Hadron physics : theory

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Outline:

conventional and exotic hadrons

their masses, decay widths and other properties

from theory

Hadrons

G=gluon, *q*=quark=*u,d,s,c,b*

Today we know (from exp and theory) that hadrons with the following minimal quark and gluon contents.

There may be more categories, but these are not reliably confirmed yet.

minimal quark (q) and gluon (G) contents

conventional hadrons	exotic hadrons	experimentaly discoveded example
$\bar{q}\ q$ meson	$q\ q\ \bar{q}\ \bar{q}$ tetraquark	$\frac{c\ c}{\bar{u}\ \bar{d}} = T_{cc}$ [LHCb 2021]
$q\ q\ q$ baryon	$q\ q\ q\ \bar{q}\ q$ pentaquark	$\frac{uud}{\bar{c}\ c} = P_c$ [LHCb 2015]
	$\bar{q}\ G\ q$ hybrid	$\bar{u}Gd = \pi_1$ [several exp.] $J^{PC} = 1^{-+}$

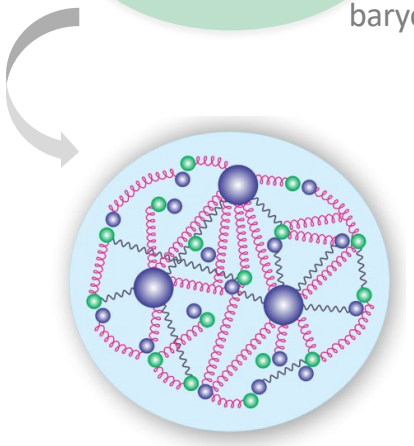


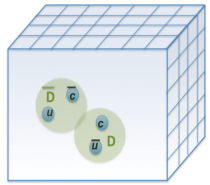
image:
https://physicstoday.scitation.org/do/10.1063/PT.5.7167/full/

QCD: $\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$ $\qquad g_s \not\ll 1$ at hadronic energy scale

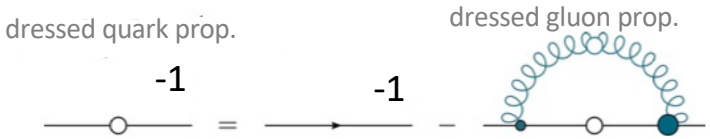
Theoretical approaches: nonperturbative approach to QCD is needed to address hadrons

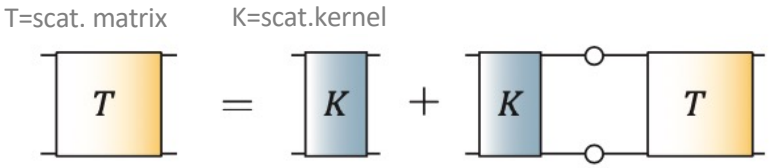
Lattice QCD $\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S^E_{QCD}/\hbar}$ $\qquad S^E_{QCD} = \int d^4 x_E\, \mathcal{L}^E_{QCD}(m_q, g_s)$

often "non-precision" studies: $m_{u/d} > m^{phy}_{u/d}$, $m_\pi > 140$ MeV



Dyson-Schwinger equations + **Bethe-Salpeter equations :** T, K computed with certain trunctions





Effective field theories : effective degrees of freedom + symmetries of QCD + expansion (in E, p, $1/m_Q$)

Quark Models

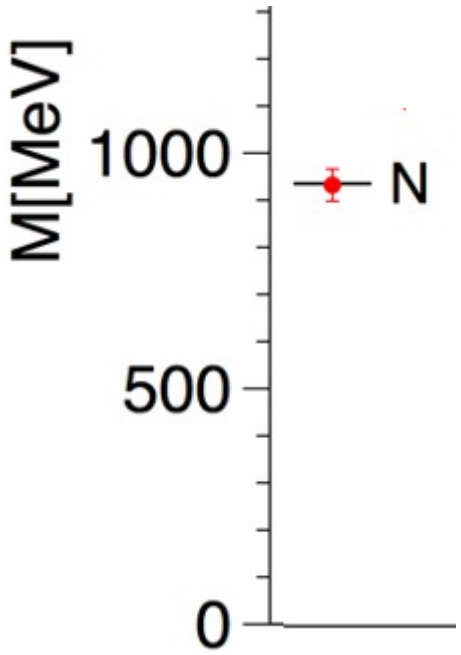
Models : effective degrees of freedom + symmetries of QCD + but not rigorous expansion (in E, p, $1/m_Q$) , ...

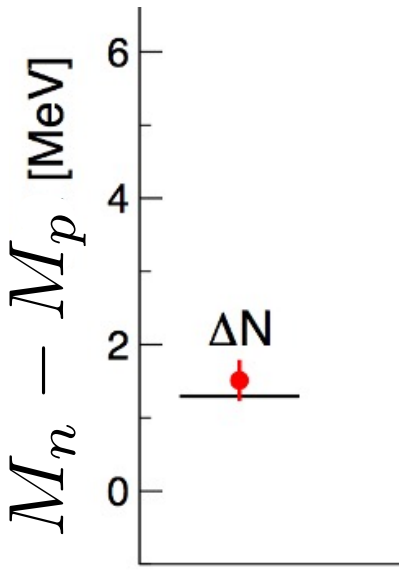
Sum rules these do not extract scattering amplitudes and do not rigorously extract resonances

This talk:

- focus on first two ab-initio approaches where results are available: strongly stable hadrons and certain strongly decaying hadrons
- solid ab-initio results not yet available for many exotic states discovered in exp: present result from other approaches

Proton and neutron mass constitute more than 99% of the bright universe mass





exp

lattice QCD
$m_u=m_d$

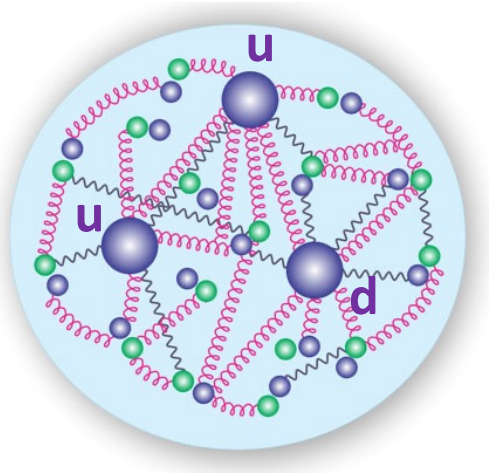
BMW collaboration
Science 322, 2008

exp

lattice QCD+QED
$m_u \neq m_d$

BMW collaboration
Science 347,2015

proton



Contribution of the Higgs mechanism

to the valence quark masses

$2m_u+m_d \cong 10$ MeV [PDG]

image:
https://physicstoday.scitation.org/do/10.1063/PT.5.7167/full/

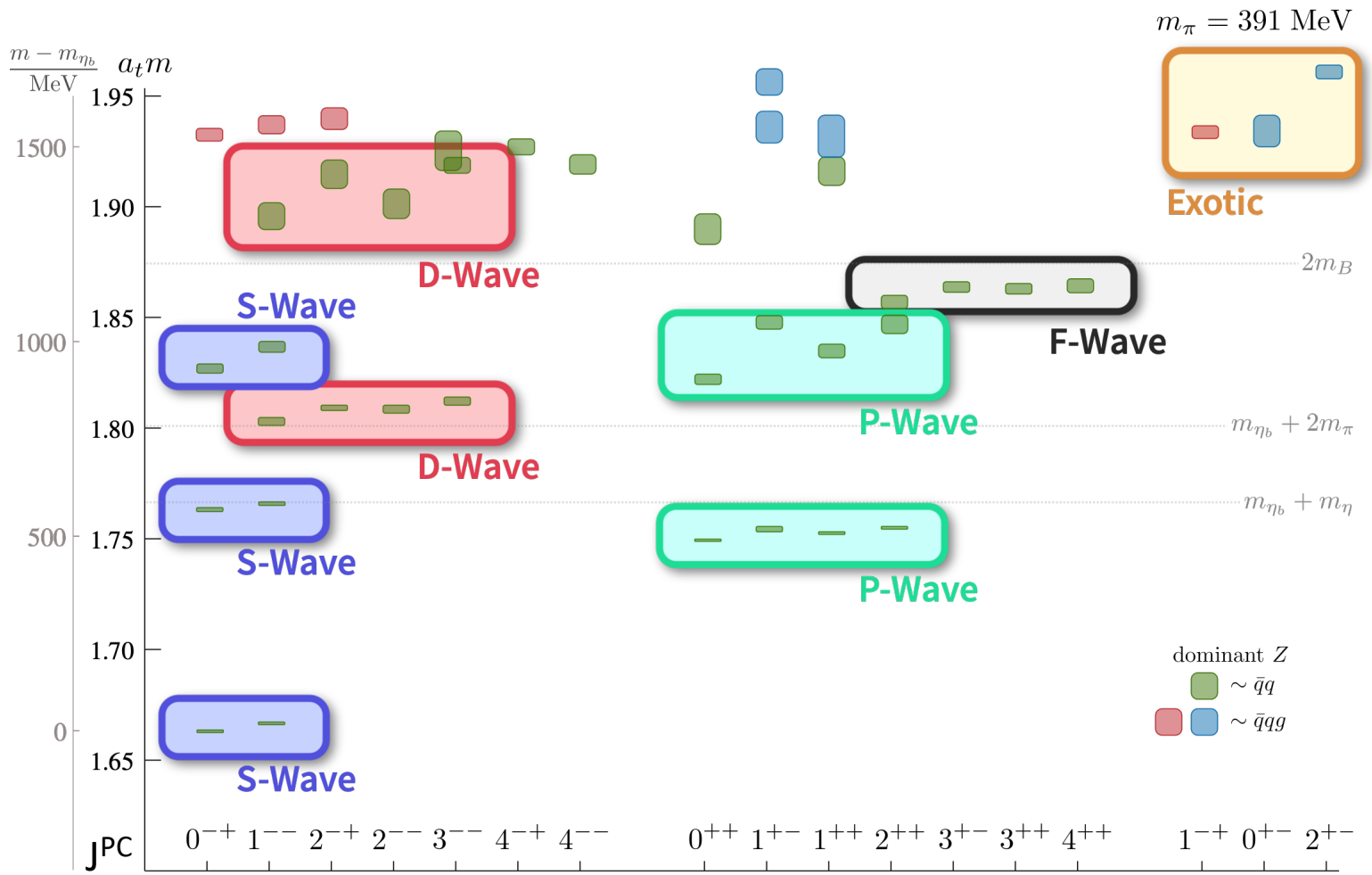
Bottomonia and bottomonium hybrids

$\bar{b}\,b$ $\bar{b}Gb$

Lattice QCD
relativistic b quarks
Ryan & Wilson (HadSpec)
2008.02656, JHEP

strongly stable below BB if
bb annihilation is omitted

$m_{hybrid} \geq 10.9$ GeV



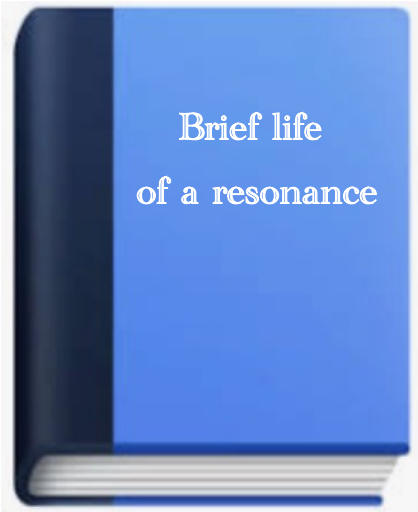
other predictions of hybrids:

from quenched lattice static potentials / EFT

Brambilla et al, 1805.07713, 1908.11699, PRD

Schlosser & Wagner, 2111.00741

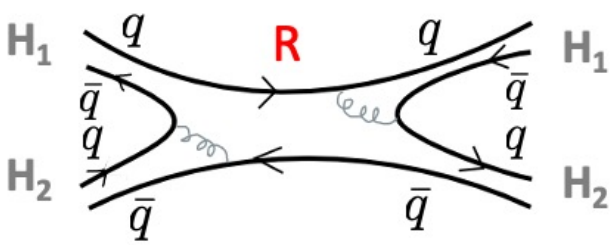
Strongly decaying hadronic resonances



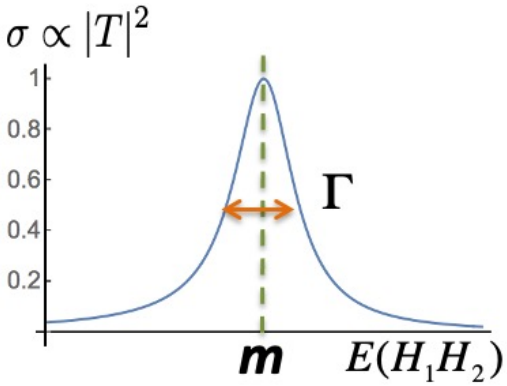
hotemoji.com

Strongly decaying resonances

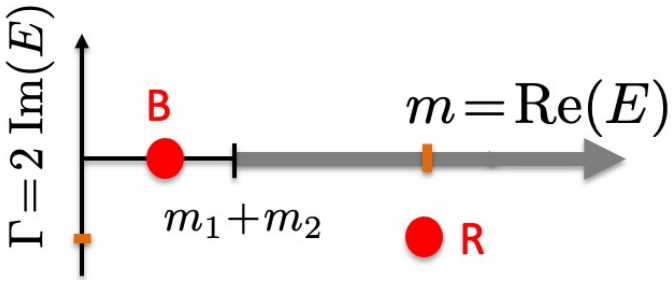
have to be extracted from the scattering of H_1H_2



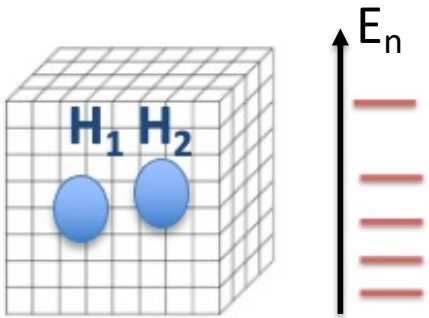
scattering matrix T(E)



Locations of poles in T(E): red



Scattering matrix T(E) from lattice QCD



many resonances and bound states extracted in this way by now
(apologies for not covering all)

real E — for real E — for complex E

$$E \rightarrow T(E) \rightarrow T(E^c)$$

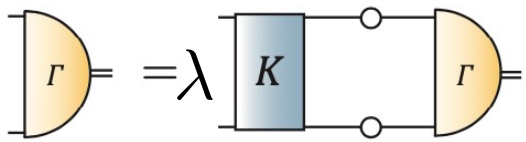
analytic relation:
Luscher 1991
generalizations by many authors

analytic contin. to complex E

Poles in T(E) from Bethe-Salpeter approach

Γ = BS amplitude of a state

K=scat.kernel



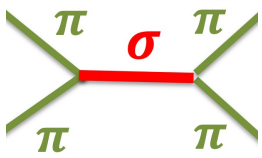
$\lambda = 1$ at $P^2 = -(m - i\frac{\Gamma}{2})^2$

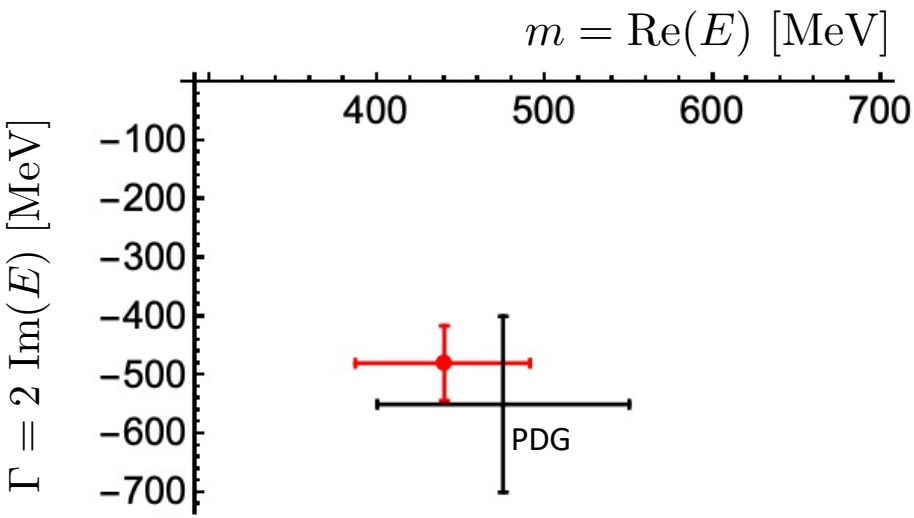
- solve this eigenvalue equation to find the resonance pole
- in practice solutions $\lambda \neq 1$ ared found for P^2 below threshold (in singularity free region) and then analytically extrapolated to find P^2 that correspond to $\lambda = 1$ above th.
- first results for resonances in QCD became available only recently

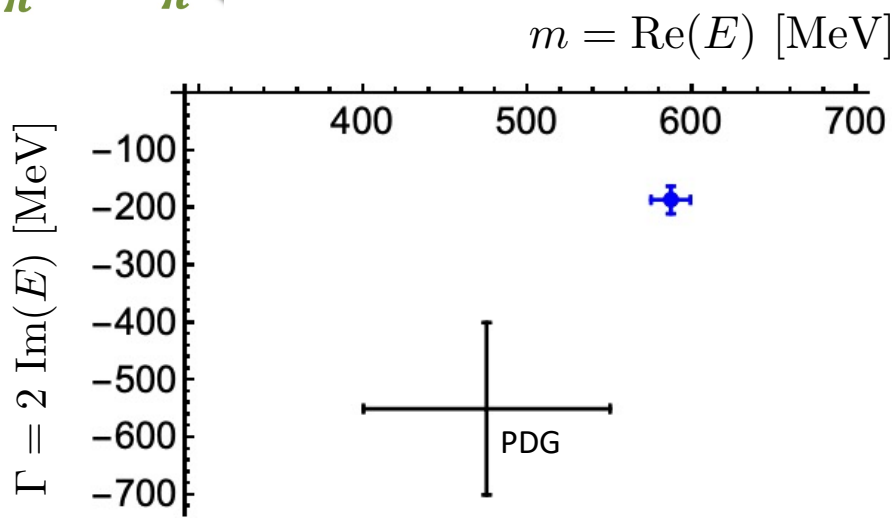
R. Williams, 1804.11161 PLB, Miramontes, Sanchis-Alepuz (2019) Eichman, Duarte, Pena, Stadler, PRD 100 (2019), Santowsky, Eichmann, Fischer, Wallbott, Williams, 2007.06495, PRD Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)

Resonance $\sigma = f_0(500)$

$\bar{q}q + \bar{q}q\bar{q}q$, $I = 0$, $J^{PC} = 0^{++}$







Lattice QCD, dynamical u&d quarks

Guo, Alexandru, Mai, Molina, Doring, 1803.02897, PRD

$$m_\pi = 227, 315 \text{ MeV} \rightarrow m_\pi^{phy}$$

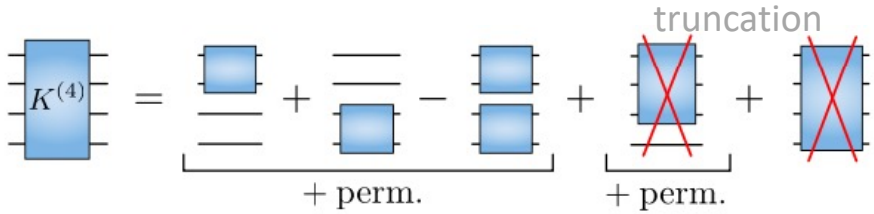
chiral unitary extrapolation

the only other lattice result for sigma pole:
Briceno, Dudek, Edwards, Wilson, 1607.05900 , PRL

See also works by RBC/UKQCD 2103.15131 PRD, Pelaez, Rodas

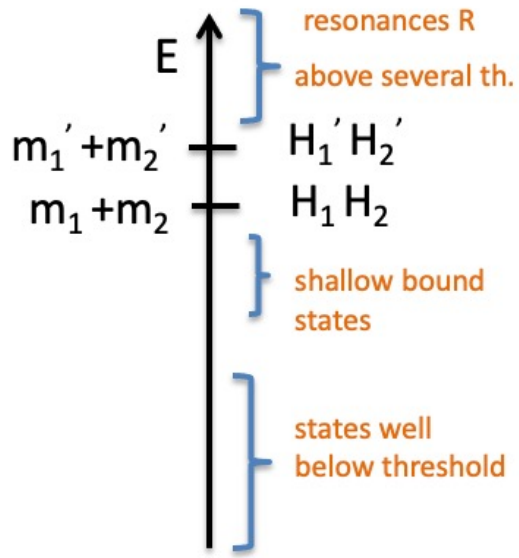
Dyson-Schwinger + Bethe-Salpeter equations

Santowsky, Eichmann, Fischer, Wallbott, Williams, 2007.06495, PRD



coupled $\bar{q}q$ and $\bar{q}q\bar{q}q$
several simplifications undertaken
the width determined from $\bar{q}q$ part
currently: only qualitative comparison to exp

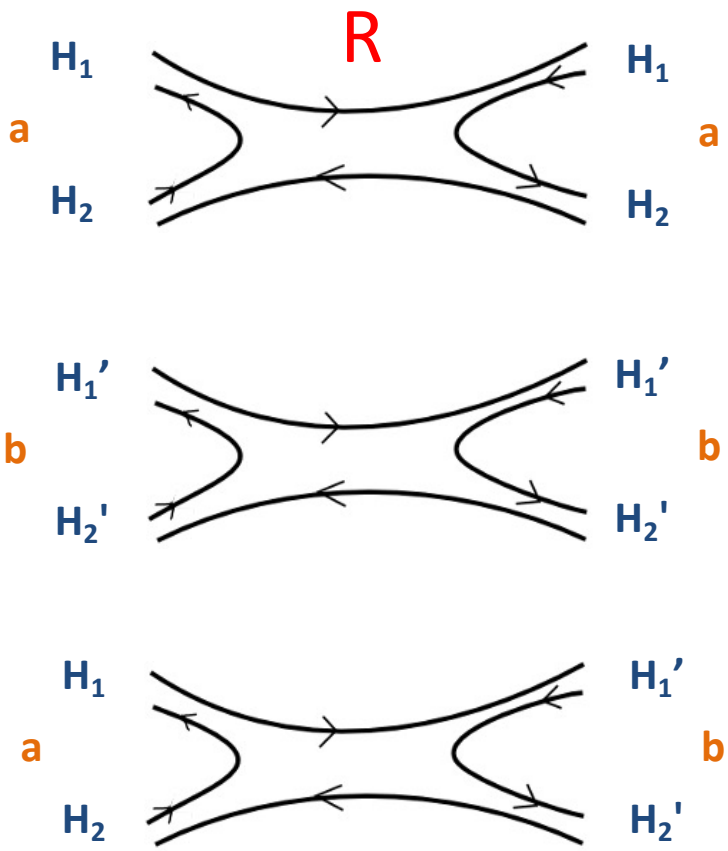
Resonances from coupled-channel scattering



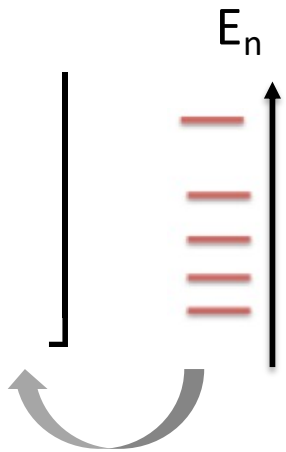
$$R \to H_1 H_2,\ H_1' H_2', \ldots$$

channel a: $H_1 H_2$

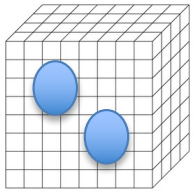
channel b: $H_1' H_2'$



$$T(E) = \begin{array}{c} \begin{array}{cc} a \to a & a \to b \end{array} \\ \begin{bmatrix} T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \end{bmatrix} \\ \begin{array}{cc} b \to a & b \to b \end{array} \end{array}$$



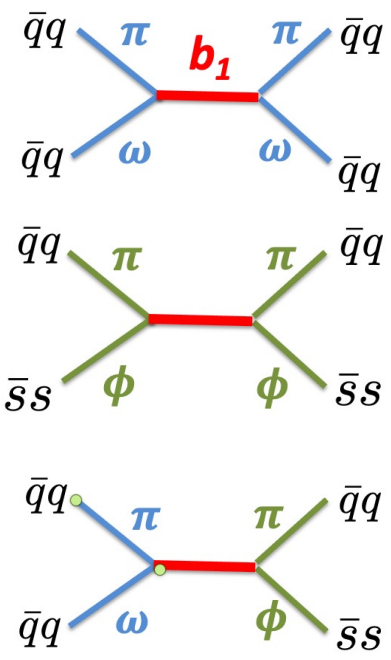
generalized. Luscher's rel.



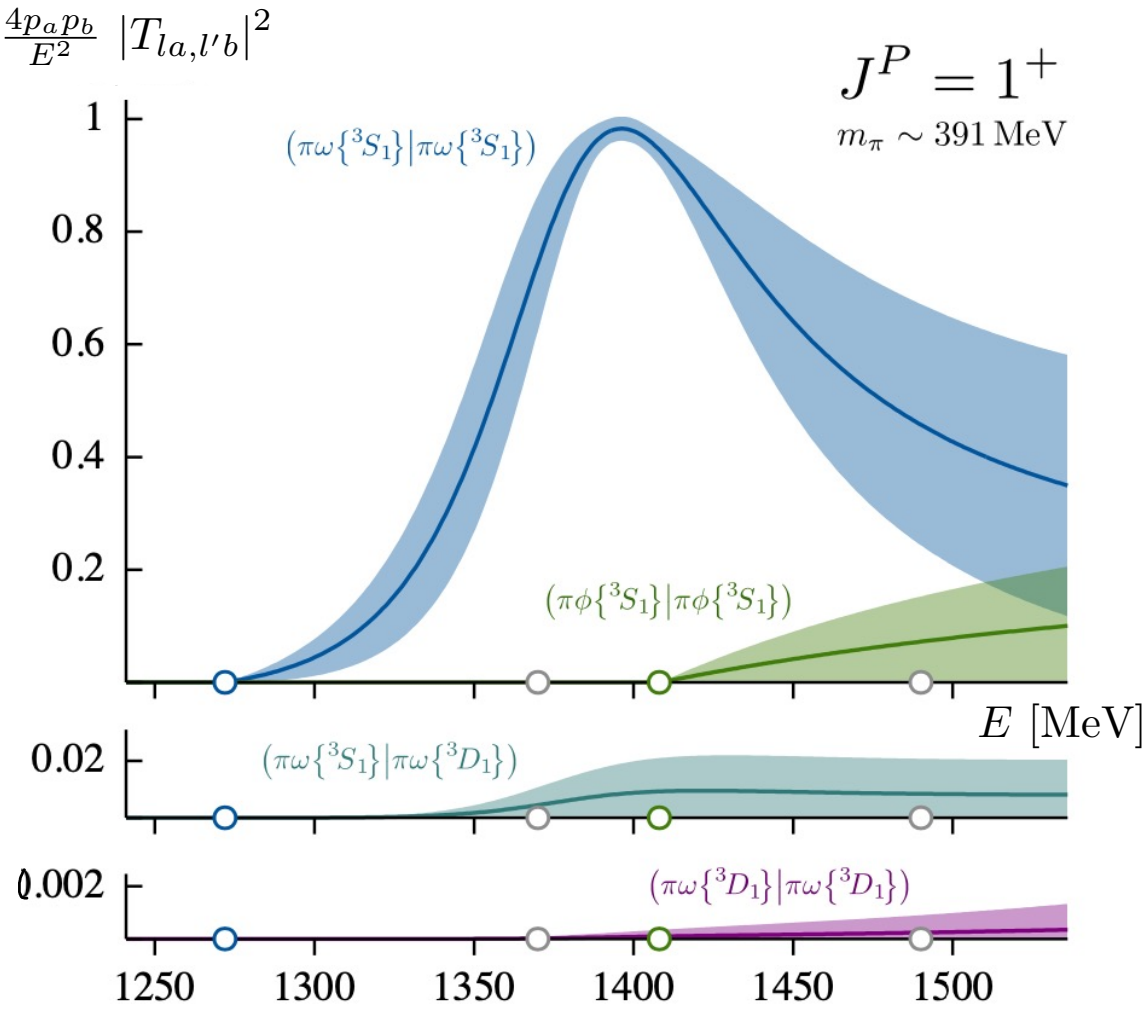
- lattice QCD studies extracted T(E) for several resonances
- most results by HadSpec. coll.

b_1 resonance from lattice

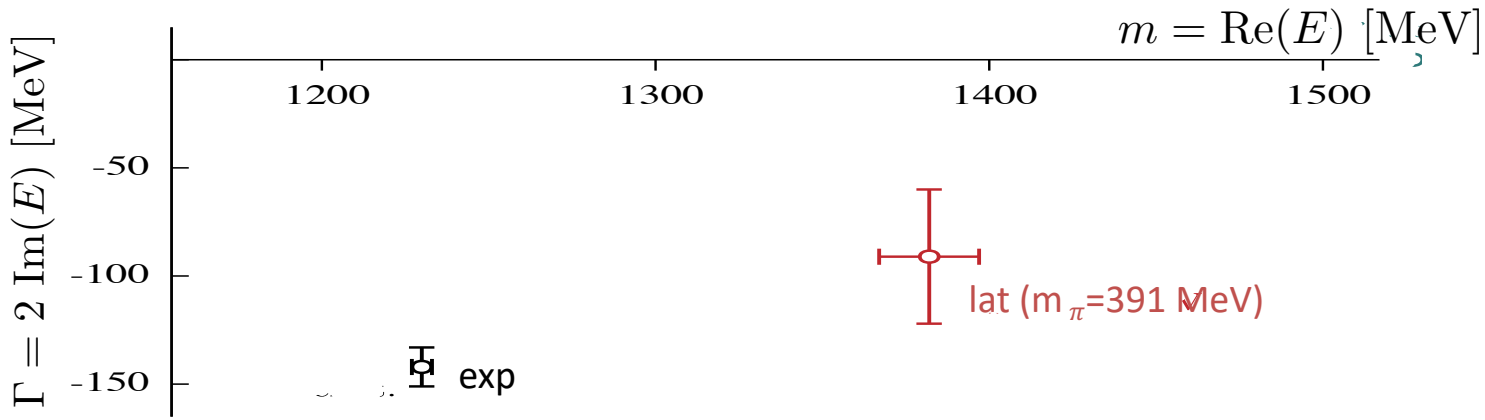
$\bar{d}u$ $J^{PC} = 1^{+-}$



q=u,d

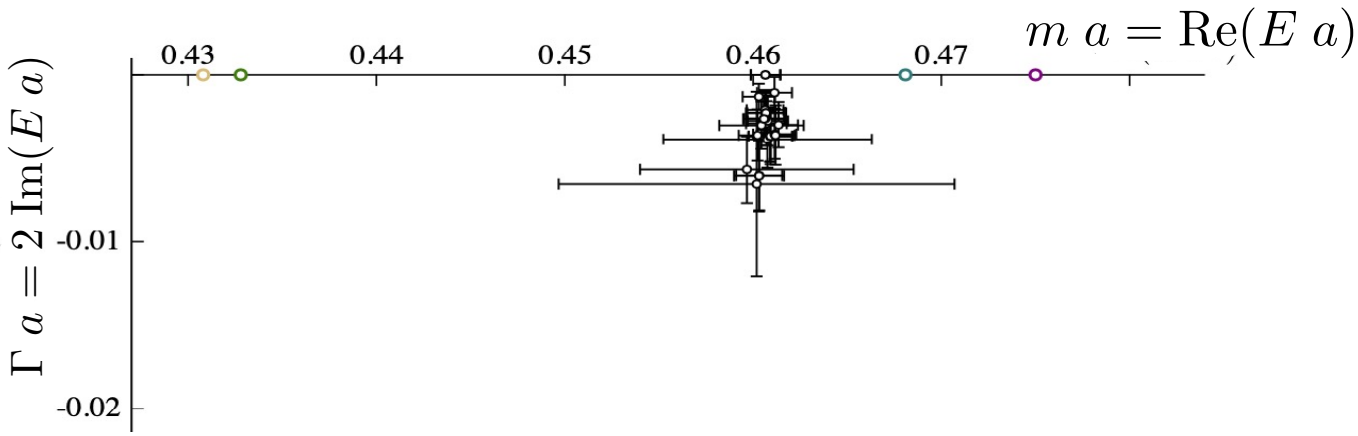


Woss et al, HadSpec, 1904.04136, PRD



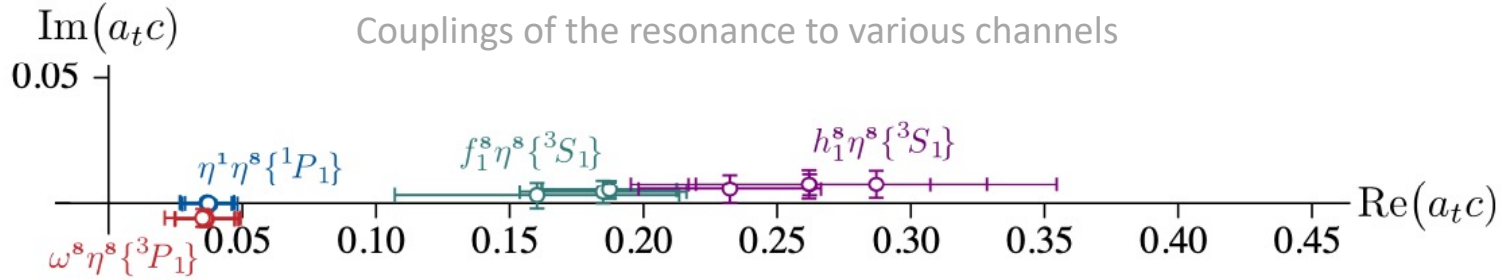
light hybrid meson π_1 from lattice

$\bar{d}Gu$ $J^{PC} = 1^{-+}$



Couplings of the resonance to various channels

$$T_{ij} \sim \frac{c_i\ c_j}{E_p^2 - E^2}$$



Woss et al. (HadSpec)

2009.10034

$m_u = m_d = m_s$, $m_\pi \approx 700\ MeV$

pheno

analysis

physical world

$\rho\ \pi$ $\eta'\ \pi$ $f_1\ \pi$ $b_1\ \pi$

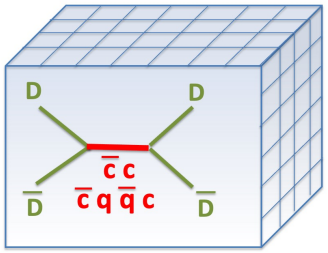
dominant coupling

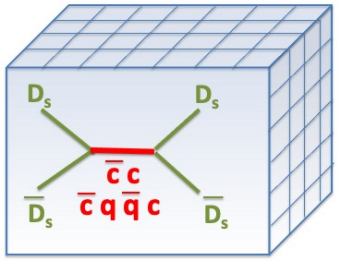
resemblence to experimental $\pi_1(1564)$: COMPASS+JPAC Rodas 1810.04171 [PRL]

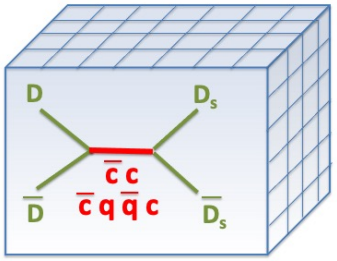
$\pi_1(1564)$ in COMPASS+JPAC replaces two older resonances $\pi_1(1400)$ and $\pi_1(1600)$

Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$







$\bar{D}_s D_s$ state

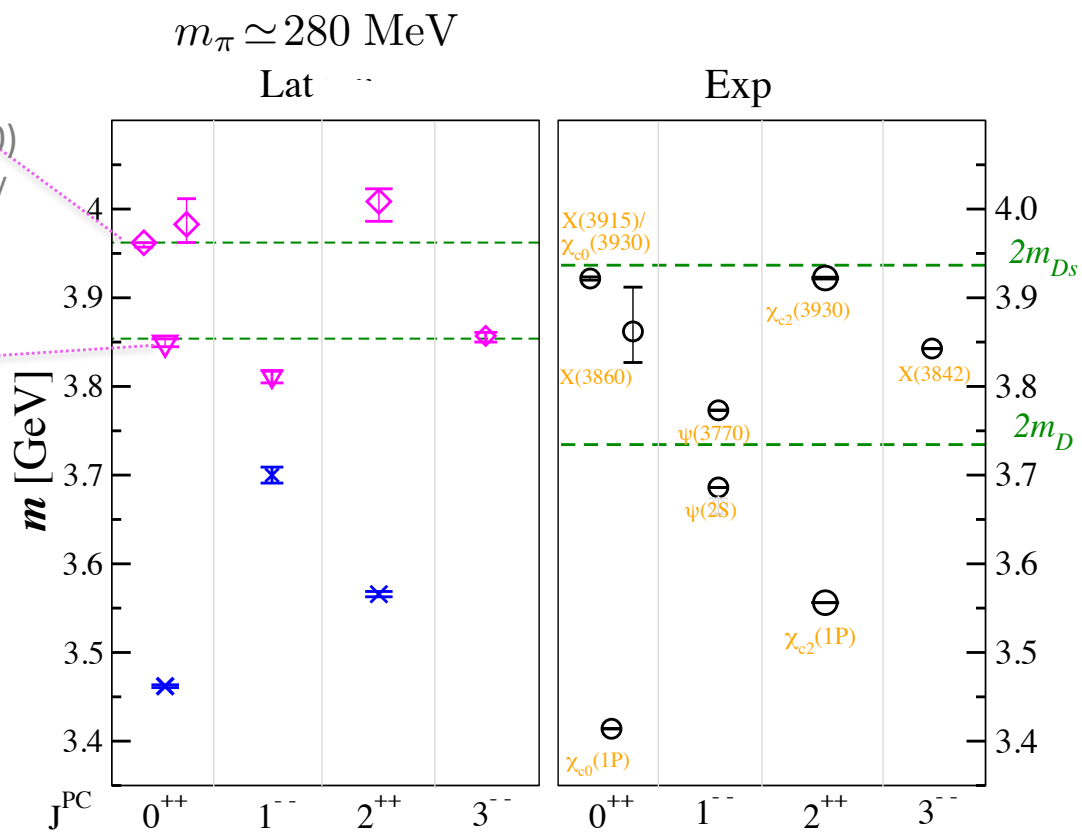
likely related to X(3915) / χ_{c0} (3930) [BaBar, LHCb 2009.00026]; explaining why it has narrow width to DD. Predicted by Lebed, Polosa 1602.08421

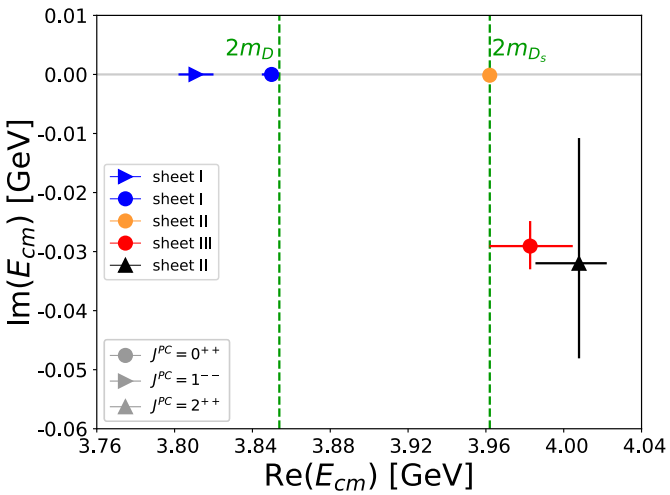
$\bar{D}D$ state

predicted in models [Oset et al, 0612179 PRD, Hildago Duque et al 1305.4487, Baru et al 1605.09649 PLB]

seen in dispersive analysis of exp. data [Deineka, Danilkin et al 2111.15033]

+ expected conventional charmonia





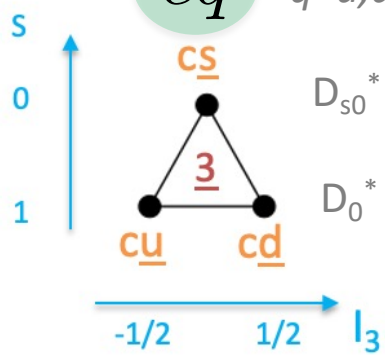
S.P. , Collins, Padmanath, Mohler, Piemonte

2011.02541 JHEP, 1905.03506 PRD, 2111.02934

Scalar heavy-light mesons $J^P = 0^+$

Conventional quark model

$c\bar{q}$ q=u,d,s



D_{s0}^*

D_0^*

new paradigm supported by:

- lattice
- effective models ChPT+HQET
- reanalysis of exp data
- states circled by blue seem to feature in the spectrum

Scattering on the lattice

S=1 Mohler et al, 1308.3175, PRL
Lang et al, 1403.8103, PRD
RQCD, 1706.01247, PRD
HadSpec 2008.06432, JHEP

S=0 Lang et al. 1208.4059, PRD
(see backup)
HadSpec, 1607.07093, JHEP
HadSpec 2102.04973, JHEP

S=-1 HadSpec, 2008.06432, JHEP

New paradigm

Du et al, 1712.07957, PRD
Albaladejo et al, 1610.06727, PLB
Lutz et al (2003), 0307133, PLB

$c\bar{q}$ + $c\bar{q}\ q\bar{q}$ q=u,d,s n=u,d

$\underline{3} \otimes 8 = \underline{3} \oplus 6 \oplus 15$ $SU(3)_F$

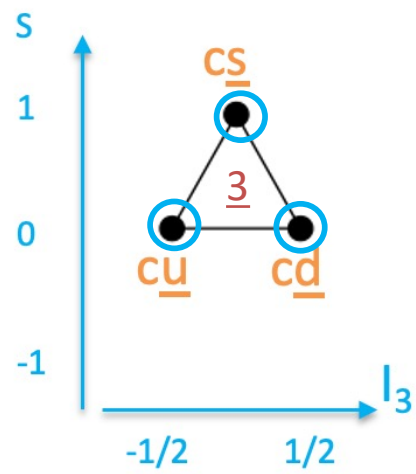
Beveren, Rupp; Dmitrasinovic

2.3 GeV

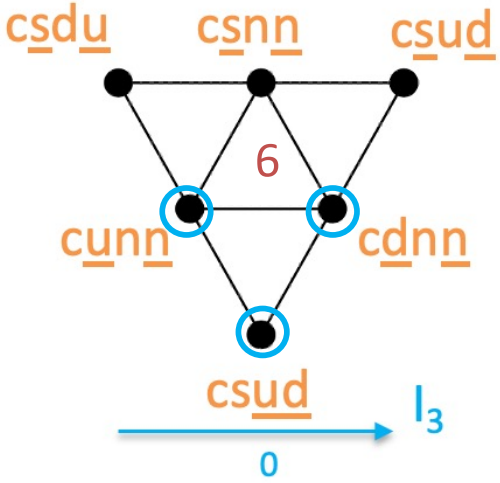
lat: 2.1-2.2 GeV (pole)

PDG: 2.3 GeV (BW)

(see backup)



mix



no state (mix with repulsive 15)

2.4-2.5 GeV
reanalysis of lat 1607.07093 by Albaladejo 1610.06727

virtual bound state
HadSpec 2008.06432
partner of X(2900) [LHCb 2009.00025] ?

Doubly heavy tetraquarks

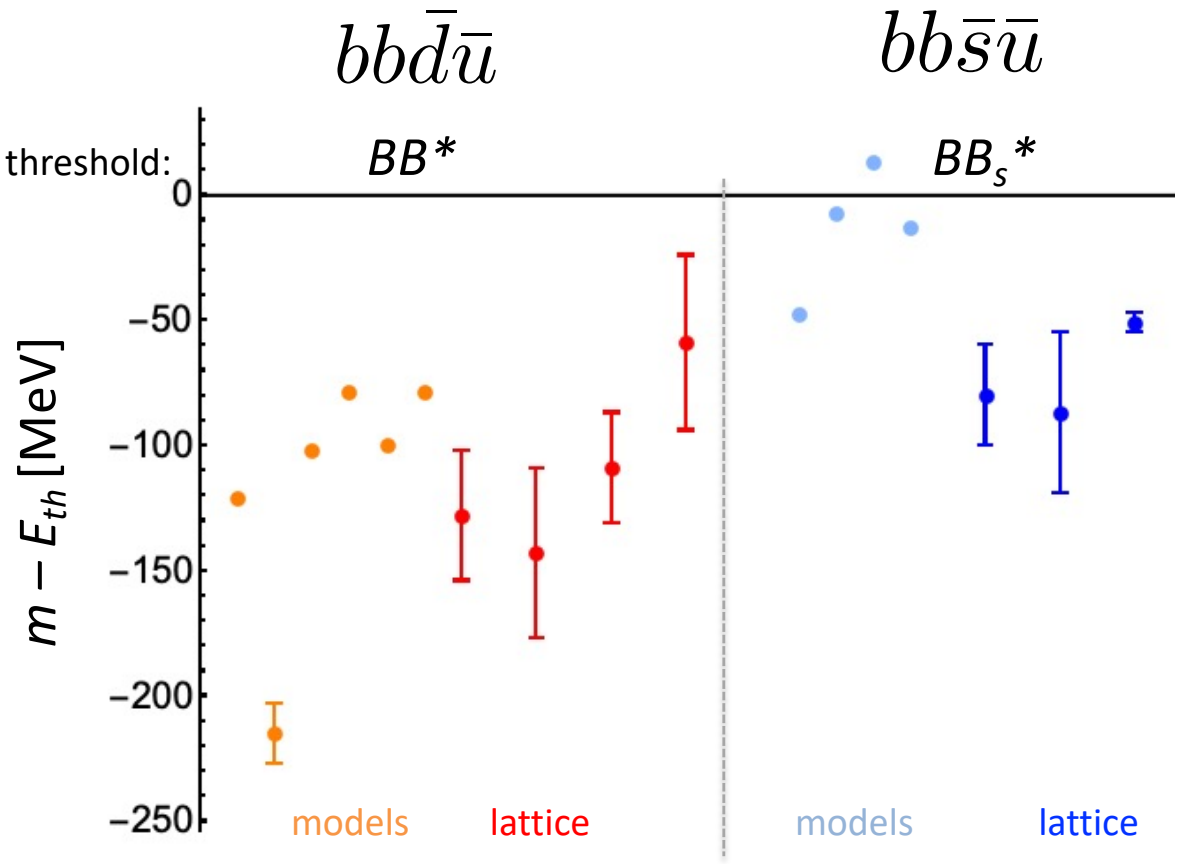
$$QQ'\bar{q}\bar{q}'$$

Q=c,b q=u,d,s

Doubly bottom tetraquarks

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

not found in exp, difficult to find



lattice	$m_{u/d}$	a [fm]
Leskovec , Pflaumer et al	$m_{u/d}$ -> $m_{u/d}^{phy}$	0.08-0.11
Junnarkar et al.	$m_{u/d}$ -> $m_{u/d}^{phy}$	a -> 0
Francis et al	$m_{u/d}$ -> $m_{u/d}^{phy}$	0.09
Bicudo et al.	$m_{u/d}$ -> $m_{u/d}^{phy}$	0.08

references from left to right

models (many more references):
Eichten and Quigg (2017) 1707.09575 PRL
Karliner and Rosner (2017) 1707.07666 PRL
Ebert et al. (2007) 0706.3853
Silvestre-Brac and Semay (1993)
Janc and Rosina (2004) hep-ph/0405208

lattice: most updated results
Leskovec, Meinel, Pflaumer, Wagner (2019) 1904.04197
Junnarkar, Mathur, Padmanth (2018) 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021) preliminary
Bicudo, Wagner et al. 1612.02758 static potentials

models (many more references)
Eichten and Quigg (2017) 1707.09575 PRL
Parket al. (2018) 1809.05257
Ebert et al. (2007) 0706.3853
Silvestre-Brac and Semay (1993)

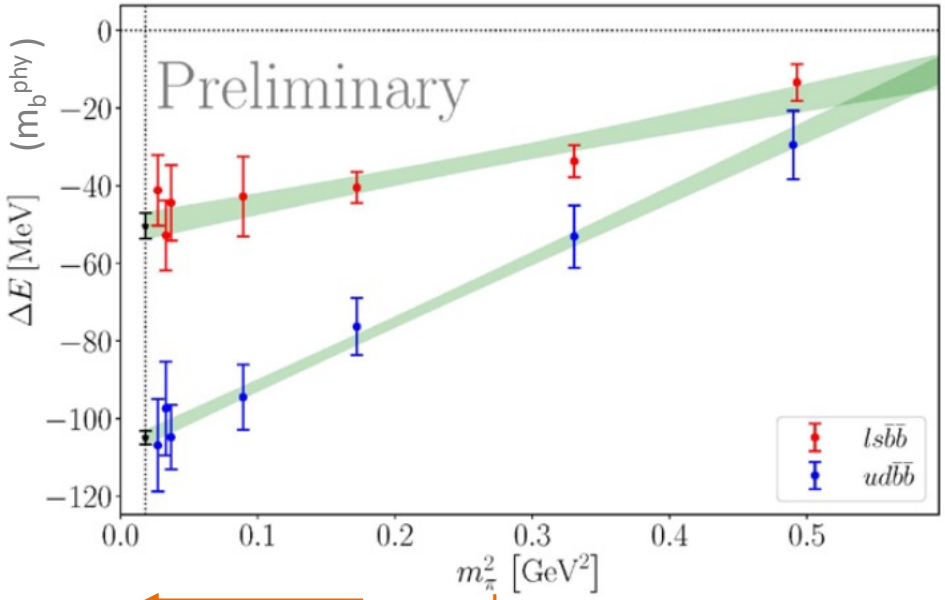
lattice: most updated results
Pflaumer, Leskovec,Meinel,Wagner (2021) 2108.10704
Junnarkar, Mathur, Padmanth (2018) 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

earlier results of Frances et al in 1810.10550, 1607.05214 PRL

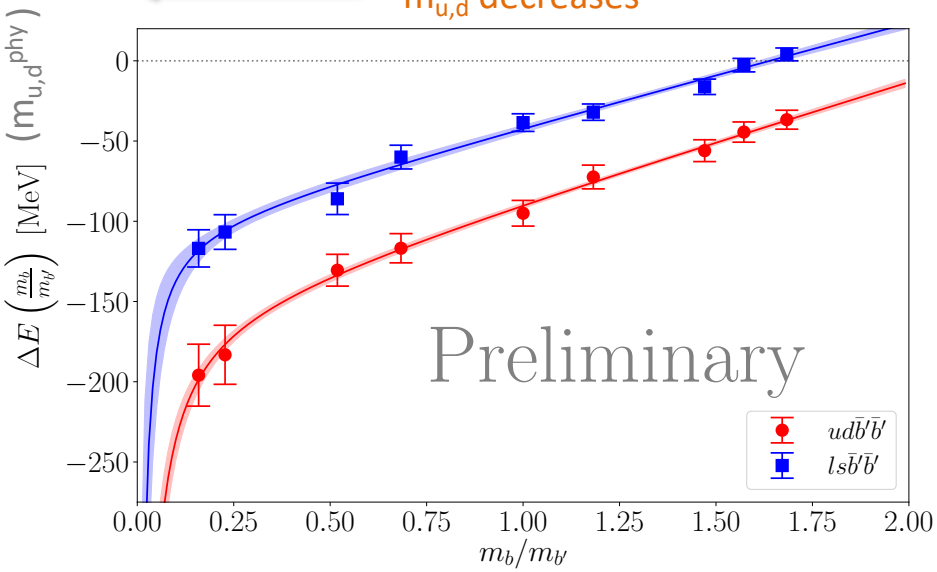
Doubly bottom tetraquarks

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



$m_{u,d}$ decreases



$m_{b'}$ increases

color Coulomb

strongly binds bb

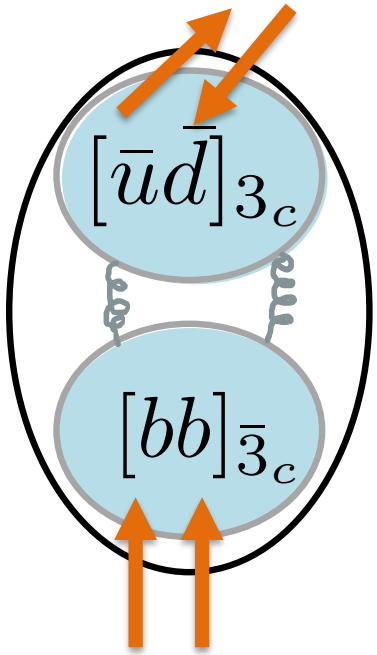
preliminary lattice results of Frances, Colquhoun, Hudspith, Maltman (2021)

@Lattice2021, Hadron2021

supports internal structure below

supported also by almost all model studes

Karliner and Rosner (2017), Janc and Rosina (2004), ...



$S^P = 0^+$

good diquark

$S^P = 1^+$

Other $QQ'\bar{q}\bar{q}'$ and J^P

No other state threo. found significantly (>40 MeV) below threshold

Two that could be near threshold

$bc\bar{d}\bar{u}$ Padmanath, Mathur 2111.01147 lattice energy extracted,

$cc\bar{d}\bar{u}$ Junnarkar, Mathur, Pad. 1810.12285 pole not established

States near or above threshold have to be identified as poles in scattering T(E): more challenging than to establish a state well below th.

Final cocnlusion not yet reached.

Other $QQ'\bar{q}\bar{q}'$ and J^P

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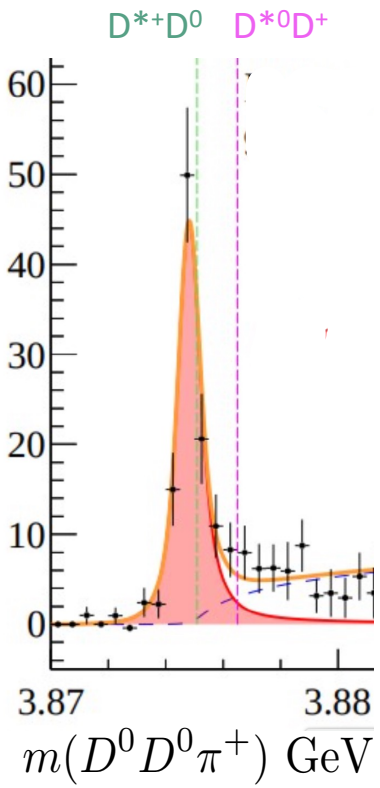
States near or above threshold have to be identified as poles in scattering T(E): more challenging than to establish a state well below th.

Final cocnlusion not yet reached.

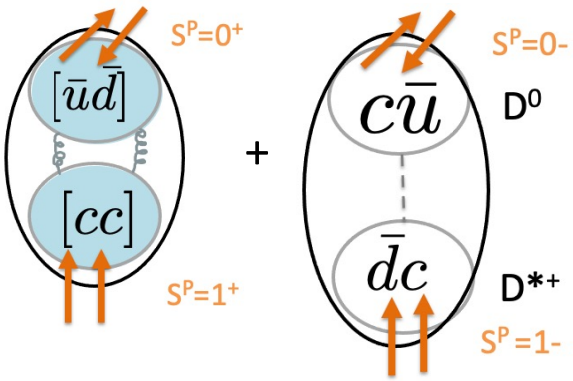
Doubly charm tetraquark T_{cc} $cc\bar{d}\bar{u}$



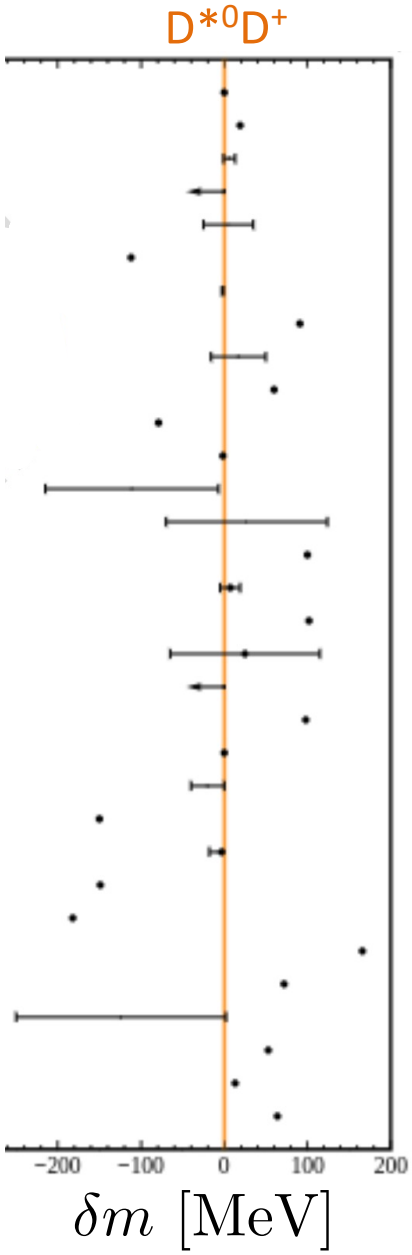
$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056



likely dominant



J. Carlson *et al.*	1987
B. Silvestre-Brac and C. Semay	1993
C. Semay and B. Silvestre-Brac	1994
S. Pepin *et al.*	1996
B. A. Gelman and S. Nussinov	2003
J. Vijande *et al.*	2003
D. Janc and M. Rosina	2004
F. Navarra *et al.*	2007
J. Vijande *et al.*	2007
D. Ebert *et al.*	2007
S. H. Lee and S. Yasui	2009
Y. Yang *et al.*	2009
G.-Q. Feng *et al.*	2013
Y. Ikeda *et al.*	2013
S.-Q. Luo *et al.*	2017
M. Karliner and J. Rosner	2017
E. J. Eichten and C. Quigg	2017
Z. G. Wang	2017
G. K. C. Cheung *et al.*	2017
W. Park *et al.*	2018
A. Francis *et al.*	2018
P. Junnarkar *et al.*	2018
C. Deng *et al.*	2018
M.-Z. Liu *et al.*	2019
G. Yang *et al.*	2019
Y. Tan *et al.*	2020
Q.-F. Lü *et al.*	2020
E. Braaten *et al.*	2020
D. Gao *et al.*	2020
J.-B. Cheng *et al.*	2020
S. Noh *et al.*	2021
R. N. Faustov *et al.*	2021

references at the back

Theoretical predictions for T_{cc} mass (I=0, $J^P=1^+$)

Doubly charm tetraquark

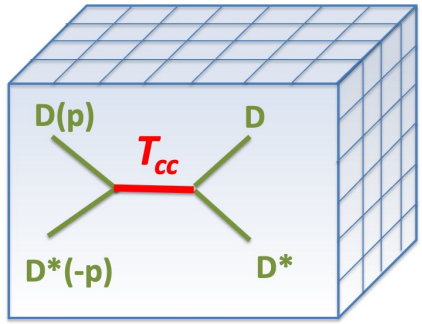
The only lattice study that extracts pole in T(E)

[Padmanath, S.P., S. Collins, preliminary 2022] , $m_\pi \approx 280$ MeV, CLS ens.

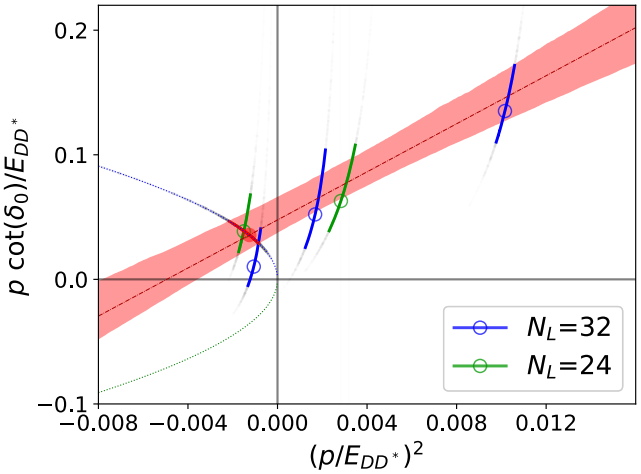
more details at the back $E_{DD^*} \equiv m_D + m_{D^*}$ *Preliminary*

$I=0,\ J^P=1^+$

$c c \bar{d} \bar{u}$



DD*



LHCb: T_{cc} : bound state p = i |p| in the limit of stable D*

T_{cc} : resonance - decays to DDπ, D* strongly decays

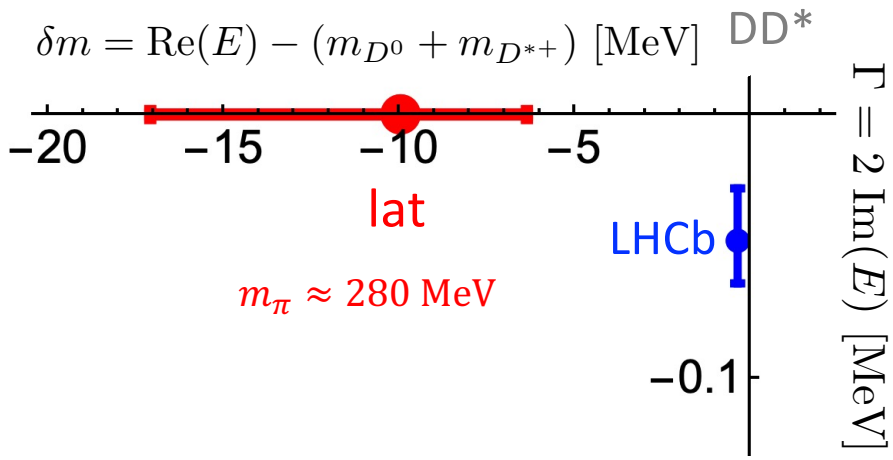
lat: T_{cc}: virtual bound state p = - i |p| for this m_π

$$E_B = -9.9\ ^{+3.6}_{-7.1}\ \text{MeV}$$

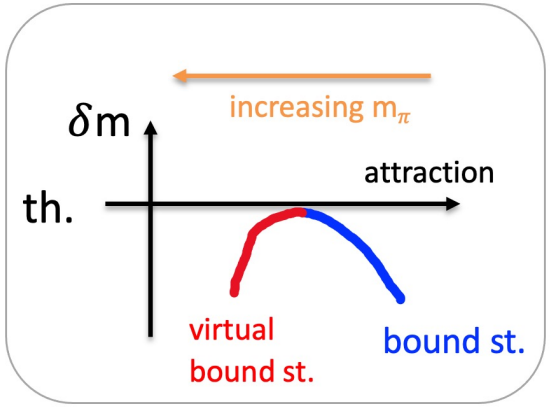
D* strongly stable for this m_π

For $m_\pi > m_\pi^{\text{phy}}$ the attraction is smaller and the LHCb bound state with δm=0.4 MeV is expected to become a virtual bound state. The lattice result confirms this.

● Pole in $T \propto (p\cot\delta_0 - ip)^{-1}$



sketch of expected behavior



Exotic hadrons with Q and $\underline{Q}$

Q=u,b

Hadrons $\bar{Q}Q\bar{q}q$, $\bar{Q}Qqqq$, $\bar{Q}Q\bar{Q}Q$

Q=c,b q=u,d,s

$\bar{b}b\bar{d}u$ $\bar{c}cuud$ $\bar{c}c\bar{c}c$

Z_b [Belle 2011] P_c [LHCb 2015] X(6900) [LHCb 2020]

GRRR!

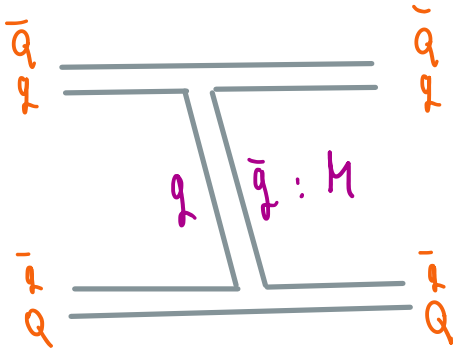
challenging for ab-initio study due to many decay channels:

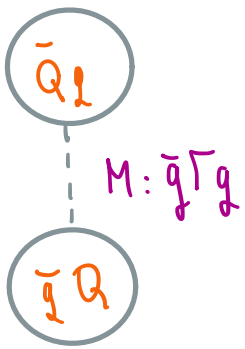
$$\bar{Q}Q\bar{q}q \to \bar{Q}q + \bar{q}Q, \quad \bar{Q}Q + \bar{q}q$$

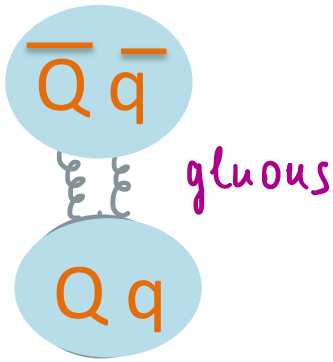
$$\bar{Q}Qqqq \to \bar{Q}q + Qqq, \quad \bar{Q}Q + qqq$$

Only partial conclusions are available from ab-initio approaches [reviewed e.g. in S.P. 2001.01767]
I'll discuss other approaches (also due to lack of time).

hadronic molecule **vs.** **diquark antidiquark**







quark spins correlated within mesons diquarks

Hadronic molecules

$(\bar{Q}q)(\bar{q}Q)$, $(\bar{Q}q)(Qqq)$

virtues:

- in line with most of exotics listed by Tamponi
- near thresholds
- interaction depends on isospin
- $\text{Br to } \bar{Q}q + \bar{q}Q \;:\; \text{large}$
  $\text{Br to } \bar{Q}Q + \bar{q}q \;:\; \text{small}$
- prevents to quick $\bar{Q}Q + \bar{q}q$

mesons exchanged (M):

- P = $\pi, ..$ L ∝ H $\partial\pi\, \gamma_5\, \bar{H}$
  V = $\rho, ..$

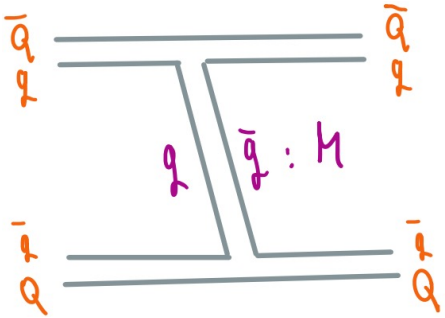
many many studies : Albaladejo, Baru, Guo, Hanhart, Oset, Meissner, Molina, Nieves et al; review 1705.00141

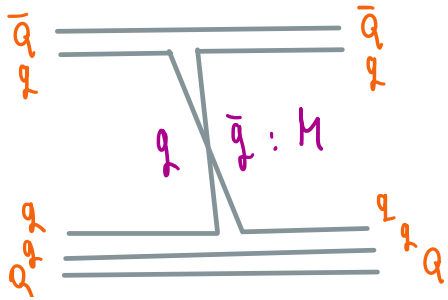
not natural candidates for molecules:

bb$\overline{ud}$, X(6900)=$\bar{c}c\bar{c}c$

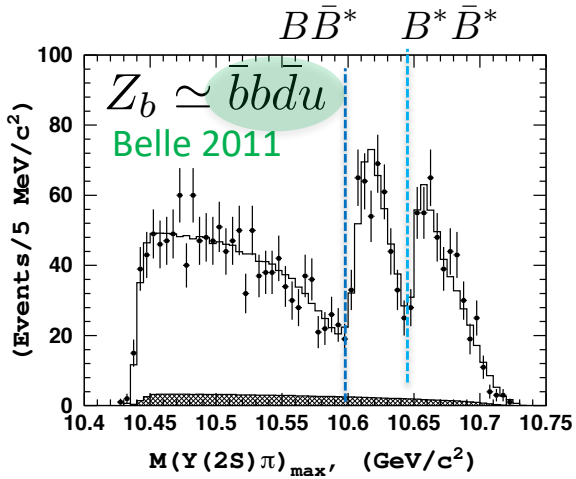
albeit see: Baru, Guo, Hanhart et al: 2107.03946, 2009.07795 PRL

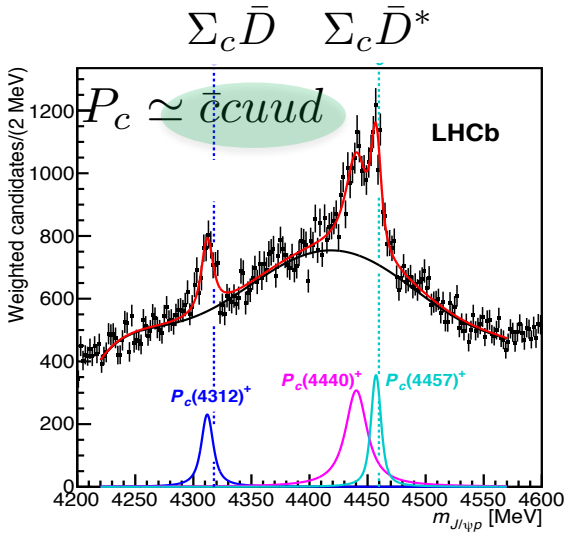
contact interactions +



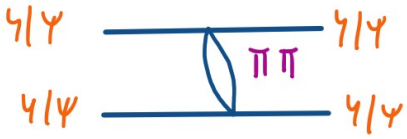


$\Sigma_c(\frac{1}{2}^+)\bar{D}(0^-) \to J^P = \frac{1}{2}^-$ $\quad \Sigma_c(\frac{1}{2}^+)\bar{D}^*(1^-) \to J^P = \frac{1}{2}^-, \frac{3}{2}^-$





Molina, Oset, et al. 1007.0573, PRL



A survey of molecules

$$\underbrace{(\bar{c}q)(\bar{q}c),\ (\bar{c}q)(cqq),}_{229\text{ molecules}}\quad \underbrace{(\bar{q}c)(\bar{q}c),\ (\bar{q}c)(cqq)}_{124\text{ molecules}}$$

q=u,d,s

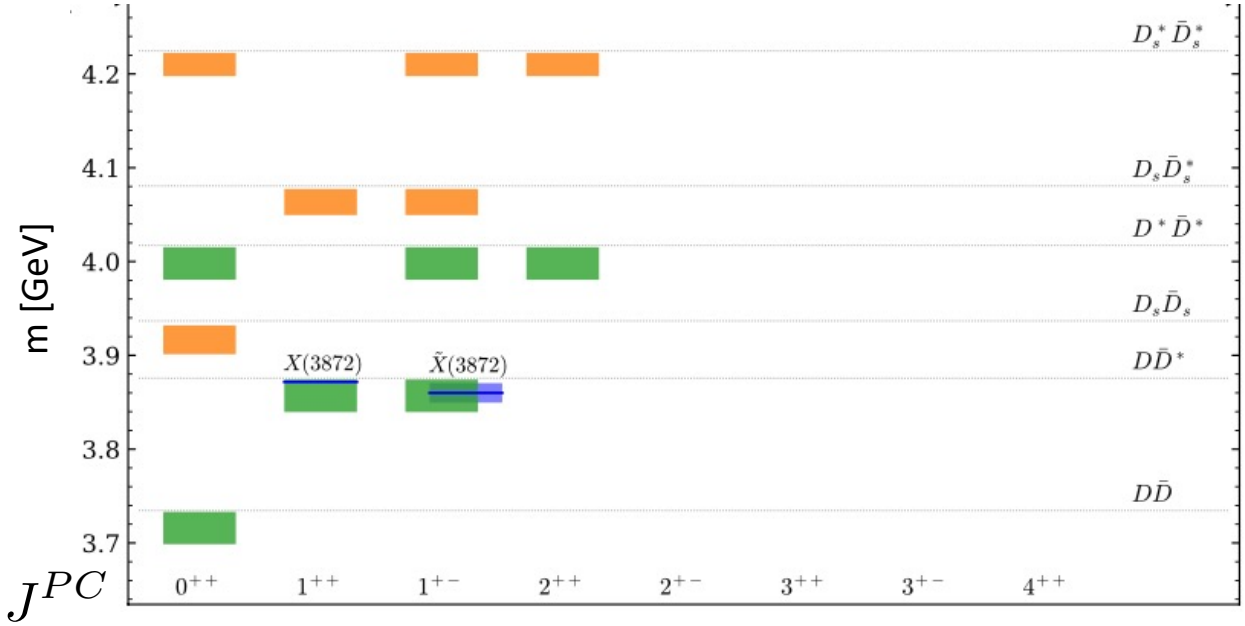
- systematic spectrum based on a single-model
- exchange of V mesons
- exchange of P omitted: subdominant near th. L ∝ H $\partial\pi\ \gamma_5\ \bar{H}$
- symmetries: ChPT, hidden local symmetry, heavy quark spin, $SU(3)_{fl}$
- no coupled-channel effects, no mixing with cc
- can provide only qualitative features

bound — virtual

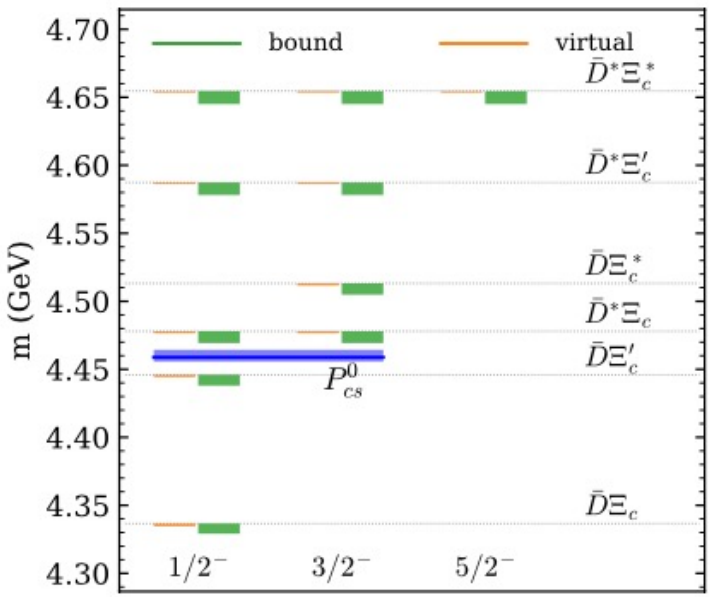
p = i |p| p = - i |p|

$$m_B = \sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2} < m_1+m_2$$

$$(\bar{c}q)(\bar{q}c),\ \ I=0, S=0$$



$$(\bar{c}u)(cds),\ \ I=0, S=1$$



Dong, F.-K. Guo, Zou, 2101.01021, 2108.02673

predictions for two values of cut-off

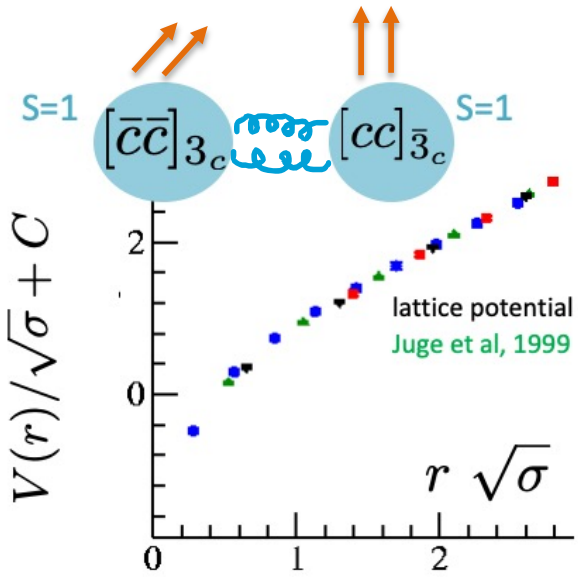
Hadrons from diquarks

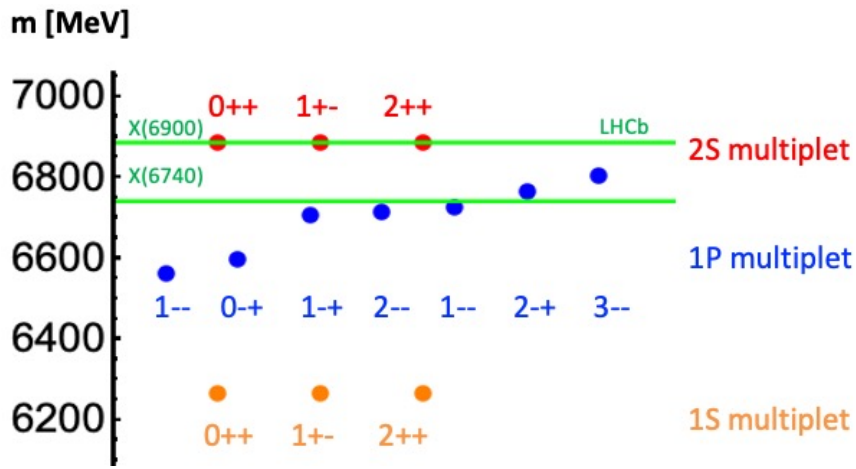
LHCb: 2006.16957

Giron, Lebed, 2008.01631 PRD X(6900) used to fix diquark mass

Bedolla, J. Ferretti, Roberts, Santopinto 1911.00960, EPJC

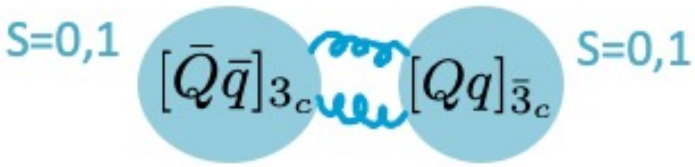
$\bar{c}\bar{c}cc$





Lattice QCD: no indication for strongly stable $\bar{b}b\bar{b}b$ [Hughes, Eichten, Davies, HPQCD, 1710.03236, PRD]

$\bar{Q}\bar{q}Qq$



many works by Jaffe, Maiani, Polosa, Riquer, Piccinini, Pilloni, Espositop, Lebed, Giron, Ferreti, Santopinto, ..

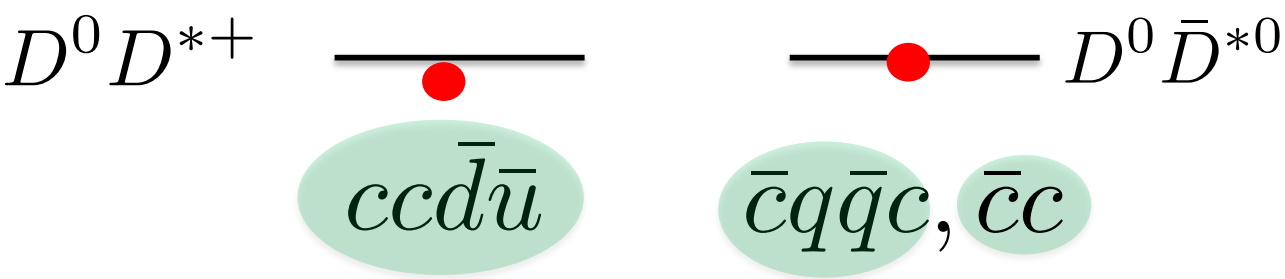
Explains several observed features.

Drawbacks:

- independent of isospin: same spectra for I=0 and 1
- sizable width to $(\bar{Q}Q)(\bar{q}q)$
- ground state J^P= 0+ (experimental states have J^P= 1+) maybe 0+ is to broad to be observed

Partial solutions to all these aspects were proposed

A puzzle comparing T_{cc} and $X(3872)$



Why both reside within 1 MeV of threshold in exp ? There are many differences ...

Similarities:

- $J^P=1^+$, I=0
- in molecular picture: attraction via light vector exchange [e.g. Guo et al. 2101.01021, 2108.02673]

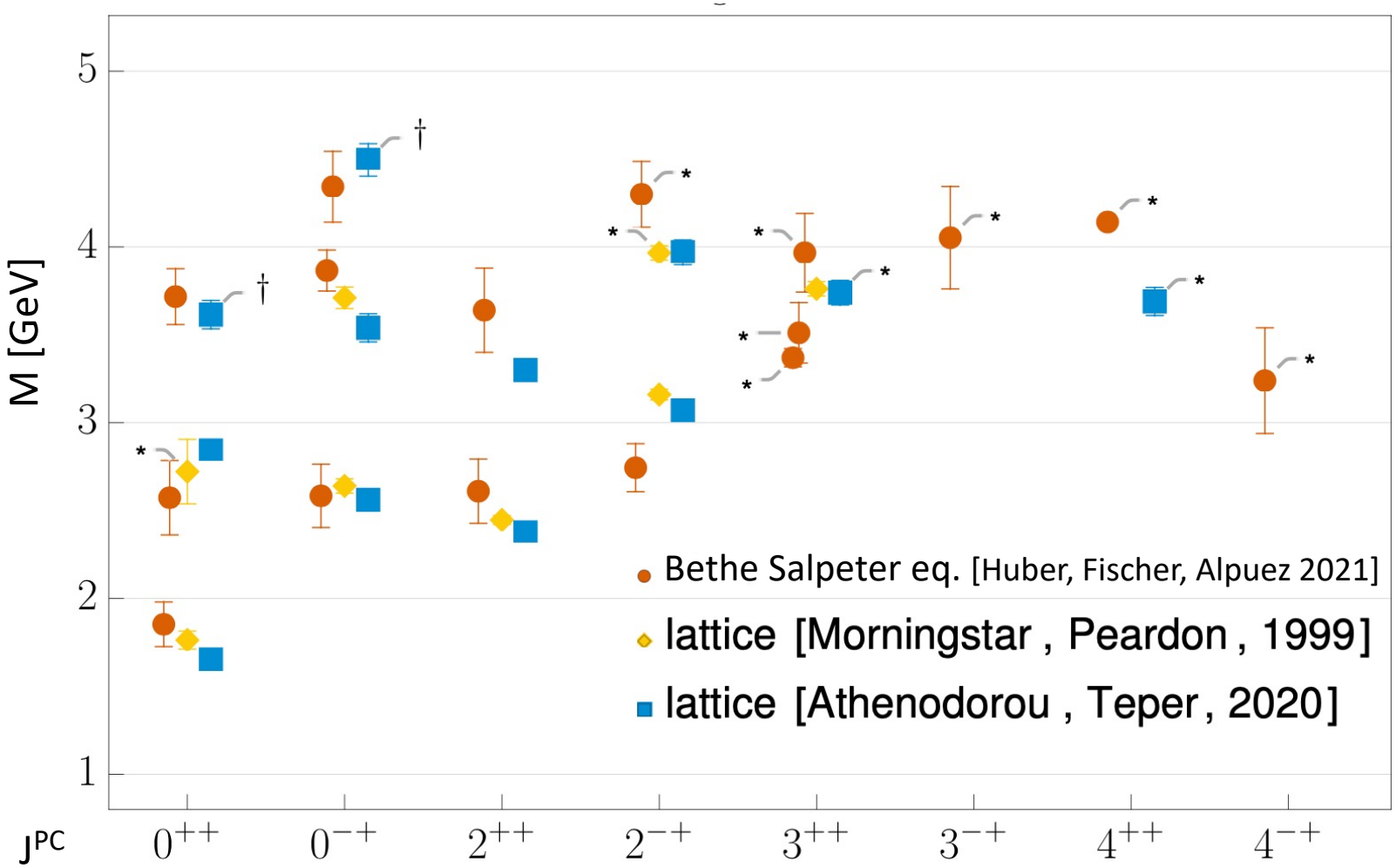
Differences:

- in molecular picture: attraction from one-pion exchange for X(3872) [Tornquist 1994]
  slight attraction from one-pion exchange for Tcc [eg. Du, Guo, Hanhart, 2110.13765]
- presence of Fock component $\underline{c}c$ for X(3872) [e.g. Padmanath, Lang, SP 1503.03257, PRD]
- presence of Fock component $[cc][\underline{ud}]$ for Tcc

Glueballs from QCD without sea-quark effects

gluons

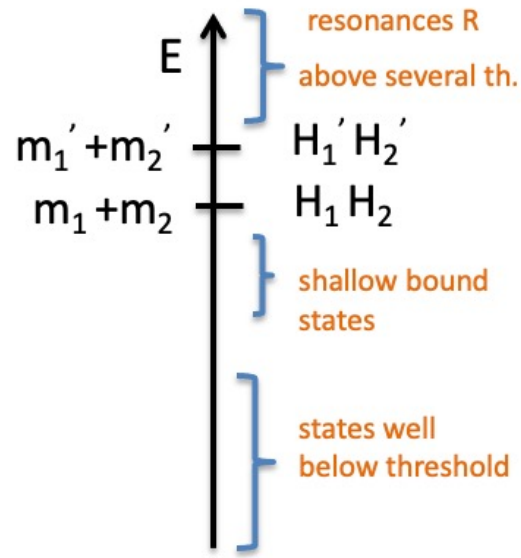
omitted to prevent glueballs from strong decay to mesons



2110.09180, EPJC

2007.06422, JHEP

Conclusions



Compliments to experiments for GREAT results !!

Status on hadron spectrum from Lattice QCD :

- strongly stable : "straighforward, done"
- strongly decaying to 1 channel "mostly done"
- 2-3 channels "challenging, some of them studied"
- > 3 channels "very challenging, mostly unexplored"
- P_c, Z_c, Z_b, $X(6900)$,... decay via many channels: reason why lattice has only partial conclusions on those
- $R \to H_1 H_2 H_3$ "very difficult" [formalism by Sharpe, Hansen, Briceno, Doring, Mai, Rusetsky, Lopez et. al]

  $a_1 \to \pi\pi\pi$ [GWU, 2112.03355] , $\pi\pi\pi$ [several groups], KKK [GWU]
- why $m_{u,d} > m_{u,d}^{phy}$? Smaller number of decay channels, smaller statistical errors on E and T(E)

Status on hadron spectrum from Bethe-Sapleter + Dyson-Schwinger equations :

strongly stable : "mostly done"

strongly decaying to 1 channel : "few resonances done over past few years"

> 2 channels: "not done yet"

Exotic hadrons: one picture can not explain all exotic hadron states; for each exotic hadron there is at least one viable picture

Theory predicts many conventional and exotic hadrons yet to be discovered

Thanks to friends and collagues

M. Padmanath, Marek Karliner , Feng-Kun Guo, Richard Lebed, Anthony Frances, Sara Collins, Mark Wagner, Luka Leskovec, Elena Sanotpinto, Christian Fischer, Sinead Ryan, Michael Doring

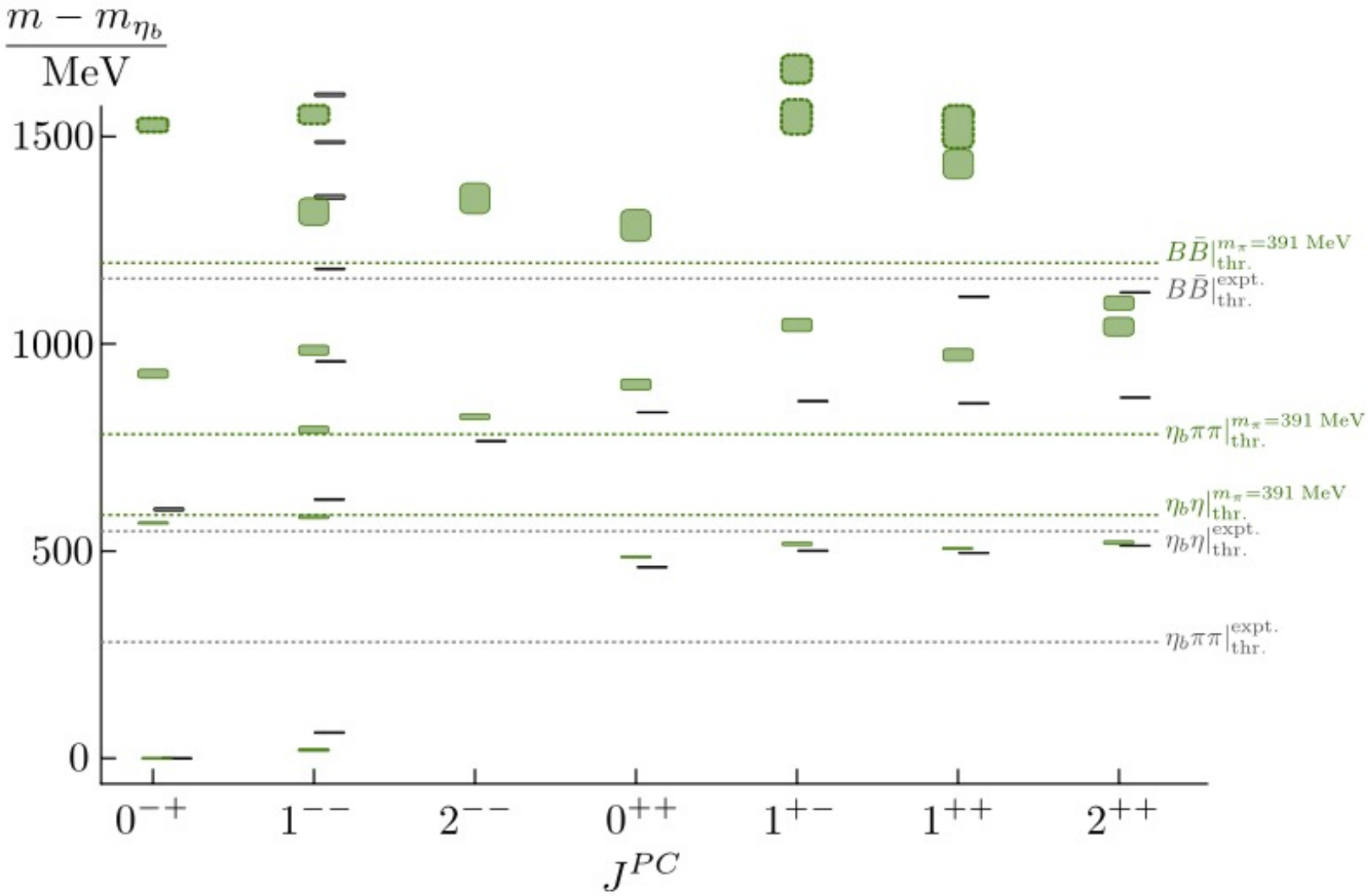
Appologies

... many interesting and relevant results could not be presented

backup

bottomonia

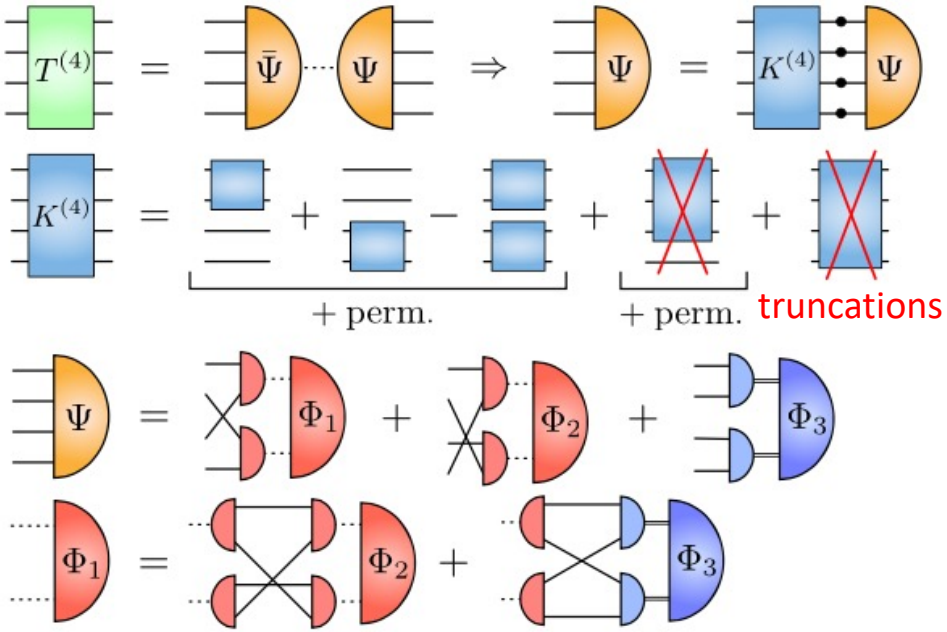
Ryan & Wilson (HadSpec)
2008.02656, JHEP



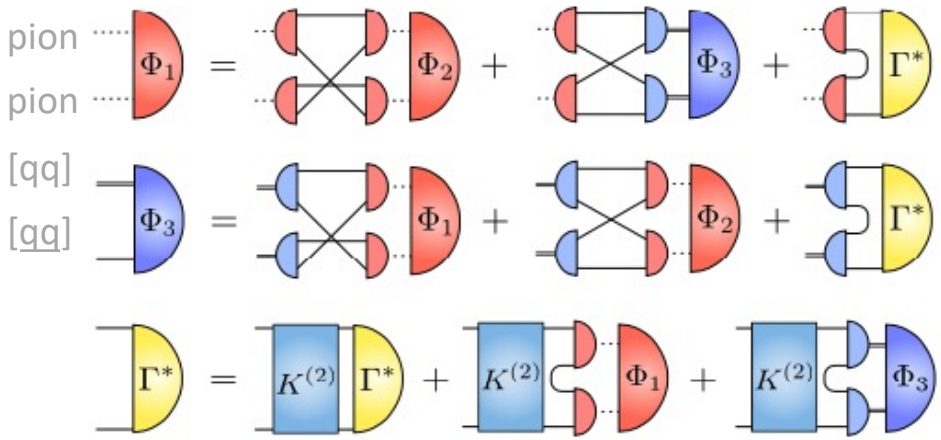
green: lat

black:exp

Sigma resonance with Bethe-Salpeter approach



Santowsky, Eichmann, Fischer, Wallbott, Williams, 2007.06495, PRD



Scalar D_0^* resonance

Puzzle: why strange and non-strage scalars mesons almost degenerate
$m[D_0^*(2300)] \approx m[D_{s0}^*(2317)]$

Mass of the lowest scalar charmed resonance (it is broad due to decay to $D\pi$)

Mass from BW fit [PDG]

$D_0^*(2300)$ MASS	2343 ± 10 MeV (S = 1.5)
$D_0^*(2300)$ WIDTH	229 ± 16 MeV

Mass from pole (lattice 2102.04973 , 1208.4059 and effective field theories) give lower value m=2.1-2.2 GeV. This makes D0* in more natural partner of $D_{s0}^*(2317)$

Comment on the D0* mass from the lattice simulation 2102.04973
The BW fit at real energies rendered m≈2.32 GeV, while the corresponding pole in the complex plan based on the same lattice data is at m ≈ 2.12 GeV.

It is not surprising that mass from BW fit and the pole differ so much in this case, since this resonance is broad and not very high above the threshold

Lattice study of a doubly charm tetraquark

$cc\bar{d}\bar{u}$ $\quad I=0,\ J^P=1^+$

[Padmanath, S.P., S. Collins, preliminary 2022] $E_{DD^*} \equiv m_D + m_{D^*}$

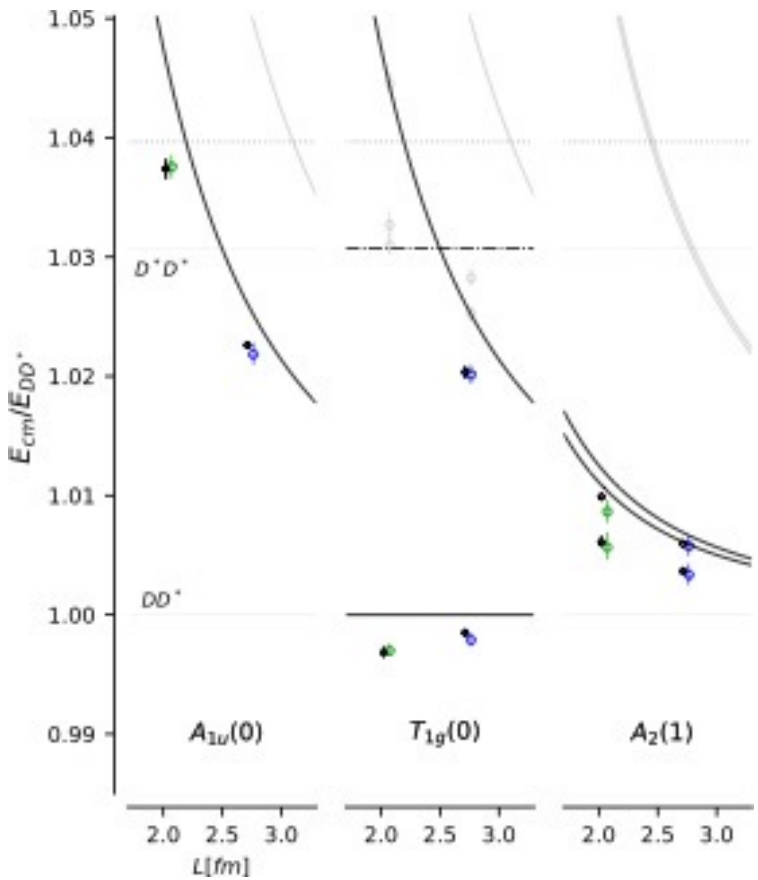
m_π [MeV]	m_K [MeV]	m_D [MeV]	m_{D^*} [MeV]
280(3)	467(2)	1927(2)	2050(2)

Preliminary

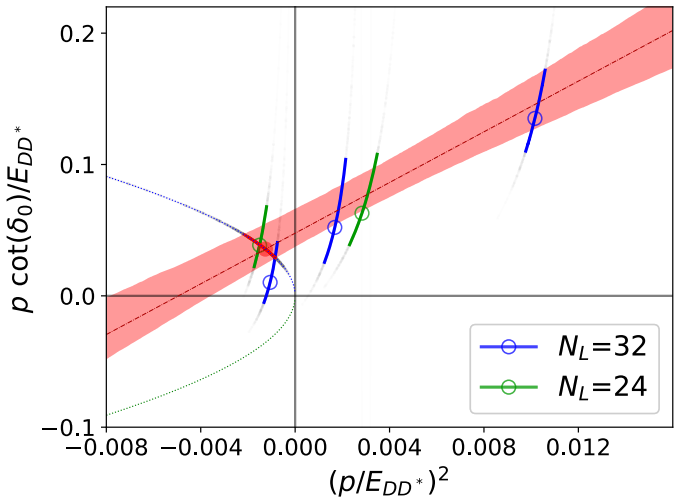
CLS ens: $m_{u,d} > m_{u,d}^{phy},\ m_s < m_s^{phy}$

$$\mathcal{O} = \sum_i D(\vec{p}_{1i}) D^*(\vec{p}_{2i})$$

P_{tot} =0 $\quad P_{tot}$ =0 $\quad P_{tot}$ =1 $2\pi/L$



Results for partial wave L=0



$$T \propto (p\cot\delta_0 - ip)^{-1}$$

$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = 1.04 \pm 0.29 \text{ fm}\ ,\ r_0 = 0.96 \pm 0.19 \text{ fm}$$

$$E_B = -9.9\ ^{+3.6}_{-7.1}\ \text{MeV}$$

Lattice eigen-energies used in fit of T(E) using L=0,1: blue, green circles

Analytic energies based on extracted T(E): black circles

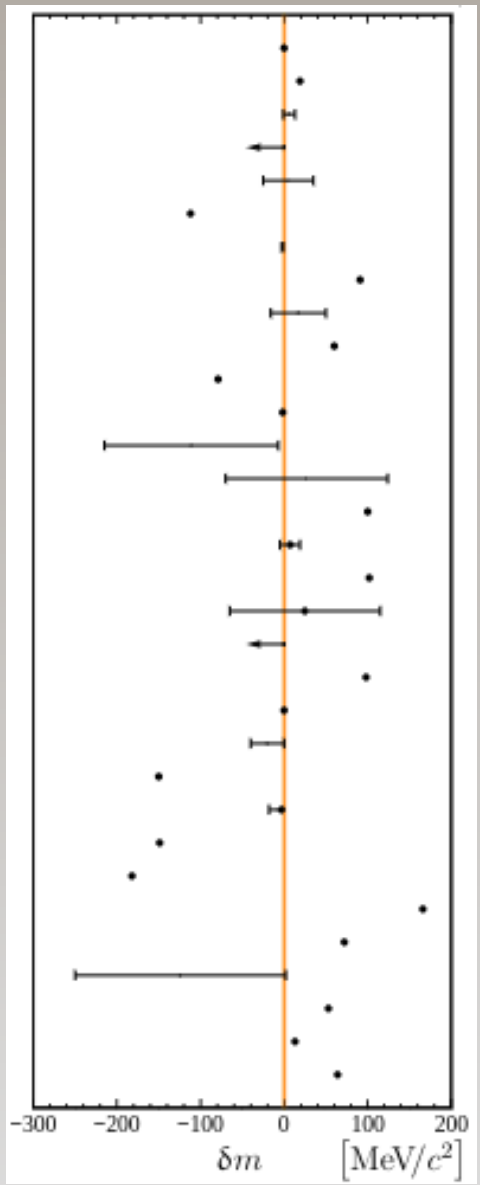
Non-interacting energies: black lines

Lattice energies not used in the fit of T(E): gray circles

Tcc theory predictions (taken from Ivan Polyakov, EPS meeting)

Theory predictions

Reference		Year	$\delta' m$ [MeV/c^2]
J. Carlson, L. Heller and J. A. Tjon	36	1987	~ 0
B. Silvestre-Brac and C. Semay	37	1993	$+19$
C. Semay and B. Silvestre-Brac	38	1994	$[-1, +13]$
S. Pepin, F. Stancu, M. Genovese and J. M. Richard	39	1996	< 0
B. A. Gelman and S. Nussinov	40	2002	$[-25, +35]$
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	41	2003	-112
D. Janc and M. Rosina	42	2004	$[-3, -1]$
F. Navarra, M. Nielsen and S. H. Lee	43	2007	$+91$
J. Vijande, E. Weissman, A. Valcarce	44	2007	$[-16, +50]$
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	45	2007	$+60$
S. H. Lee and S. Yasui	46	2009	-79
Y. Yang, C. Deng, J. Ping and T. Goldman	47	2009	-1.8
G.-Q. Feng, X.-H. Guo and B.-S. Zou	48	2013	-215
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	49	2013	$[-70, +124]$
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	50	2017	$+100$
M. Karliner and J. Rosner	51	2017	$7 \pm 12 \rightarrow 1$
E. J. Eichten and C. Quigg	52	2017	$+102$
Z. G. Wang	53	2017	$+25 \pm 90$
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	54	2017	$\lesssim 0$
W. Park, S. Noh and S. H. Lee	55	2018	$+98$
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	56	2018	~ 0
P. Junnarkar, N. Mathur and M. Padmanath	57	2018	$[-40, 0]$
C. Deng, H. Chen and J. Ping	58	2018	-150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	59	2019	-3^{+4}_{-15}
G. Yang, J. Ping and J. Segovia	60	2019	-149
Y. Tan, W. Lu and J. Ping	61	2020	-182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	62	2020	$+166$
E. Braaten, L.-P. He and A. Mohapatra	63	2020	$+72$
D. Gao , D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	64	2020	$[-250, +2]$
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	65	2020	$+53$
S. Noh, W. Park and S. H. Lee	66	2021	$+13$
R. N. Faustov, V. O. Galkin and E. M. Savchenko	67	2021	$+64$



Ivan Polyakov, Syracuse University 17

Tcc theory predictions (taken from Ivan Polyakov, EPS meeting)

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Charmonium resonances from lattice

$$\Gamma \equiv g^2 p_D^{2l+1}/m^2$$

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2011.02541 JHEP, 1905.03506 PRD, 2111.02934