

Hadron Spectroscopy

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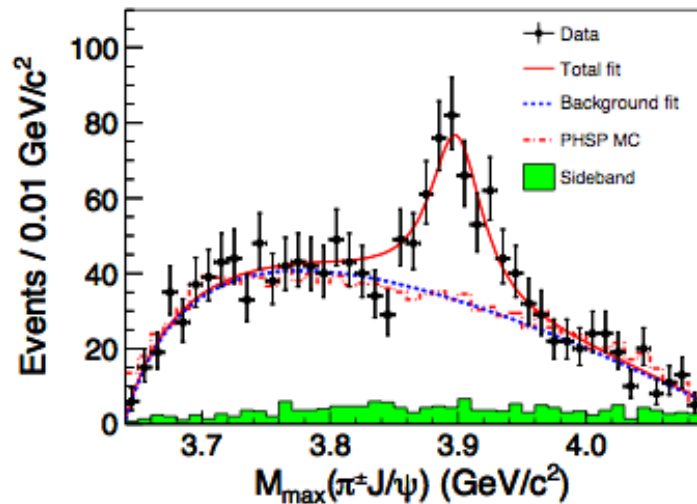
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Lattice 2014, June 2014, Columbia University, New York City



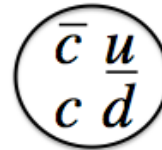
Disclaimer

- the organizers asked me to concentrate on **mesons**, in particular to exotic quarkonium-like states (XYZ) that have been experimentally confirmed during 2013/2014



[BESIII, 2013, 1303.5949, PRL]

$$Z_c^+(3900) \rightarrow J/\psi \pi^+ \underline{c} \underline{c} \underline{d} u$$



such states
confirmed by
BeSI, Belle, LHCb, Cleo-c

- I will review also other topics in hadron spectroscopy to the extent possible

Outline

- **Methods** (most commonly used ones)
 - ✧ States well below strong decay threshold
 - ✧ Excited states: single-hadron approximation
 - ✧ Near-threshold states
 - ✧ Resonances
- **Related studies**

Methods

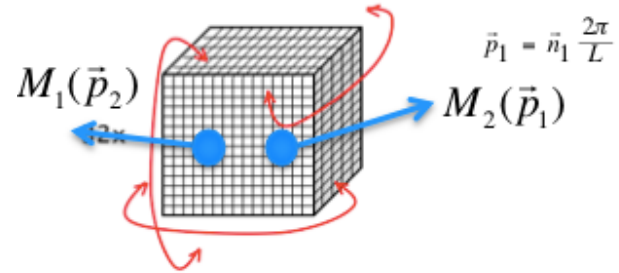
(most commonly used ones)

Discrete energy spectrum from correlators

Meson(like) system with given J^{PC} (or irrep.) is created by

$$\mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 q)_{\vec{p}_2}, \quad [\bar{q}\Gamma_3 \bar{q}][q\Gamma_4 q], \dots$$

$$M_1(\vec{p}_1) M_2(\vec{p}_2)$$



$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

$$= \sum_n \langle 0 | \mathcal{O}_i | n \rangle e^{-E_n t} \langle n | \mathcal{O}_j^\dagger | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t} \quad Z_i^n = \langle 0 | \mathcal{O}_i | n \rangle$$

All physical states with given J^{PC} (or irrep) appear as E_n in principle (and are mixtures of) :

- "single-particle" states
- "two-particle" states: for periodic BC $E(L) = \sqrt{m_1^2 + \vec{p}_1^2} + \sqrt{m_2^2 + \vec{p}_2^2} + \Delta E, \quad \vec{p}_1 = \frac{2\pi}{L} \vec{n}_1 \quad \vec{p}_2 = \frac{2\pi}{L} \vec{n}_2$
- three particle states.... (not yet in practice)

E_n and overlaps Z are extracted from variational method [C. Michael 1985, Luscher, Wolff, 1990, Blossier 2009]

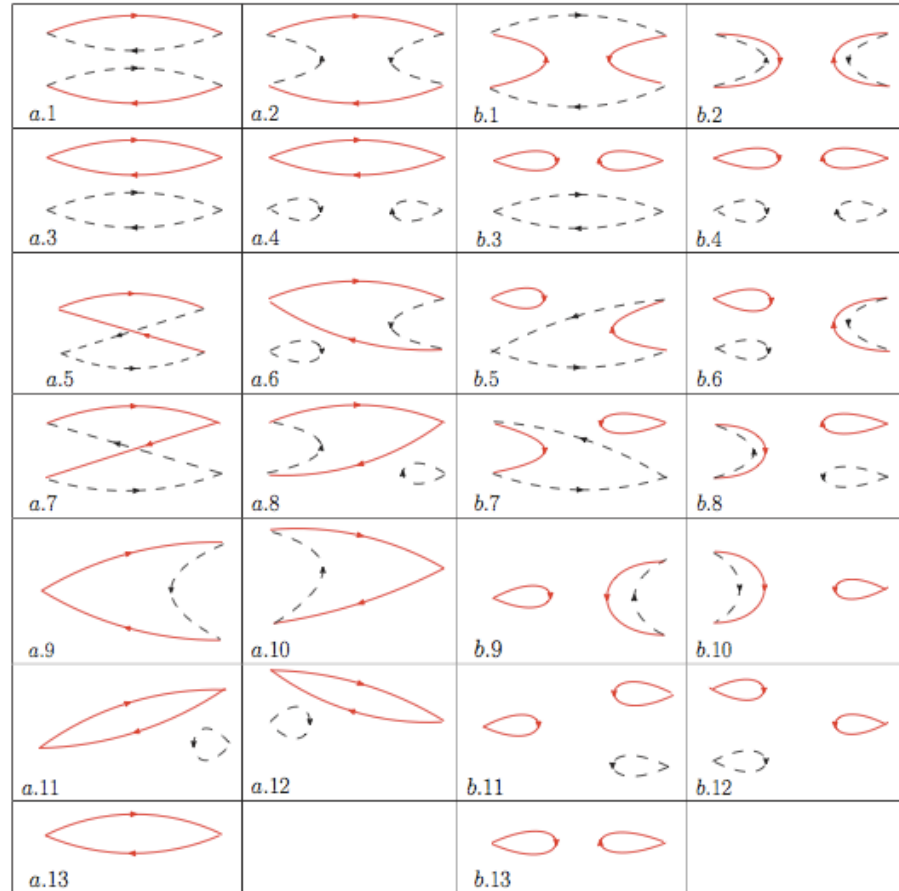
$$\lambda^{(n)}(t) \propto e^{-E_n t} [1 + O(e^{-\Delta E t})] \quad Z_j^{(n)}(t) = e^{E_n t/2} \frac{|C_{jk}(t) u_k^{(n)}(t)|}{|C(t)^{\frac{1}{2}} u^{(n)}(t)|} \quad C(t) u^{(n)}(t) = \lambda^{(n)}(t) C(t_0) u^{(n)}(t)$$

Wick contractions require all-to-all methods

- Example of Wick contractions needed for the $X(3872)$ channel

$$\mathcal{O}: \bar{c} c, (\bar{c}u)(\bar{u}c), (\bar{c}c)(\bar{u}u)$$

- all-to-all methods are needed and widely used by now
- Examples:
 - distillation method
[Peardon et al, JHEP 2009]
 - stochastic distillation method
[Morningstar et al, PRD 2011]
 - a number of others



c quark
u,d quarks

[S.P. and L. Leskovec,
Phys. Rev. Lett. 2013]

Extracting information on strong interactions

- Lüscher-type relation: input E^{cm} [Lüscher 1991]

$$\tan \delta(p) = \frac{\sqrt{\pi} p L}{2 \mathcal{Z}_{00} \left(1; \left(\frac{pL}{2\pi}\right)^2\right)}$$

in favorable cases ($P=0$): one equation for one unknown $\delta_l(E^{\text{cm}})$
in less favorable case ($P \neq 0$, $\theta \neq 0$, coupled ch.): one equation for several unknowns $\delta_l^a(E^{\text{cm}})$
encouraging that HSC managed to extract T-matrix for coupled $K\pi$, $K\eta$ [Dudek et al, HSC, 1406.4158], Wilson
extensions and references reviewed by Briceño and Yamazaki, plenaries

$$\det \left(F^{-1} + i\mathcal{M} \right) = 0$$

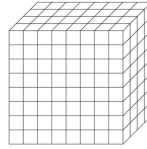
$\searrow$
 $\delta_l^a(E^{\text{cm}})$
- finite-volume Hamiltonian EFT: input E [Hall et al, 1303.4157, PRD], Leinweber
fit E with λ of Hamiltonian EFT and extract parameters of Hamiltonian
- HALQCD method: [Ishii et al., PLB712, 437 (2012)] members of HALQCD
determine $V(r)$ between two mesons and extract $\delta(E)$ by solving Schrodinger eq.
- possibility of rigorously extracting info from overlaps has not been fully explored
so far used mostly at the intuitive level
considerations in this direction may turn out fruitful

$$Z_i^n = \langle 0 | \mathcal{Q}_i | n \rangle$$

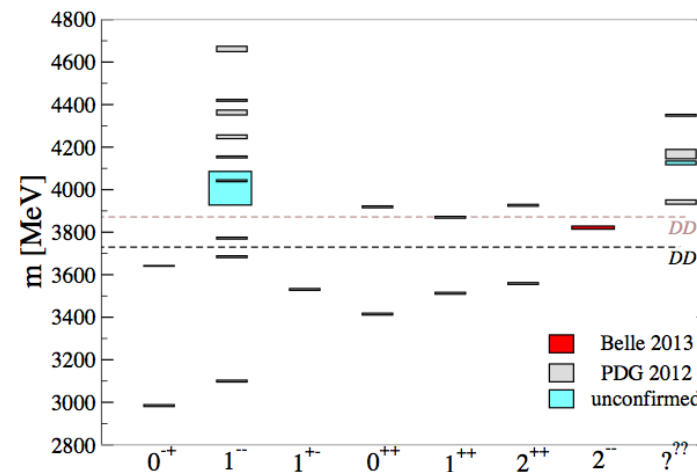
Precision spectrum: States well below strong decay threshold

States well below threshold

- no two-particle states nearby: standard procedure applicable
- $m=E$ (for $P=0$)
- extrapolation $a \rightarrow 0$, $L \rightarrow \infty$
- simulation at m_q^{phy} or extrapolation/interpolation $m_q \rightarrow m_q^{\text{phy}}$
- particular care needed for discretization errors related to am_c and am_b : complementary methods give compatible results for $a \rightarrow 0$
- many precision lattice results available !



Charmonium : cc



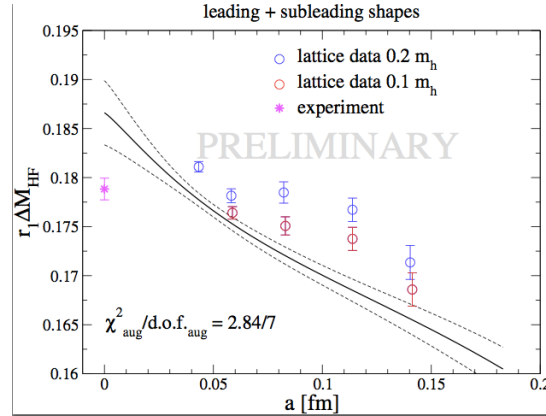
- mixing with light hadrons is omitted for all hidden charm states that I will present !
- the rigorous treatment which goes beyond that still presents an unsolved challenge
 - (i) noise in disconnected diagrams, (ii) mixing with a number of lighter states

Low lying charmonium

many results available, only few examples shown

$$M_{J/\psi} - M_{\eta_c}$$

$$118.1 \pm 2.1^{+1.5}_{-4.0} \text{ MeV}$$



band indicates uncertainty
related to omission of
charm annihilation
[Levkova, DeTar]

$$M_{J/\psi} - M_{\eta_c}$$

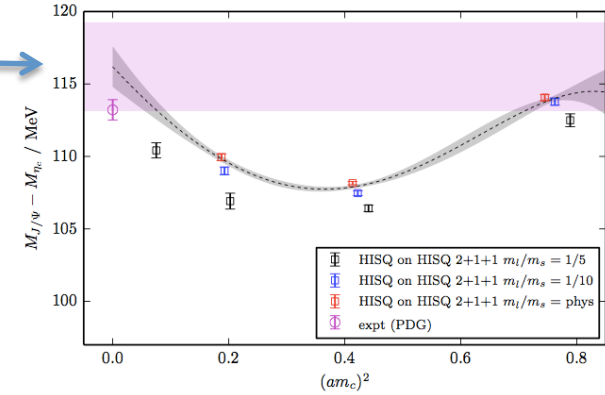
$$116.2 \pm 1.4 \pm 2.8 \text{ MeV}$$

$$M_{1\bar{S}} = \frac{1}{4}[M_{\eta_c} + 3M_{J/\psi}]$$

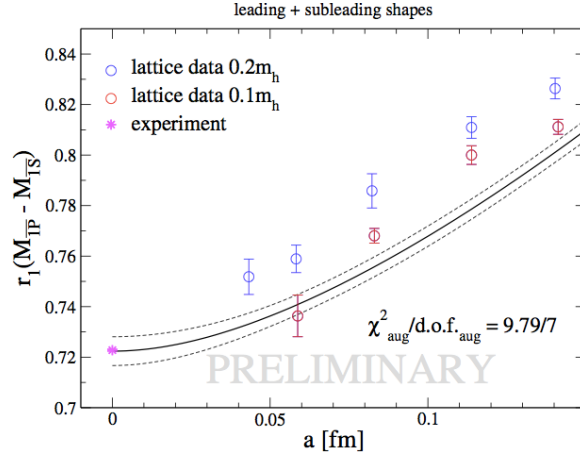
$$M_{1\bar{S}} = \frac{1}{4}[M_{\eta_{2S}} + 3M_{\psi_{2S}}]$$

$$M_{1\bar{P}} = \frac{1}{9}[M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}}]$$

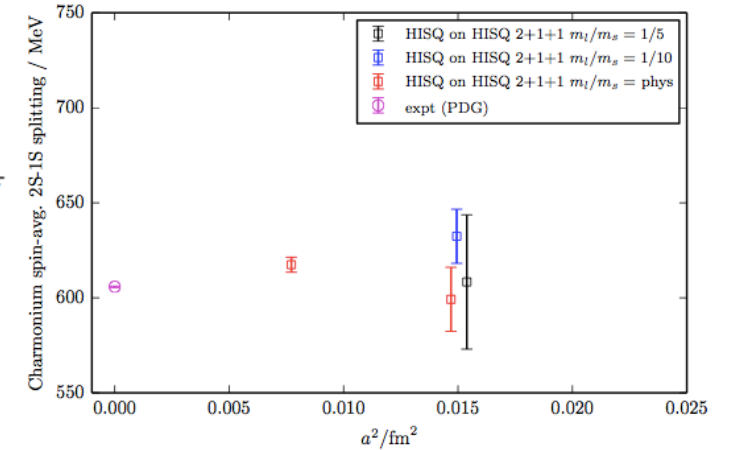
$$[M_{J/\psi} - M_{\eta_c}]^{\text{exp}} = 113.2 \pm 0.7 \text{ MeV}$$



$$M_{1\bar{P}} - M_{1\bar{S}}$$



$$M_{2\bar{S}} - M_{1\bar{S}}$$

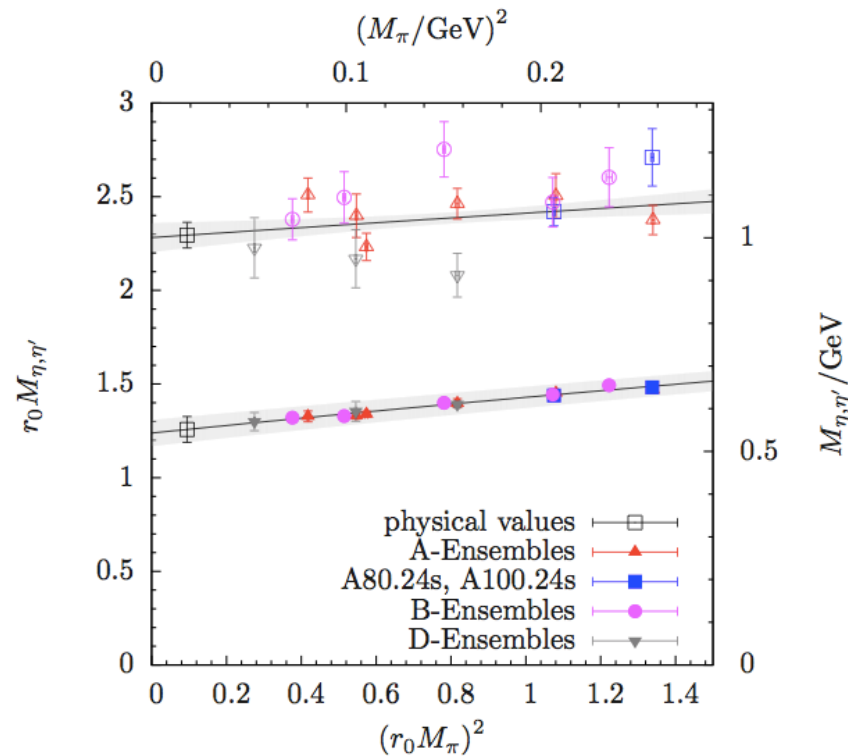


uncertainty from scale setting and disconnected
diagrams not taken into account in above plots

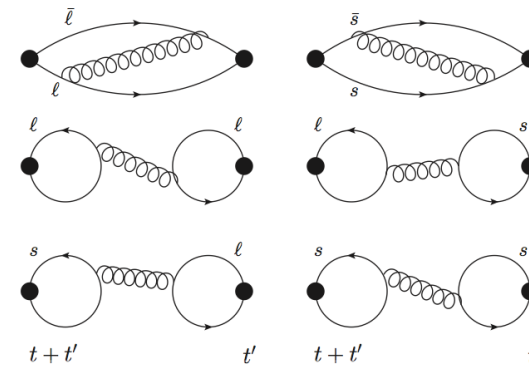
Fermilab/MILC, Daniel Mohler, Monday 17h30

HPQCD/MILC, Ben Galloway, Tuesday 15h35

η and η'



- both very narrow
- two-meson strong decay modes negligible



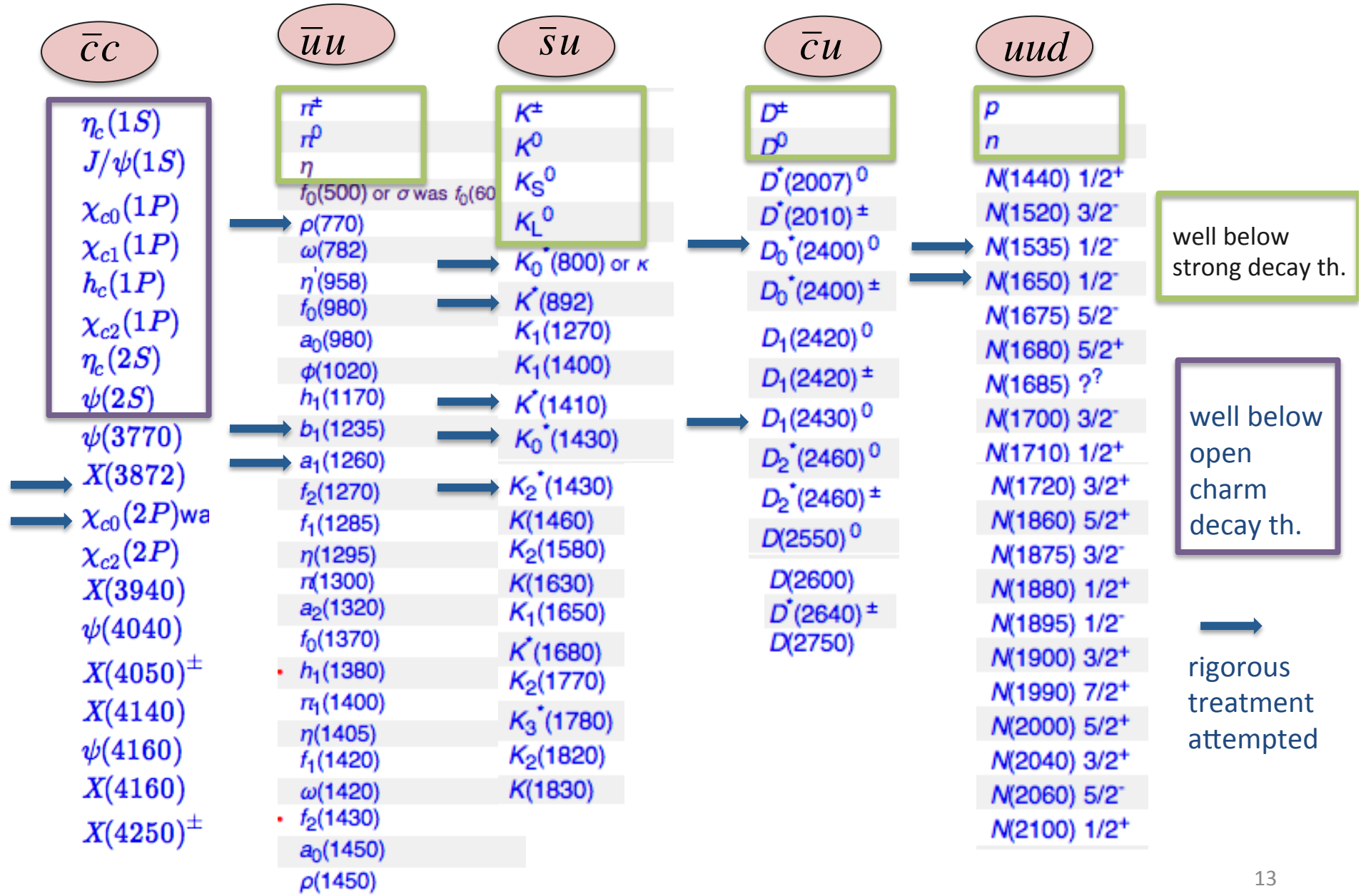
[C. Michael, K. Ottnad, C. Urbach, ETMC, 1310.1207, Phys. Rev. Lett. 2013]

"Non-precision" spectrum: states near or above threshold

only one or few $a, L, m_{u/d}$

limits $a \rightarrow 0, L \rightarrow \infty, m_{u/d} \rightarrow m_{u/d}^{\text{phy}}$ usually not performed

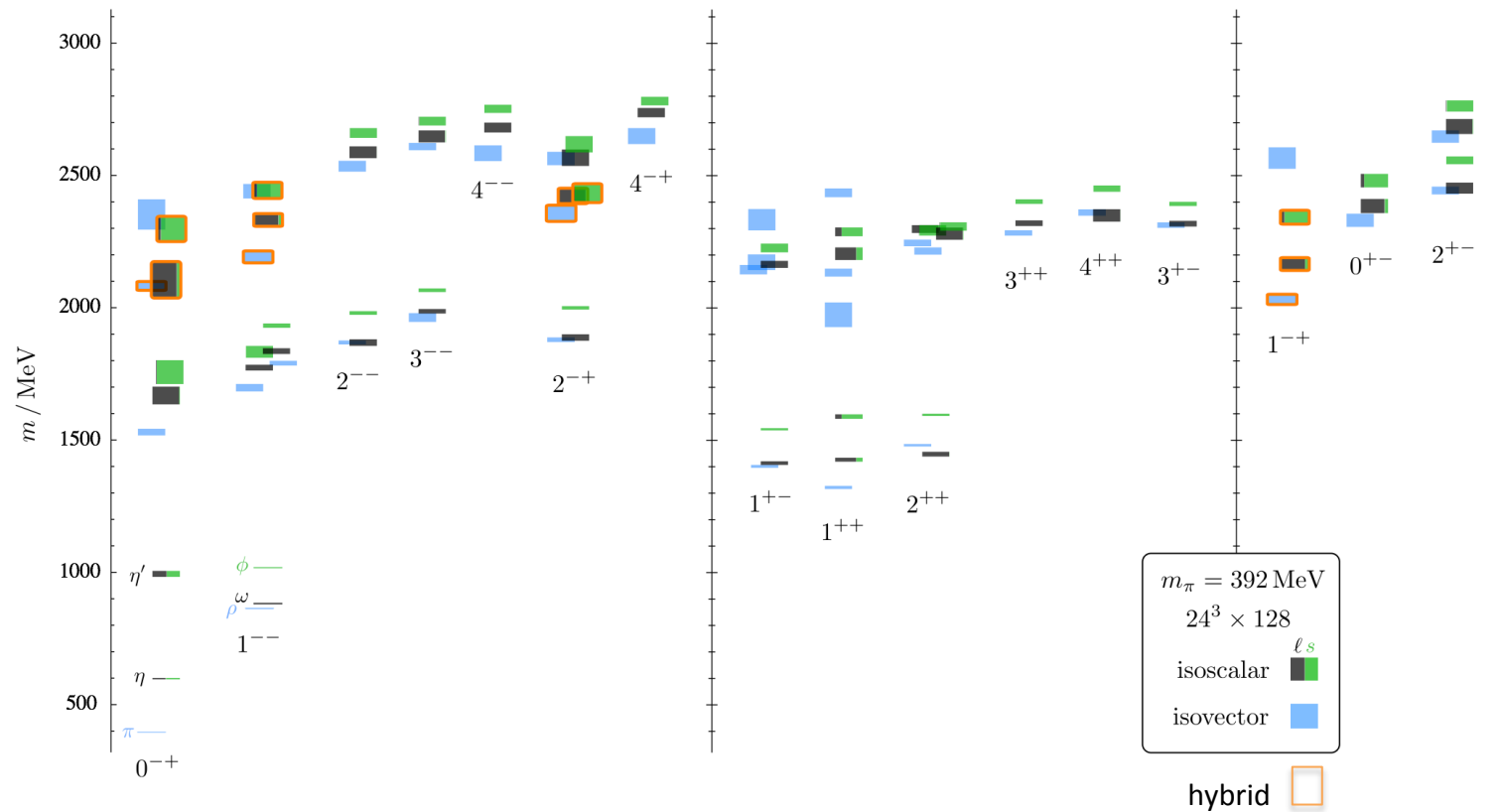
Almost all hadrons are near or below threshold



States near or above threshold: single-hadron approximation

- only interpolating fields $\mathcal{O} \approx \bar{q} q$
- assumptions: all energy levels correspond to "one-particle" states
no two-particle state is seen
 $m=E$ (for $P=0$)
these are strong assumptions ...
but results still present valuable reference point

Isoscalar mesons : single hadron approximation



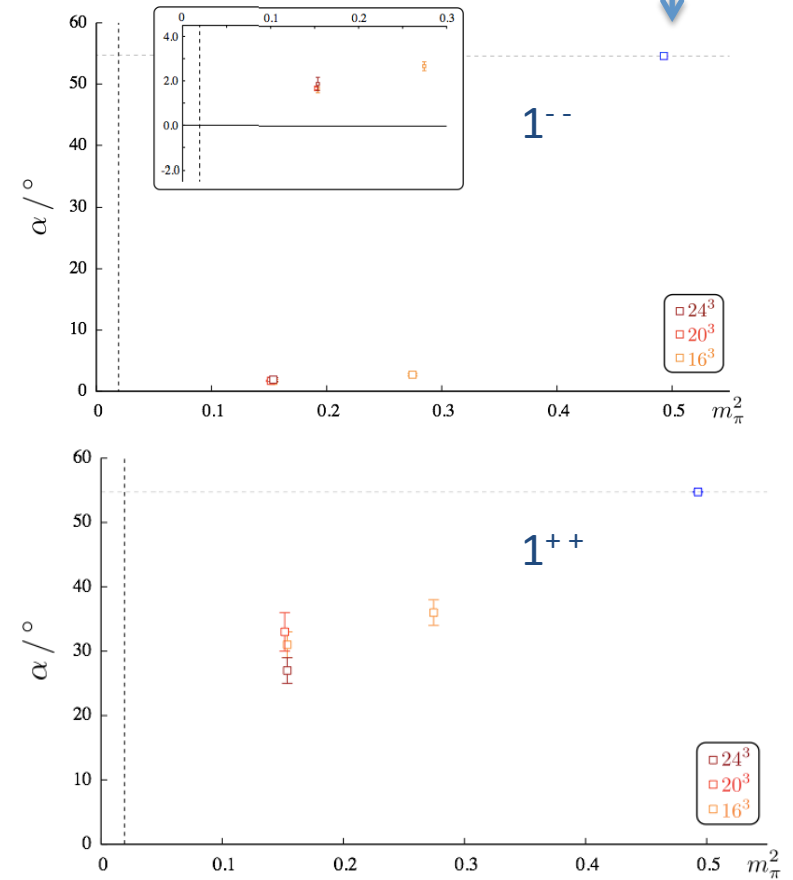
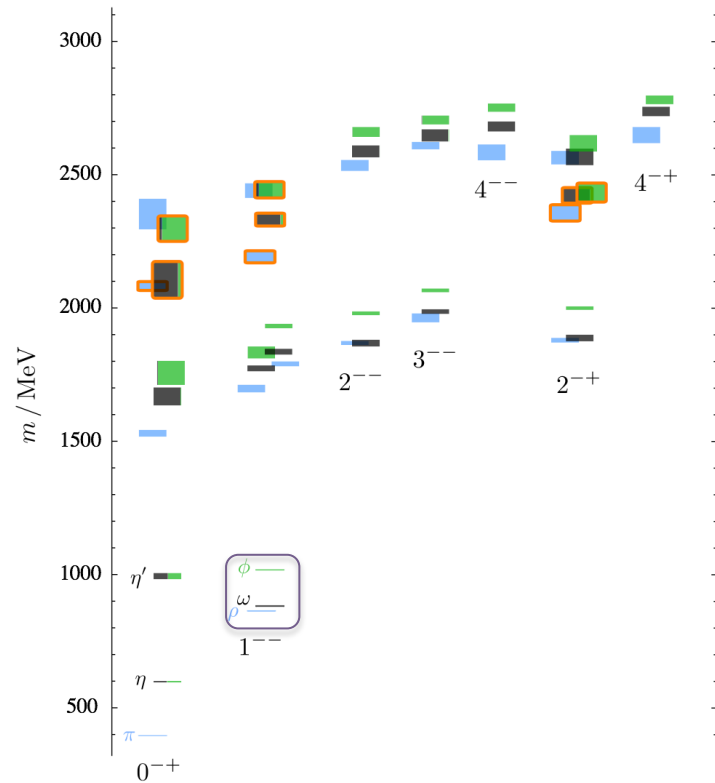
$$\begin{pmatrix} |a\rangle \\ |b\rangle \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} |\ell\rangle \\ |s\rangle \end{pmatrix}$$

$$|\ell\rangle \equiv \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$$

$$|s\rangle \equiv |s\bar{s}\rangle$$

[HSC : Dudek, Edward, Guo, Thomas: 1309.2608, PRD]

Isoscalar mesons: mixing angle



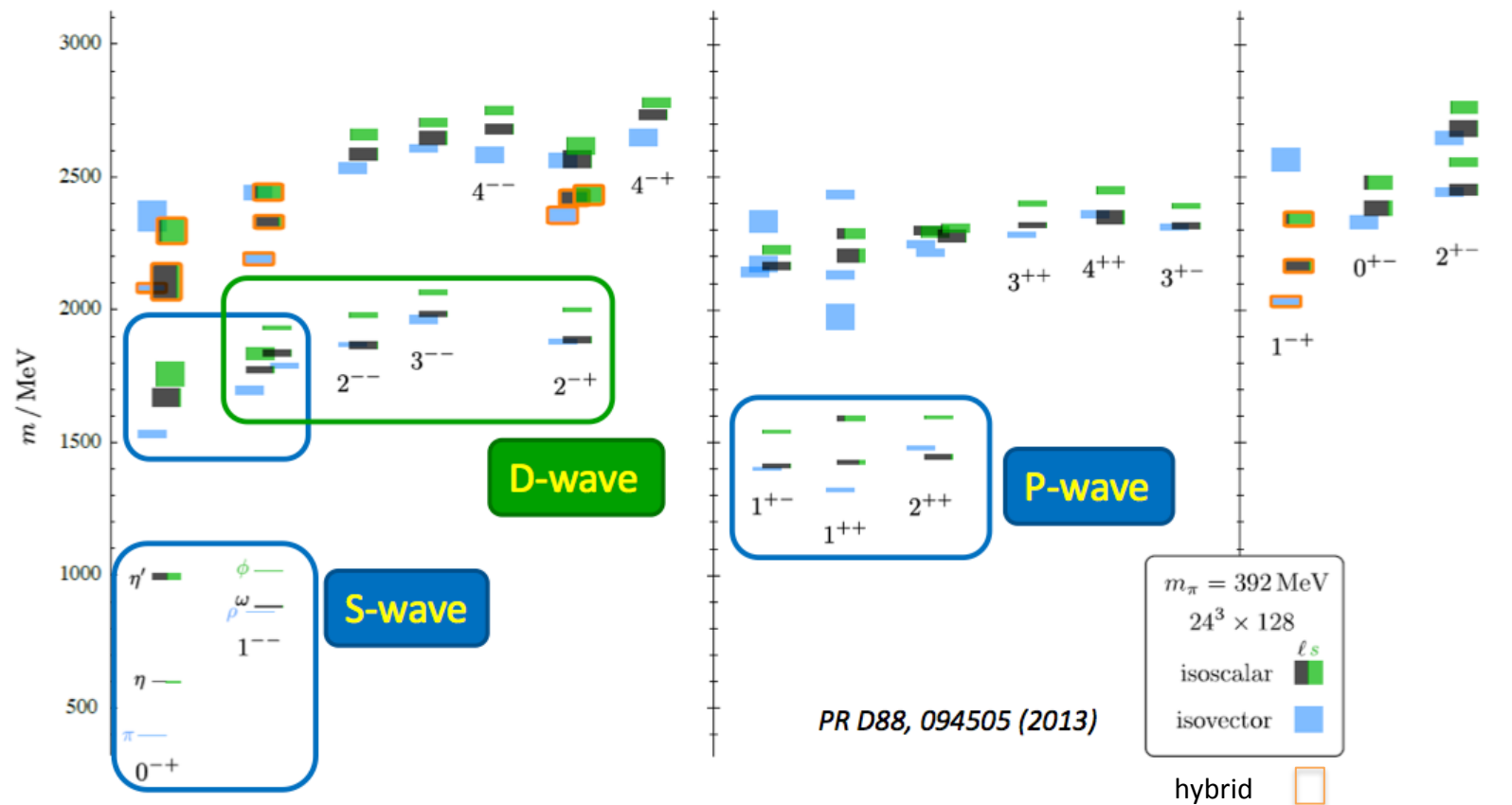
$$\begin{pmatrix} |a\rangle \\ |b\rangle \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} |\ell\rangle \\ |s\rangle \end{pmatrix}$$

$$|\ell\rangle \equiv \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$$

$$|s\rangle \equiv |s\bar{s}\rangle$$

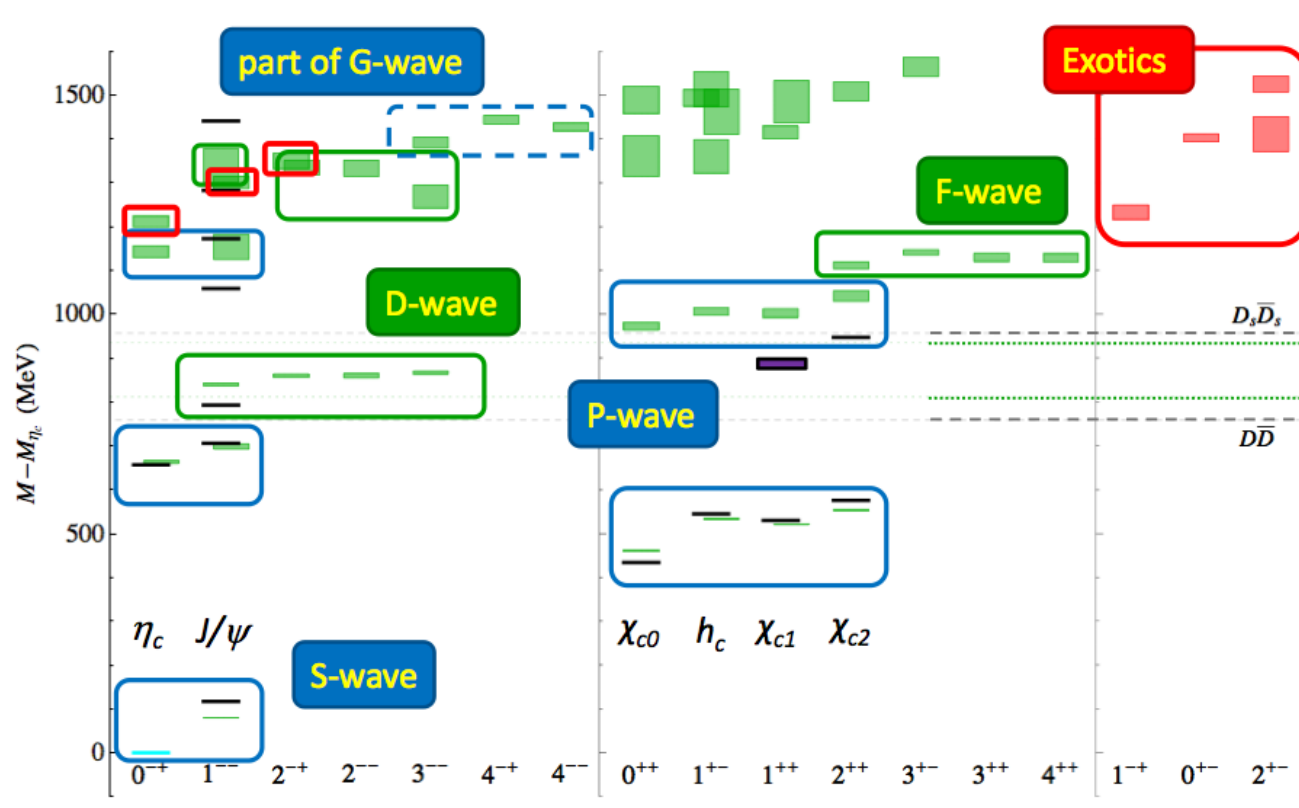
[HSC : Dudek, Edward, Guo, Thomas: 1309.2608, PRD]

Isoscalar mesons: multiplets from overlaps



[HSC : Dudek, Edward, Guo, Thomas: 1309.2608, PRD]

cc spectrum: single hadron approximation



D and D_s mesons:
[G. Moir et al., HSC :
1301.7670, JHEP]

[HSC , L. Liu et al: 1204.5425, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- reliable J^{PC} determination
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:

some of them have exotic J^{PC}
large overlap with $O = \underline{q} F_{ij} q$



Beyond single hadron approximation

- most of the effort in this direction
- one can not expect plots with a number of multiplets soon

States near threshold

Most of interesting states are found near threshold in experiment !

Z_c^+ , Z_b^+ , $X(3872)$, $D_{s0}^*(2317)$, $\Lambda(1405)$

Challenges for the lattice community: quarkonium-like states

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C -parity is given for the neutral members of the corresponding isotriplets.

State	M , MeV	Γ , MeV	J^{PC}	Process (mode)	Experiment ($\#\sigma$)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \rightarrow K(\pi^+\pi^-J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \rightarrow (\pi^+\pi^-J/\psi) \dots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \rightarrow (\pi^+\pi^-J/\psi) \dots$	LHCb [997, 998] (np)	2012	Ok
				$B \rightarrow K(\pi^+\pi^-\pi^0J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \rightarrow K(\gamma J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5)	2005	Ok
				$B \rightarrow K(\gamma \psi(2S))$	LHCb [1003] (> 10)		
$Z_c(3885)^+$ $Z_c(3900)^+$	3883.9 ± 4.5 3891.2 ± 3.3	25 ± 12 40 ± 8	1^{+-} $?^{? -}$	$B \rightarrow K(D\bar{D}^*)$	BaBar [1002] (3.6), Belle [1001] (0.2)	2008	NC!
				$Y(4260) \rightarrow \pi^-(D\bar{D}^*)^+$	LHCb [1003] (4.4)		
				$Y(4260) \rightarrow \pi^-(\pi^+J/\psi)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
					BES III [1006] (np)	2013	NC!
					BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
					T. Xiao <i>et al.</i> [CLEO data] [1009] (>5)		
$Z_c(4020)^+$ $Z_c(4025)^+$ $Z_b(10610)^+$	4022.9 ± 2.8 4026.3 ± 4.5 10607.2 ± 2.0	7.9 ± 3.7 24.8 ± 9.5 18.4 ± 2.4	$?^{? -}$ $?^{? -}$ 1^{+-}	$Y(4260, 4360) \rightarrow \pi^-(\pi^+h_c)$	BES III [1010] (8.9)	2013	NC!
				$Y(4260) \rightarrow \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
				$\Upsilon(10860) \rightarrow \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \rightarrow \pi^-(\pi^+h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \rightarrow \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
				$\Upsilon(10860) \rightarrow \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \rightarrow \pi^-(\pi^+h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \rightarrow \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!

QCD and strongly coupled gauge theories: challenges and perspectives

[review: Brambilla et al., 1404.3723]

N. Brambilla^{†,1} S. Eidelman^{†,2,3} P. Foka^{†,4} S. Gardner^{†,5} A.S. Kronfeld^{†,6}
M.G. Alford^{†,7} R. Alkofer^{†,8} M. Butenschön^{†,9} T.D. Cohen^{†,10} J. Erdmenger^{†,11} L. Fabbietti^{†,12}
M. Faber^{†,13} J.L. Goity^{†,14,15} B. Ketzer^{†,16} H.W. Lin^{†,17} F.J. Llanes-Estrada^{†,18}
H.B. Meyer^{†,19} P. Pakhlov^{†,19,20} E. Pallante^{†,21} M.I. Polikarpov^{†,19,20} H. Satzjian^{†,22}
A. Schmitt^{†,23} W.M. Snow^{†,24} A. Vairo^{†,1} R. Vogt^{†,25,26} A. Vuorinen^{†,27} H. Wittig^{†,18}
P. Arnold^{†,28} P. Christakoglou^{†,29} P. Di Nezza^{†,30} Z. Fodor^{†,31,32,33} X. Garcia i Tormo^{†,34} R. Höllwieser^{†,13}
M.A. Janik^{†,35} A. Kalweit^{†,36} D. Keane^{†,37} E. Kiritsis^{†,38,39,40} A. Mischke^{†,41} R. Mizuk^{†,19,42}
G. Odyniec^{†,43} K. Papadodimas^{†,21} A. Pich^{†,44} R. Pittau^{†,45} J.-W. Qiu^{†,46,47} G. Ricciardi^{†,48,49}
C.A. Salgado^{†,50} K. Schwenzer^{†,7} N.G. Stefanis^{†,51} G.M. von Hippel^{†,18} and V.I. Zakharov^{†,19}

More challenges: quarkonium-*like* states above threshold

TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C -parity is given for the neutral members of the corresponding is triplets.

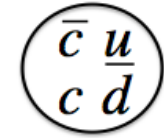
State	M , MeV	Γ , MeV	J^{PC}	Process (mode)	Experiment ($\# \sigma$)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \rightarrow K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \rightarrow e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \rightarrow e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \rightarrow J/\psi(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \rightarrow (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \rightarrow (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \rightarrow K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \rightarrow K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9), LHCb [1062] (1.4), CMS [1063] (>5)	2009	NC!
					D0 [1064] (3.1)		
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \rightarrow (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \rightarrow (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \rightarrow J/\psi(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \rightarrow K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \rightarrow K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \rightarrow (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11)	2005	Ok
					Belle [1008, 1056] (15), BES III [1007] (np)		
				$e^+e^- \rightarrow (f_0(980)J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \rightarrow (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \rightarrow (\gamma X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \rightarrow K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0), CMS [1063] (>3), D0 [1064] (np)	2011	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \rightarrow e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^-\psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \rightarrow K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4)	2007	Ok
					LHCb [1077] (13.9)		
				$\bar{B}^0 \rightarrow K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \rightarrow (\Lambda_c^+ \bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^-\psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \rightarrow (B^{(*)}_s \bar{B}^{(*)}_s(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \rightarrow (\pi\pi\Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \rightarrow (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \rightarrow (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \rightarrow (\eta\Upsilon(1S, 2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \rightarrow (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

All these believed
NOT to be QQ !

[review:
Brambilla et al.,
1404.3723]

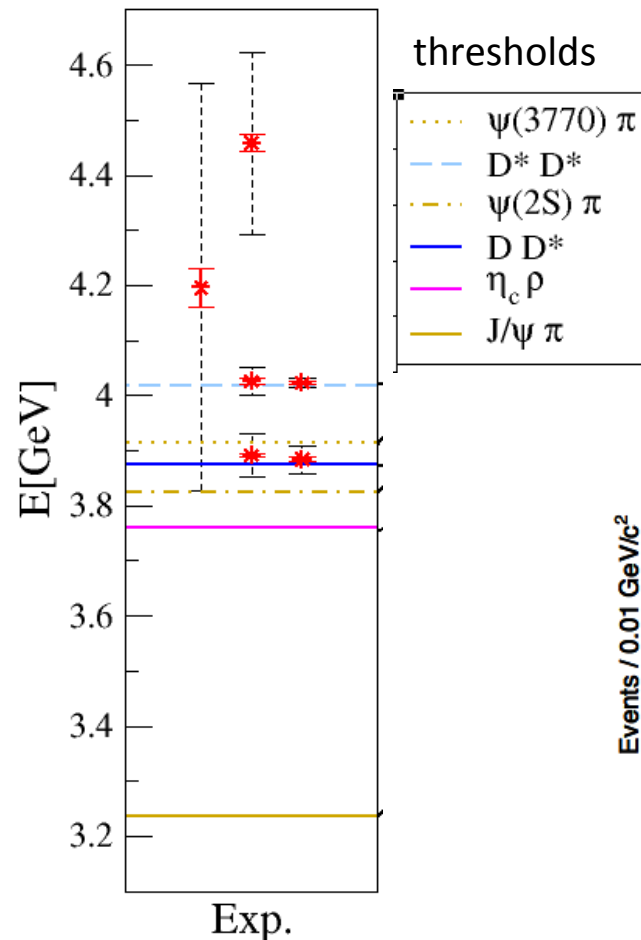
Charged charmonium-like Z_c^+ (manifestly exotic)

Charged charmonium Z_c^+ : experimental status

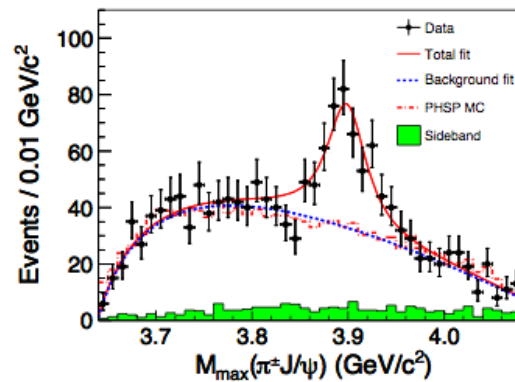


[review: Brambilla et al., 1404.3723]

candidates with
preferred
 $|G=1^+, J^{PC}=1^{+-}$
(C is for neutral partner)

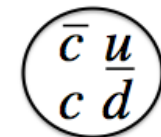


particle	C	J^P	decay	year	coll
$Z^+(4430)$	-	1^+	$\psi(2S) \pi^+$	2008	Belle, BABAR, LHCb
$Z_c^+(3900)$	-	?	$J/\psi \pi^+$	2013	BESIII, Belle, CLEOc
$Z_c^+(3885)$	-	1^+	$(DD^*)^+$	2013	BESIII
$Z_c^+(4020)$	-	?	$h_c(1P) \pi^+$	2013	BESIII
$Z_c^+(4025)$	-	?	$(D^* D^*)^+$	2013	BES III
$Z^+(4200)$	-	1^+	$J/\psi \pi^+$	2014	Belle
$Z^+(4050)$	+	?	$\chi_{c1} \pi^+$	2008	Belle
$Z^+(4250)$	+	?	$\chi_{c1} \pi^+$	2008	Belle



[BESIII, 2013, 1303.5949, PRL]

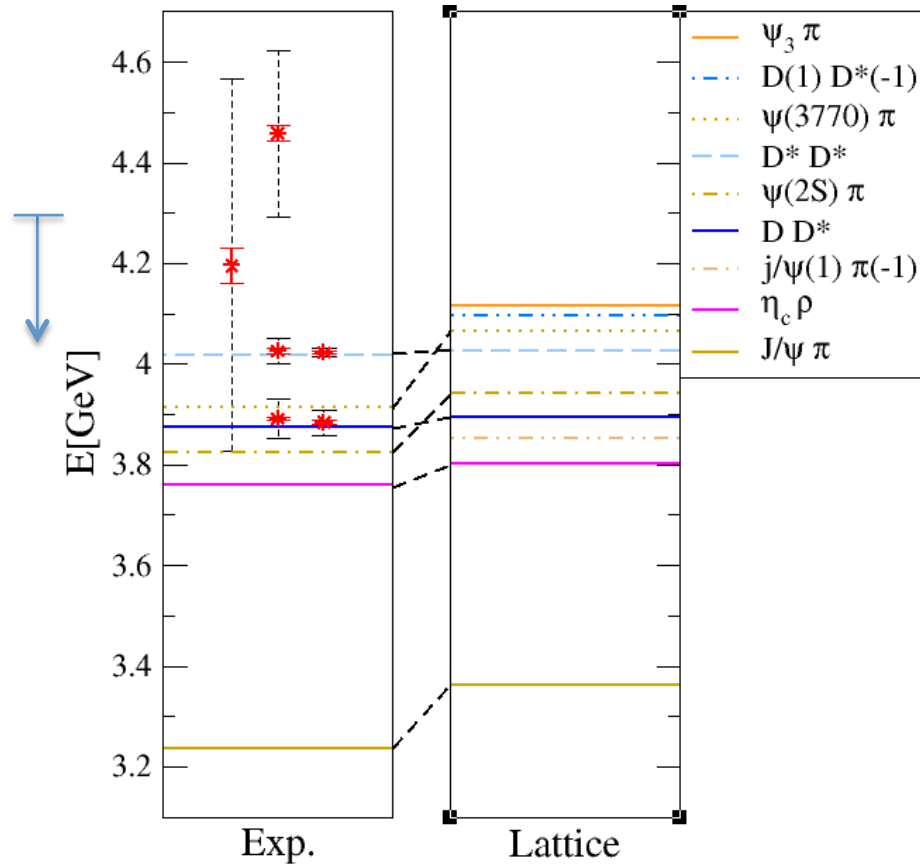
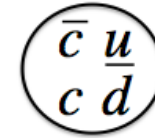
$$Z_c^+(3900) \rightarrow J/\psi \pi^+ \quad \bar{c} c \quad u \bar{d}$$



Question for our community:

Does QCD support existence of such states?

Towards evidence for Z_c^+ from lattice: $I^G=1^+, J^{PC}=1^{+-}$



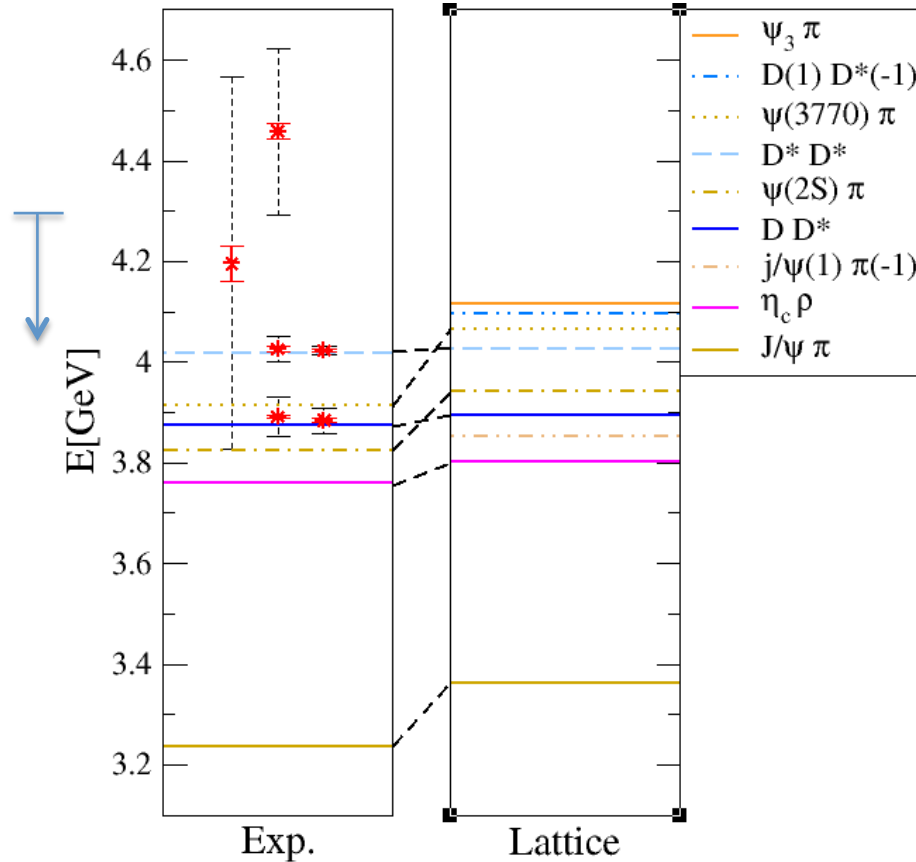
- search for Z_c^+ with $E < 4.3$ GeV
- horizontal lines correspond to energies of all two-particle states with $E < 4.3$ GeV on this lattice
- There would be many more two-particle states for larger L !
- 9 two-particle states are expected

[S.P., Lang, Leskovec, Mohler, 1405.7623]

Wilson Clover, $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$

Towards evidence for Z_c^+ from lattice: $I^G=1^+, J^{PC}=1^{+-}$

$$\begin{pmatrix} \bar{c} & u \\ c & \bar{d} \end{pmatrix}$$



Meson-meson interpolators:

$$\mathcal{O}_1^{\psi(0)\pi(0)} = \bar{c}\gamma_i c(0) \bar{d}\gamma_5 u(0),$$

$$\mathcal{O}^{\psi(1)\pi(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k) \bar{d}\gamma_5 u(-e_k),$$

$$\mathcal{O}^{\eta_c(0)\rho(0)} = \bar{c}\gamma_5 c(0) \bar{d}\gamma_i u(0),$$

$$\mathcal{O}_1^{D(0)D^{*(0)}} = \bar{c}\gamma_5 u(0) \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\},$$

$$\mathcal{O}^{D^{*(0)}D^{*(0)}} = \epsilon_{ijk} \bar{c}\gamma_j u(0) \bar{d}\gamma_k c(0),$$

and 9 others ..

Diquark antidiquark interpolators

(expected to couple particularly well to exotic state but couple also to two-meson st.):

$$\mathcal{O}_1^{4q} \approx [\bar{c} C \gamma_5 \bar{d}]_{3_c} [c \gamma_i C u]_{\bar{3}_c}$$

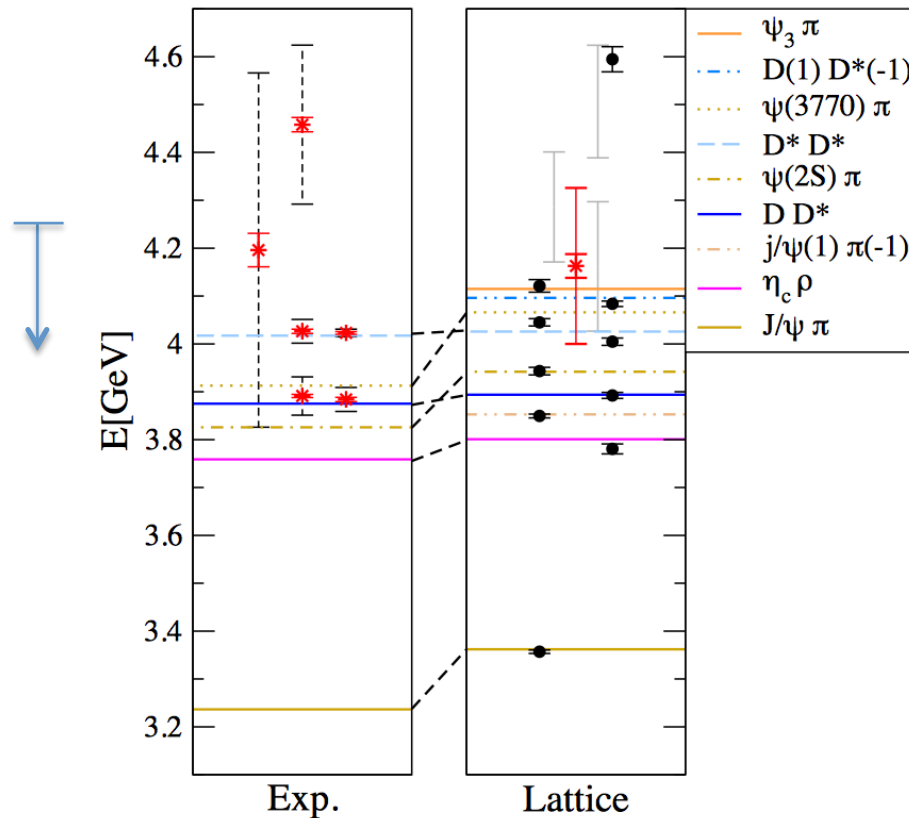
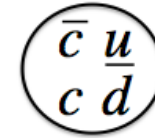
$$\mathcal{O}_2^{4q} \approx [\bar{c} C \bar{d}]_{3_c} [c \gamma_i \gamma_5 C u]_{\bar{3}_c}$$

and 2 others ..

[S.P., Lang, Leskovec, Mohler, 1405.7623]

Wilson Clover, $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$

Evidence for Z_c^+ from lattice: $I^G=1^+, J^{PC}=1^{+-}$



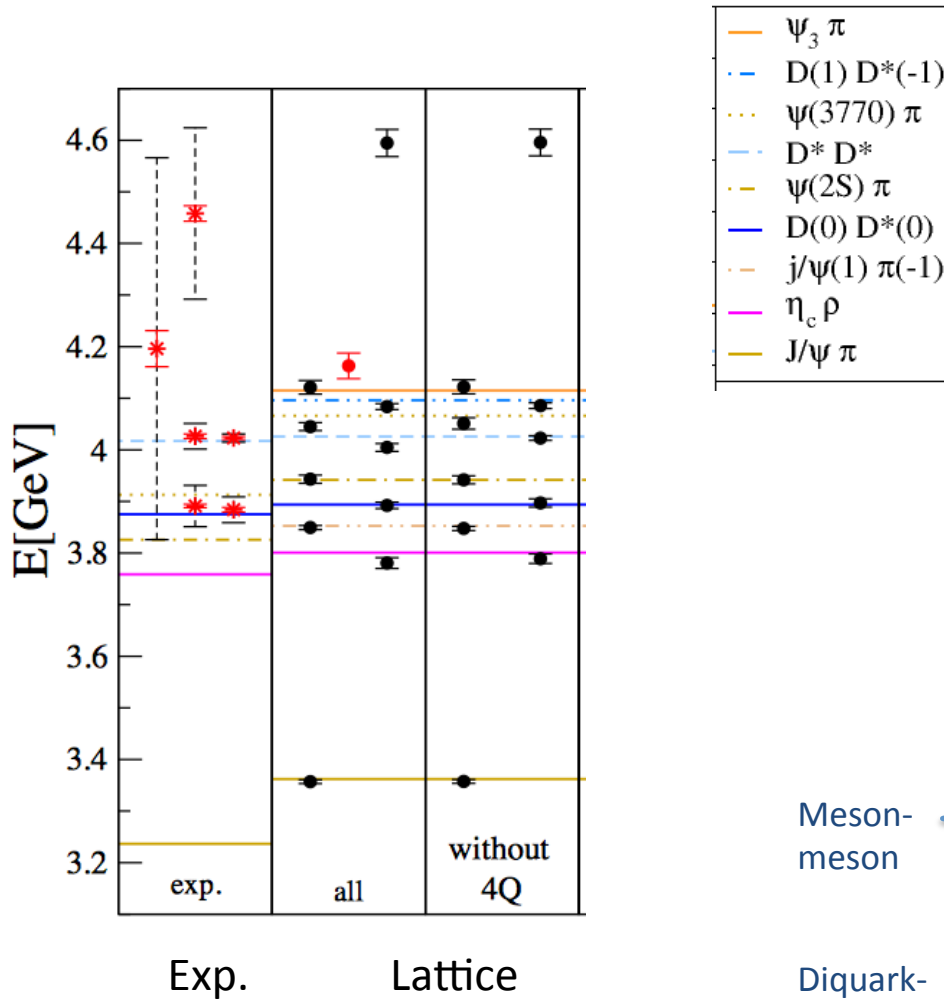
- Black circles: two-meson states
- Red asterix: candidate for Z_c^+
(the smaller error is statistical, the larger corresponds to systematics)
- 9 two meson states below 4.3 GeV
- an additional state found
- since we exhausted all two meson-states below 4.3 GeV, it is a candidate for an exotic Z_c^+ .

[S.P., Lang, Leskovec, Mohler, 1405.7623]

$m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$

Additional supporting evidence for Z_c^+

$$\begin{pmatrix} \bar{c} & u \\ c & d \end{pmatrix}$$



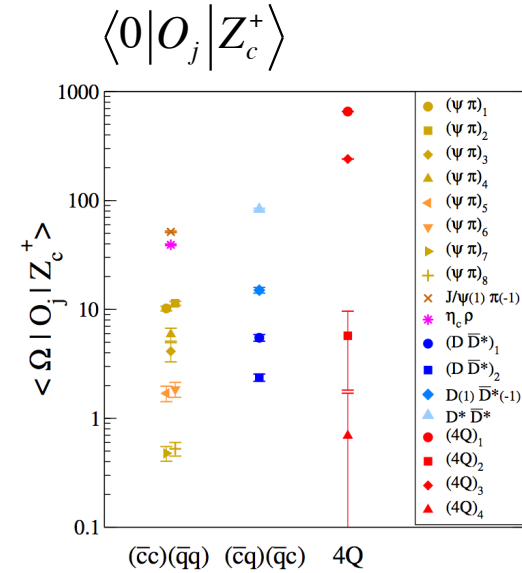
[S.P., Lang, Leskovec, Mohler, 1405.7623]

Meson-

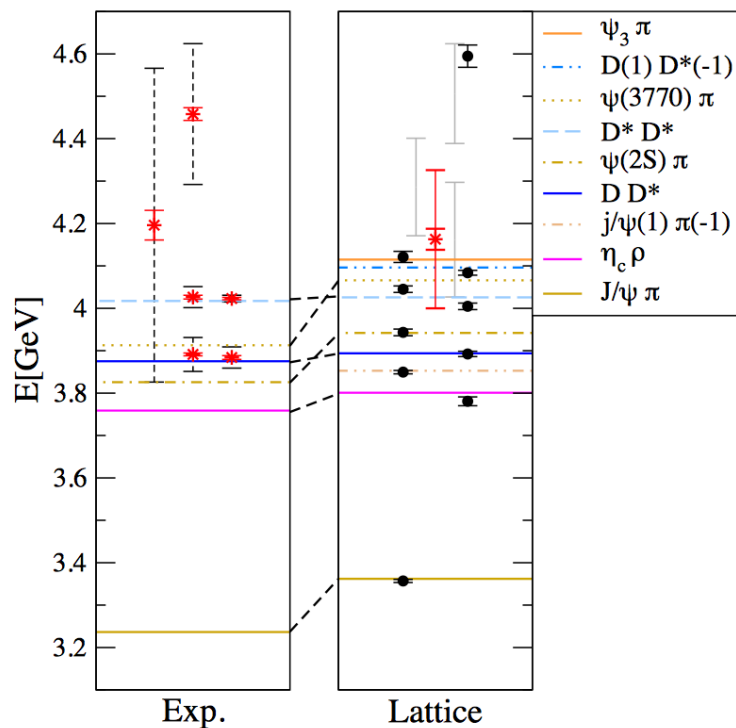
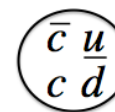
Diquark-
antidiquark
4Q

$$\begin{aligned} \mathcal{O}_1^{\psi(0)\pi(0)} &= \bar{c}\gamma_i c(0) \bar{d}\gamma_5 u(0), \\ \mathcal{O}_1^{\psi(1)\pi(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k) \bar{d}\gamma_5 u(-e_k), \\ \mathcal{O}^{\eta_c(0)\rho(0)} &= \bar{c}\gamma_5 c(0) \bar{d}\gamma_i u(0), \\ \mathcal{O}_1^{D(0)D^*(0)} &= \bar{c}\gamma_5 u(0) \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\ \mathcal{O}^{D^*(0)D^*(0)} &= \epsilon_{ijk} \bar{c}\gamma_j u(0) \bar{d}\gamma_k c(0), \\ &\text{and 9 others ..} \end{aligned}$$

$$\begin{aligned} \mathcal{O}_1^{4q} &\approx [\bar{c} C \gamma_5 \bar{d}]_{3_c} [c \gamma_i C u]_{\bar{3}_c} \\ \mathcal{O}_2^{4q} &\approx [\bar{c} C \bar{d}]_{3_c} [c \gamma_i \gamma_5 C u]_{\bar{3}_c} \\ &\text{and 2 others ..} \end{aligned}$$



Comparing to experimental Z_c^+ candidates



Luka Leskovec, Friday 15h15

Hadron Spectroscopy

- Challenge: this problem would have to be ideally treated as 6-coupled channels, but rigorous Luscher-type treatment is not realistic in the near future ☹
- Smart ideas for improvement along these welcome!

Nearby experimental candidates:

$Z_c^+(4020)$, $\Gamma = 7.9 \pm 3.7$ MeV **BESIII 2013**

$Z_c^+(4025)$, $\Gamma = 24.8 \pm 9.5$ MeV **BESIII 2013**

$Z_c^+(4200)$, $\Gamma = 370 \pm 110$ MeV **Belle, Moriond 2014**

Lattice ($m_\pi = 266$ MeV, $N_f = 2$) :

$m(Z_c^+) = 4.16$ GeV

± 0.163 GeV $\pm O(\Gamma)$

[S.P., Lang, Leskovec, Mohler, 1405.7623]

Other searches with no Z_c^+ candidate (yet)

(1) Search for $Z_c^+(4430)$ in $D^* \underline{D}_1$ scattering near threshold

3 quenched asymmetric lattices

phase shift extracted with help of asymmetric boxes

[G.Z. Meng et al, CLQCD, 0905.0752, PRD 2009]

(2) First search for $Z_c^+(3900)$

$\psi \pi$ and $D \underline{D}^*$ interpolators, no diquark antidiquark

$m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$

only two-meson states found, no candidate for Z_c^+

[S.P. & L. Leskovec, 1308.2097, Phys. Lett. B]

(3) $\Psi \pi$ and $D \underline{D}^*$ interpolators, no diquark antidiquark

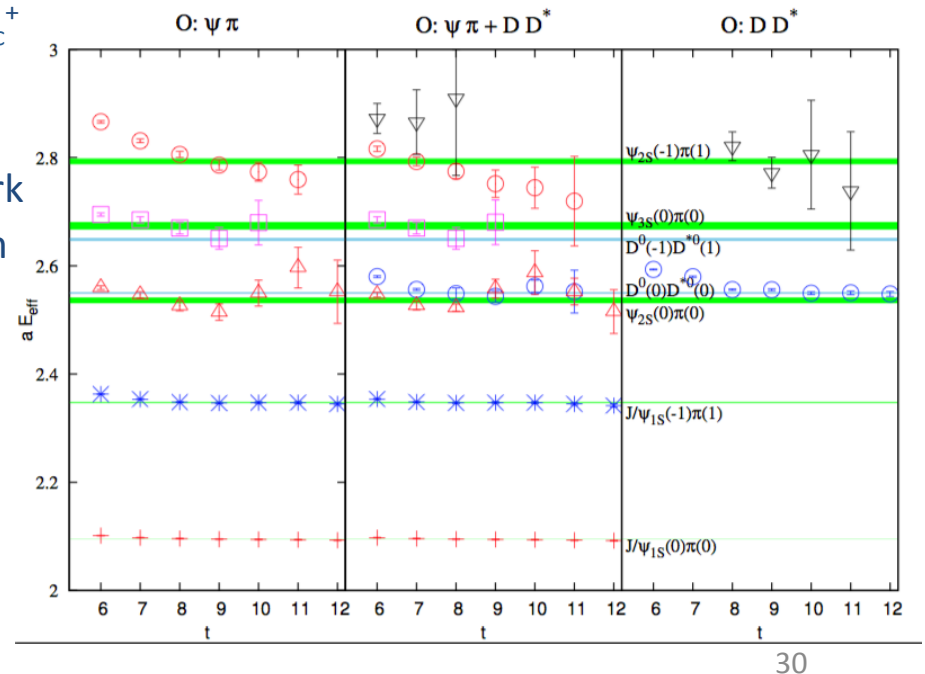
HISQ quarks, $m_u = m_d = m_s/5$, $16^3 \times 48$, $a=0.15$ fm

only two-meson states, no candidate for Z_c^+

[C. DeTar, Song-haeng Lee,

private communication]

C. DeTar, Poster Session

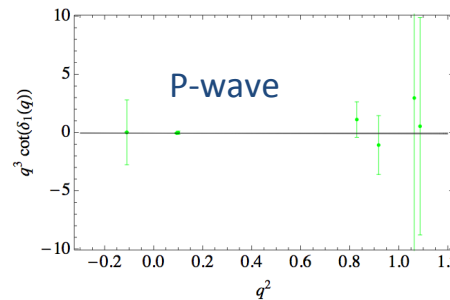
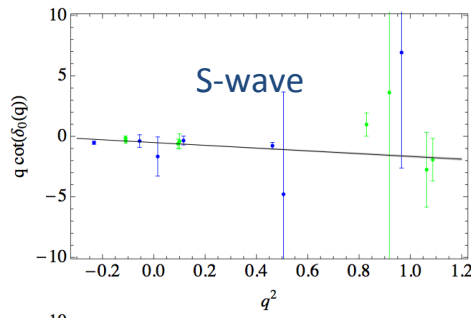


Other searches with no Z_c^+ candidate (yet)

(4) Search for resonance in $D\bar{D}^*$ scattering near threshold $E \sim 3.9$ GeV

- just $D D^*$ interpolators, no $\psi \pi$ interpolators
- twisted mass quarks, $m_\pi = 300, 420, 485$ MeV, $32^3 \times 64$
- partially twisted BC for u, d (not for c) and take care about s - p mixing when present
- the authors conclude that no Z_c^+ candidate is found near $D\bar{D}^*$ threshold

[Y. Chen et al, 1403.1318, CLQCD coll, Phys. Rev. D] L. Liu: Parallel, Hadron Spectrum, Friday, 14h55

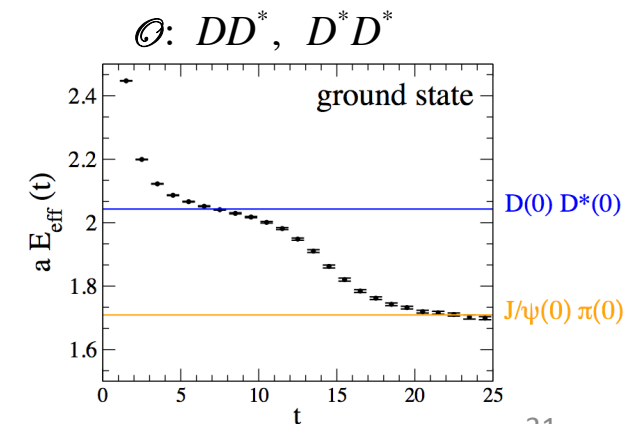


$$p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

	$\mu = 0.003$	$\mu = 0.006$	$\mu = 0.008$
a_0 [fm]	-0.67(1)	-2.1(1)	-0.51(7)
r_0 [fm]	-0.78(3)	-0.27(7)	0.82(27)

$m_\pi = 300$ MeV

- Cautionary remark and lesson based on experience from [S.P., Lang, Leskovec, Mohler, 1405.7623]
 - conclusions based on $D D^*$ interpolators may not be reliable
 - m_{eff} is dropping down to the true ground state $\psi \pi$
 - $\psi \pi$ interpolators (and probably some others) needed

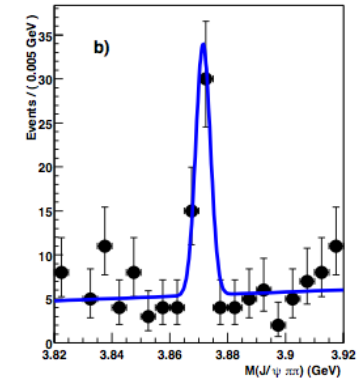


X(3872) , $J^{PC}=1^{++}$, charmonium-like

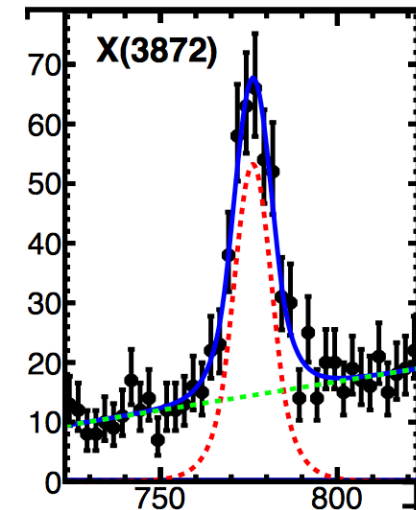
- First charmonium-like state discovered [Belle, PRL, 2003]
- sits within 1 MeV of $D^0\bar{D}^{0*}$ threshold
8 MeV below D^+D^{*-} threshold } isospin breaking effects may be important
- believed to have a large molecular $D^0\bar{D}^{0*}$ Fock component
- $\Gamma < 1.2$ MeV
- decays to $I=0, 1$ equally important

$$X(3872) \rightarrow J/\psi \omega \quad (I=0)$$

$$X(3872) \rightarrow J/\psi \rho \quad (I=1)$$

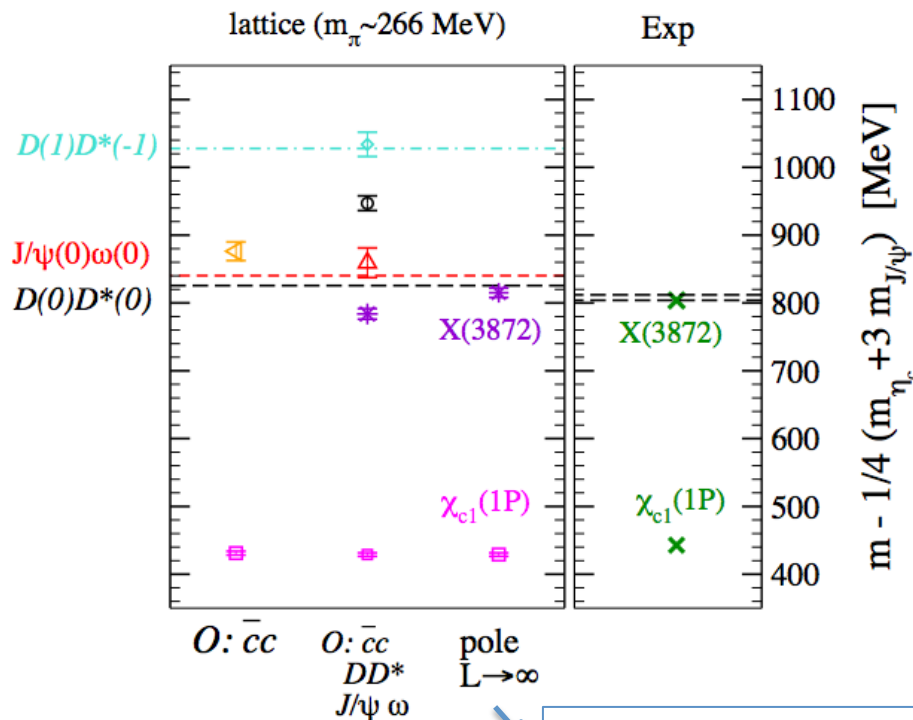


[LHCb, PRL 2013]

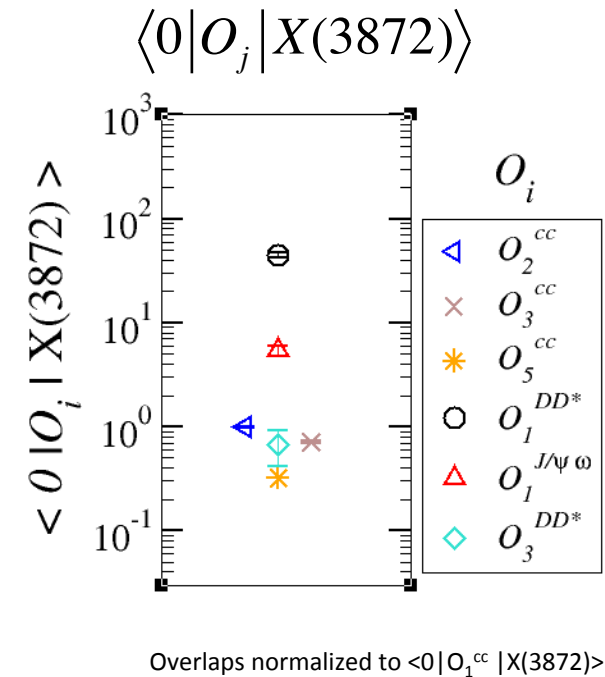


Evidence for X(3872) : $J^{PC}=1^{++}, I=0$

$\mathcal{O} : \bar{c} c, DD^*, J/\psi \omega$



- δ_0 for DD^* extracted using Luscher's rel. and interpolated near threshold
- pole in T-matrix $T \propto [\cot \delta - i]^{-1} = \infty$ found just below DD^* threshold.



X(3872)	$m - (m_{D0} + m_{D0^*})$
lat	-11 ± 7 MeV
exp	-0.14 ± 0.22 MeV

[S.P. and L. Leskovec : 1307.5172, Phys. Rev. Lett.]

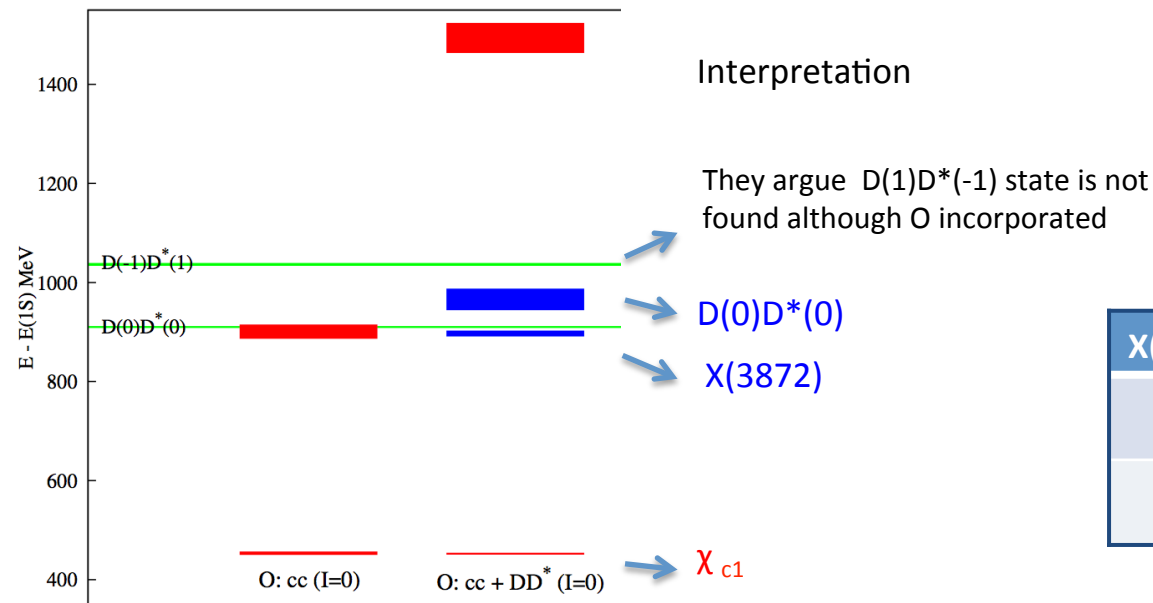
$m_{\pi} \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$

New evidence for X(3872) : $J^{PC}=1^{++}, I=0$

$\mathcal{O}$: $\bar{c} c, DD^*$

HISQ quarks, $m_u = m_d = m_s/5, 16^3 \times 48, a = 0.15$ fm

[C. DeTar, Song-haeng Lee] C. DeTar, Poster Session



X(3872)	$m - (m_{D^0} + m_{D^{0*}})$
lat	-13 ± 6 MeV
exp	-0.14 ± 0.22 MeV

Possible direction to improve on X(3872):

- larger volumes since molecule may be of considerable size
- isospin breaking on the lattice

remember: sits 1 MeV of $D^0\bar{D}^{0*}$ threshold and 8 MeV below D^+D^{*-} threshold, decays to $I=0,1$

Related analytical studies

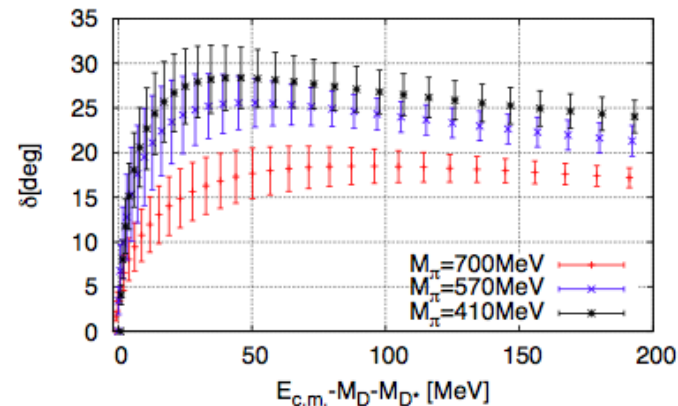
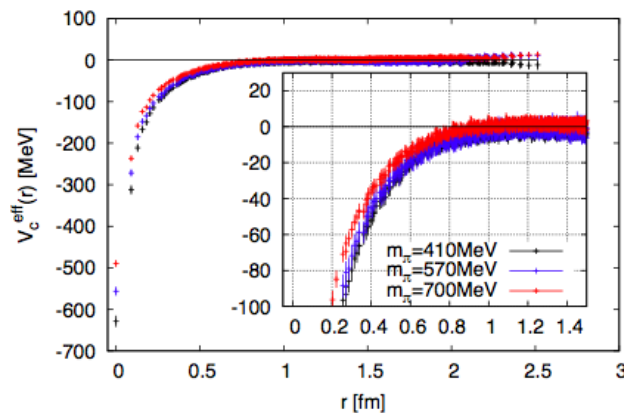
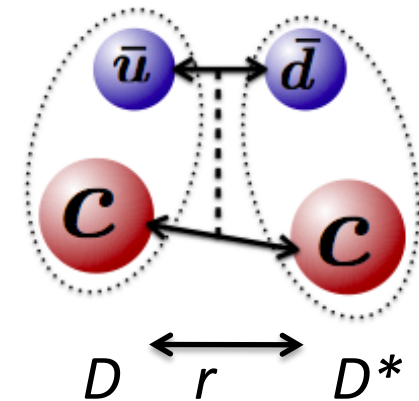
- *Light quark mass dependence of the $X(3872)$ in XEFT*
[M. Jansen, H.-W. Hammer, Yu Jia , 1310.6937, Phys. Rev. D]
- *Strategies for an accurate determination of the $X(3872)$ energy from QCD lattice simulations*
[E. J. Garzon, R. Molina, A. Hosaka, E. Oset, 1310.0972, Phys. Rev. D]
- *Hidden charm molecules in a Finite Volume*
[M. Albaladejo, C. Hidalgo-Duque, J. Nieves, E. Oset, 1312.5339]

Searches for double charm tetraquark with $J^P=1^+, I=0$

(1) HALQCD method [Ishii et al., PLB712, 437 (2012)]

- potential between D and D^* , and corresponding phase shift
- $m_\pi \approx 410$ - 700 MeV, $L \approx 2.9$ fm, $N_f=2+1$
- potential is attractive, no bound tetraquark state found

[Y. Ikeda, HALQCD coll., 1311.6214, Phys. Lett. B 2014]



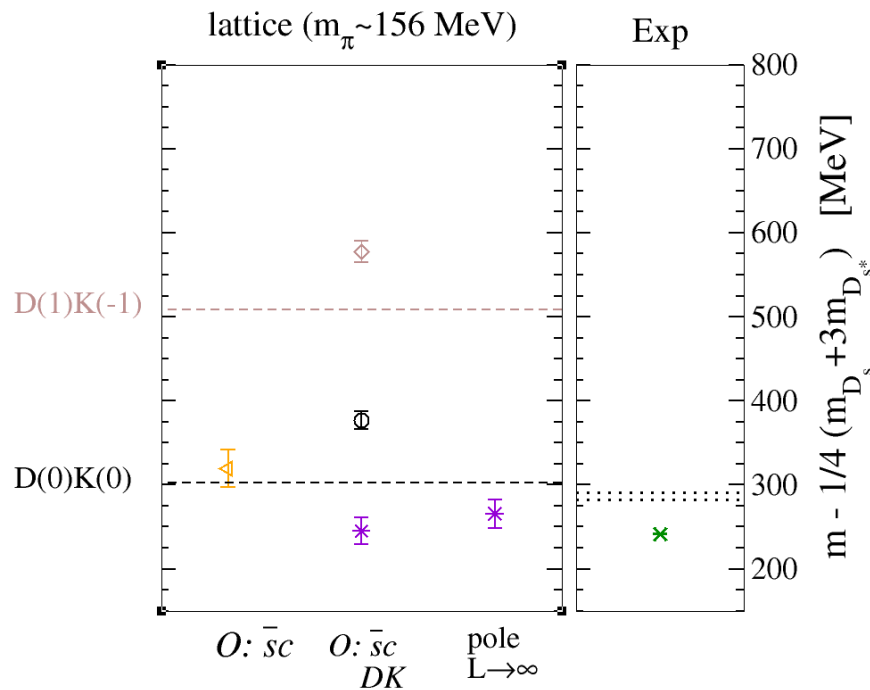
(2) variational method with DD^* , D^*D^* and tetraquark interpolators

preliminary results do not lead yet to the conclusion on existence of these states

Andrea Guerrieri, Wednesday 12h10, Hadron Spectrum

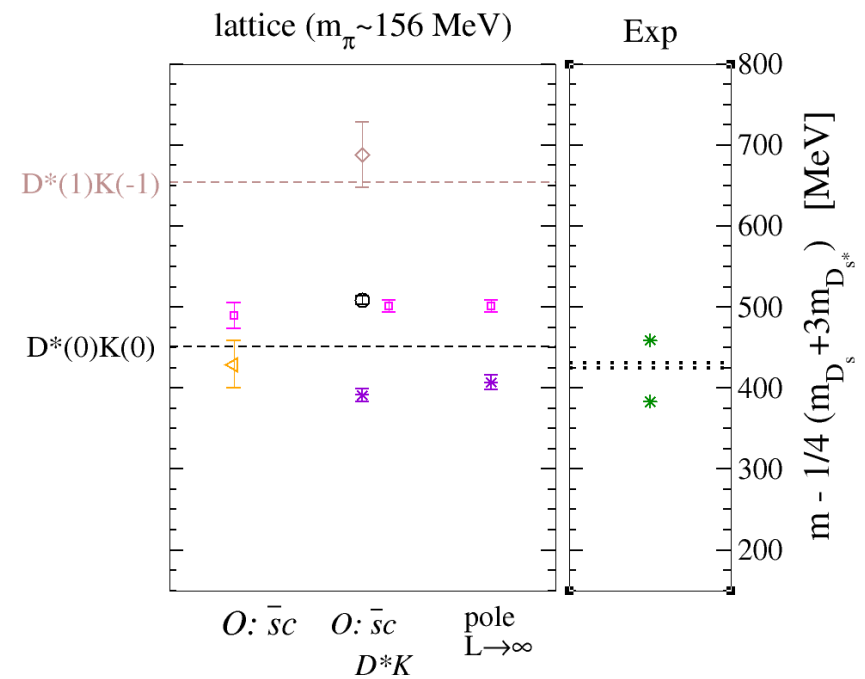
D_s states near DK and D^*K thresholds

DK in s-wave and $D_{s0}^*(2317)$ bound state



[D. Mohler, C. Lang, L. Leskovec, S.P. ,
R. Woloshyn, 1308.3175, Phys. Rev. Lett.]

D^*K s-wave and $D_{s1}(2460)$, $D_{s1}(2536)$



[C. Lang, L. Leskovec, D. Mohler, S.P. ,
R. Woloshyn, 1403.8103]

Wilson Clover, $m_\pi \approx 156$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$

$$a_0 = -1.33 \pm 0.20 \text{ fm}$$

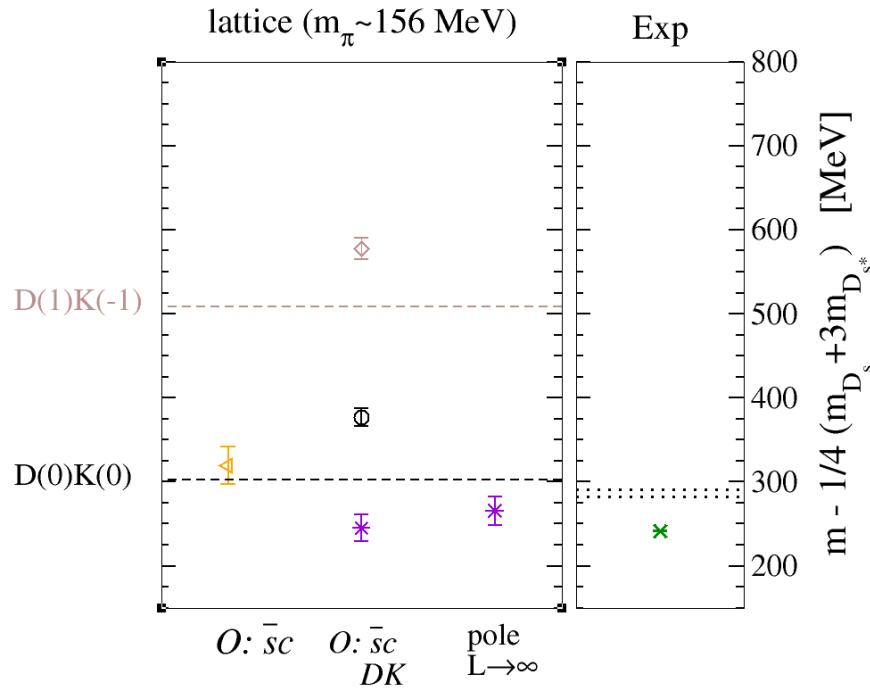
$$r_0 = 0.27 \pm 0.17 \text{ fm}$$

- phase shift for DK or D*K $\tan \delta(p) = \frac{\sqrt{\pi} p L}{2 \mathcal{Z}_{00} \left(1; \left(\frac{pL}{2\pi} \right)^2 \right)}$
- parametrization of phase shift near th. $p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$
- poles in S-matrix correspond to bound states $T \propto \frac{1}{\cot \delta - i} = \infty$

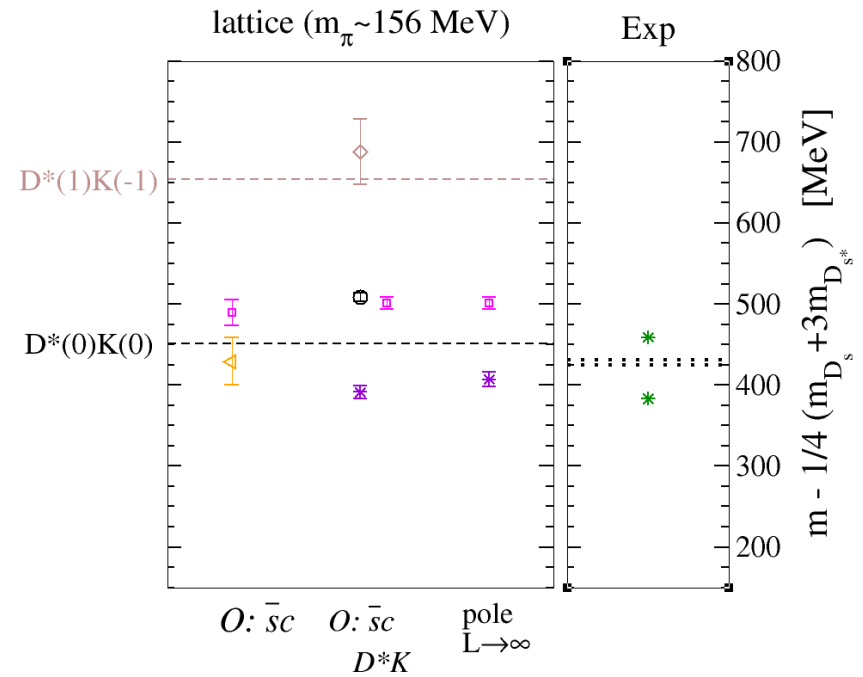
$$a_0 = -1.11 \pm 0.11 \text{ fm}$$

$$r_0 = 0.10 \pm 0.10 \text{ fm}$$

DK s-wave and $D_{s0}^*(2317)$ bound state



D*K s-wave and $D_{s1}(2460)$, $D_{s1}(2536)$



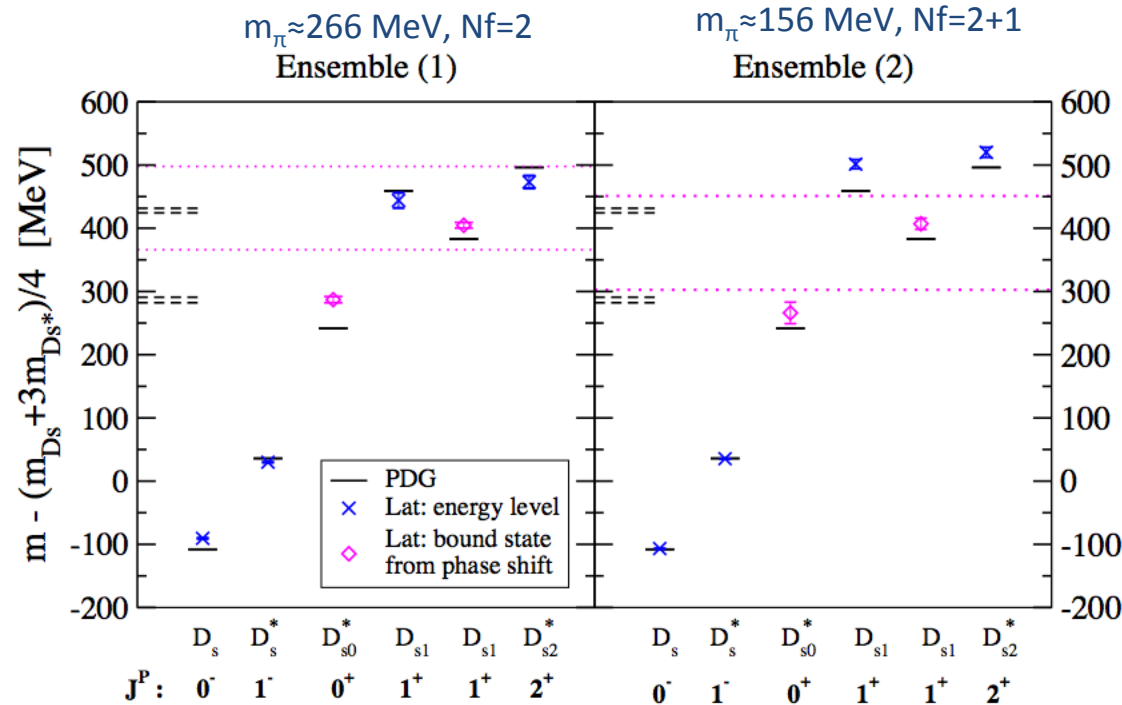
for $m_c = \infty$: $D_{s1}(2536)$ does not couple to s-wave
[Isgur Wise 1991]

[D. Mohler, C. Lang, L. Leskovec, S.P. ,
R. Woloshyn, 1308.3175, Phys. Rev. Lett.]

[C. Lang, L. Leskovec, D. Mohler, S.P. ,
R. Woloshyn, 1403.8103]

$m_\pi \approx 156 \text{ MeV}$, $L \approx 2.9 \text{ fm}$, $N_f = 2+1$

D_s and D spectrum



D_s mesons (near-threshold)

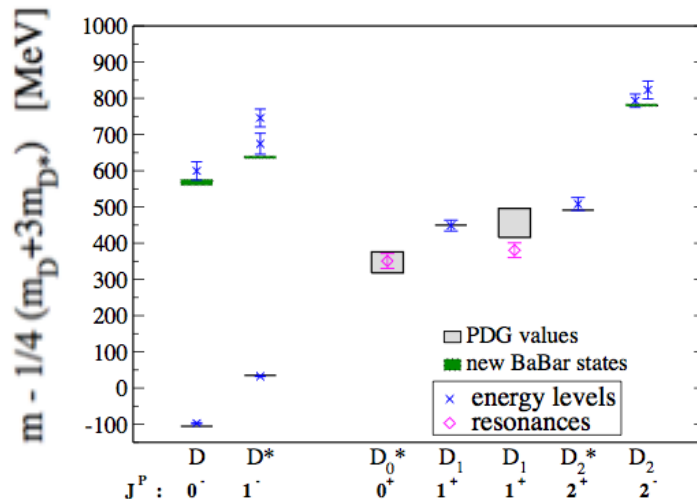
[C. Lang, L. Leskovec, D. Mohler, S.P., R. Woloshyn: PRL 2013, 1403.8103]

these results

C. B. Lang, Monday, 17h50

preliminary results for DK and $D\pi$

S. Ryan, Thursday 16h15



D mesons (resonances)

[D. Mohler, S.P., R. Woloshyn: PRD 2013]

Charmed scalar meson "puzzle" revisited

• why do these scalar partners have mass so close ?

$D_0^*(2400)$: $M \approx 2318$ MeV $\Gamma \approx 267$ MeV $\bar{c}u$ or $\bar{c}u\bar{s}s$?

$D_{s0}^*(2317)$: $M \approx 2318$ MeV $\Gamma \approx 0$ MeV $\bar{c}s$ or $\bar{c}s[\bar{u}u + \bar{d}d]$?

1) is D_0^* mass pushed up : valence $\bar{s}s$ pair ??

✗

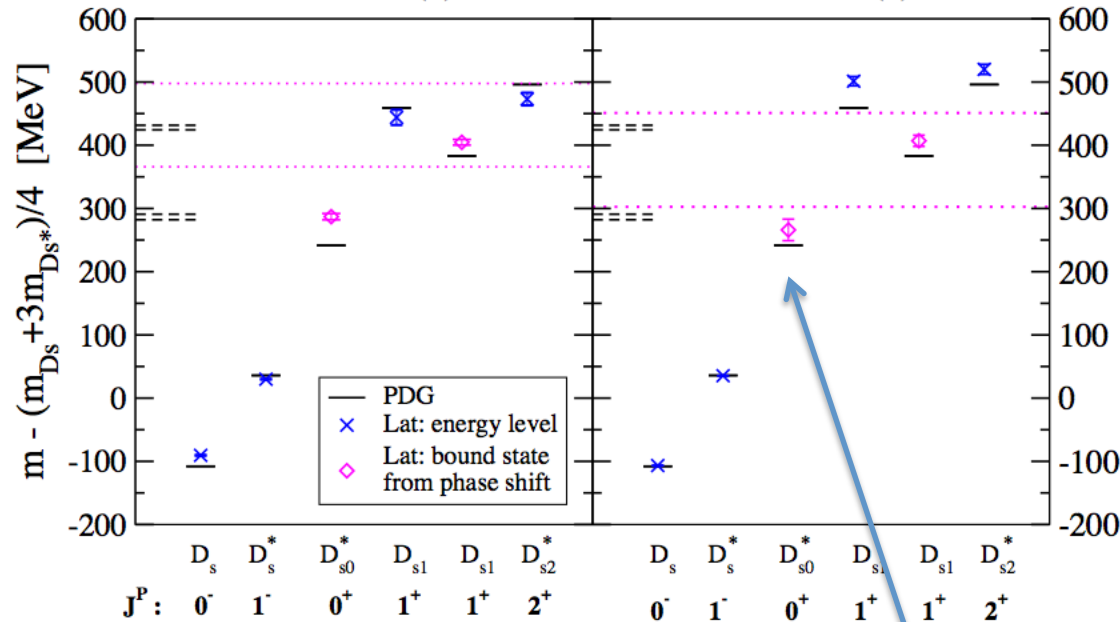
2) is D_{s0}^* mass pushed down : effect of DK threshold ??

✓

D_s and D spectrum

$m_\pi \approx 266$ MeV, $N_f=2$
Ensemble (1)

$m_\pi \approx 156$ MeV, $N_f=2+1$
Ensemble (2)



D_s mesons (near-threshold)

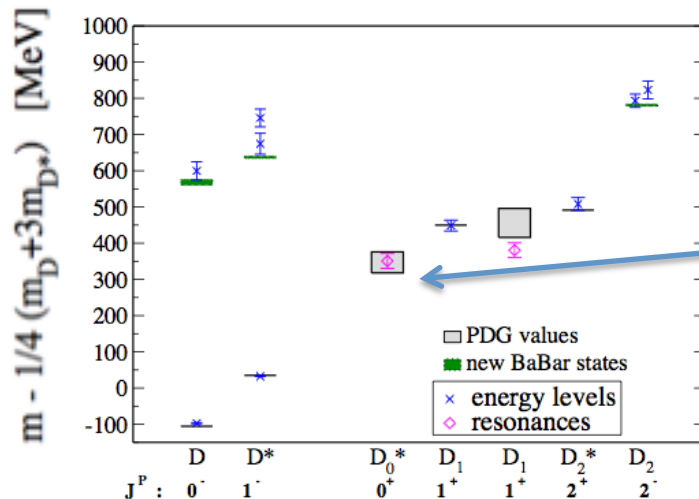
[C. Lang, L. Leskovec, D. Mohler, S.P. ,
R. Woloshyn: PRL 2013, 1403.8103]

these results

C. B. Lang, Monday, 17h50

preliminary results for DK and $D\pi$

S. Ryan, Thursday 16h15



D mesons

(resonances)

[D. Mohler, S.P. ,

R. Woloshyn:

PRD 2013]

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1) is D_0^* mass pushed up : valence $\bar{s}s$ pair ??

✗

2) is D_{s0}^* mass pushed down : effect of DK threshold ??

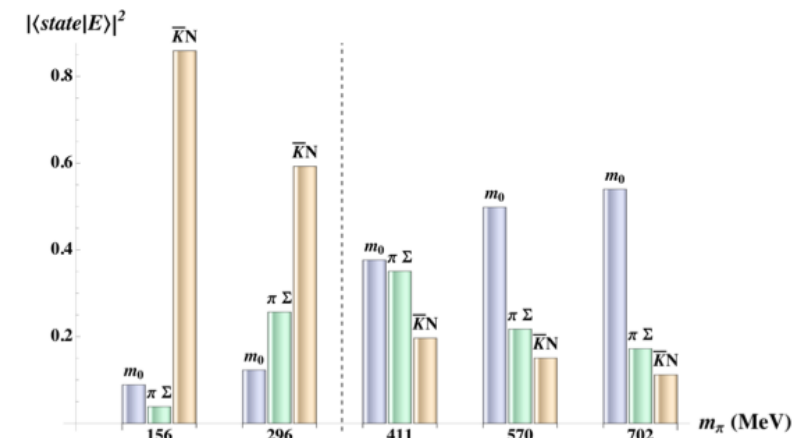
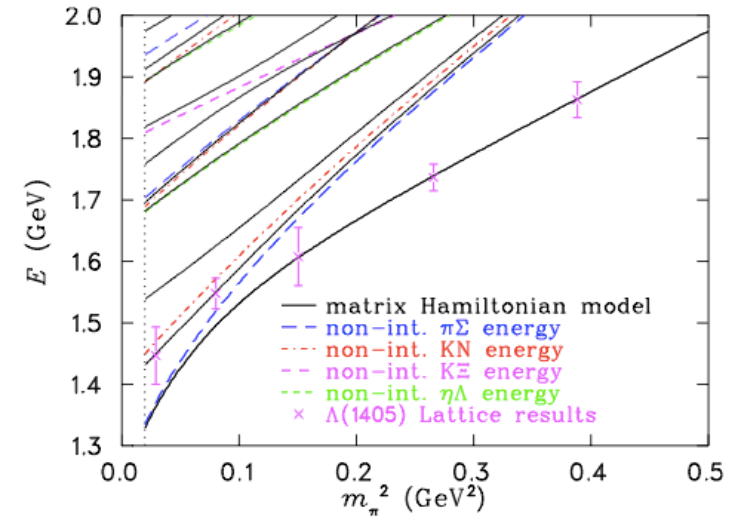
✓

Composition of $\Lambda(1405)$, $J^P = \frac{1}{2}^-$

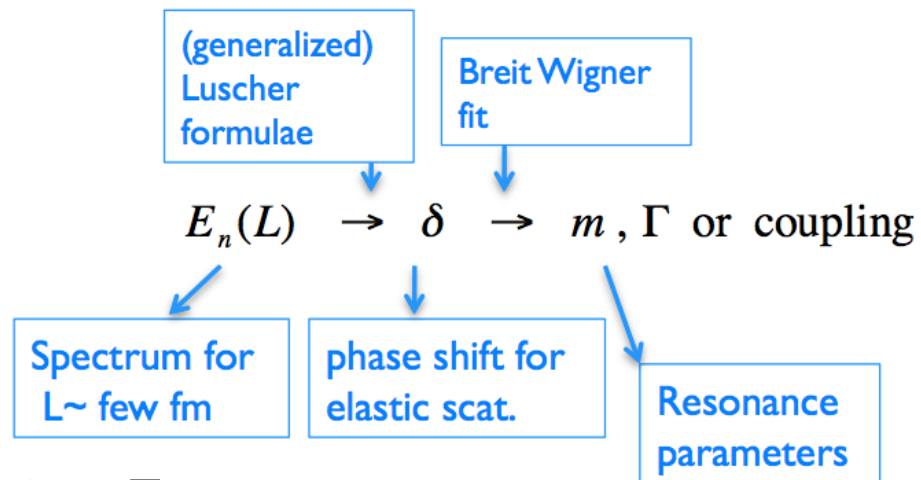
Derek Leinweber, Wednesday, Hadron Spectroscopy

- exp: resonance in $\pi\Sigma$ located below $\bar{K}N$ th.
- ground state energy determined from lat. using $O = uds$ and represented by pink crosses
- ground state E fitted with the eigenvalue of finite volume Hamiltonian EFT and parameters extracted [Hall et al, 1303.4157, PRD]
- Hamiltonian EFT describes interactions between uds , $\pi\Sigma$, $\bar{K}N$, $K\Xi$, $\eta\Lambda$
- composition of eigenstate is extracted from EFT
- authors conclude that $\Lambda(1405)$ is dominated by $\bar{K}N$ at the physical quark mass
- PACS-CS conf, $m_\pi = 150-700$ MeV, $L = 2.9$ fm

Effect of including $N\pi$ interpolators for other channels
 Waseem Kamleh, Wednesday, Hadron Spectrum
 Valentina Verduci, Poster Session

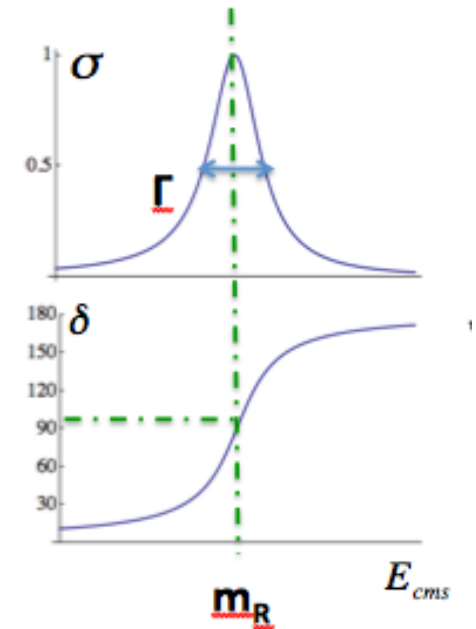
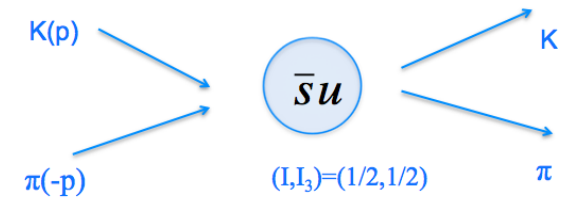


Mesons above threshold – resonances



$$\mathcal{O} = \bar{q}\Gamma q,$$

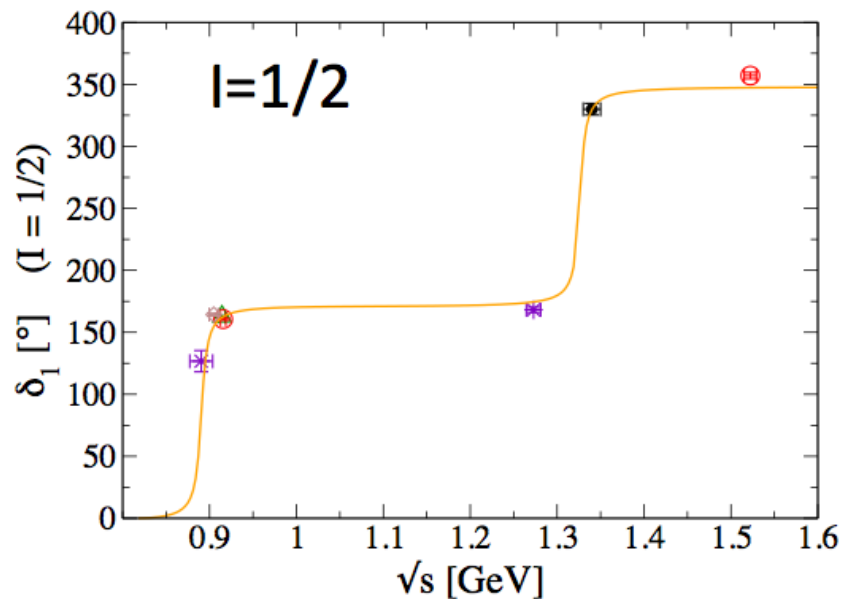
$$(\bar{q}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 q)_{\vec{p}_2} = M_1(\vec{p}_1) M_2(\vec{p}_2)$$



ρ resonance in $\pi\pi$ reviewed by Takeshi Yamazaki, plenary talk

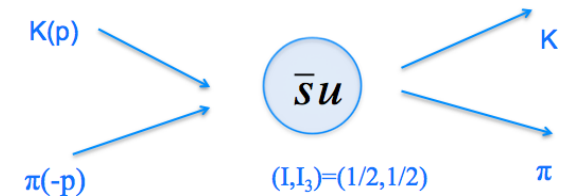
$K^*(892)$ resonance $K\pi$ reviewed by Takeshi Yamazaki, plenary talk

$K\pi$, $I=1/2$: p-wave phase shift



[S.P. ,Lang, Leskovec, Mohler, 1307.0736, PRD 2013]

$m_\pi \approx 266 \text{ MeV}$



$$\text{BW : } \delta = \text{acot} \frac{m_R^2 - E_{cms}^2}{m_R \Gamma}$$

$$\Gamma[K^* \rightarrow K\pi] = \frac{g^2}{6\pi} \frac{p^{*3}}{s}$$

	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6
exp	891.66 ± 0.26	5.72 ± 0.06

Resonances in $K\pi$, $K\eta$ coupled channels

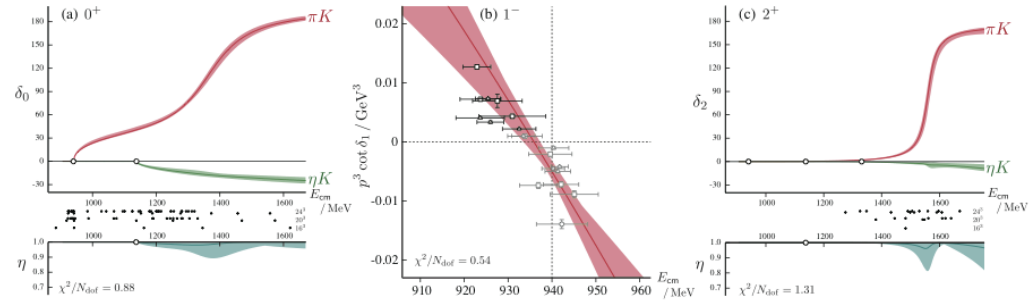
discussed by Yamazaki, Briceno, Wilson at Lat 14

- qq , $K\pi$, $K\eta$ interpolators
- a number of different $0 < P \leq 2$
- for each E_n : one determinant equation for many unknowns
- T-matrix parametrized

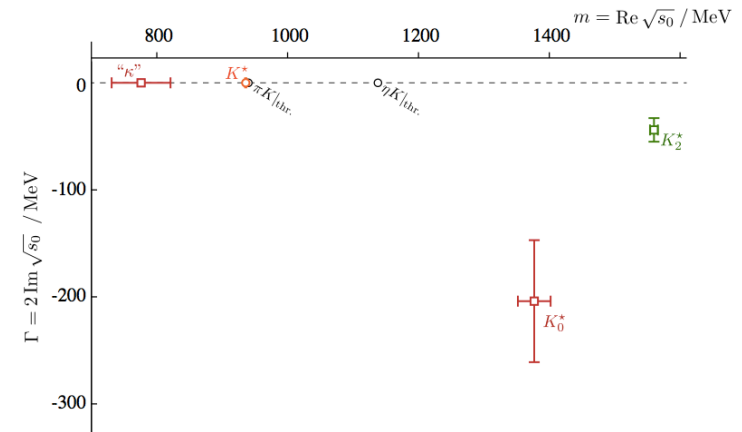
to get around this problem

- the location of poles of T-matrix in complex plane is given below
- $K^*(892)$ and κ are below threshold for this m_π
- K_0^* , K_2^* are resonances
- $m_\pi = 391$ MeV, $N_L = 16, 20, 24$

[Dudek, Edwards, Thomas, Wilson, HSC, 1406.4158]



$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, \quad t_{ij} = \frac{\sqrt{1-\eta^2} e^{i(\delta_i + \delta_j)}}{2\sqrt{\rho_i \rho_j}}$$



$$\det \left[\delta_{ij} \delta_{JJ'} + i \rho_i t_{ij}^{(J)}(E_{\text{cm}}) \left(\delta_{JJ'} + i \mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L) \right) \right] = 0,$$

location of poles in T matrix in complex plane

D-meson resonances in $D\pi$ and $D^*\pi$

discussed by Daniel Mohler, plenary at Lattice 2012

$$\Gamma(E) \equiv g^2 \frac{p}{E^2}$$

g is compared to exp instead of Γ (Γ depends on phase sp. and m_π)

$J^P=0^+ : D \pi$

$D_0^*(2400)$	$m - 1/4(m_D+3m_{D^*})$	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

$J^P=1^+ : D^* \pi$

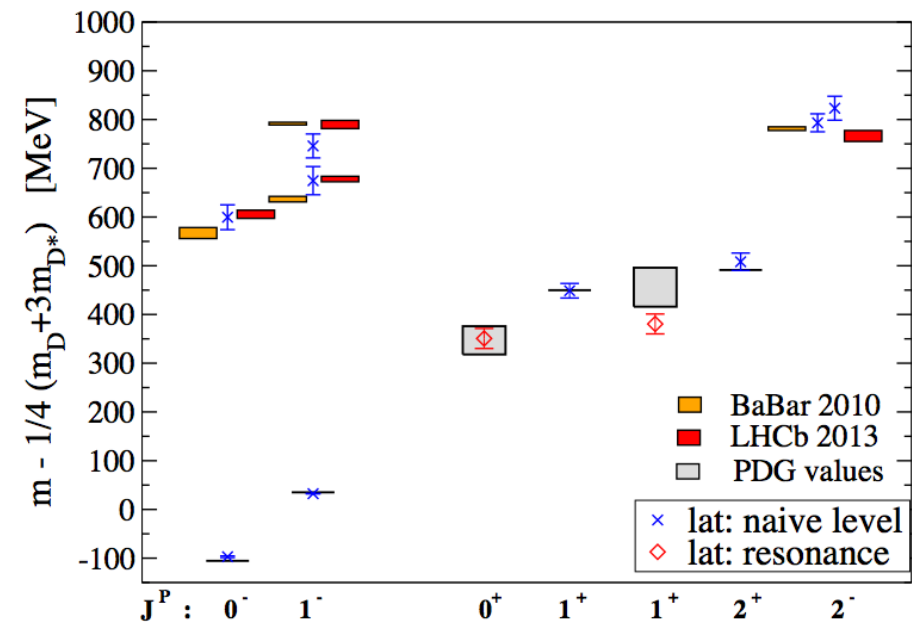
(analysis of spectrum in this case is based on an assumption given in paper below)

$D_1(2430)$	$m - 1/4(m_D+3m_{D^*})$	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

first lattice result for strong decay width of a hadron containing charm quark

[D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD]

- $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$



Lightest axial resonances $a_1(1260)$ and $b_1(1235)$

- Simulating scattering:
 $\rho \pi$ in 1^{++} channel to extract a_1
 $\omega \pi$ in 1^{+-} channel to extract b_1
- $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$, $P=0$ [Lang, Leskovec, Mohler, S.P., 1401.2088, JHEP]

resonance	$a_1(1260)$			$b_1(1235)$	
quantity	$m_{a_1}^{\text{res}}$ [GeV]	$g_{a_1 \rho \pi}$ [GeV]	$a_{l=0}^{\rho \pi}$ [fm]	$m_{b_1}^{\text{res}}$ [GeV]	$g_{b_1 \omega \pi}$ [GeV]
lat	1.435(53)($^{+0}_{-109}$)	1.71(39)	0.62(28)	1.414(36)($^{+0}_{-83}$)	input
exp	1.230(40)	1.35(30)	-	1.2295(32)	0.787(25)

$$\Gamma(E) \equiv g^2 \frac{p}{E^2}$$

- ρ and ω assumed to be stable which is a good approximation for given simulation parameters
- going beyond that approximation will be very challenging
- analytical study of a_1 for unstable ρ : [Roca, Oset, 1201.0438]
- analytical studies of 3-particles:
[Hansen, Sharpe 1311.4848; Polejaeva, Rusetsky, 1203.1241; Briceno, Davoudi, 1212.3398]

Isovectors including meson-meson interpolators

- anisotropic, $m_\pi=240, 390$ MeV, $N_L=24,32$
- a number of qq and MM interpolators for a number of u,d,s channels
- stochastic distillation
- results for p channel shown

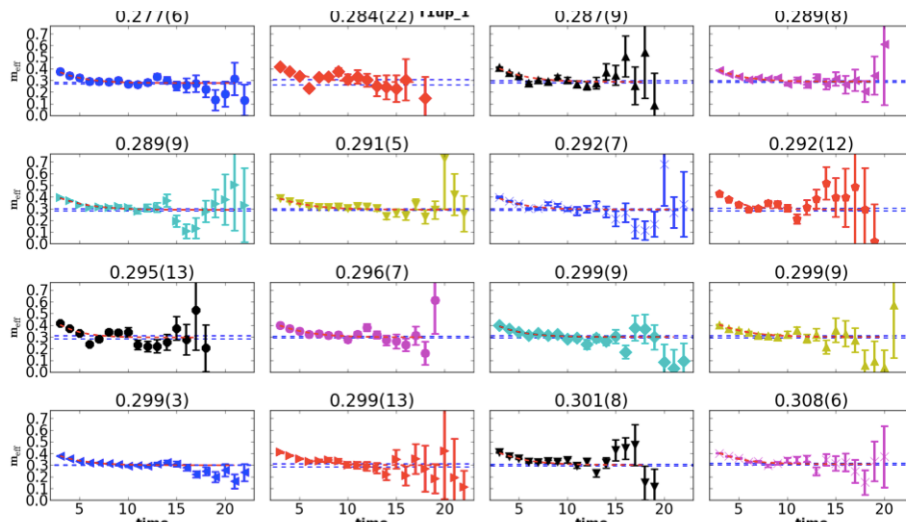
Morningstar, Wednesday, Hadron Spectroscopy

• numbers of operators for $I = 1, S = 0, P = (0, 0, 0)$ on 24^3 lattice

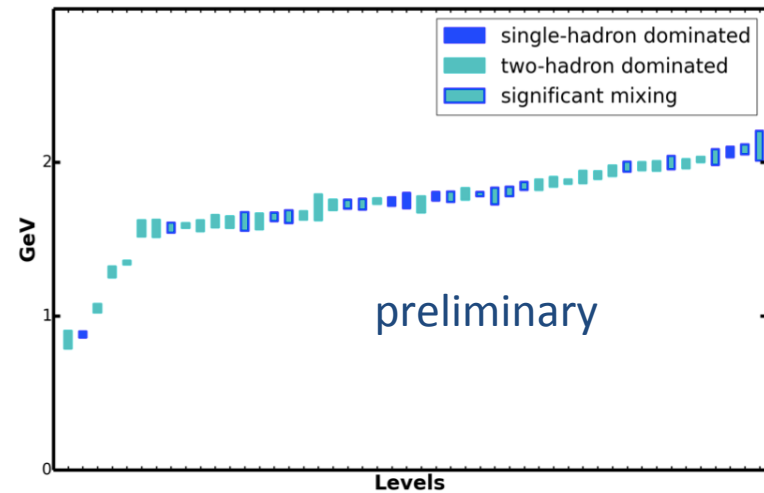
$(24^2 390)$	A_{1g}^+	A_{1u}^+	A_{2g}^+	A_{2u}^+	E_g^+	E_u^+	T_{1g}^+	T_{1u}^+	T_{2g}^+	T_{2u}^+
SH	9	7	13	13	9	9	14	23	15	16
" $\pi\pi$ "	6	12	2	6	8	9	15	17	10	12
" $\eta\pi$ "	2	10	8	4	8	11	21	14	14	13
" $\phi\pi$ "	2	10	8	4	8	11	23	3	14	13
" $K\bar{K}$ "	0	4	1	4	1	4	8	10	4	6
Total	19	43	32	31	34	44	81	67	57	60

effective masses $\tilde{m}^{\text{eff}}(t)$ for levels 16 to 31

$32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV



E_n in p channel for approx. 50 levels



Related topics

- Meson mass decomposition

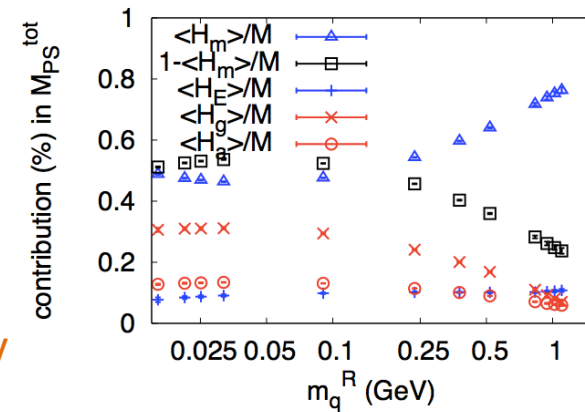
Yi-Bo Yang et al, XQCD coll., 1405.4440, Yang, Tuesday

$$M = -\langle T_{44} \rangle = \langle H_E \rangle + \langle H_m \rangle + \langle H_g \rangle + \langle H_a \rangle$$

$$H_E = \sum_{u,d,s,\dots} \int d^3x \bar{\psi} (D \cdot \gamma) \psi$$

$$H_m = \sum_{u,d,s,\dots} \int d^3x m \bar{\psi} \psi$$

$$H_g = \int d^3x \frac{1}{2} (B^2 - E^2),$$



- Extended QCD

David Kaplan, 1306.5818

$$\mathcal{D} = \not{D} + \not{\psi} + i\not{a}\gamma_5 + 2(\Phi P_+ + \Phi^\dagger P_-)$$

$$S_{\text{XQCD}} = N_c \int d^4x \left[\bar{\psi} (\mathcal{D} + m) \psi + S_{\text{YM}} + \lambda^2 \left(\text{Tr} \Phi^\dagger \Phi + \frac{1}{2} \text{Tr} [\mathbf{v}_\mu^2 + \mathbf{a}_\mu^2] \right) \right]$$

$$\int e^{-N_c \lambda^2 \int d^4x [\text{Tr} \Phi^\dagger \Phi + \frac{1}{2} \text{Tr} (\mathbf{v}_\mu \mathbf{v}_\mu + \mathbf{a}_\mu \mathbf{a}_\mu)]} \det [\mathcal{D} + m] = \mathcal{N} \det [\not{D} + m]$$

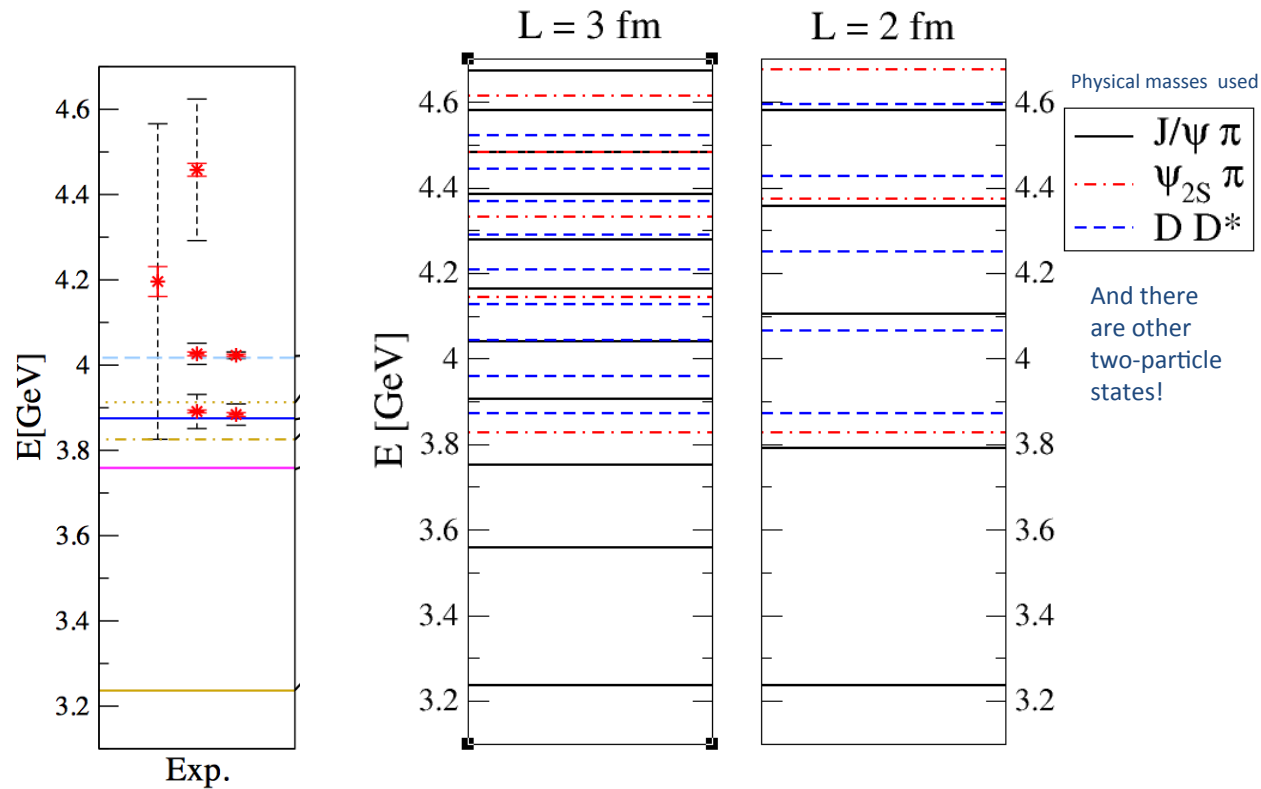
Exactly equivalent to QCD

limit $N_c \rightarrow \infty$: direct connection with nonrelativistic quark models with constituent quark mass respects chiral symmetry giving massless pion in chiral limit

Challenges: two examples

Challenge : precision simulation of Z_c^+

On larger volume: more two particle states



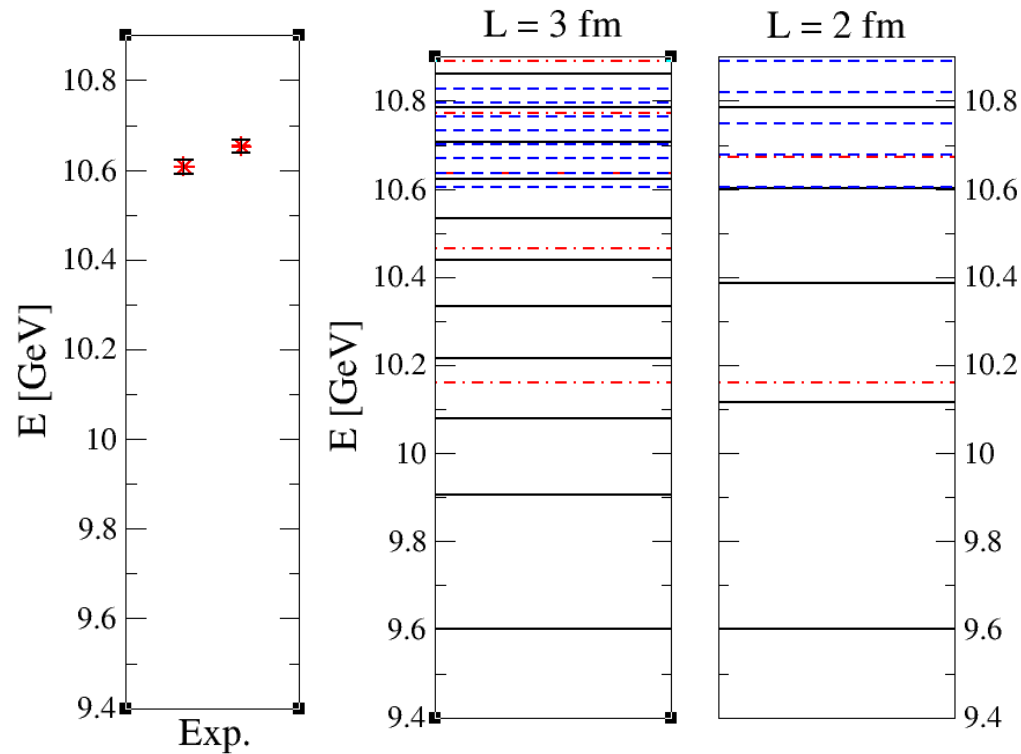
$$E(L) = \sqrt{m_1^2 + \vec{p}_1^2} + \sqrt{m_2^2 + \vec{p}_2^2} + \Delta E$$

$$\vec{p}_1 = \frac{2\pi}{L} \vec{n}_1 \quad \vec{p}_2 = \frac{2\pi}{L} \vec{n}_2$$

Rigorous treatment very challenging: at least 6 two-particle channels coupled !!

Another challenge: Z_b^+

On larger volume: more two-particle states

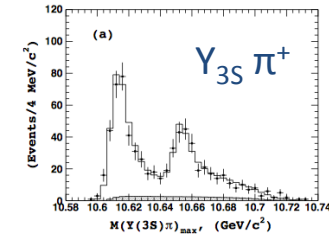
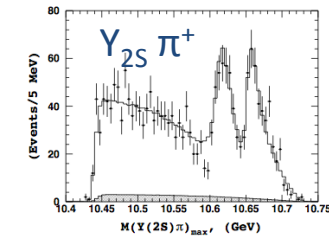
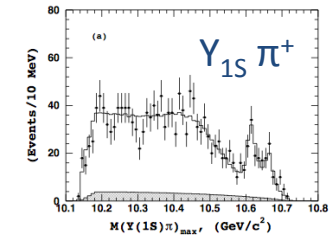


And there are other two-particle states!

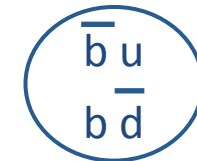
$$E(L) = \sqrt{m_1^2 + \vec{p}_1^2} + \sqrt{m_2^2 + \vec{p}_2^2} + \Delta E$$

$$\vec{p}_1 = \frac{2\pi}{L} \vec{n}_1 \quad \vec{p}_2 = \frac{2\pi}{L} \vec{n}_2$$

Belle 2011



$Z_b^+ \rightarrow Y_{nS} \pi^+$



Rigorous treatment very challenging: at least 6 two-particle channels coupled !!

Conclusions

Recent developments in hadron spectroscopy (with emphasis on mesons):

- below threshold states treated with unprecedented accuracy
- extensive results for multiplets within single-hadron approximation
- first rigorous treatments of near-threshold states:
evidences for Z_c^+ , $X(3872)$, $D_{s0}^*(2317)$, $\Lambda(1405)$
- a number of resonances studied rigorously:
 ρ , κ , K^* , K_0^* , K_2^* , D_0^* , D_1 , a_1 , b_1
- coupled inelastic problem treated in QCD for the first time (to my knowledge):
 $K\pi$, $K\eta$: κ , K^* , K_0^* , K_2^*

Conclusions

Recent developments in hadron spectroscopy (with emphasis on mesons):

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 $K\pi$, $K\eta$: κ , K^* , K_0^* , K_2^*



Conclusions

Many exciting challenges remain !

✧ An urgent example: quarkonium-like states

- can one afford to study them on larger volumes given the increasing number of two particle states?
- how about interesting states that lie even higher above threshold(s) ?
- rigorous treatment near multiple thresholds?

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M , MeV	Γ , MeV	J^{PC}	Process (mode)	Experiment (# σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \rightarrow K(\pi^+ \pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \rightarrow (\pi^+ \pi^- J/\psi) \dots$	CDF [994, 995] [11.6], D0 [996] (5.2)	2003	Ok
				$B \rightarrow K(\pi^+ \pi^- J/\psi) \dots$	LHCb [997, 998] (np)	2012	Ok
				$B \rightarrow K(\pi^+ \pi^- \pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \rightarrow K(\gamma J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5)	2005	Ok
				$B \rightarrow K(\gamma \psi(2S))$	LHCb [1003] (> 10)		
					BaBar [1002] (3.6), Belle [1001] (0.2)	2008	NC!
					LHCb [1003] (4.4)		
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$B \rightarrow K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?}$	$Y(4260) \rightarrow \pi^- (D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
				$Y(4260) \rightarrow \pi^- (\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
					T. Xiao <i>et al.</i> [CLEO data] [1009] (>5)		
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?}$	$Y(4260) \rightarrow \pi^- (\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?}$	$Y(4260) \rightarrow \pi^- (D^* \bar{D}^*)^+$	Belle [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$T(10860) \rightarrow \pi^- (\pi^+ \Upsilon(1S, 2S, 3S))$	Belle [1012-1014] (>10)	2011	Ok
				$T(10860) \rightarrow \pi^- (\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$T(10860) \rightarrow \pi^- (\pi^+ B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$T(10860) \rightarrow \pi^- (\pi^+ \Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$T(10860) \rightarrow \pi^- (\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$T(10860) \rightarrow \pi^- (B^* \bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!

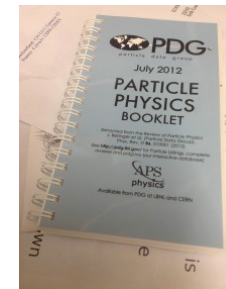


TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M , MeV	Γ , MeV	J^{PC}	Process (mode)	Experiment (# σ)	Year	Status
$Y(3015)$	3018.4 ± 1.9	20 ± 5	0^{2-}	$B \rightarrow K(\omega J/\psi)$	Belle [1001] (8), Belle [1003, 1001] (19)	2004	Ok
				$e^+ e^- \rightarrow e^+ e^- (\omega J/\psi)$	Belle [1002] (7.7), BaBar [1003] (7.6)	2009	Ok
$X_c(2P)$	3027.2 ± 2.6	24 ± 6	2^{++}	$e^+ e^- \rightarrow e^+ e^- (DD)$	Belle [1004] (5.3), BaBar [1005] (5.8)	2005	Ok
$X(3040)$	$3042.2^{+2.2}_{-2.1}$	37^{+17}_{-12}	$?^{?}$	$e^+ e^- \rightarrow J/\psi (D\bar{D}^*)$	Belle [1006, 1007] (6)	2005	NC!
$Y(4008)$	4001 ± 42	255 ± 42	1^{--}	$e^+ e^- \rightarrow (\pi^+ \pi^- J/\psi)$	Belle [1008, 1009] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+ e^- \rightarrow (D^{*+} D^{*-} (e))$	PDG [1]	1979	Ok
				$e^+ e^- \rightarrow (\eta J/\psi)$	Belle [1007] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-18}	82^{+11}_{-10}	$?^{?}$	$B^0 \rightarrow K^+ (\pi^- \pi^+ \chi_{c1})$	Belle [1008] (5.0), BaBar [1009] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?}$	$B^0 \rightarrow K^+ (\phi J/\psi)$	CDF [1000] (5.0), Belle [1001] (1.9), LHCb [1002] (1.4), CMS [1003] (>5)	2009	NC!
					DO [1064] (3.1)		
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+ e^- \rightarrow (D^{*+} D^{*-} (e))$	PDG [1]	1979	Ok
				$e^+ e^- \rightarrow (\eta J/\psi)$	Belle [1007] (6.3)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+119}_{-55}	$?^{?}$	$e^+ e^- \rightarrow J/\psi (D^* \bar{D}^*)$	Belle [1008] (5.3)	2007	NC!
$Z(4200)^+$	4196^{+45}_{-35}	370^{+110}_{-60}	1^{+-}	$B^0 \rightarrow K^+ (\pi^- J/\psi)$	Belle [1008] (7.2)	2014	NC!
$Z(4200)^+$	4246^{+45}_{-35}	177^{+60}_{-40}	$?^{?}$	$B^0 \rightarrow K^+ (\pi^- \pi^+ \chi_{c1})$	Belle [1008] (5.0), BaBar [1009] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	198 ± 12	1^{--}	$e^+ e^- \rightarrow (\pi^+ \pi^- J/\psi)$	BaBar [1006, 1007] (8), CLEO [1008, 1009] (11)	2005	Ok
					Belle [1008, 1009] (10), BES III [1007] (np)		
				$e^+ e^- \rightarrow (J_s(3885) J/\psi)$	BaBar [1007] (np), Belle [1008] (np)	2012	Ok
				$e^+ e^- \rightarrow (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+ e^- \rightarrow (\pi^- X(3885)^+)$	BES III [1007] (5.3)	2013	NC!
$Y(4274)$	4290 ± 20	35 ± 16	$?^{?}$	$B^0 \rightarrow K^+ (\phi J/\psi)$	CDF [1001] (3.1), LHCb [1002] (1.0), CMS [1003] (>5), DO [1064] (np)	2011	NC!
$X(4350)$	$4350.6^{+1.1}_{-1.1}$	13^{+18}_{-10}	0^{2-}	$e^+ e^- \rightarrow e^+ e^- (\phi J/\psi)$	Belle [1071] (3.9)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+ e^- \rightarrow (\pi^+ \pi^- \psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4438 ± 15	166^{+25}_{-20}	1^{+-}	$B^0 \rightarrow K^+ (\pi^- \psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4)	2007	Ok
					LHCb [1077] (13.0)		
				$B^0 \rightarrow K^+ (\pi^- J/\psi)$	Belle [1008] (4.0)	2014	NC!
$X(4630)$	4624^{+11}_{-11}	92^{+11}_{-10}	1^{--}	$e^+ e^- \rightarrow (\Lambda_2 \bar{\Lambda}_2)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4660 ± 10	53 ± 14	1^{--}	$e^+ e^- \rightarrow (\pi^+ \pi^- \psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$T(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+ e^- \rightarrow (B_c^{(v)} B_c^{(v)*} (e))$	PDG [1]	1985	Ok
				$e^+ e^- \rightarrow (\pi^+ \Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+ e^- \rightarrow (J_s(3885) \Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+ e^- \rightarrow (\chi_{c1}(10610, 10600))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+ e^- \rightarrow (\eta \Upsilon(1S, 2S))$	Belle [945] (10)	2012	Ok
				$e^+ e^- \rightarrow (\pi^+ \pi^- \Upsilon(1J))$	Belle [945] (9)	2012	Ok
$Y_1(10888)$	10888.4 ± 3.0	$30.7^{+2.9}_{-2.8}$	1^{--}	$e^+ e^- \rightarrow (\pi^+ \pi^- \Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

✧ Can we understand almost complete absence of exotics in most of other meson systems?

✧ Looking forward to new challenges ...



Thanks to those who sent me the material

Christine Davies, Carleton DeTar, Derek
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Morningstar, Raul Briceno, Takeshi
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Thomas

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Daniel Mohler	(Fermilab)
Luka Leskovec	(Ljubljana)
Richard Woloshyn	(Vancouver)

Apologies to those whose work I was not
able to present.



Backup slides

Interpolators in Zc channel

[S.P., Lang, Leskovec, Mohler, 1405.7623]

$$\mathcal{O}_1 = \mathcal{O}_1^{\psi(0)\pi(0)} = \bar{c}\gamma_i c(0) \bar{d}\gamma_5 u(0) ,$$

$$\mathcal{O}_2 = \mathcal{O}_2^{\psi(0)\pi(0)} = \bar{c}\gamma_i \gamma_t c(0) \bar{d}\gamma_5 u(0) ,$$

$$\mathcal{O}_3 = \mathcal{O}_3^{\psi(0)\pi(0)} = \bar{c} \overleftrightarrow{\nabla}_j \gamma_i \overrightarrow{\nabla}_j c(0) \bar{d}\gamma_5 u(0) ,$$

$$\mathcal{O}_4 = \mathcal{O}_4^{\psi(0)\pi(0)} = \bar{c} \overleftrightarrow{\nabla}_j \gamma_i \gamma_t \overrightarrow{\nabla}_j c(0) \bar{d}\gamma_5 u(0) ,$$

$$\mathcal{O}_5 = \mathcal{O}_5^{\psi(0)\pi(0)} = |\epsilon_{ijk}| |\epsilon_{klm}| \bar{c} \gamma_j \overleftrightarrow{\nabla}_l \overrightarrow{\nabla}_m c(0) \bar{d}\gamma_5 u(0) ,$$

$$\mathcal{O}_6 = \mathcal{O}_6^{\psi(0)\pi(0)} = |\epsilon_{ijk}| |\epsilon_{klm}| \bar{c} \gamma_t \gamma_j \overleftrightarrow{\nabla}_l \overrightarrow{\nabla}_m c(0) \bar{d}\gamma_5 u(0) ,$$

$$\mathcal{O}_7 = \mathcal{O}_7^{\psi(0)\pi(0)} = R_{ijk} Q_{klm} \bar{c} \gamma_j \overleftrightarrow{\nabla}_l \overrightarrow{\nabla}_m c \bar{d}\gamma_5 u(0) ,$$

$$\mathcal{O}_8 = \mathcal{O}_8^{\psi(0)\pi(0)} = R_{ijk} Q_{klm} \bar{c} \gamma_t \gamma_j \overleftrightarrow{\nabla}_l \overrightarrow{\nabla}_m c \bar{d}\gamma_5 u(0) .$$

$$\mathcal{O}_9 = \mathcal{O}^{\psi(1)\pi(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k) \bar{d}\gamma_5 u(-e_k) ,$$

$$\mathcal{O}_{10} = \mathcal{O}^{\eta_c(0)\rho(0)} = \bar{c}\gamma_5 c(0) \bar{d}\gamma_i u(0) ,$$

$$\mathcal{O}_{11} = \mathcal{O}_1^{D(0)D^*(0)} = \bar{c}\gamma_5 u(0) \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\} ,$$

$$\mathcal{O}_{12} = \mathcal{O}_2^{D(0)D^*(0)} = \bar{c}\gamma_5 \gamma_t u(0) \bar{d}\gamma_i \gamma_t c(0) + \{\gamma_5 \leftrightarrow \gamma_i\} ,$$

$$\mathcal{O}_{13} = \mathcal{O}^{D(1)D^*(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_5 u(e_k) \bar{d}\gamma_i c(-e_k) + \{\gamma_5 \leftrightarrow \gamma_i\} ,$$

$$\mathcal{O}_{14} = \mathcal{O}^{D^*(0)D^*(0)} = \epsilon_{ijk} \bar{c}\gamma_j u(0) \bar{d}\gamma_k c(0) ,$$

$$\mathcal{O}_{15} = \mathcal{O}_1^{4q} = N_L^3 \epsilon_{abc} \epsilon_{ab'c'} (\bar{c}_b C \gamma_5 \bar{d}_c c_{b'} \gamma_i C u_{c'} - \bar{c}_b C \gamma_i \bar{d}_c c_{b'} \gamma_5 C u_{c'})$$

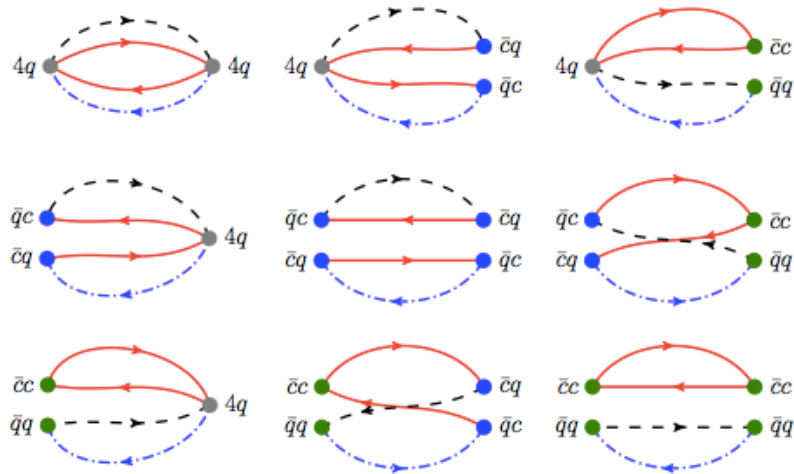
$$\mathcal{O}_{16} = \mathcal{O}_2^{4q} = N_L^3 \epsilon_{abc} \epsilon_{ab'c'} (\bar{c}_b C \bar{d}_c c_{b'} \gamma_i \gamma_5 C u_{c'} - \bar{c}_b C \gamma_i \gamma_5 \bar{d}_c c_{b'} C u_{c'})$$

$$\mathcal{O}_{17} = \mathcal{O}_3^{4q} = \mathcal{O}_1^{4q}(N_v=32) ,$$

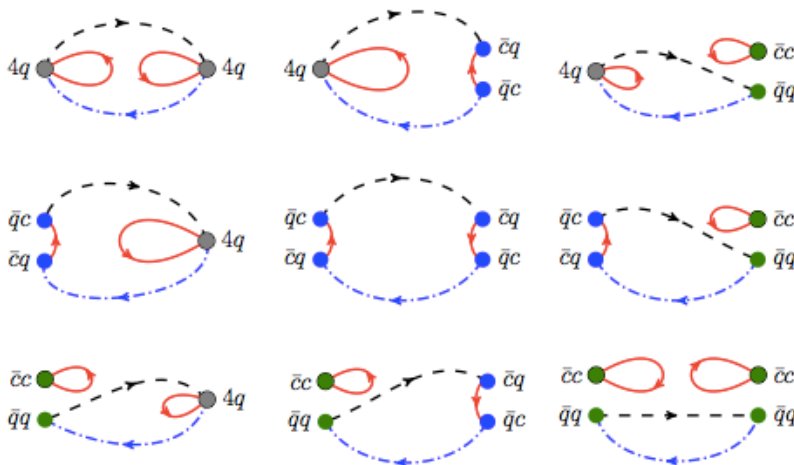
$$\mathcal{O}_{18} = \mathcal{O}_4^{4q} = \mathcal{O}_2^{4q}(N_v=32) .$$

Wick contractions for Zc^+

a

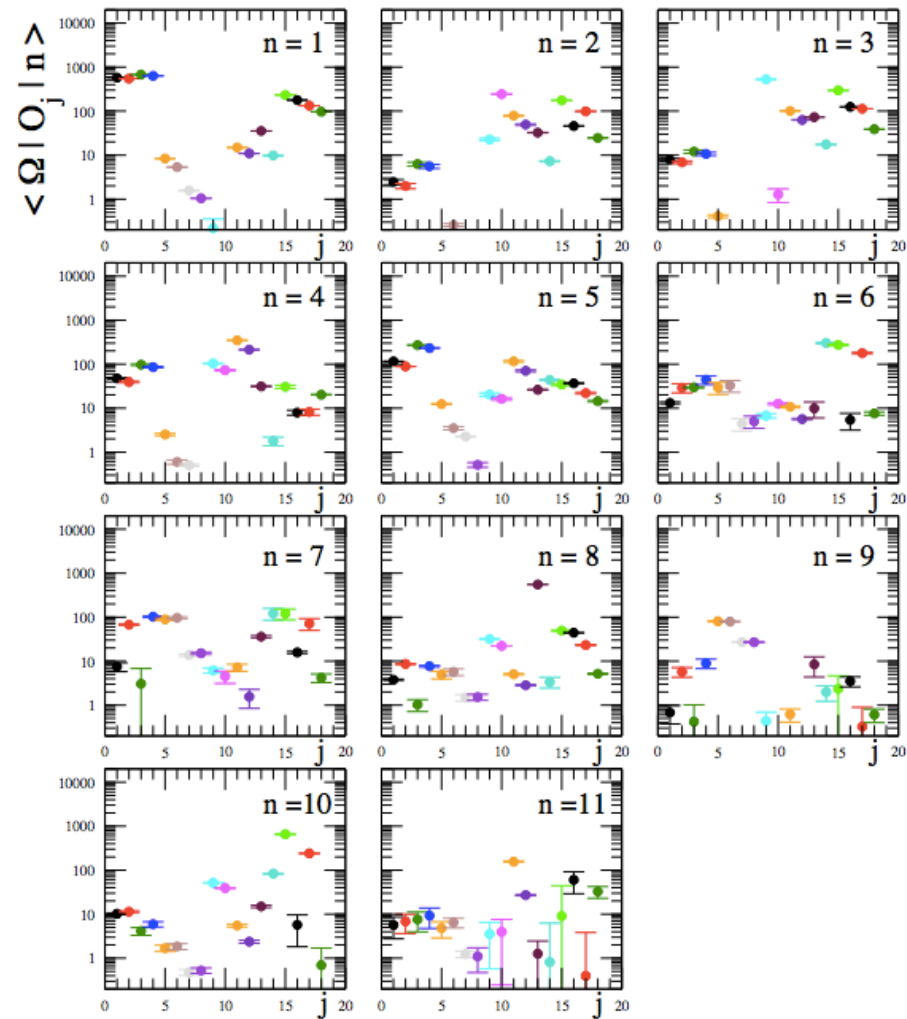
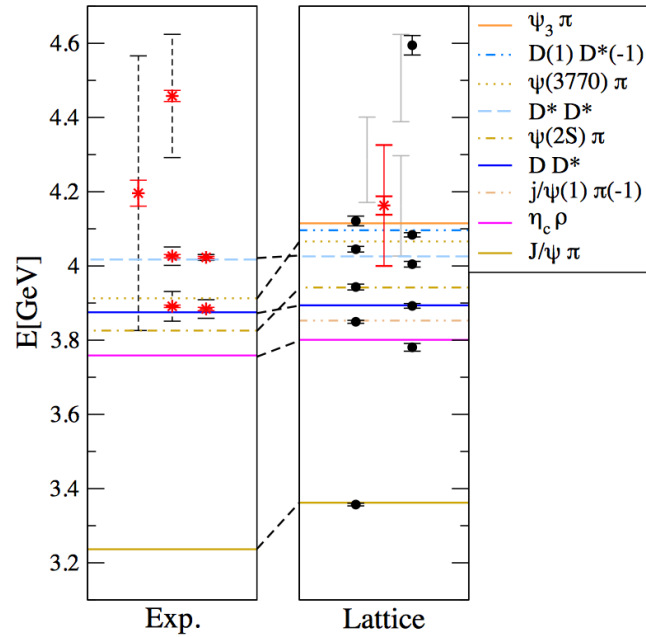


b



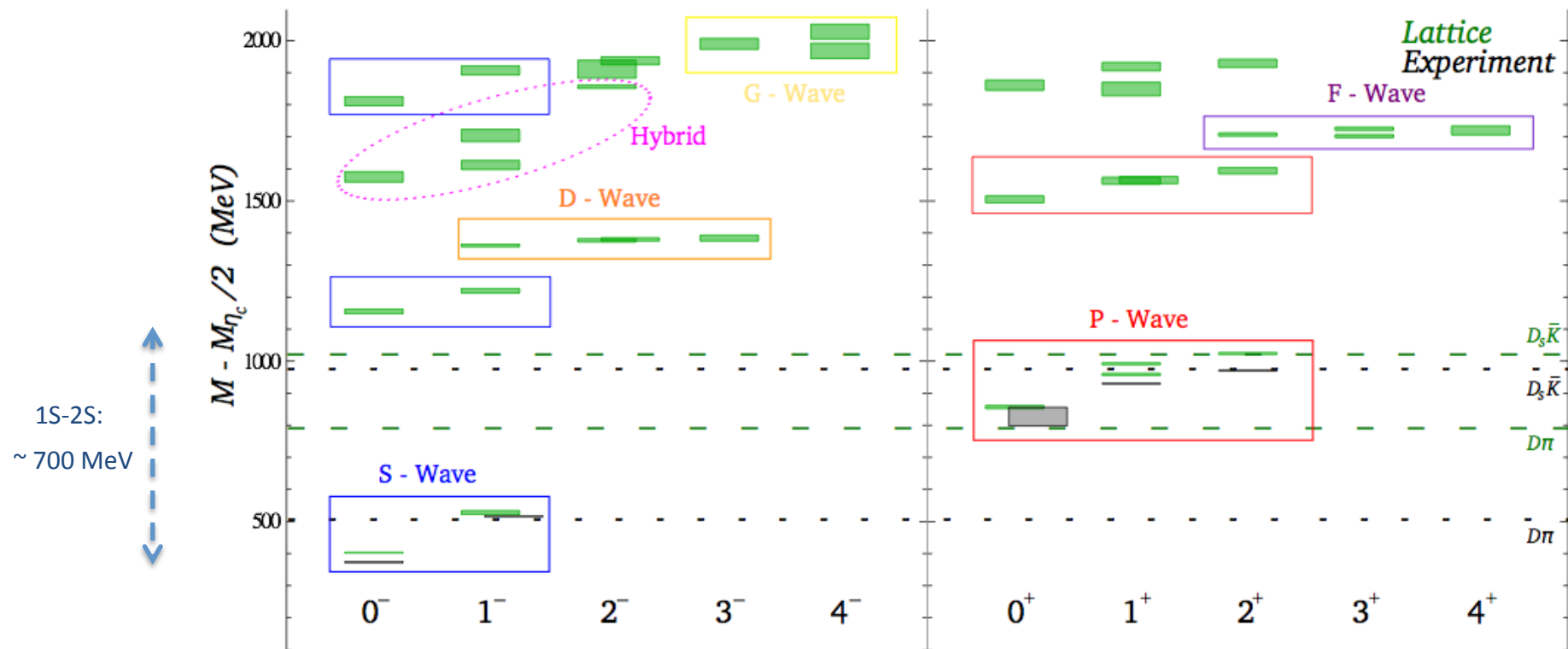
[S.P., Lang, Leskovec, Mohler,
1405.7623]

Overlaps of all states in Z_c^+ channel



[S.P., Lang, Leskovec, Mohler, 1405.7623]

D spectrum: single hadron approximation

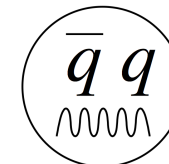


[G. Moir et al, HSC (Hadron Spectrum Coll.): 1301.7670, JHEP]

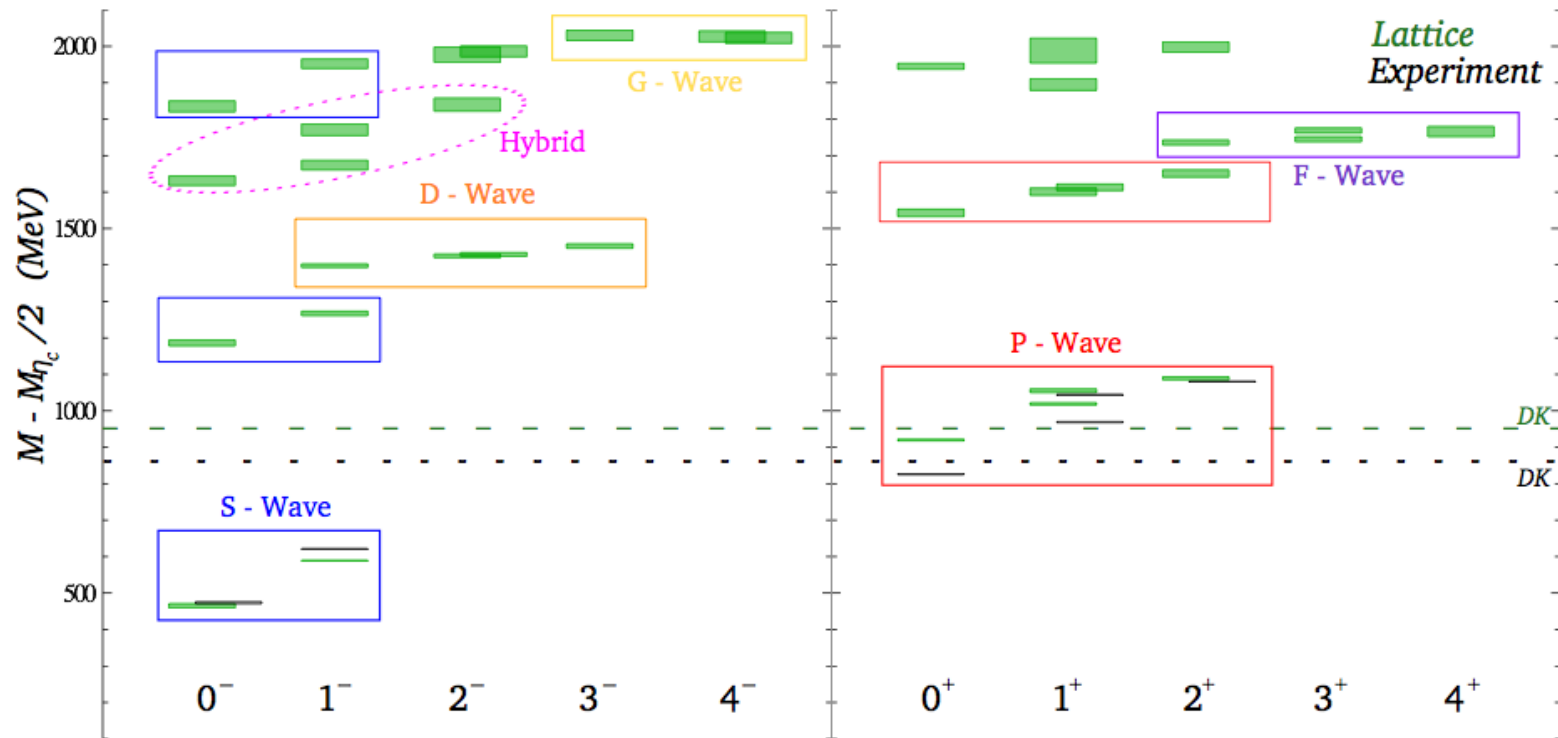
- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- reliable J^P determination; many excited states
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:

large overlap with $O = \bar{q} F_{ij} q$
gluonic tensor $F_{ij} = [D_i, D_j]$



D_s spectrum: single hadron approximation



[G. Moir et al., HSC : 1301.7670, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- reliable J^{PC} determination
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:

large overlap with $O = \bar{q} F_{ij} q$
 gluonic tensor $F_{ij} = [D_i, D_j]$

