Charmonium(like) and charmed states from lattice QCD

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in collaboration with

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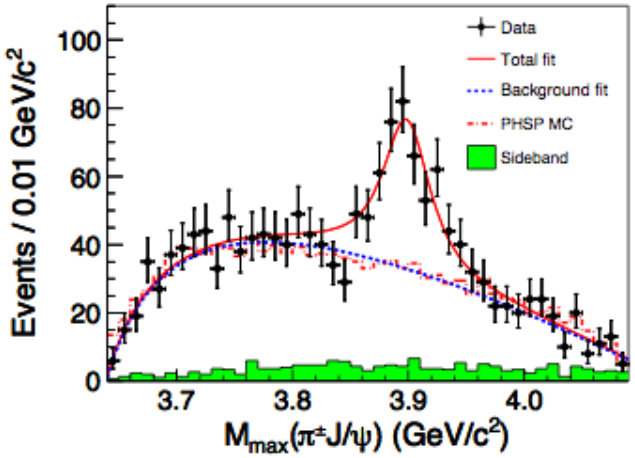
FERMILAB Graz Ljubljana Vancouver

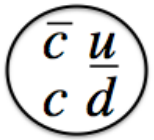
Renewed motivation: exotics from BESIII



[BESIII, 2013, arXiv:1303.5949, PRL]

$Z_c^+(3900) \rightarrow J/\Psi\ \pi^+$

$\underline{c}c\ \underline{d}u$



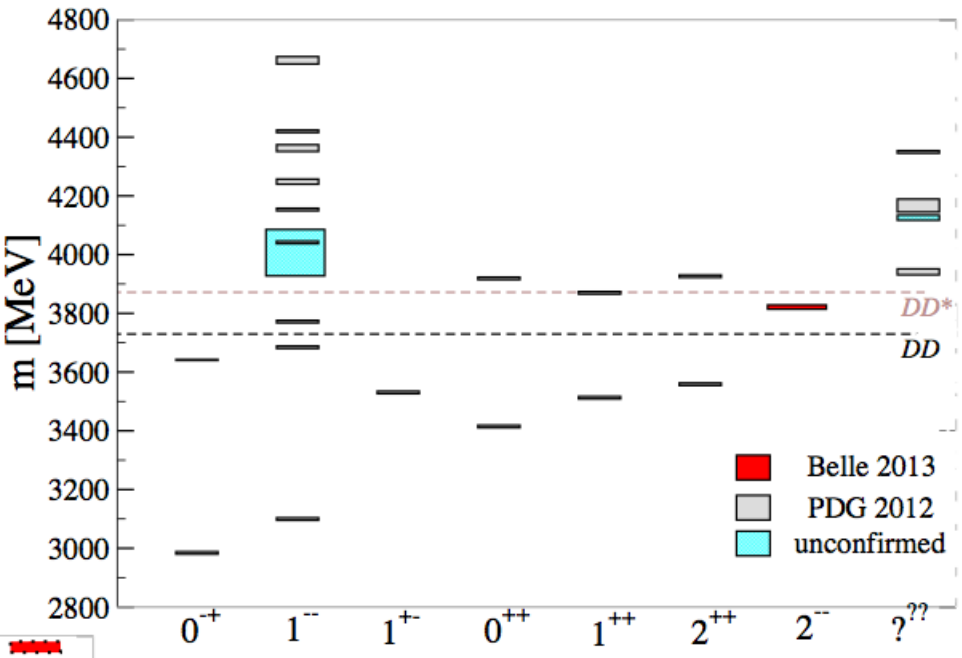
particle	decay	year	coll
$Z^+(4430)$	$\psi(2S)\ \pi^+$	2008	Belle, ~~BABAR~~
$Z^+(4050)$, $Z^+(4250)$	$\chi_{c1}\ \pi^+$	2008	Belle, unconfirmed
$Z_c^+(3900)$	$J/\psi\ \pi^+$	2013	BESIII, Belle, CLEOc
$Z_c^+(3885)$	$(D\ D^*)^+$	2013	BESIII
$Z_c^+(4020)$	$h_c(1P)\ \pi^+$	2013	BESIII
$Z_c^+(4025)$	$(D^*\ D^*)^+$	2013	BES III

same ?

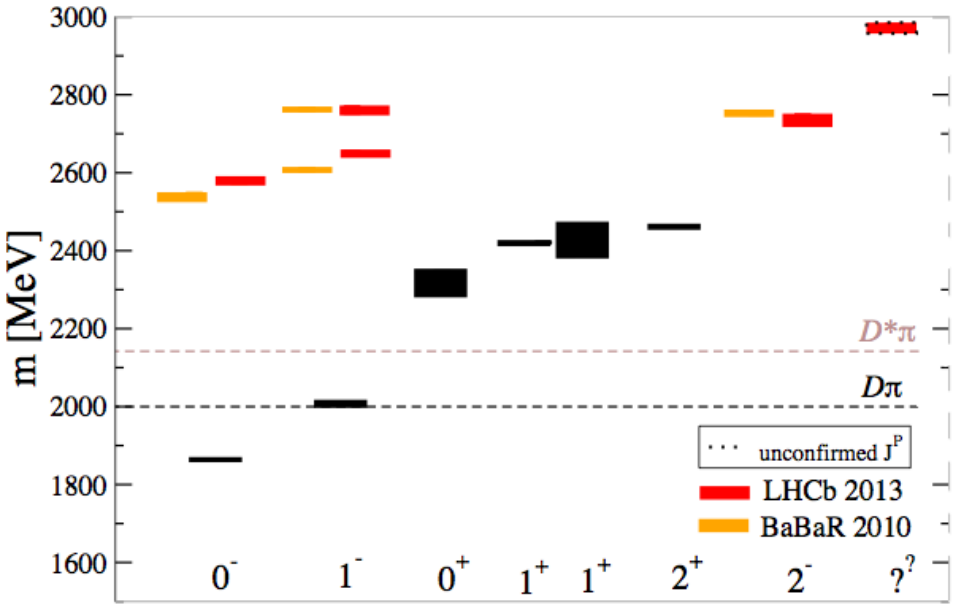
same ?

Experimental status

charmonium (like)



D-mesons



cc 2^{--}: Belle, 1304.3975, PRL

D : LHCb, 1307.4556

Lattice status – general overview: D, D_s , $\underline{c}c$

- States well below strong decay threshold:

  proper treatment & precision calculations already available for some time

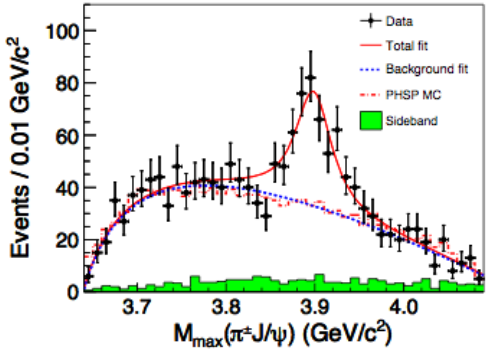
- States near threshold and resonances above threshold:

  ★ until 2012: **single-meson approximation**: - effect of threshold not taken into account
  - strong decays of states ignored

  exception: [Bali, Ehmann, Collins, 2011]

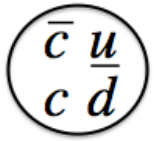
  ★ 2012, 2013, ...: first exploratory simulations with **rigorous treatment**

Charged charmonium-like and bottomonium–like states happen to lie near thresolds



[BESIII, 2013, arXiv:1303.5949]

Strong motivation to treat near-threshold state properly on the lattice



$Z_c^+(3900) \to J/\Psi\ \pi^+$

$\underline{c}c\ \ \underline{d}u$

particle	decay	year	coll	near th.
$Z^+(4430)$	$\psi(2S)\ \pi^+$	2008	Belle, ~~BABAR~~	$D^*\ D_1$
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$Z_c^+(4020)$	$h_c(1P)\ \pi^+$	2013	BESIII	D^*D^*
$Z_c^+(4025)$	$(D^*\ D^*)^+$	2013	BES III	D^*D^*

Overview

"pedestrian" review:
S. P., 1310.4354
plenary @ CHARM 13

Spectrum of $\underline{c}c$(like), D, D_s states from lattice QCD:

- States well bellow threshold

- Excited states:

  ★ single-meson approximation

  ★ rigorous treatment:

  (1) states near threshold

  (2) search for exotic states

  (3) resonances (above threshold)

  ★ indirect method & EFT

Non-perturbative method: QCD on lattice

$$L_{QCD} = -\tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s,c,b,t} \bar{q} i\gamma_\mu (\partial^\mu + i g_s G^\mu_a T^a) q - m_q \bar{q} q$$

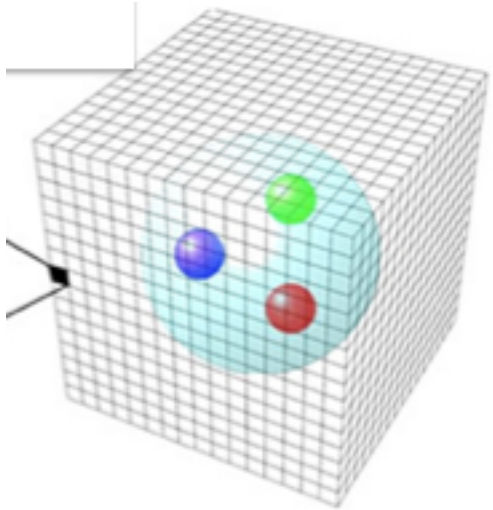
input : g_s , m_s^{fiz} , m_c^{fiz} , $m_{u,d} = 3.6 \times m_{u,d}^{fiz}$

$m_\pi = 266$ MeV , $m_\pi^{fiz} = 140$ MeV

output : hadron properties

hadron interactions (if we are lucky)

a



$V = N_L^3 \times N_T = 16^3 \times 32$

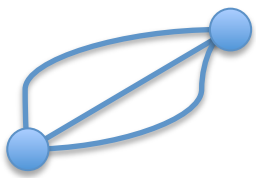
$a = 0.12$ fm

(for results shown)

Evaluation of Feynman path integrals in discretized space-time

quantum m.

$$\int Dx \, e^{i S/\hbar}$$



$S = \int dt \, L[x(t)]$

quantum field theory

$$\int DG \, Dq \, D\bar{q} \, e^{i S_{QCD}/\hbar}$$

$$S_{QCD} = \int d^4x \, L_{QCD}[G(x), q(x), \bar{q}(x)]$$

Discrete energy spectrum from correlators

$$\langle C \rangle \propto \int DG\, Dq\, D\bar{q}\; C(q,\bar{q},G)\, e^{i S_{QCD}/\hbar}, \quad S_{QCD} = \int d^4x\, L_{QCD}$$

Example: meson channel with given J^{PC}

$$\mathcal{O} = \bar{q}\Gamma q, \quad \bar{q}\Gamma' q, \quad (\bar{q}\Gamma_1 q)(\bar{q}\Gamma_2 q), \ldots$$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\; \mathcal{O}_j^{+}(0) | 0 \rangle$$
$$= \sum_n \langle 0 | \mathcal{O}_i | n \rangle\; e^{-E_n t}\, \langle n | \mathcal{O}_j^{+} | 0 \rangle = \sum_n A_{ij}^n\, e^{-E_n t}$$

All physical states appear as energy levels E_n in principle : single particle, two-particle,...

examples:

$J^{PC} = 0^{-+}, I = 1: \quad \pi,\ \pi(1400), \pi\pi\pi$

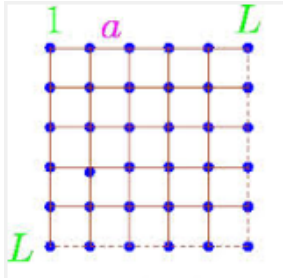
$J^{PC} = 1^{--}, I = 1: \quad \rho,\ \rho(1450),\ \pi\pi$

$J^{PC} = 1^{++},\ \bar{c}c: \quad \chi_{c1},\ X(3872),\ DD^*$

$J^{PC} = 1^{+-},\ \bar{c}c\bar{d}u: \ Z_c^{+}(3900),\ J/\psi\pi^{+},\ DD^*$

"Precision" spectrum:
States well below strong decay threshold

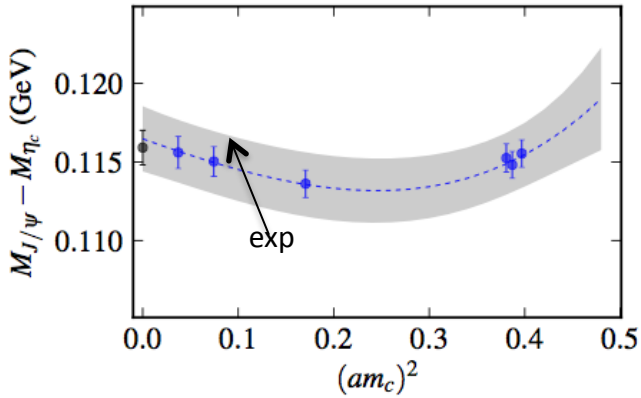
States well below threshold



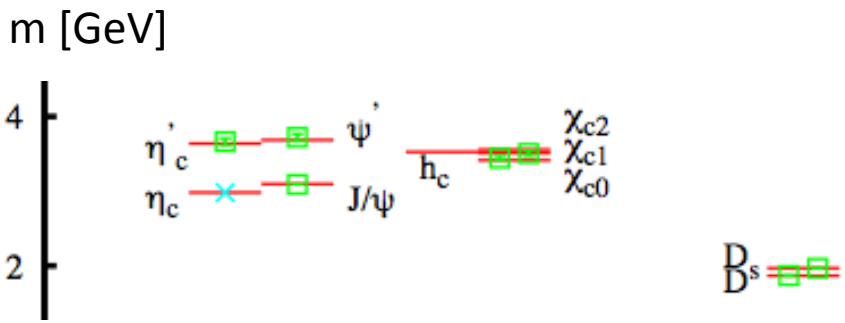
Lattice QCD already determined masses of these states very reliably and precisely O(10 MeV):

- $\boxed{m=E \quad (\text{for } P=0)}$
- extrapolation : $a \to 0$, $L \to \infty$
- extrapolation or interpolation : $m_q \to m_q^{phy}$
- particular care needed for am_c discretization errors: several complementary methods give compatible results

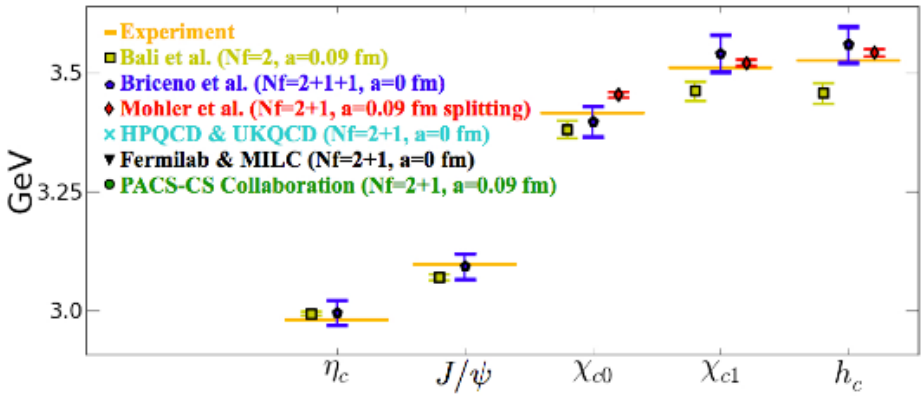
[HPQCD: 1208.2855, PRD]



[HPQCD: 1207.5149, PRD]



[Briceno, Lin, Bolton,1207.3536, PRD]



"Non-precision" spectrum: excited states

only one or two a, L, $m_{u/d}$

limits $a \to 0$, $L \to \infty$, $m_{u/d} \to m_{u/d}^{phy}$ usually not performed

Excited states: single-meson approximation

- only interpolating fields $\mathcal{O} \approx \bar{q}\, q$
- assumptions: all energy levels correspond to "one-particle" states

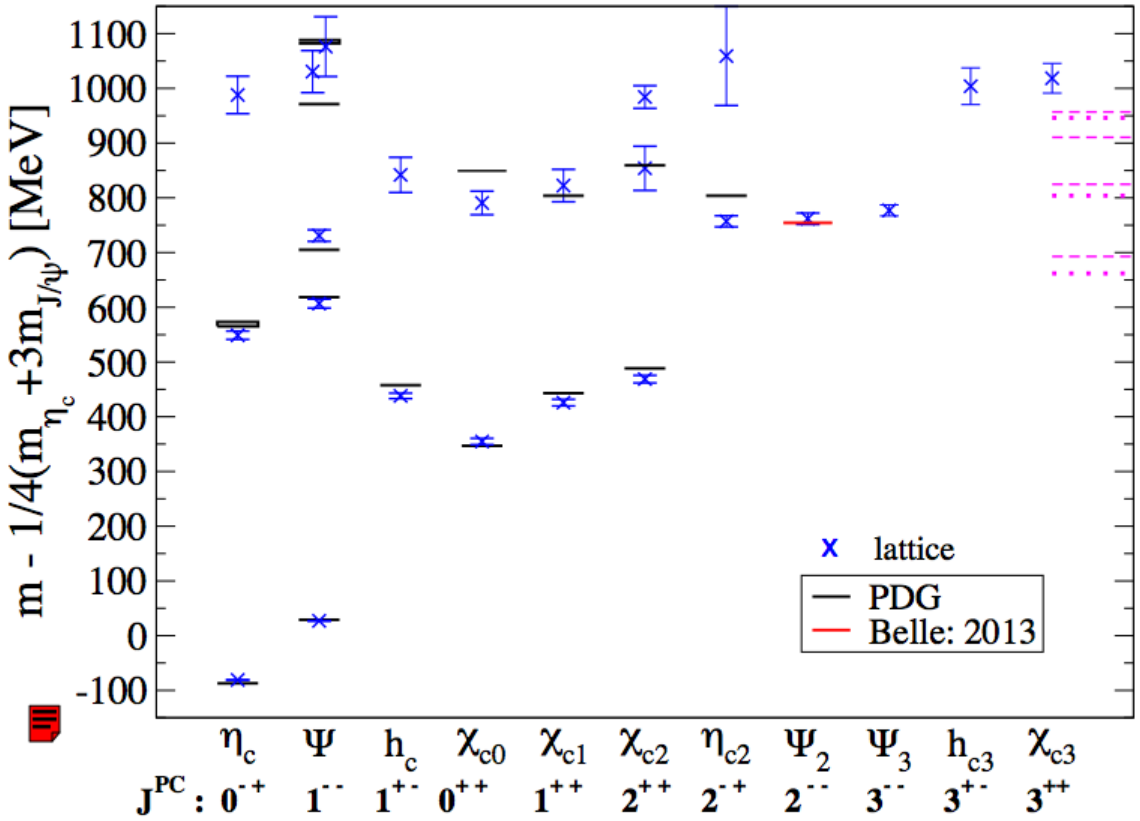
  none of the levels corresponds to multi-particle state

  m=E (for P=0)

  these are strong assumptions ...

$\bar{c}c$ spectrum: single meson approx.

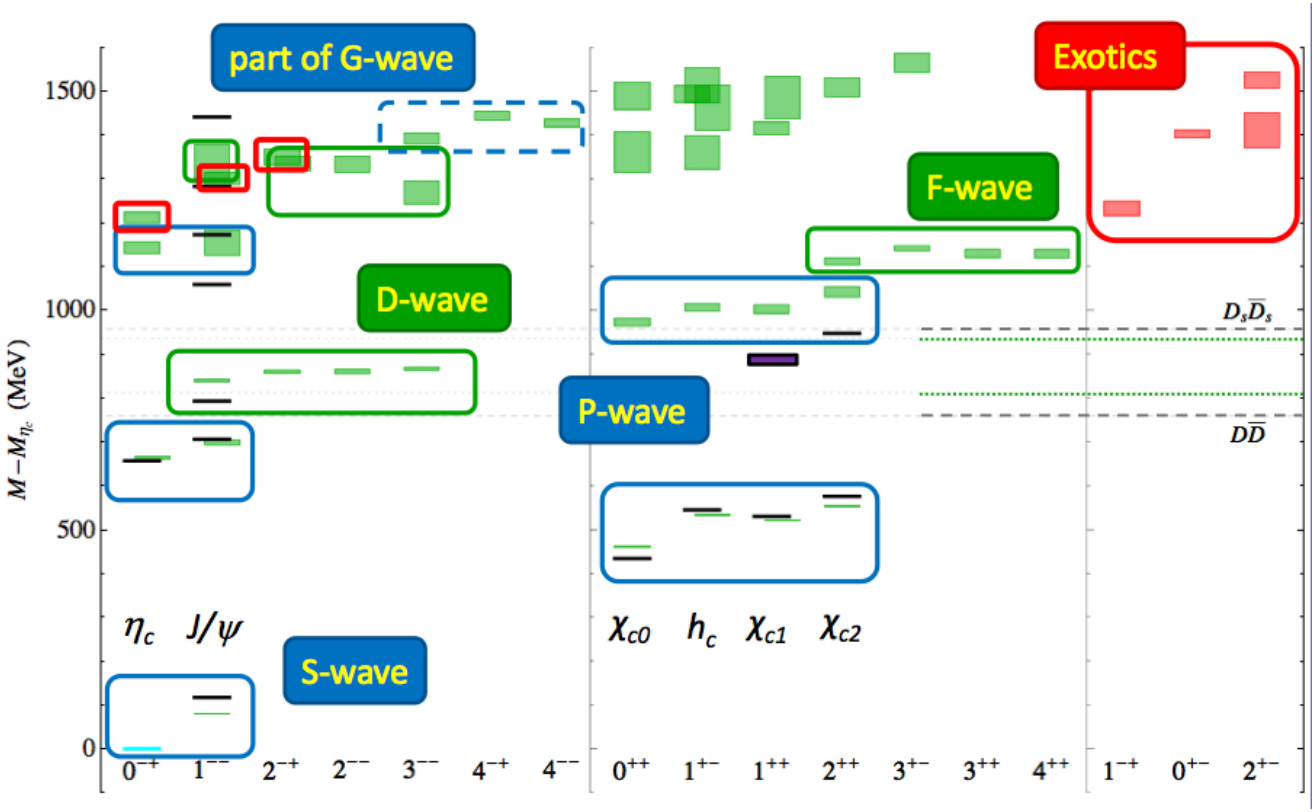
$m-m_{ref}$ compared between lat and exp in order to cancel leading am_c discretization effects



D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD:

- $m_\pi \approx 266$ MeV, $L \approx 2$ fm, Nf=2
- crosses: naive lat, diamonds: rigorous lat, lines & boxes: exp

$\underline{c}c$ spectrum: single meson approx.



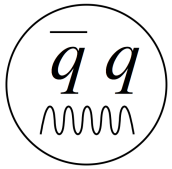
HSC , L. Liu et al: 1204.5425, JHEP:

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, Nf=2+1
- reliable J^{PC} determination
- identification with $n\ ^{2S+1}L_J$ multiplets using $\langle O|n\rangle$
- green: lat, black: exp

Hybrids:

some of them have exotic J^{PC}

large overlap with $O = \underline{q}\ F_{ij}\ q$

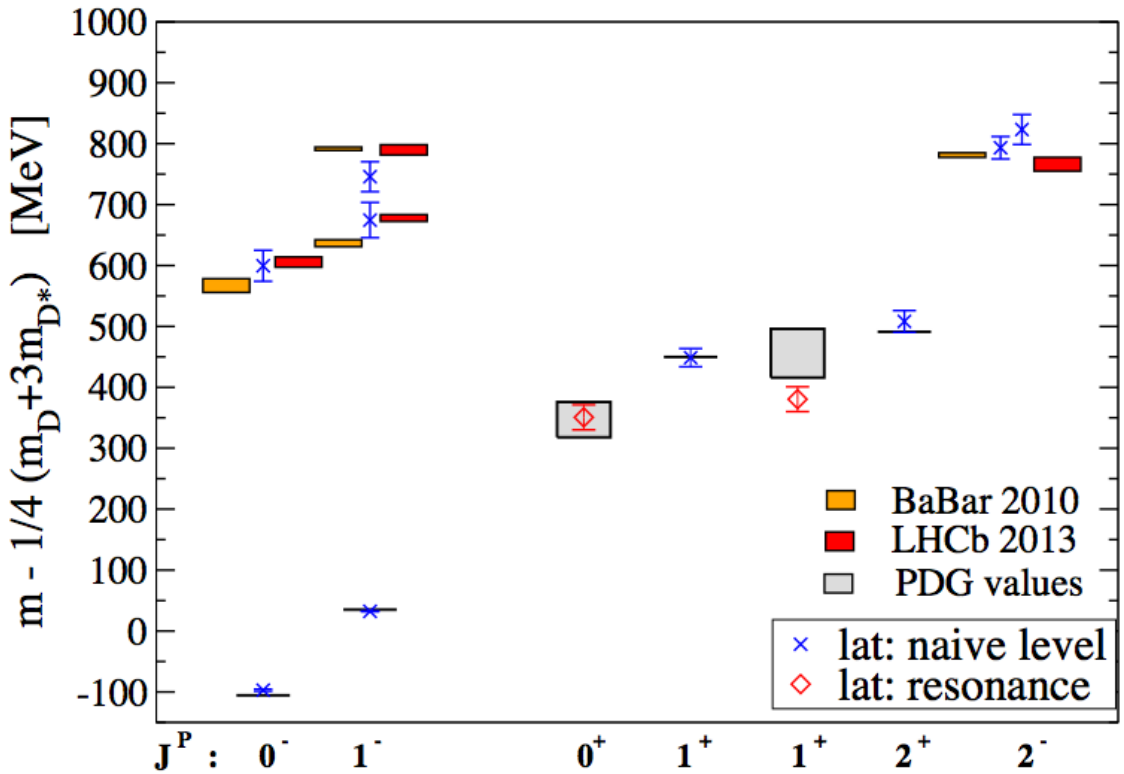


D spectrum: single-meson approx.

$m-m_{ref}$ compared between lat and exp in order to cancel leading am_c discretization effects

1S-2S splitting:

~ 700 MeV



red diamonds: rigorous treatment: discussed later

D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD:

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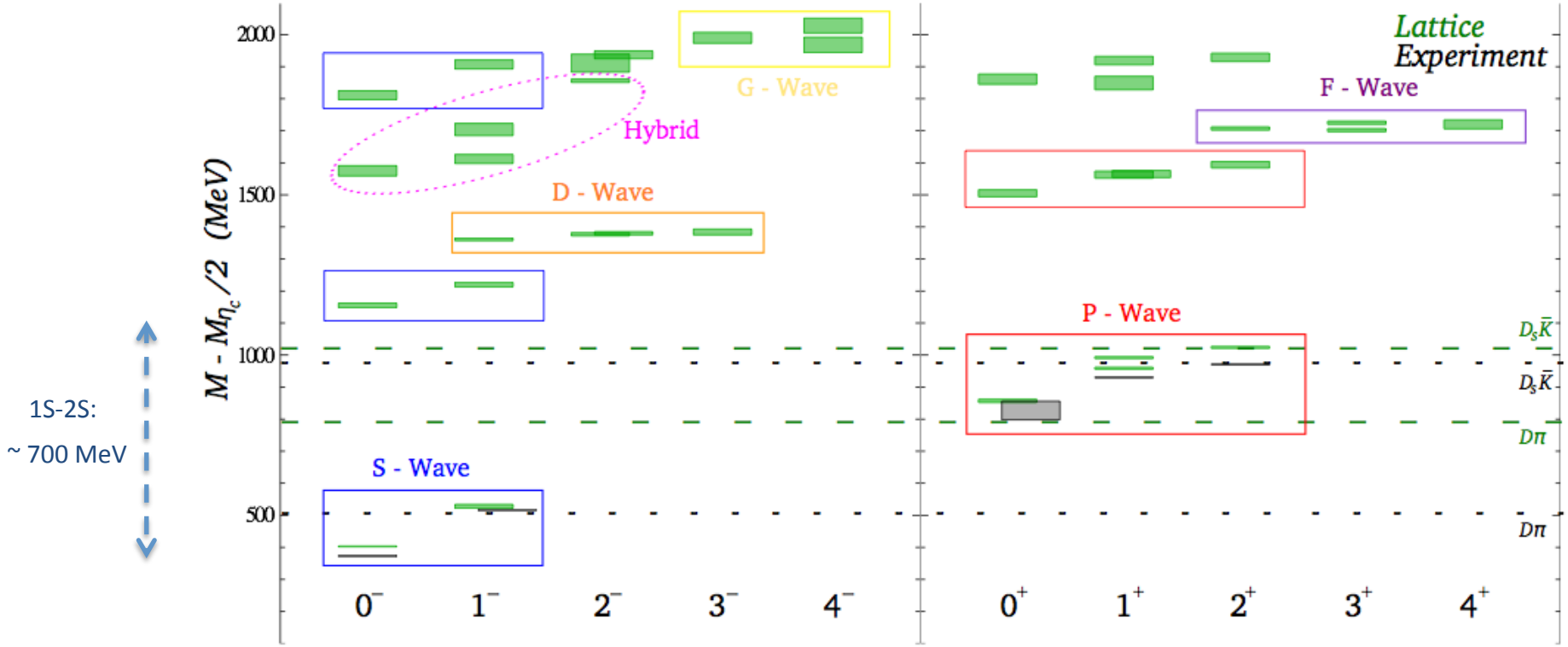
D spectrum: single meson approx.



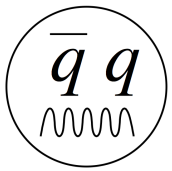
G. Moir et al, HSC (Hadron Spectrum Coll.): 1301.7670, JHEP:

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, Nf=2+1
- reliable J^P determination; many excited states
- identification with $n\ ^{2S+1}L_J$ multiplets using $\langle O|n\rangle$
- green: lat, black: exp

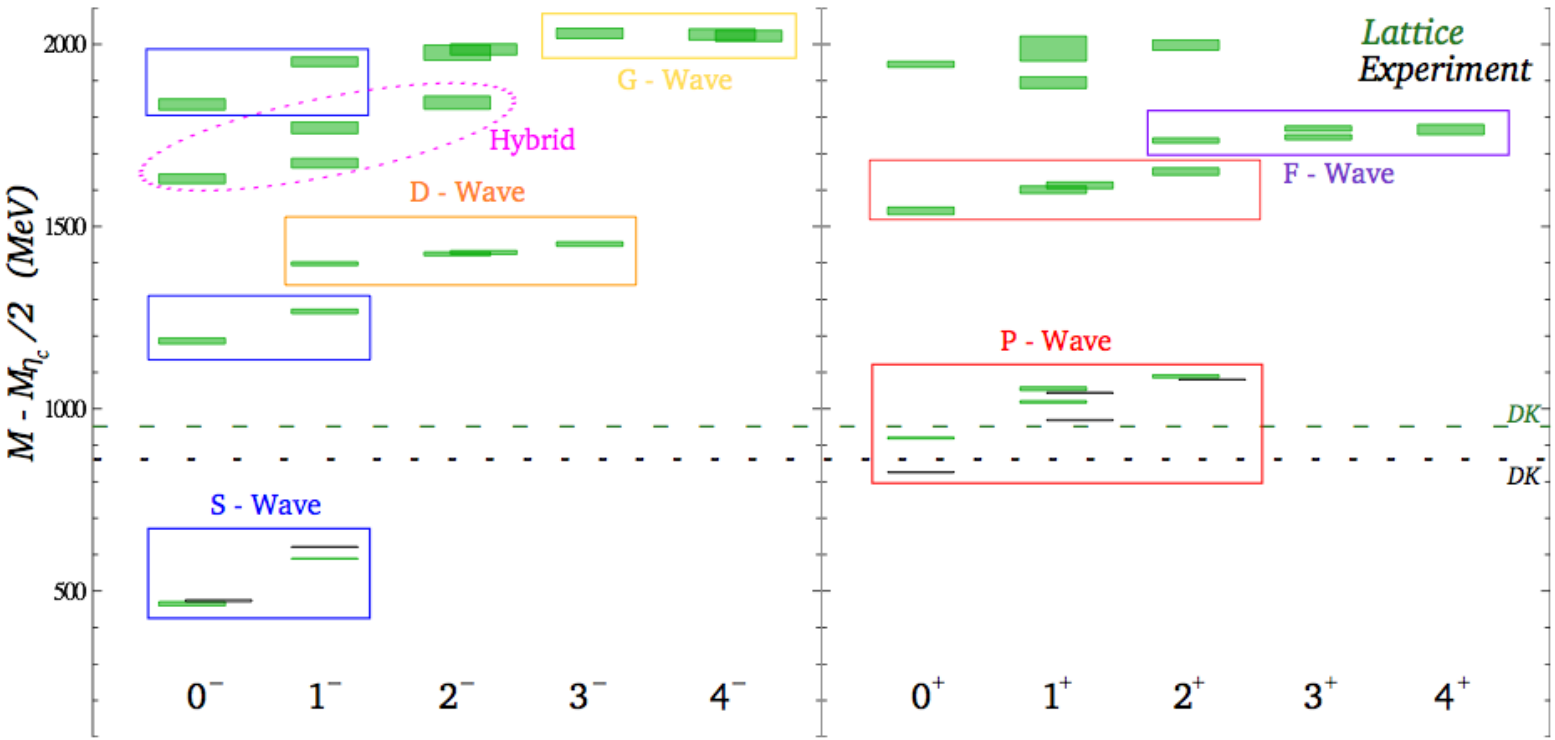
Hybrids:

large overlap with $O = \underline{q}\ F_{ij}\ q$

gluonic tensor $F_{ij} = [D_i, D_j]$



D_s spectrum: single meson approx.



G. Moir et al., HSC : 1301.7670, JHEP:

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, Nf=2+1
- reliable J^{PC} determination
- identification with n $^{2S+1}L_J$ multiplets using $\langle O|n\rangle$
- green: lat, black: exp

Hybrids:

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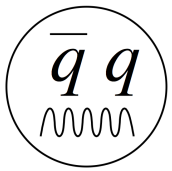
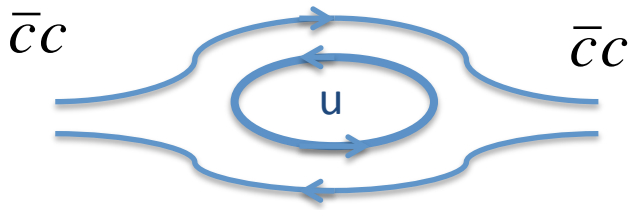
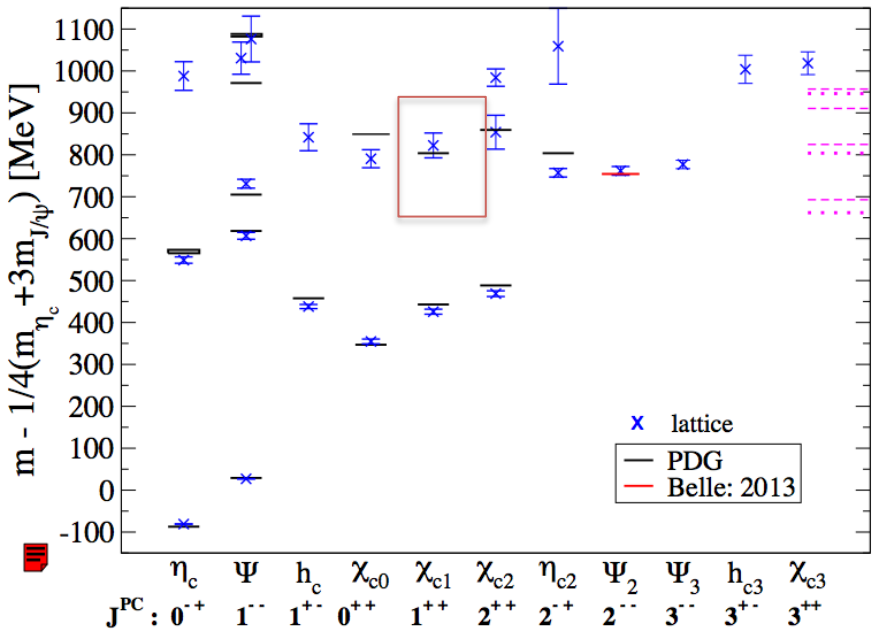


Single meson approximation: one of the problems



$$D^0\overline{D}^{0*}$$



DD*

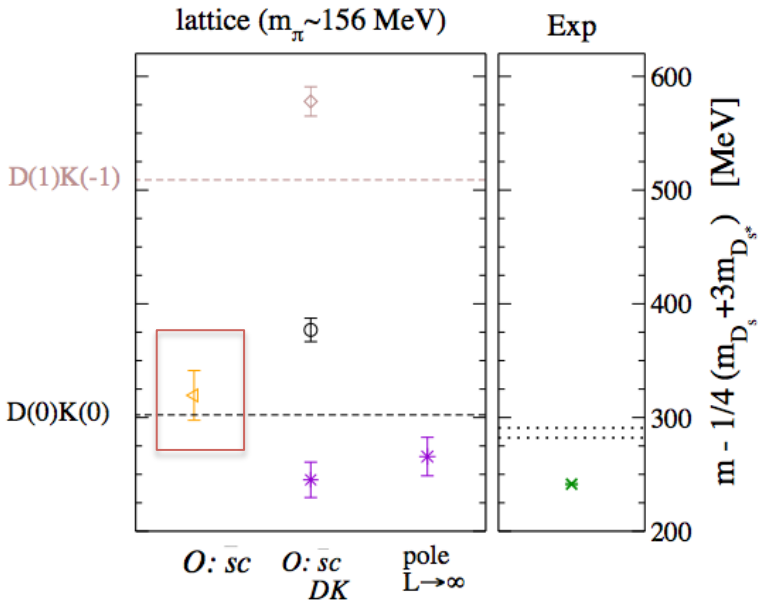
Examples:

- X(3872) channel $\underline{c}c$ with $J^{PC}=1^{++}$

  Is the level X(3872) or perhaps D(0)D*(0) ?

- $D_{s0}(2317)$ channel $\underline{s}c$ with $J^P=0^+$

  Is the level $D_{s0}(2317)$ or perhaps D(0)K(0) ?



Excited states: rigorous treatment (1) states near threshold

note: most of interesting states are found near threshold:

$D_{s0}^{*}(2317)$, $X(3872)$, $Z_c^{+}(3900)$, Z_b^{+}

$D_{s0}^*(2317)$ $J^P=0^+$

- $D_{s0}(2317)$ was theoretically expected above DK threshold, but it was experimentally found ~50 MeV below threshold

- why do these scalar partners have mass so close ?

$$D_{s0}(2317): \quad M \approx 2318\ MeV \quad \Gamma \approx 0\ MeV \qquad \bar{c}s \ \text{ or } \ \bar{c}s[\bar{u}u + \bar{d}d] \ ?$$

$$D_0^*(2400): \quad M \approx 2318\ MeV \quad \Gamma \approx 267\ MeV \quad \bar{c}u \ \text{ or } \ \bar{c}u\bar{s}s\ ?$$

- popular phenomenological explanation: DK threshold pushes D_{s0} mass down

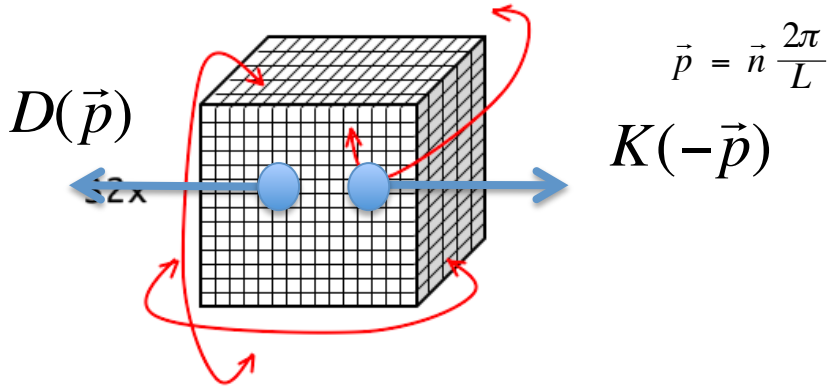
- take into account the effect of DK threshold in simulation for the first time

Basics of rigorous treatment example: $D_{s0}^*(2317)$ with $J^P=0^+$

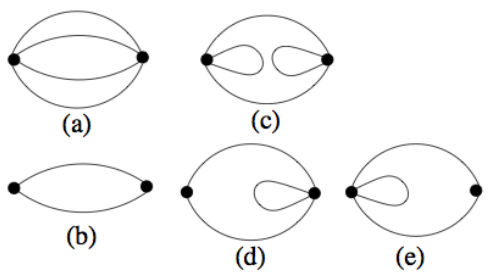
Aims to extract also two-meson states E_n

$$\mathcal{O} = \bar{s}\, c$$

$$\mathcal{O} = DK \approx [\bar{d}\gamma_5 c]\ [\bar{s}\gamma_5 d]$$



$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^+(0) | 0 \rangle$$



We use distillation method

[Peardon et al. 2009] to evaluate C_{ij}

Extract E_n from $C_{ij}(t)$: variational method

$$C_{ij}(t) = \sum_n A_n^{ij}\, e^{-E_n t}$$

due to strong int.

$$\vec{p} = \vec{n}\,\frac{2\pi}{L} \qquad E(L) = \sqrt{m_D^2 + \vec{p}^{\,2}} + \sqrt{m_K^2 + (-\vec{p})^2} + \Delta E$$

Energy levels that appear in addition to these discrete two particles states correspond to <u>bound states</u> or <u>resonances</u>

$D_{s0}^*(2317)$ and DK scattering ($J^P=0^+$)

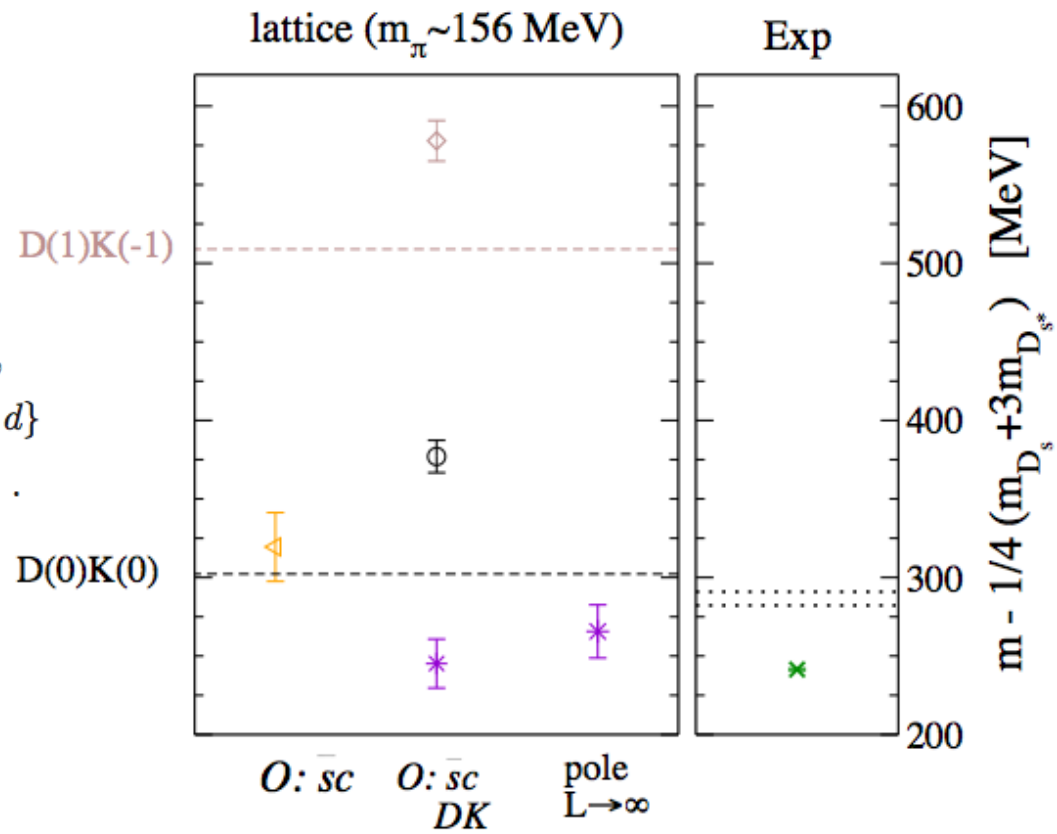
$\mathcal{O}: \bar{s}\, c, \quad DK \approx [\bar{d}\gamma_5 c]\,[\bar{s}\gamma_5 d]$

$$O_{1-4}^{qq} = \bar{s}\, M\, c$$

$$O_1^{DK} = [\bar{s}\gamma_5 u]\,(p=0)\,[\bar{u}\gamma_5 c]\,(p=0) + \{u \to d\}\ ,$$

$$O_2^{DK} = [\bar{s}\gamma_t\gamma_5 u]\,(p=0)\,[\bar{u}\gamma_t\gamma_5 c]\,(p=0) + \{u \to d\}$$

$$O_3^{DK} = \sum_{p=\pm e_{x,y,z}\ 2\pi/L} [\bar{s}\gamma_5 u]\,(p)\,[\bar{u}\gamma_5 c]\,(-p) + \{u \to d\}\ .$$



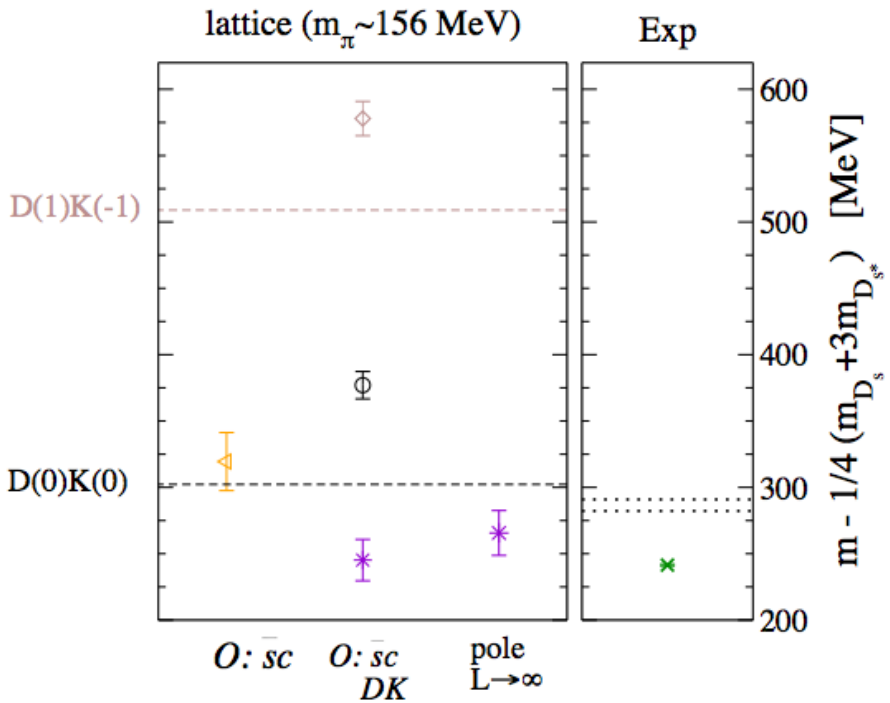
Candidate for $D_{s0}^*(2317)$ is found in addition to the DK states for the first time.

D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn:

1308.3175, PRL : $m_\pi \approx 156$ MeV, L≈2.9 fm, Nf=2+1

$D_{s0}^*(2317)$ and DK scattering

$\mathcal{O}: \bar{s}\, c, \quad DK \approx [\bar{d}\gamma_5 c]\,[\bar{s}\gamma_5 d]$



D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn: 1308.3175, PRL : m_π≈156 MeV, L≈2.9 fm, Nf=2+1

- M. Luscher, 80': $\boxed{E \rightarrow \delta(E)}$

  phase shift for DK scattering in s-wave

- δ for DK scattering in s-wave

  extracted using Luscher's relation

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = -1.33 \pm 0.20 \text{ fm}$$

$$r_0 = 0.27 \pm 0.17 \text{ fm}$$

a_0<0 indicates a state below th.

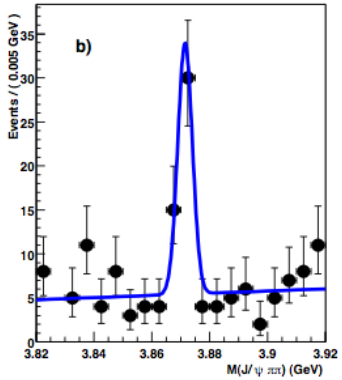
- relation above gives pole position and the mass of $D_{s0}^*(2317)$

$$S \propto [\cot\delta - i]^{-1} = \infty, \quad \cot\delta(p_{BS}) = i$$

$$m_{D_{s0}}^{lat, L\to\infty} = E_D(p_{BS}) + E_K(p_{BS})$$

$D_{s0}^*(2317)$	m - ¼ (m_{Ds}+3m_{Ds^*})
lat	266 ± 16±4 MeV
exp	241.45 ± 0.6 MeV

X(3872): experimental facts



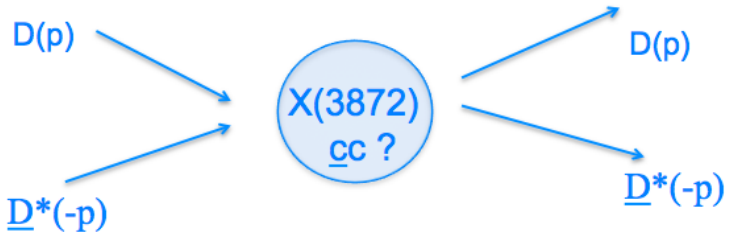
- first observed in 2003 [Belle PRL 2003]
- **J^{PC}=1^{++}** [LHCb, 2013]
- sits within 1 MeV of D^0D^{0*} threshold
- selected decays

X(3872) → J/Ψ ω (I=0)

X(3872) → J/Ψ ρ (I=1)

X(3872): interpolators $J^{PC}=1^{++}$ (T_1^{++}) , P=0, I=0,1

$$\mathcal{O}: \ \bar{c}\, c, \quad D\bar{D}^*, \quad J/\psi\,\omega$$



$$O^{\bar{c}c}_{1-8} = \bar{c}\ \hat{M}_i\ c(0) \qquad\qquad (\text{only } I=0)$$

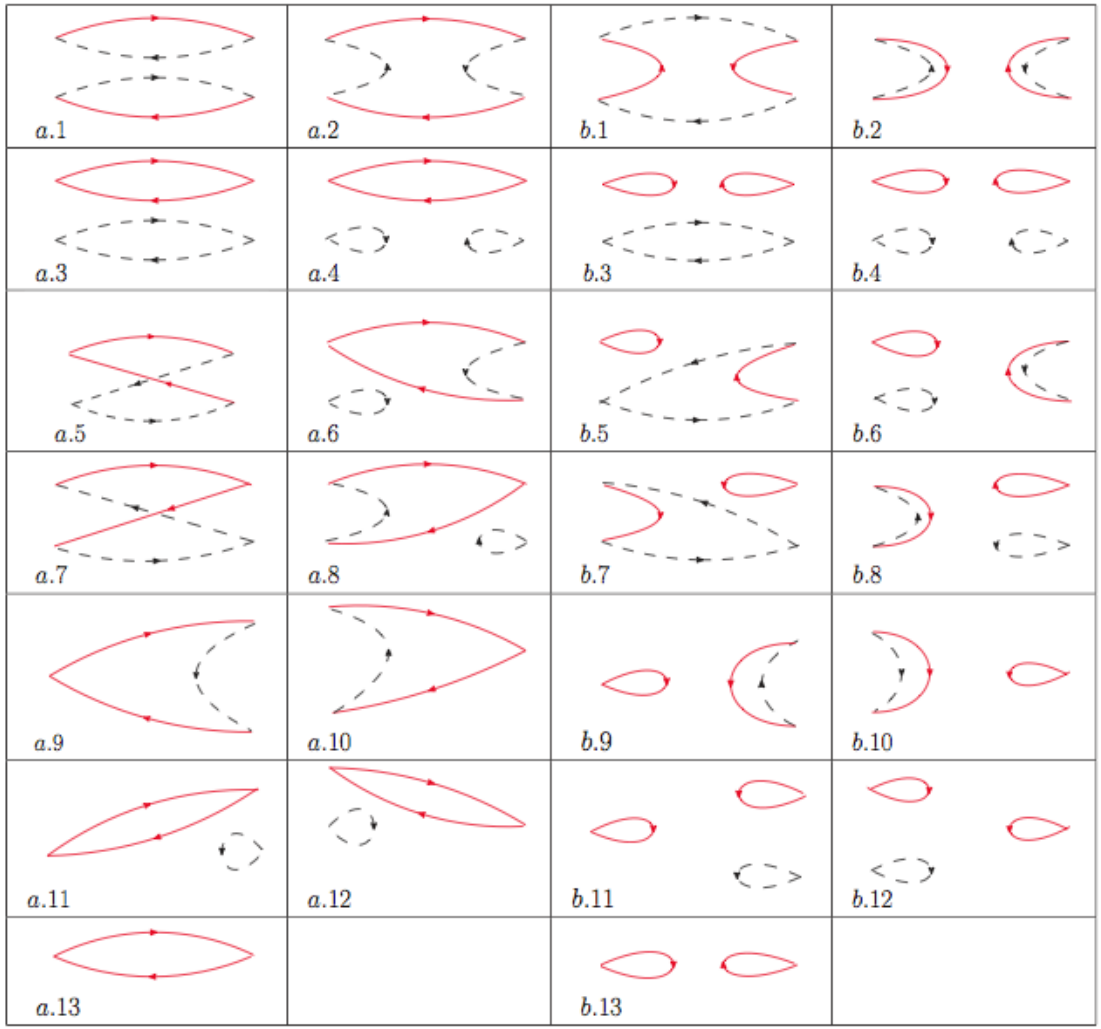
$$O_1^{DD^*} = [\bar{c}\gamma_5 u(0)\ \bar{u}\gamma_i c(0) - \bar{c}\gamma_i u(0)\ \bar{u}\gamma_5 c(0)] + f_I\{u \to d\}$$
$$O_2^{DD^*} = [\bar{c}\gamma_5\gamma_t u(0)\ \bar{u}\gamma_i\gamma_t c(0) - \bar{c}\gamma_i\gamma_t u(0)\ \bar{u}\gamma_5\gamma_t c(0)] + f_I\ \{u \to d\}$$
$$O_3^{DD^*} = \sum_{e_k=\pm e_{x,y,z}} [\bar{c}\gamma_5 u(e_k)\ \bar{u}\gamma_i c(-e_k) - \bar{c}\gamma_i u(e_k)\ \bar{u}\gamma_5 c(-e_k)] + f_I\ \{u \to d\}$$

$$O_1^{J/\psi V} = \epsilon_{ijk}\ \bar{c}\gamma_j c(0)\ [\bar{u}\gamma_k u(0) + f_I\ \bar{d}\gamma_k d(0)]$$
$$O_2^{J/\psi V} = \epsilon_{ijk}\ \bar{c}\gamma_j\gamma_t c(0)\ [\bar{u}\gamma_k\gamma_t u(0) + f_I\ \bar{d}\gamma_k\gamma_t d(0)]$$

$$I=0: \quad f_I = 1, \quad V = \omega$$
$$I=1: \quad f_I = -1, \quad V = \rho$$

S. P. and L. Leskovec : 1307.5172, PRL

X(3872) : $J^{PC}=1^{++}$ [LHCb 2013] , I=0: Wick contractions



$$\mathcal{O}: \bar{c}\, c, \quad DD^*, \quad J/\psi\,\omega$$

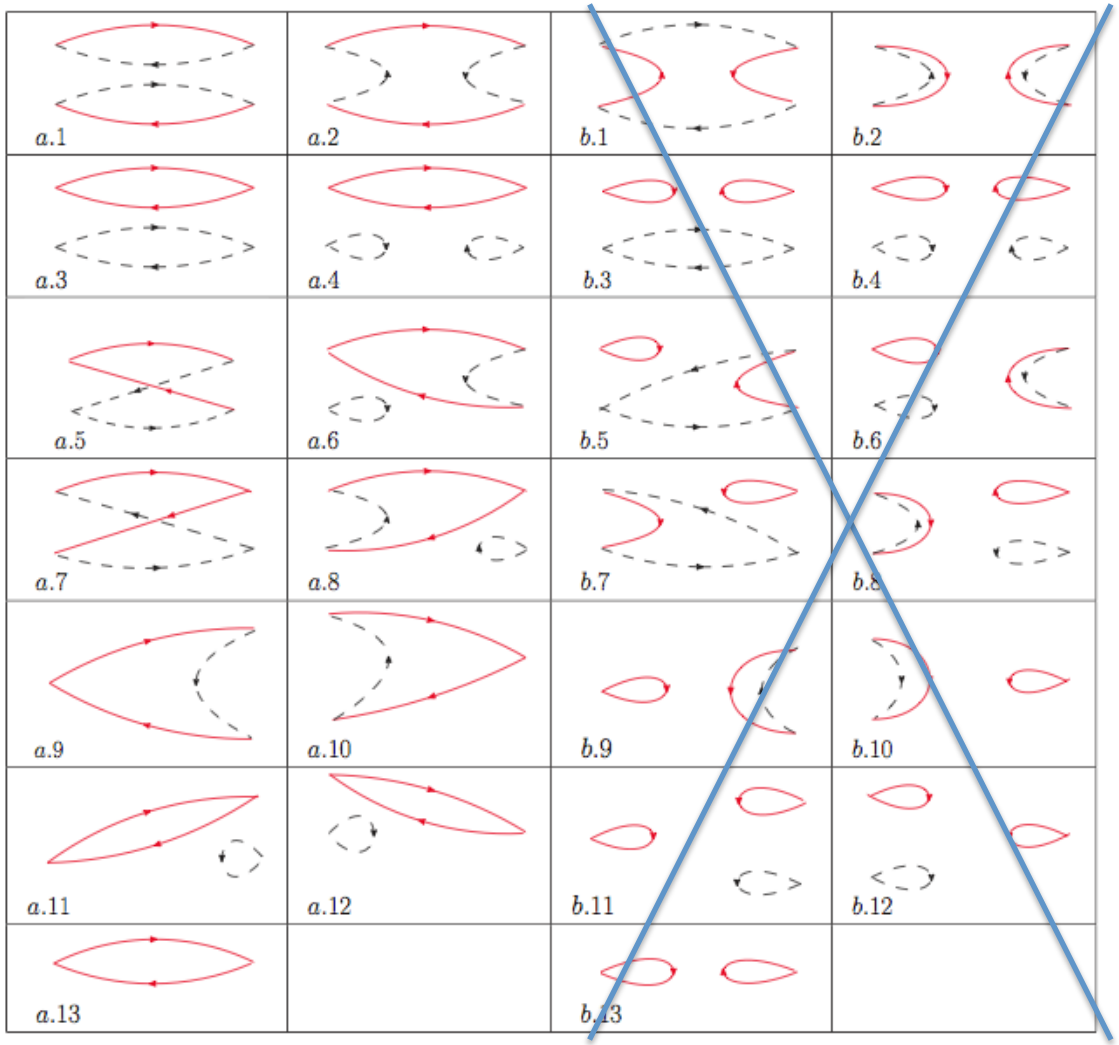
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^+(0) | 0 \rangle$$

- we calculate all Wick contractions

c quark

u,d quark

X(3872) : $J^{PC}=1^{++}$ [LHCb 2013] , I=0: Wick contractions



$\mathcal{O}: \bar{c}\, c, \quad DD^*, \quad J/\psi\, \omega$

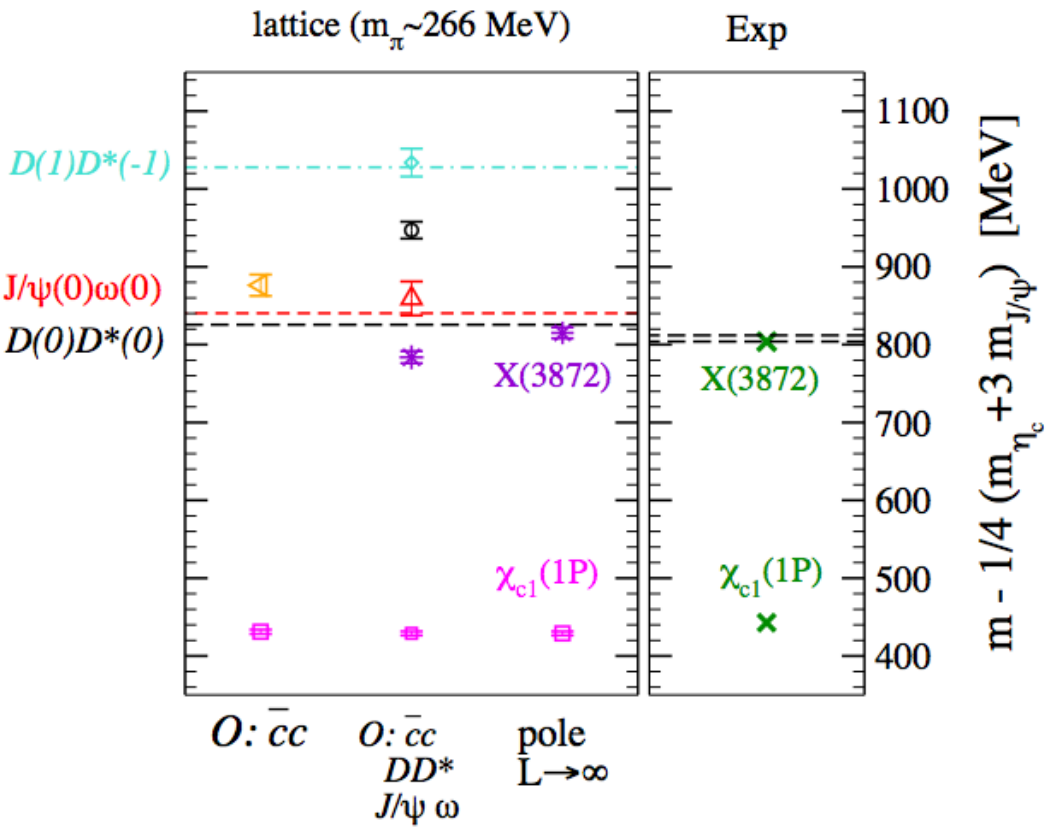
- we calculate all Wick contractions
- results are based only on 13 Wick contractions in Fig. a (where c propagates from source to sink)
- the effect of remaining ones suppressed by OZI rule [see also Levkova, DeTar 2011]
- their effect will be addressed on follow-up analysis

c quark

u,d quark

X(3872) : $J^{PC}=1^{++}$ [LHCb 2013] , I=0

$\mathcal{O}: \bar{c}\,c, \quad DD^*, \quad J/\psi\,\omega$



- δ for DD* scattering in s-wave

  extracted using Luscher's relation

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = -1.7 \pm 0.4 \text{ fm}$$

$$r_0 = 0.5 \pm 0.1 \text{ fm}$$

large and a_0<0 indicates a state slightly below DD* threshold: X(3872)

- pole position gives mass of X(3872)

$$S \propto [\cot\delta - i]^{-1} = \infty, \quad \cot\delta(p_{BS}) = i$$

$$m_X^{lat, L\to\infty} = E_D(p_{BS}) + E_{D^*}(p_{BS})$$

X(3872)	m - (m_{D0}+m_{D0*})
lat	- 11 ± 7 MeV
exp	- 0.14 ± 0.22 MeV

Candidate for X(3872) is found in addition to the expected two-particle states for the first time.

S. P. and L. Leskovec : 1307.5172, PRL

m_π≈266 MeV, L≈2 fm, Nf=2

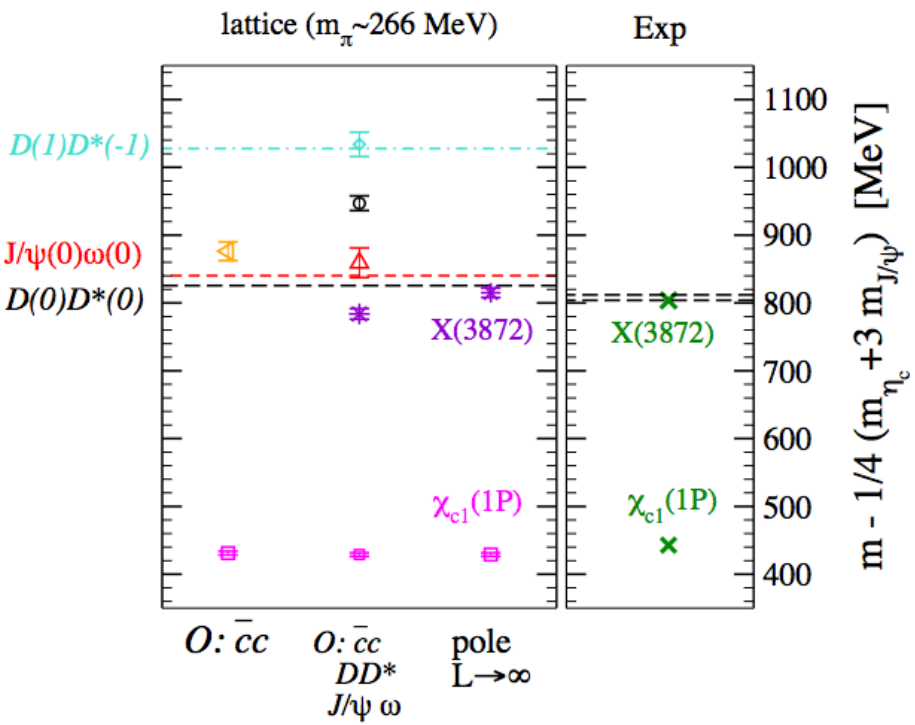
lat: simulations on larger L required

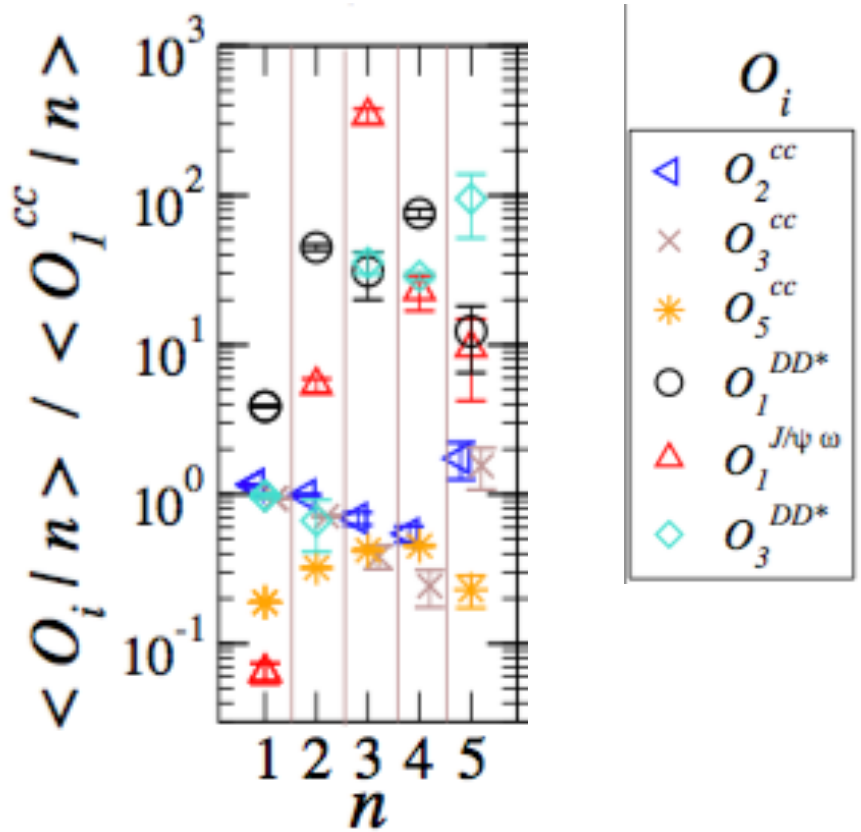
exp: Tomaradze et al., 1212.4191

Composition of established X(3872) with I=0

write two interp.

- it has sizable coupling with $\bar{c}c$ as well as DD^* interpolating fields
- overlaps of X with interpolators $\langle O_i | X(3872) \rangle$



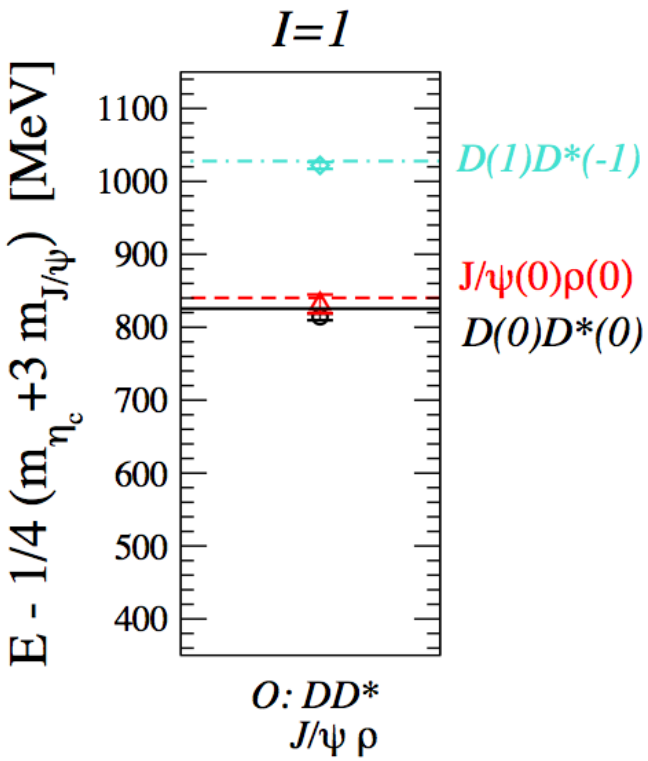


S. P. and L. Leskovec : 1307.5172, PRL

m_π≈266 MeV, L≈2 fm, Nf=2

exp: X(3872) → J/Ψ ρ (I=1)

Search for X(3872) with $J^{PC}=1^{++}$ and I=1



Only expected two-particle states observed.
No candidate for X(3872) found.

In agreement with two interpretations:

(1) X(3872) pure I=0

isospin breaking happens only in decay

X(3872) → J/Ψ ρ (I=1)

isospin breaking: $D^0 D^{0*}$, D^+D^{-*} splitting

(2) $X(3872) = a_{I=0}|DD^*\rangle_{I=0} + a_{I=1}|DD^*\rangle_{I=1}$

$$a_{I=1}(m_u = m_d) = 0$$
$$a_{I=1}(m_u \neq m_d) << a_{I=0}$$

In simulation: $m_u=m_d$

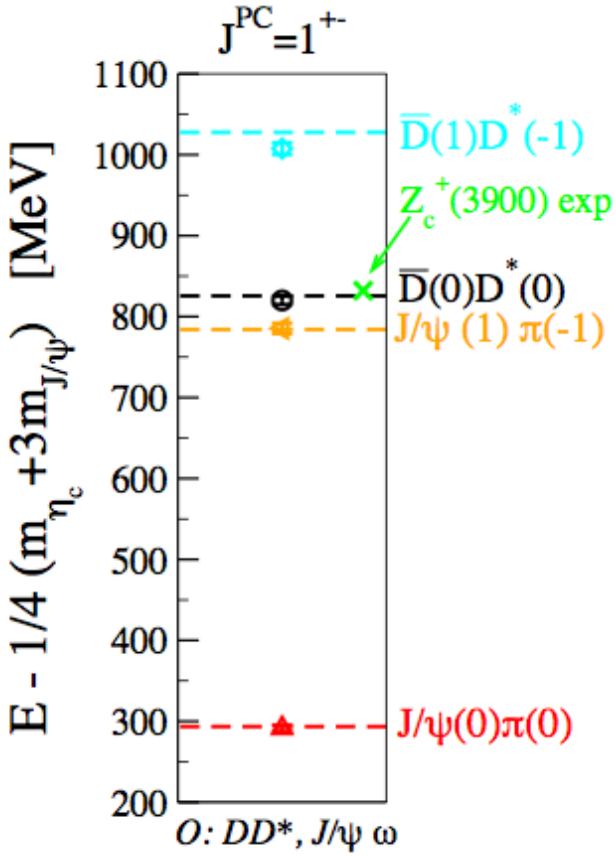
S. P. and L. Leskovec : 1307.5172, PRL

$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

(2) Searches for exotic states:
rigorous treatment

Search for $Z_c^+(3900)$ in $J^{PC}=1^{+-}$, I=1 channel

$\mathcal{O}: DD^*, \quad J/\psi\ \pi$



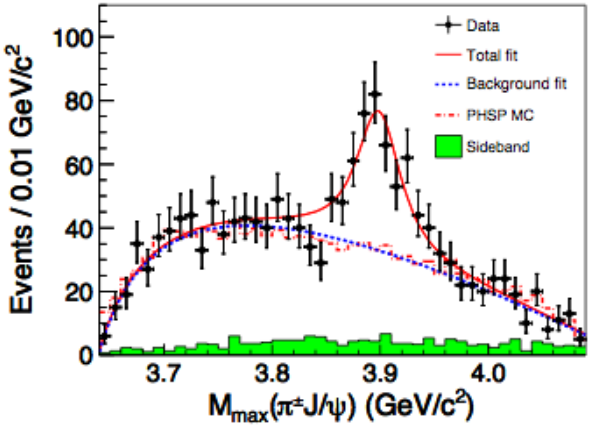
$$Z_c^+(3900) \to J/\psi\ \pi^+$$

$$J^{PC} = 1^{-+} \qquad \bar{c}c\ \bar{d}u$$

if $Z_c(3900)=Z_c(3885)$

[BesIII, arXiv:1310.1163]

[BesIII, Belle, CleoC, 2013]



Only expected two-particle states observed.

No candidate for $Z_c^+(3900)$ with $J^{PC}=1^{+-}$ is found.

- Possible reasons:

✧ perhaps $J^{PC}\neq 1^{+-}$ if $Z_c(3900)\neq Z_c(3885)$

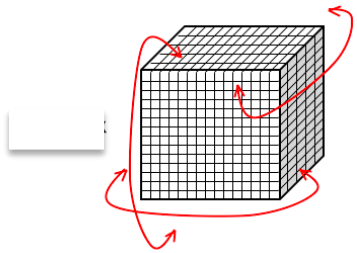
✧ perhaps our interpolators (all of scat. type) are not diverse enough : calls for further simulations

✧ ??

S. P. and L. Leskovec : 1308.2097, PLB

$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

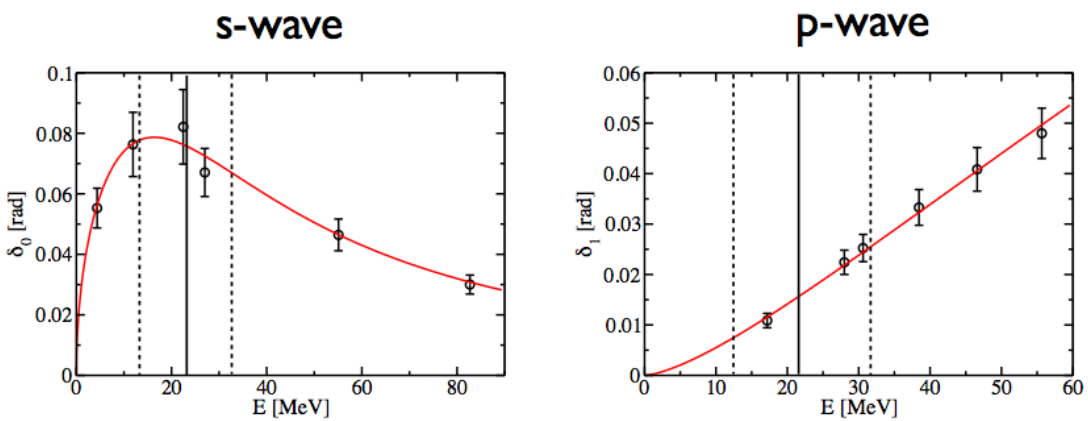
Search for Y(4140) in J/Ψ Φ scattering



Experiment:

- Y(4140) found in J/Ψ Φ , Γ ≈ 11 MeV [CDF 2009]
  not seen in $D_s\underline{D}_s$
- not seen by Belle, LHCb

J/Ψ Φ scattering phase shift [radians]



S. Ozaki and S. Sasaki, 1211.5512, PRD

m_π≈156 MeV, L≈2.9 fm, Nf=2+1

Lattice:

- method to get δ at more E:

twisted BC for valence q.

$$q(x+L) = e^{i\theta}\, q(x)$$

instead of periodic BC (conventional)

$$q(x+L) = q(x)$$

- conclusion:

no resonant structure found at energies reported by CDF

- Caveats:

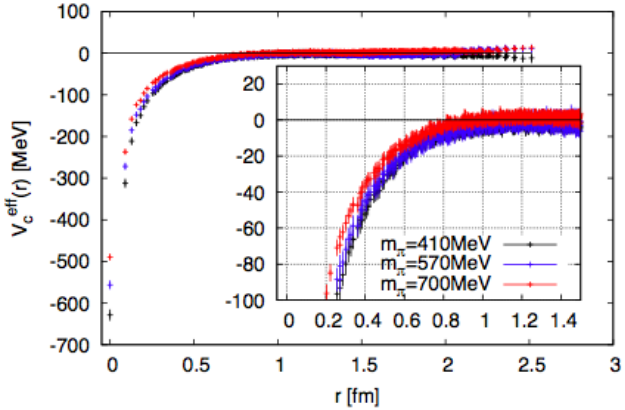
★ s-quark annihilation ignored

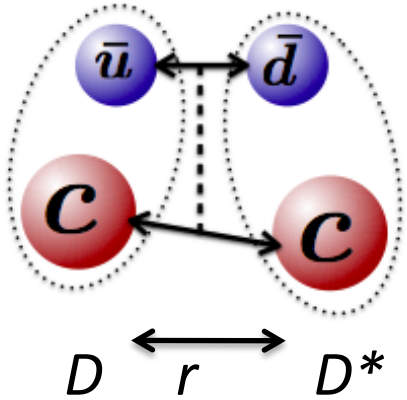
★ twisting is partial: only on valence quarks

Search for bound double charm tetraquark with $J^P=1^+$, I=0

(1) determine potential between D and D*

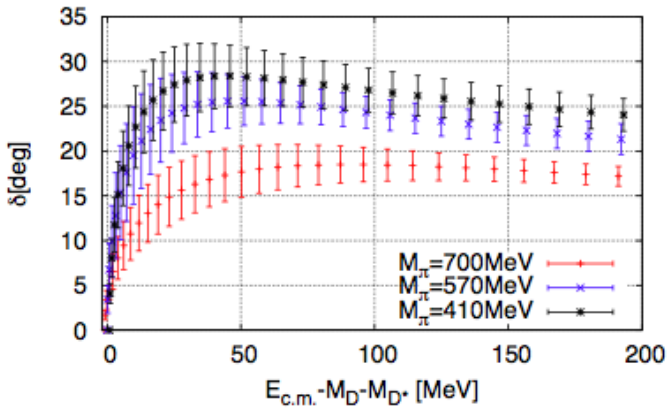
at distance r: HALQCD method: Ishii et al., PLB712, 437 (2012)





(2) Solve Schrodinger equation with given V(r)

and determine DD* scattering phase shift



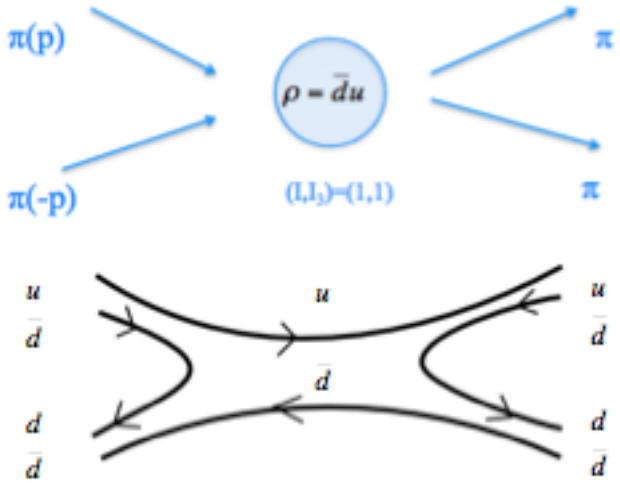
Conclusion:

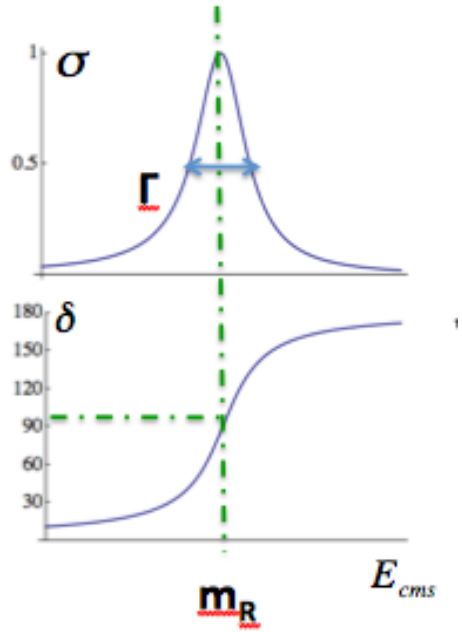
- **potential is attractive**
- **no bound tetraquark state** at simulated m_π
- in case of one bound state one would expect δ(E=0)=π due to Levinson's theorem

Y. Ikeda et al, HAL QCD coll. , 2013, private com.

m_π≈410-700 MeV, L≈2.9 fm, Nf=2+1

Excited states: rigorous treatment (3) resonances





Almost all hadrons are **hadronic resonances** (decay strongly)

$\bar{c}c$	$\bar{u}u$	$\bar{s}u$	$\bar{c}u$	uud
$\eta_c(1S)$	$\pi^\pm$	$K^\pm$	$D^\pm$	p
$J/\psi(1S)$	π^0	K^0	D^0	n
$\chi_{c0}(1P)$	η	$K_S{}^0$	$D^*(2007)^0$	$N(1440)\ 1/2^+$
$\chi_{c1}(1P)$	$f_0(500)$ or σ was $f_0(60$	$K_L{}^0$	$D^*(2010)^\pm$	$N(1520)\ 3/2^-$
$h_c(1P)$	$\rho(770)$	$K_0^*(800)$ or κ	$D_0^*(2400)^0$	$N(1535)\ 1/2^-$
$\chi_{c2}(1P)$	$\omega(782)$	$K^*(892)$	$D_0^*(2400)^\pm$	$N(1650)\ 1/2^-$
$\eta_c(2S)$	$\eta'(958)$	$K_1(1270)$	$D_1(2420)^0$	$N(1675)\ 5/2^-$
$\psi(2S)$	$f_0(980)$	$K_1(1400)$	$D_1(2420)^\pm$	$N(1680)\ 5/2^+$
$\psi(3770)$	$a_0(980)$	$K^*(1410)$	$D_1(2430)^0$	$N(1685)\ ?^?$
$X(3872)$	$\phi(1020)$	$K_0^*(1430)$	$D_2^*(2460)^0$	$N(1700)\ 3/2^-$
$\chi_{c0}(2P)$wa	$h_1(1170)$	$K_2^*(1430)$	$D_2^*(2460)^\pm$	$N(1710)\ 1/2^+$
$\chi_{c2}(2P)$	$b_1(1235)$	$K(1460)$	$D(2550)^0$	$N(1720)\ 3/2^+$
$X(3940)$	$a_1(1260)$	$K_2(1580)$	$D(2600)$	$N(1860)\ 5/2^+$
$\psi(4040)$	$f_2(1270)$	$K(1630)$	$D^*(2640)^\pm$	$N(1875)\ 3/2^-$
$X(4050)^\pm$	$f_1(1285)$	$K_1(1650)$	$D(2750)$	$N(1880)\ 1/2^+$
$X(4140)$	$\eta(1295)$	$K^*(1680)$		$N(1895)\ 1/2^-$
$\psi(4160)$	$\pi(1300)$	$K_2(1770)$		$N(1900)\ 3/2^+$
$X(4160)$	$a_2(1320)$	$K_3^*(1780)$		$N(1990)\ 7/2^+$
$X(4250)^\pm$	$f_0(1370)$	$K_2(1820)$		$N(2000)\ 5/2^+$
	$h_1(1380)$	$K(1830)$		$N(2040)\ 3/2^+$
	$\pi_1(1400)$			$N(2060)\ 5/2^-$
	$\eta(1405)$			$N(2100)\ 1/2^+$
	$f_1(1420)$			
	$\omega(1420)$			
	$f_2(1430)$			
	$a_0(1450)$			
	$\rho(1450)$			

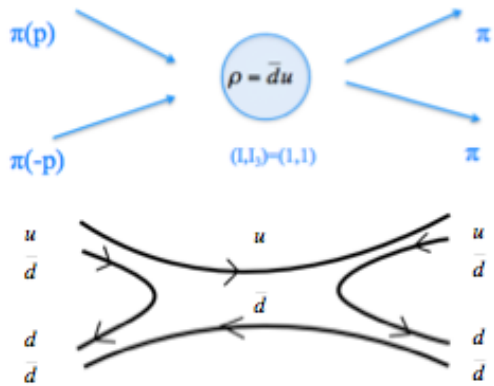
- ❑ stable on strong decay: ab-initio OK
- ❑ others decay strongly; <u>hadronic resonances</u>

only resonance simulated properly by several collab. (first simulation: CP-PACS 2007)

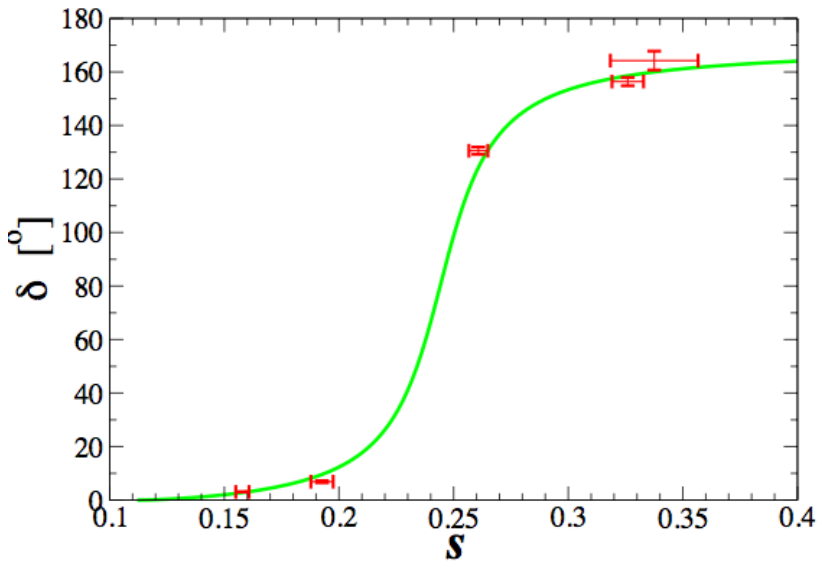
SP and coll. (Lang, Mohler, Leskovec, Woloshyn) (2011- 2013)

Verduci, Lang (2012)

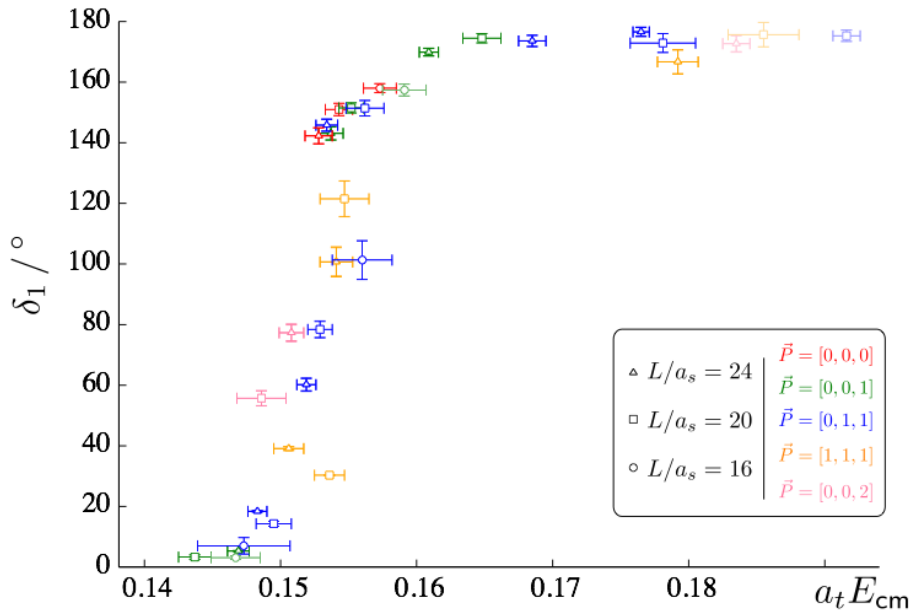
ρ resonance



P≠0: $s=E^2-P^2$, Luscher-type relation: $\boxed{s \rightarrow \delta(s)}$



[Lang, Mohler, S.P. ,Vidmar, PRD 2011]
$m_\pi \approx 266$ MeV

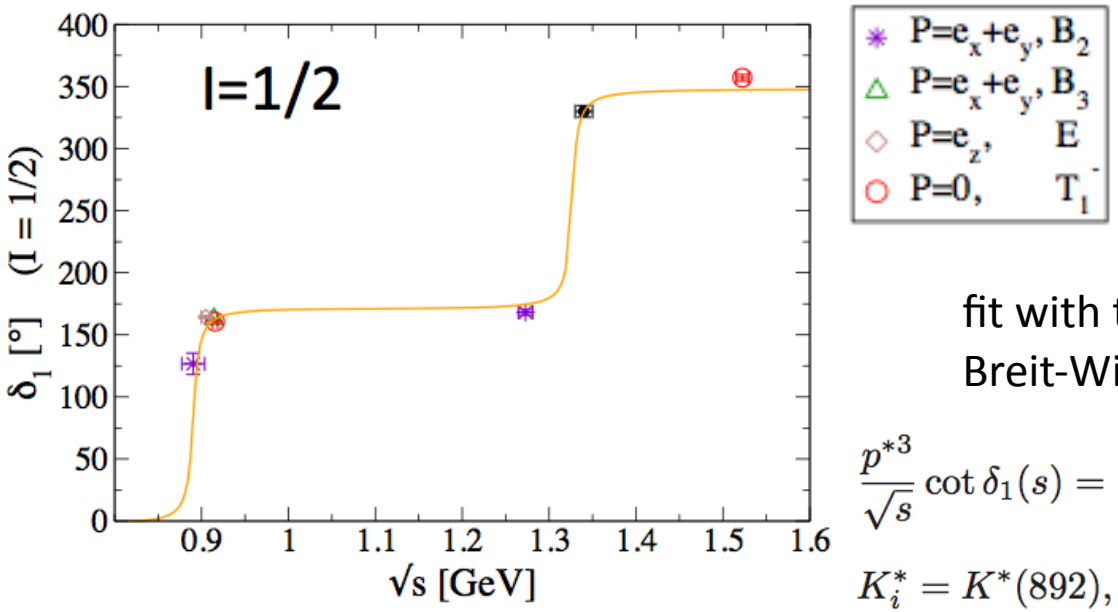


Simulation also by CP-PACS, PACS-CS, QCDSF, ETMC

[HSC, PRD 2013]
$m_\pi \approx 400$ MeV

K*(892) resonance: first lat deterimation of width

Kπ , I=1/2: p-wave phase shift



fit with two elastic
Breit-Wigner resonances

$$\frac{p^{*3}}{\sqrt{s}}\cot\delta_1(s) = \left[\sum_{K_i^*}\frac{g^2_{K_i^*}}{6\pi}\frac{1}{m^2_{K_i^*}-s}\right]^{-1}$$

$$K_i^* = K^*(892),\ K^*(1410)\ .$$

$$\Gamma[K^*\to K\pi] = \frac{g^2}{6\pi}\frac{p^{*3}}{s}$$

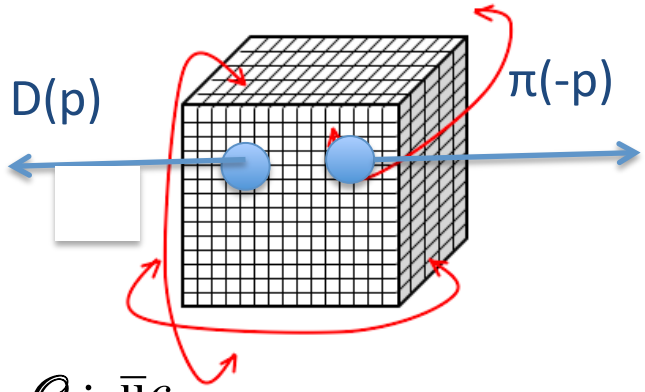
	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]	$m_{K^*(1410)}$ [GeV]	$g_{K^*(1410)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6	1.33 ± 0.02	input
exp	891.66 ± 0.26	5.72 ± 0.06	1.414 ± 0.0015	1.59 ± 0.03

[S.P. ,Lang, Leskovec, Mohler, 1307.0736, PRD]
$m_\pi \approx 266$ MeV

$D_0^*(2400)$ resonance in Dπ scattering: $J^P=0^+$, I=1/2

"rigorous" treatment illustrated on this example

$$p = n\frac{2\pi}{L}$$



$\mathcal{O}: \ \bar{u}c$

$$D(\vec{p})\pi(-\vec{p}) \approx [\bar{d}\gamma_5 c]\ [\bar{u}\gamma_5 d]$$

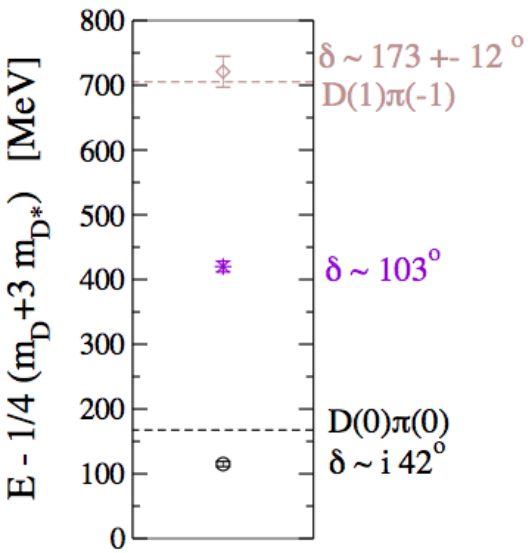
All states with $J^P=0^+$ appear in lat. spectrum:

- $D_o^*(2400)$
- D(p) π(-p) with p=n 2π/L : "two-particle" states

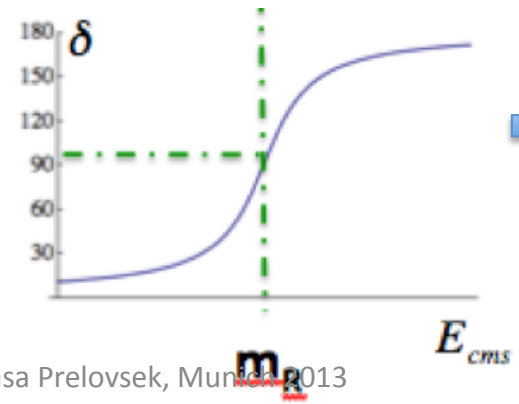
horizontal lines indicate their energies in absence of interaction

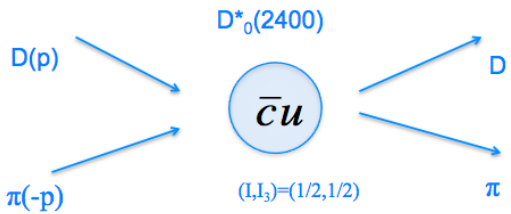
Rigorous relation [M. Luscher , 1991]:

E → δ(E) phase shift for Dπ scattering in s-wave



$$\text{BW}: \quad \delta = \text{acot}\,\frac{m_R^2 - E_{cms}^2}{m_R\,\Gamma}$$





m and Γ

for $D_0^*(2400)$

D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD

D-meson resonance masses and widths

$$\Gamma(E) \equiv g^2 \frac{p}{E^2}$$

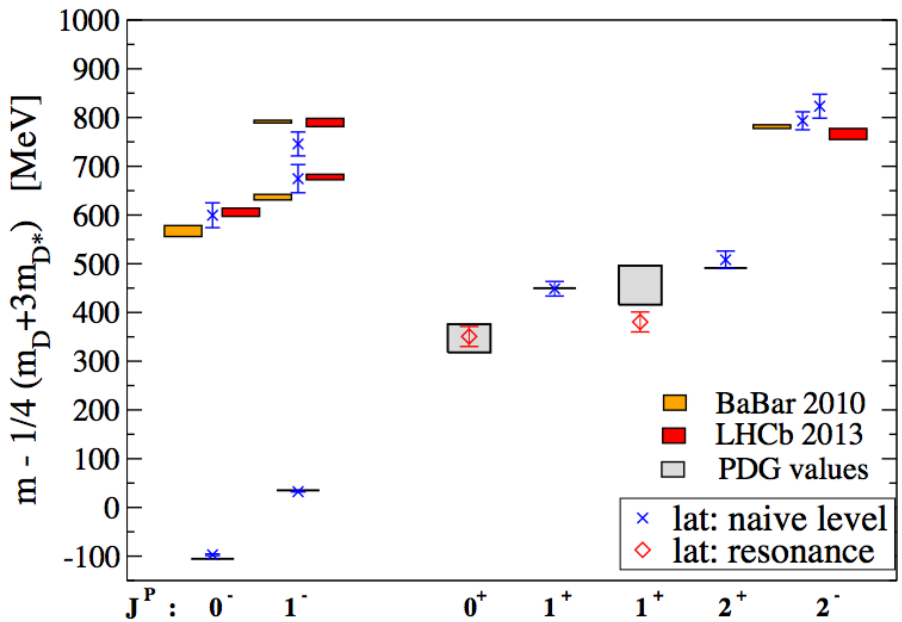
g is compared to exp instead of Γ (Γ depends on phase sp. and m_π)

$J^P=0^+$: D π

$J^P=1^+$: D^* π (analysis of spectrum in this case is based on an assumption given in paper below)

$D_0^*(2400)$	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

$D_1(2430)$	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV



first lattice result for strong decay width of a hadron containing charm quark

[D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD]

- m_π≈266 MeV, L≈2 fm, Nf=2

Dπ scattering : I=1/2, s-wave, $J^P=0^+$

Puzzle

$$D_0^*(2400): \quad M \approx 2318\ MeV \quad \Gamma \approx 267\ MeV \quad \bar{c}u \ \text{ or } \ \bar{c}u\bar{s}s\ ?$$

$$D_{s0}(2317): \quad M \approx 2318\ MeV \quad \Gamma \approx 0\ MeV \quad \bar{c}s \ \text{ or } \ \bar{c}s[\bar{u}u + \bar{d}d]\ ?$$

Our resulting D0*(2400) mass is in favorable agreement with exp without valence $s\bar{s}$ pair.

$J^{PC}=0^{++}$ charmonuim resonance(s): χ_{c0}' ?

PRELIMINARY

By simulating DD scattering in s-wave we find:

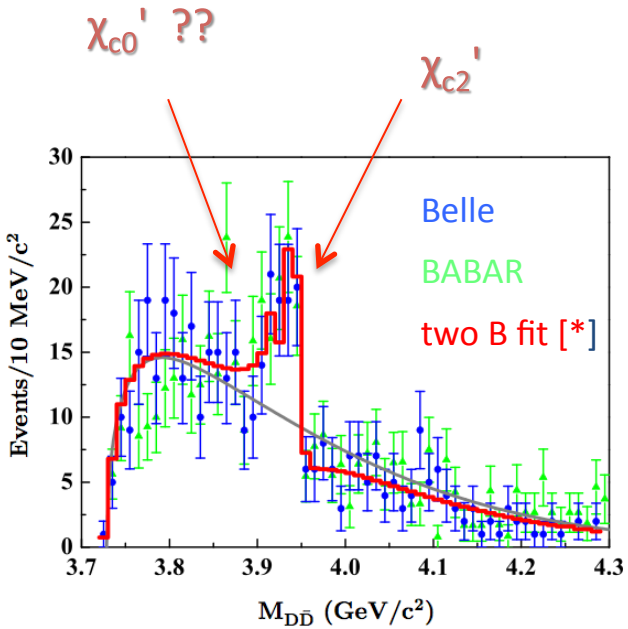
(1) narrow resonance in DD scattering [we call it χ_{c0}']

$$m[\chi_{c0}'] = 3932 \pm 25 \ \ \text{MeV}$$

$$\Gamma[\chi_{c0}' \to \bar{D}D] = 36 \pm 17 \ \ \text{MeV}$$

PDG12: χ_{c0}'=X(3915) ?! Why no X(3915)→DD in exp ?!

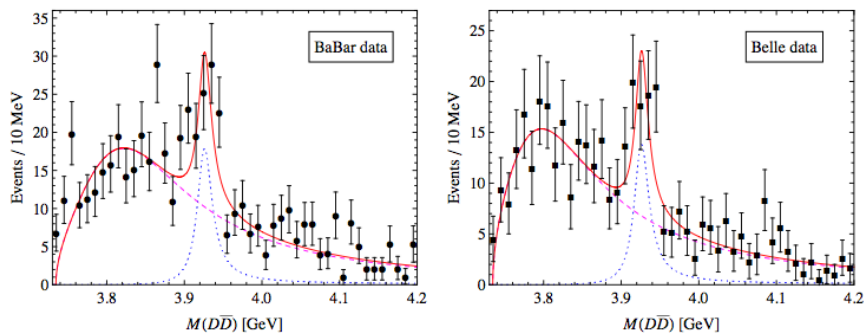
perhaps there is a hit of it [D. Chen et al, 1207.3561, PRD]



(2) additional enhancement of σ(DD) near th. :

could it be related to broad structures ?

[see also F. Guo, U. Meissner, 1208.1134, PRD]



S.P. , L. Leskovec and D. Mohler,

1310.8127, Lat 2013 proc:

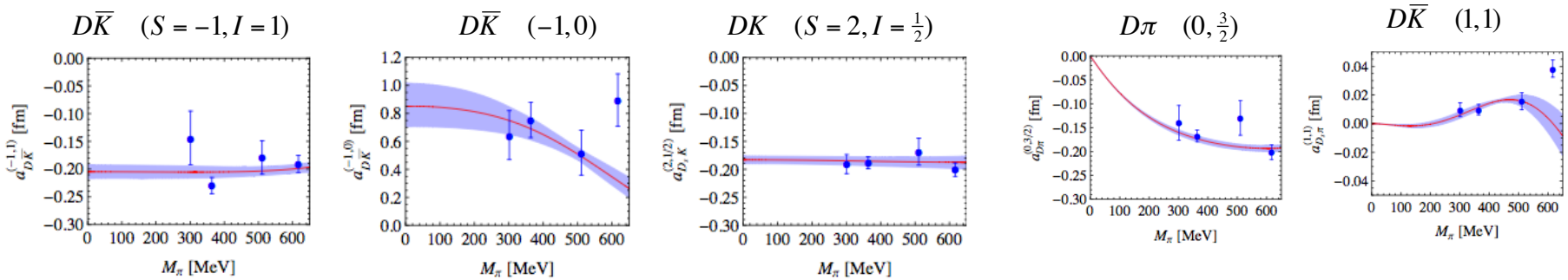
- m_π≈266 MeV, L≈2 fm, Nf=2

Indirect method combined with EFT

Indirect study of $D_{s0}^*(2317)$ channel

(1) Five channels that do not include Wick contractions  are simulated

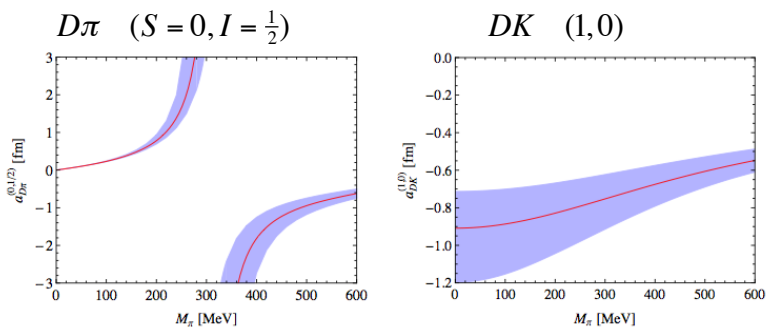
(2) Scattering lengths $a = \lim_{p\to 0} \frac{\tan\delta(p)}{p}$ for four m_π extracted



(3) simultaneous fit using SU(3) unitarized ChPT is performed and LEC's are determined

(4) using these LEC's indirect predictions for:

- scattering length of two resonant-channels with contractions 
- DK (S=1,I=0): pole in the first Riemann sheet found



$D_{s0}^*(2317)$	m	$\Gamma\,[D_{s0}^* \to D_s \pi]$
indirect lat	$2315\ ^{+18}_{-28}$ MeV	133±22 keV
exp	2317.8 ±0.6 MeV	< 3.8 MeV

L. Liu, Orginos, Guo, Hanhart, Meissner, 1208.4535, PRD, m_π≈300-620 MeV, Nf=2+1

Conclusions & outlook

Present status of lattice results for D, D_s, $\underline{c}c$ spectra **:**

- states well below strong decay threshold determined reliably and with good precision

- excited states: single-meson approximation

  spectra with a number of full qq multiplets and hybrids calculated during 2012, 2013

- excited states: rigorous treatment: first simulations during 2012, 2013

  ★ $D_0^*(2400)$, $D_1(2430)$, $D_{s0}^*(2317)$, X(3872) identified

  ★ $Z_c^+(3900)$, Y(4140), $cc\underline{ud}$ not (yet) found

Precision simulations of these channels will have to be performed in the future.

Conclusions & outlook

Outlook for lattice simulations of D, D_s, $\bar{c}c$ spectra **:**

Which excited states can one treat rigorously in the near future?

- states not to far above strong decay threshold that have one (dominant) decay mode

  example: $Z_c^+(3900)$ is less challenging than $Z^+(4430)$

- states that are *not* accompanied by many lower states of the same quantum number

  example: higher lying 1^- charmonium states would be very challenging for rigorous treatment

Lots of exciting experimental results prompt for lots of exciting lattice simulations

in the near future, encouraged by the pioneering exploratory steps made during the last year!

Backup slides

Lattice simulation

Two ensembles:

A. Hasenfratz

PACS-CS

ID	$N_L^3 \times N_T$	N_f	a[fm]	L[fm]	#configs	m_π[MeV]
(1)	$16^3 \times 32$	2	0.1239(13)	1.98	279	266(3)(3)
(2)	$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)

On both ensembles:

- dynamical u, d, (s) , valence u,d,s : Improved Wilson Clover
- valence c: Fermilab method [El-Khadra et al. 1997]

- dispersion relation for mesons containing charm

$$E(p) = M_1 + \frac{p^2}{2M_2} - \frac{a^3 W_4}{6}\sum_i p_i^4 - \frac{(p^2)^2}{8M_4^3} + \dots$$

- m_s set using ϕ
- m_c set using

$$\tfrac{1}{4}[M_2(\eta_c) + 3M_2(J/\psi)]_{lat} = \tfrac{1}{4}[M(\eta_c) + 3M(J/\psi)]_{\exp}$$

- distillation method:

(1) conventional distillation method [Peardon et al. (2009)]

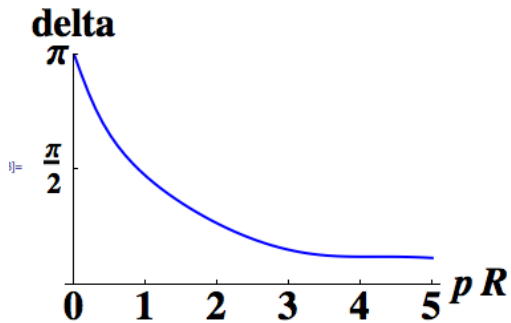
(2) stochastic version of distillation method [Morningstar et al. (2012)]

Identification of shallow bound state and Levinson's theorem

application to lattice[Sasaki, Yamazaki, 2006]

- example: non-rel. QM scattering with square-well (3D) potential
  radius R ; V_0 is such that it contains N=1 bound state

- Levinson's theorem: delta(0)=N π
  N=number of bound states



- applications to case of DK scattering:

one DK bound st $D_{s0}(2317)$ ➔ delta(0)=π and falls at small p ➔ negative a_0

- on lattice: negative a_0 ➔ positive E shift

- up-shifted scattering state was observed also in the deuterium channel (pn)
  [NPLQCD:1301.5790, PACS-CS PRD84 (2011) 054506]