Charmonium(like) and charmed states from lattice QCD

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in collaboration with

D. Mohler, C.B. Lang, L. Leskovec, R. Woloshyn

FERMILAB, Graz, Ljubljana, Vancouver
Renewed motivation: exotics from BESIII

\[ Z_c^+(3900) \rightarrow J/\psi \, \pi^+ \,_{cc}^u_{du} \]

<table>
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Same?
Experimental status

D-mesons

charmonium (like)

cc 2−: Belle, 1304.3975, PRL
D−: LHCb, 1307.4556
Lattice status – general overview: $D, D_s, \bar{c}c$

- States well below strong decay threshold:
  - proper treatment & precision calculations already available for some time

- States near threshold and resonances above threshold:
  - until 2012: **single-meson approximation**: effect of threshold not taken into account
    - strong decays of states ignored
  - exception: [Bali, Ehmann, Collins, 2011]
  - 2012, 2013, ...: first exploratory simulations with **rigorous treatment**
Charged charmonium-like and bottomonium-like states happen to lie near thresholds.

Strong motivation to treat near-threshold state properly on the lattice.

$Z_c^+(3900) \rightarrow J/\Psi \pi^+$

c c d

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[Sasa Prelovsek, Munich 2013]
Overview

Spectrum of $c\bar{c}$ (like), $D$, $D_s$ states from lattice QCD:

- States well bellow threshold

- Excited states:
  - ★ single-meson approximation
  - ★ rigorous treatment:
    - (1) states near threshold
    - (2) search for exotic states
    - (3) resonances (above threshold)

★ indirect method & EFT

"pedestrian" review:
S. P., 1310.4354
plenary @ CHARM 13
Non-perturbative method: QCD on lattice

\[ L_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G^a_\mu T^a) q - m_q \bar{q} q \]

input: \( g_s, m_s^{\text{fiz}}, m_c^{\text{fiz}}, m_{u,d} = 3.6 \times m_{u,d} \)
\[ m_\pi = 266 \text{ MeV}, \quad m_\pi^{\text{fiz}} = 140 \text{ MeV} \]

output: hadron properties
hadron interactions (if we are lucky)

Evaluation of Feynman path integrals in discretized space-time

quantum m.
\[ \int Dx \: e^{i S / \hbar} \]
\[ S = \int dt \: L[x(t)] \]

quantum field theory
\[ \int DG \: Dq \: D\bar{q} \: e^{i S_{QCD} / \hbar} \]
\[ S_{QCD} = \int d^4x \: L_{QCD}[G(x), q(x), \bar{q}(x)] \]

Sasa Prelovsek, Munich 2013
Discrete energy spectrum from correlators

\[ \langle C \rangle \propto \int D G D q D \bar{q} \ C(q, \bar{q}, G) \ e^{i S_{QCD} / \hbar}, \quad S_{QCD} = \int d^4 x \ L_{QCD} \]

Example: meson channel with given \( J^{PC} \)

\[ \Theta = \bar{q} \Gamma q, \quad \bar{q} \Gamma' q, \quad (\bar{q} \Gamma_1 q)(\bar{q} \Gamma_2 q), \ldots \]

\[ C_{ij}(t) = \langle 0 | \Theta_i(t) \Theta_j^+(0) | 0 \rangle = \sum_n \langle 0 | \Theta_i | n \rangle \ e^{-E_n t} \langle n | \Theta_j^+ | 0 \rangle = \sum_n A_{ij}^n \ e^{-E_n t} \]

All physical states appear as energy levels \( E_n \) in principle: single particle, two-particle,...

examples:

\( J^{PC} = 0^{++}, I = 1: \quad \pi, \ \pi(1400), \pi\pi\pi \)

\( J^{PC} = 1^{--}, I = 1: \quad \rho, \ \rho(1450), \pi\pi \)

\( J^{PC} = 1^{++}, \bar{c}c: \quad \chi_{c1}, \ X(3872), \ DD^* \)

\( J^{PC} = 1^{+-}, \bar{c}cd\bar{u}: \quad Z_{c}^+(3900), \ J/\psi \pi^+, \ DD^* \)
"Precision" spectrum:
States well below strong decay threshold
Lattice QCD already determined masses of these states very reliably and precisely $O(10\text{ MeV})$:

- **$m=E$ (for $P=0$)**
- Extrapolation: $a\to 0$, $L\to\infty$
- Extrapolation or interpolation: $m_q \to m_q^{\text{phy}}$
- Particular care needed for $a m_c$ discretization errors: several complementary methods give compatible results
"Non-precision" spectrum: excited states

only one or two $a, L, m_{u/d}$

limits $a \to 0, L \to \infty, \ m_{u/d} \to m_{u/d}^{\text{phy}}$ usually not performed
Excited states: single-meson approximation

- only interpolating fields
  \[ \Theta \approx \bar{q} q \]
- assumptions: all energy levels correspond to "one-particle" states
  none of the levels corresponds to multi-particle state
  \( m = E \) (for \( P = 0 \))
  these are strong assumptions ...
m-m_{ref} compared between lat and exp in order to cancel leading $a m_c$ discretization effects

D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD:

- $m_\pi \approx 266$ MeV, L$\approx$ 2 fm, Nf=2
- crosses: naive lat, diamonds: rigorous lat, lines & boxes: exp
HSC, L. Liu et al: 1204.5425, JHEP:

- $m_n \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- Reliable $J^{PC}$ determination
- Identification with $n^{2S+1}L_J$ multiplets using $<O|n>$
- Green: lat, black: exp

Hybrids:
- Some of them have exotic $J^{PC}$
- Large overlap with $O = q F_{ij} q$

Sasa Prelovsek, Munich 2013
D. Mohler, S.P., R. Woloshyn: 1208.4059, PRD:

- $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$
- crosses: naive lat, diamonds: rigorous lat, lines & boxes: exp

$m-m_{\text{ref}}$ compared between lat and exp in order to cancel leading $a m_c$ discretization effects

1S-2S splitting: $\sim 700$ MeV

red diamonds: rigorous treatment: discussed later
G. Moir et al, HSC (Hadron Spectrum Coll.): 1301.7670, JHEP:

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2 + 1$
- reliable $J^p$ determination; many excited states
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:

large overlap with $O = q F_{ij} q$

gluonic tensor $F_{ij} = [D_i, D_j]$
G. Moir et al., HSC: 1301.7670, JHEP:

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Hybrids:
large overlap with $O=q F_{ij} \bar{q}$
gluonic tensor $F_{ij}=[D_i, D_j]$
Examples:

• $X(3872)$ channel $\bar{c}c$ with $J^{PC}=1^{++}$
  
  Is the level $X(3872)$ or perhaps $D(0)D^*(0)$?

• $D_{s0}(2317)$ channel $sc$ with $J^P=0^+$
  
  Is the level $D_{s0}(2317)$ or perhaps $D(0)K(0)$?
Excited states: rigorous treatment

(1) states near threshold

note: most of interesting states are found near threshold:

$$D_{s0}^\ast(2317), \ X(3872), \ Z_c^+ (3900), \ Z_b^+$$
\( \text{**D}_{s0}(2317) \quad J^P=0^+ \)

- \( \text{D}_{s0}(2317) \) was theoretically expected above DK threshold, but it was experimentally found ~50 MeV below threshold

- why do these scalar partners have mass so close?

\[
\text{D}_{s0}(2317): \quad M \approx 2318 \text{ MeV} \quad \Gamma \approx 0 \text{ MeV} \quad \bar{c}s \text{ or } \bar{c}s[\bar{u}u + \bar{d}d] \ ?
\]

\[
\text{D}_{0}^*(2400): \quad M \approx 2318 \text{ MeV} \quad \Gamma \approx 267 \text{ MeV} \quad \bar{c}u \text{ or } \bar{c}u\bar{s}s \ ?
\]

- popular phenomenological explanation: DK threshold pushes \( \text{D}_{s0} \) mass down

- take into account the effect of DK threshold in simulation for the first time

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Basics of rigorous treatment
example: $D_{s0}^{*}(2317)$ with $J^P=0^+$

Aims to extract also two-meson states $E_n$

$\mathcal{O} = \bar{s} \, c$
$\mathcal{O} = DK \approx [\bar{d} \gamma_5 c] \, [\bar{s} \gamma_5 d]$

$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \, \mathcal{O}_j^+(0) | 0 \rangle$

We use distillation method [Peardon et al. 2009] to evaluate $C_{ij}$

$D(\vec{p})$
$K(-\vec{p})$

$\bar{p} = \frac{2\pi}{L}$

Extract $E_n$ from $C_{ij}(t)$: variational method

$C_{ij}(t) = \sum_n A_n^{ij} \, e^{-E_n t}$ due to strong int.

$E(L) = \sqrt{m_D^2 + \vec{p}^2} + \sqrt{m_K^2 + (-\vec{p})^2} + \Delta E$

Energy levels that appear in addition to these discrete two particles states correspond to bound states or resonances
\( \Theta : \bar{s} c, \ DK \approx [\bar{d}\gamma_5 c] [\bar{s}\gamma_5 d] \)

\[
\begin{align*}
O_{1-4}^{qq} &= \bar{s} M c \\
O_1^{DK} &= [\bar{s}\gamma_5 u] (p = 0) [\bar{u}\gamma_5 c] (p = 0) + \{u \rightarrow d\}, \\
O_2^{DK} &= [\bar{s}\gamma_5 \gamma_5 u] (p = 0) [\bar{u}\gamma_5 \gamma_5 c] (p = 0) + \{u \rightarrow d\}, \\
O_3^{DK} &= \sum_{p=\pm e_x, e_y, e_z} [\bar{s}\gamma_5 u] (p) [\bar{u}\gamma_5 c] (-p) + \{u \rightarrow d\}.
\end{align*}
\]

**Candidate for \( D_{s0}^* (2317) \) is found** in addition to the DK states for the first time.

D. Mohler, C. Lang, L. Leskovec, S.P., R. Woloshyn:

1308.3175, PRL : \( m_\pi \approx 156 \) MeV, \( L \approx 2.9 \) fm, \( N_f = 2+1 \)
$D_{s0}^*(2317)$ and DK scattering

$\Theta : \bar{s}c$, $DK \approx [\bar{d}\gamma_5 c] [\bar{s}\gamma_5 d]$

- M. Luscher, 80': $E \rightarrow \delta(E)$
  phase shift for DK scattering in s-wave

- $\delta$ for DK scattering in s-wave
  extracted using Luscher's relation

  \[ pcot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 \]

  $a_0 = -1.33 \pm 0.20$ fm
  $r_0 = 0.27 \pm 0.17$ fm

  $a_0 < 0$ indicates a state below th.

- relation above gives pole position and
  the mass of $D_{s0}^*(2317)$

  \[ S \propto [\cot\delta - i]^{-1} = \infty, \quad \cot\delta(p_{BS}) = i \]

  \[ m_{D_{s0}}^{lat, L \rightarrow \infty} = E_D(p_{BS}) + E_K(p_{BS}) \]

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<tr>
<th>$D_{s0}^*(2317)$</th>
<th>$m - \frac{1}{4} (m_{D_s} + 3m_{D_{s*}})$</th>
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<tbody>
<tr>
<td>lat</td>
<td>266 $\pm 16\pm4$ MeV</td>
</tr>
<tr>
<td>exp</td>
<td>241.45 $\pm 0.6$ MeV</td>
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D. Mohler, C. Lang, L. Leskovec, S.P., R. Woloshyn:
1308.3175, PRL : $m_\pi \approx 156$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$

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X(3872): experimental facts

- first observed in 2003 [Belle PRL 2003]
- $J^{PC}=1^{++}$ [LHCb, 2013]
- sits within 1 MeV of $D^0D^{0*}$ threshold
- selected decays
  
  $X(3872) \rightarrow J/\Psi \omega \ (I=0)$
  
  $X(3872) \rightarrow J/\Psi \rho \ (I=1)$
$X(3872)$: interpolators \quad J^{PC}=1^{++} \ (T_1^{++}) \ , \ P=0, \ I=0,1

$\Theta : \bar{c} \ c, \ \bar{D} \bar{D}^*, \ J/\psi \omega$

\[
O^\bar{c}c_{i-8} = \bar{c} \hat{M}_i \ c(0) \quad \text{(only } I = 0) \\
O^{DD^*}_1 = [\bar{c}\gamma_5 u(0) \ \bar{u}\gamma_i c(0) - \bar{c}\gamma_i u(0) \ \bar{u}\gamma_5 c(0)] + f_I \{u \rightarrow d\} \\
O^{DD^*}_2 = [\bar{c}\gamma_5 \gamma_t u(0) \ \bar{u}\gamma_i c(0) - \bar{c}\gamma_i \gamma_t u(0) \ \bar{u}\gamma_5 c(0)] + f_I \{u \rightarrow d\} \\
O^{DD^*}_3 = \sum_{e_k = \pm e_{x,y,z}} [\bar{c}\gamma_5 u(e_k) \ \bar{u}\gamma_i c(-e_k) - \bar{c}\gamma_i u(e_k) \ \bar{u}\gamma_5 c(-e_k)] + f_I \{u \rightarrow d\}
\]

$O^{J/\psi V}_1 = \epsilon_{ijk} [\bar{c}\gamma_j \gamma_t c(0) + f_I \ \bar{d}\gamma_k \gamma_t d(0)]$ \\
$O^{J/\psi V}_2 = \epsilon_{ijk} [\bar{c}\gamma_j \gamma_t c(0) + f_I \ \bar{d}\gamma_k \gamma_t d(0)]$

$I = 0: \quad f_I = 1, \quad V = \omega$ \\
$I = 1: \quad f_I = -1, \quad V = \rho$
\( X(3872) : \quad J^{PC}=1^{++} \quad [\text{LHCb 2013}], \quad I=0: \text{Wick contractions} \)

\[ \mathcal{O} : \bar{c} c, \quad DD^*, \quad J/\psi \omega \]

\[ C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^+(0) | 0 \rangle \]

- we calculate all Wick contractions
$X(3872):$ $J^{PC}=1^{++}$ [LHCb 2013], $I=0$: Wick contractions

- we calculate all Wick contractions
- results are based only on 13 Wick contractions in Fig. a (where $c$ propagates from source to sink)
- the effect of remaining ones suppressed by OZI rule [see also Levkova, DeTar 2011]
- their effect will be addressed on follow-up analysis
**X(3872): J^{PC}=1^{++} [LHCb 2013], I=0**

\[ \Theta: \bar{c}c, \quad DD^*, \quad J/\psi \omega \]

- \( \delta \) for \( DD^* \) scattering in s-wave extracted using Luscher's relation

\[
p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2
\]

\[ a_0 = -1.7 \pm 0.4 \text{ fm} \]

\[ r_0 = 0.5 \pm 0.1 \text{ fm} \]

large and \( a_0<0 \) indicates a state slightly below \( DD^* \) threshold: \( X(3872) \)

- Pole position gives mass of \( X(3872) \)

\[ S \propto [\cot \delta - i]^{-1} = \infty, \quad \cot \delta(p_{BS}) = i \]

\[ m_{X_{lat.,L\to\infty}} = E_D(p_{BS}) + E_{D^*}(p_{BS}) \]

---

**Candidate for X(3872) is found** in addition to the expected two-particle states for the first time.

S. P. and L. Leskovec: 1307.5172, PRL

\( m_\pi = 266 \text{ MeV}, \quad L=2 \text{ fm}, \quad N_f=2 \)

---

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<td>lat</td>
<td>- 11 ± 7 MeV</td>
</tr>
<tr>
<td>exp</td>
<td>- 0.14 ± 0.22 MeV</td>
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lat: simulations on larger L required

exp: Tomaradze et al., 1212.4191

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Composition of established $X(3872)$ with $I=0$

- it has sizable coupling with $\bar{c}c$ as well as $DD^*$ interpolating fields
- overlaps of $X$ with interpolators $\left\langle O_i \right| X(3872) \right\rangle$

S. P. and L. Leskovec: 1307.5172, PRL
$m_\pi = 266$ MeV, $L=2$ fm, $N_f=2$
Search for $X(3872)$ with $J^{PC}=1^{++}$ and $I=1$

Only expected two-particle states observed. No candidate for $X(3872)$ found.

In agreement with two interpretations:

1. $X(3872)$ pure $I=0$
   
   isospin breaking happens only in decay

   $X(3872) \rightarrow J/\Psi \rho \ (I=1)$

   isospin breaking: $D^0 D^{0*}, D^+ D^{-*}$ splitting

2. $X(3872) = a_{I=0} \langle DD^* \rangle_{I=0} + a_{I=1} \langle DD^* \rangle_{I=1}$

   $a_{I=1}(m_u = m_d) = 0$

   $a_{I=1}(m_u \neq m_d) \ll a_{I=0}$

In simulation: $m_u = m_d$

S. P. and L. Leskovec: 1307.5172, PRL

$m_\pi = 266$ MeV, $L=2$ fm, $N_f=2$
(2) Searches for exotic states: rigorous treatment
Search for $Z_c^{+}(3900)$ in $J^{PC}=1^{+-}$, $l=1$ channel

$\Theta: DD^*, J/\psi \pi$

$Z_c^+(3900) \rightarrow J/\psi \pi^+$

$J^{PC} = 1^{+-} \quad \bar{c}c \bar{d}u$

if $Z_c(3900) = Z_c(3885)$

[BesIII, Belle, CleoC, 2013]

Only expected two-particle states observed.

**No candidate for $Z_c^{+}(3900)$ with $J^{PC}=1^{+-}$ is found.**

- Possible reasons:
  - perhaps $J^{PC} \neq 1^{+-}$ if $Z_c(3900) \neq Z_c(3885)$
  - perhaps our interpolators (all of scat. type) are not diverse enough: calls for further simulations
  - ??
Search for $Y(4140)$ in $J/\psi\Phi$ scattering

**Experiment:**
- $Y(4140)$ found in $J/\psi\Phi$, $\Gamma \approx 11$ MeV [CDF 2009]
- not seen in $D_sD_s$
- not seen by Belle, LHCb

**Lattice:**
- method to get $\delta$ at more $E$:
  - twisted BC for valence $q$.
  
  \[ q(x + L) = e^{i\theta} q(x) \]

  instead of periodic BC (conventional)
  
  \[ q(x + L) = q(x) \]

- conclusion:
  - no resonant structure found at energies reported by CDF

**Caveats:**
- $s$-quark annihilation ignored
- twisting is partial: only on valence quarks

S. Ozaki and S. Sasaki, 1211.5512, PRD

$m_\pi \approx 156$ MeV, $L \approx 2.9$ fm, $N_f=2+1$
Conclusion:

• potential is attractive
• no bound tetraquark state at simulated $m_{\pi}$
• in case of one bound state one would expect $\delta(E=0) = \pi$ due to Levinson's theorem


$m_{\pi} \approx 410-700$ MeV, $L \approx 2.9$ fm, $N_f=2+1
Excited states: rigorous treatment
(3) resonances
Almost all hadrons are **hadronic resonances** (decay strongly)

- $\bar{c}c$
  - $\eta_c(1S)$
  - $J/\psi(1S)$
  - $\chi_{c0}(1P)$
  - $\chi_{c1}(1P)$
  - $h_c(1P)$
  - $\chi_{c2}(1P)$
  - $\eta_c(2S)$
  - $\psi(2S)$
  - $\psi(3770)$
  - $X(3872)$
  - $\chi_{c0}(2P)_{\omega\varepsilon}$
  - $\chi_{c2}(2P)$
  - $X(3940)$
  - $\psi(4040)$
  - $X(4050)^{\pm}$
  - $X(4140)$
  - $\psi(4160)$
  - $X(4160)$
  - $X(4250)^{\pm}$

- $\bar{u}u$
  - $\pi^\pm$
  - $\eta$
  - $\rho(770)$
  - $\omega(782)$
  - $\eta(958)$
  - $f_0(980)$
  - $a_0(980)$
  - $\phi(1020)$
  - $h_1(1170)$
  - $b_1(1235)$
  - $a_0(1260)$
  - $\rho(1450)$

- $\bar{s}u$
  - $K^\pm$
  - $K^0$
  - $K_0^0$
  - $K_0^*$
  - $K_1(1430)$

- $\bar{c}u$
  - $D^+$
  - $D^0$
  - $D^*_0(2400)^0$
  - $D_1(2420)^+$
  - $D_2^*(2460)^0$
  - $D_2^*(2460)^+$

- $uud$
  - $p$
  - $n$
  - $N(1440)_{1/2^+}$
  - $N(1520)_{3/2^+}$

- Others decay strongly; **hadronic resonances**
  - $N(1535)_{1/2^+}$
  - $N(1650)_{1/2^+}$
  - $N(1675)_{5/2^+}$
  - $N(1680)_{5/2^+}$
  - $N(1685)_{??}$
  - $N(1700)_{3/2^+}$
  - $N(1710)_{1/2^+}$
  - $N(1720)_{3/2^+}$
  - $N(1860)_{5/2^+}$
  - $N(1875)_{3/2^+}$
  - $N(1880)_{1/2^+}$
  - $N(1895)_{1/2^+}$
  - $N(1900)_{3/2^+}$
  - $N(1990)_{7/2^+}$
  - $N(2000)_{5/2^+}$
  - $N(2040)_{3/2^+}$
  - $N(2060)_{5/2^+}$
  - $N(2100)_{1/2^+}$

- Only resonance simulated properly by several collab. (first simulation: CP-PACS 2007)
- SP and coll. (Lang, Mohler, Leskovec, Woloshyn) (2011-2013)
- Verduci, Lang (2012)

- Stable on strong decay: ab-initio OK

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$P \neq 0: s = E^2 - P^2$, Luscher-type relation: $s \rightarrow \delta(s)$

$\rho$ resonance

$[\text{Lang, Mohler, S.P. ,Vidmar, PRD 2011}]$
$m_\pi \approx 266\text{ MeV}$

Simulation also by CP-PACS, PACS-CS, QCDSF, ETMC

$[\text{HSC, PRD 2013}]$
$m_\pi \approx 400\text{ MeV}$

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**K*(892) resonance:** first lattice determination of width

**Kπ, I=1/2: p-wave phase shift**

![Graph showing the fit with two elastic Breit-Wigner resonances](image)

\[
\frac{p^3}{\sqrt{s}} \cot \delta_1(s) = \left[ \sum \frac{g_{K_i}^2}{6\pi} \frac{1}{m_{K_i}^2 - s} \right]^{-1}
\]

\[
K_i^* = K^*(892), \ K^*(1410) .
\]

\[
\Gamma[K^* \to K\pi] = \frac{g^2 p^3}{6\pi s}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_{K^*(892)}</td>
<td>891 ± 14</td>
<td>MeV</td>
</tr>
<tr>
<td>g_{K^*(892)}</td>
<td>5.7 ± 1.6</td>
<td>no unit</td>
</tr>
<tr>
<td>m_{K^*(1410)}</td>
<td>1.33 ± 0.02</td>
<td>GeV</td>
</tr>
<tr>
<td>g_{K^*(1410)}</td>
<td>input</td>
<td>no unit</td>
</tr>
</tbody>
</table>

m_π ≈ 266 MeV

[S.P. Lang, Leskovec, Mohler, 1307.0736, PRD]
D_0^*(2400) resonance in Dπ scattering: J^P=0^+, I=1/2

"rigorous" treatment illustrated on this example

All states with J^P=0^+ appear in lat. spectrum:

• D_0^*(2400)

• D(p) π(-p) with p=n 2π/L : "two-particle" states
  horizontal lines indicate their energies in absence of interaction

Rigorous relation [M. Luscher, 1991]:
E → δ(E)  phase shift for Dπ scattering in s-wave

E - 1/4 (m_D+3 m_D^*) [MeV]  δ ~ 173^o +/- 12^o  D(1)π(-1)

δ ~ 103^o  D(0)π(0)

δ ~ i 42^o  D(0)π(0)

BW:  δ = acot \( \frac{m_R^2 - E_{\text{cms}}^2}{m_R \Gamma} \)

D(p) π(-p) ≈ [\overline{d} \gamma_5 c] [\overline{u} \gamma_5 d]

D. Mohler, S.P., R. Woloshyn: 1208.4059, PRD  Sasa Prelovsek, Munich 2013
D-meson resonance masses and widths

\[ \Gamma(E) \equiv g^2 \frac{P}{E^2} \]

g is compared to \( \exp \) instead of \( \Gamma \) (\( \Gamma \) depends on phase sp. and \( m_\pi \))

\[ J^P=0^+ : D \pi \]

<table>
<thead>
<tr>
<th>( D_0^* (2400) )</th>
<th>( m - \frac{1}{4}(mD + 3mD^*) )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat</td>
<td>351 ± 21 MeV</td>
<td>2.55 ± 0.21 GeV</td>
</tr>
<tr>
<td>exp</td>
<td>347 ± 29 MeV</td>
<td>1.92 ± 0.14 GeV</td>
</tr>
</tbody>
</table>

\[ J^P=1^+ : D^* \pi \] (analysis of spectrum in this case is based on an assumption given in paper below)

<table>
<thead>
<tr>
<th>( D_1 (2430) )</th>
<th>( m - \frac{1}{4}(mD + 3mD^*) )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat</td>
<td>381 ± 20 MeV</td>
<td>2.01 ± 0.15 GeV</td>
</tr>
<tr>
<td>exp</td>
<td>456 ± 40 MeV</td>
<td>2.50 ± 0.40 GeV</td>
</tr>
</tbody>
</table>

first lattice result for strong decay width of a hadron containing charm quark

[D. Mohler, S.P., R. Woloshyn: 1208.4059, PRD]

- \( m_\pi = 266 \) MeV, \( L = 2 \) fm, \( N_f = 2 \)

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\( \text{Dπ scattering : I}=1/2, \text{ s-wave, } J^P=0^+ \)

**Puzzle**

\[ D_0^*(2400): \quad M \approx 2318 \text{ MeV} \quad \Gamma \approx 267 \text{ MeV} \quad \bar{c}u \quad \text{or} \quad \bar{c}u\bar{s}s \quad ? \]

\[ D_{s0}(2317): \quad M \approx 2318 \text{ MeV} \quad \Gamma \approx 0 \text{ MeV} \quad \bar{c}s \quad \text{or} \quad \bar{c}s[\bar{u}u + \bar{d}d] \quad ? \]

Our resulting \( D_0^*(2400) \) mass is in favorable agreement with exp without valence \( ss \) pair.
\( J^{PC}=0^{++} \) charmonium resonance(s): \( \chi_{c0}' \) ?

PRELIMINARY

By simulating DD scattering in s-wave we find:

(1) narrow resonance in DD scattering [we call it \( \chi_{c0}' \)]

\[
m[\chi_{c0}'] = 3932 \pm 25 \text{ MeV}
\]

\[
\Gamma[\chi_{c0}' \to DD] = 36 \pm 17 \text{ MeV}
\]

PDG12: \( \chi_{c0}'=\chi(3915) \) ?! Why no \( X(3915) \to DD \) in exp ?!

perhaps there is a hit of it [D. Chen et al, 1207.3561, PRD]

(2) additional enhancement of \( \sigma(DD) \) near th.: could it be related to broad structures ?

[see also F. Guo, U. Meissner, 1208.1134, PRD]

S.P. , L. Leskovec and D. Mohler,

1310.8127, Lat 2013 proc:

- \( m_\pi \approx 266 \text{ MeV}, L=2 \text{ fm}, Nf=2 \)
Indirect method combined with EFT
Indirect study of $D_{s0}^*(2317)$ channel

(1) Five channels that do not include Wick contractions are simulated

(2) Scattering lengths $a = \lim_{p \to 0} \frac{\tan \delta(p)}{p}$ for four $m_\pi$ extracted

(3) Simultaneous fit using SU(3) unitarized ChPT is performed and LEC's are determined

(4) Using these LEC's indirect predictions for:
- scattering length of two resonant-channels with contractions
- $DK$ ($S=1, I=0$): pole in the first Riemann sheet found

<table>
<thead>
<tr>
<th>$D_{s0}^*(2317)$</th>
<th>$m$</th>
<th>$\Gamma [D_{s0}^* \rightarrow D_s \pi]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>indirect lat</td>
<td>2315 $^{+18}_{-28}$ MeV</td>
<td>133$^{\pm 22}$ keV</td>
</tr>
<tr>
<td>exp</td>
<td>2317.8 ±0.6 MeV</td>
<td>&lt; 3.8 MeV</td>
</tr>
</tbody>
</table>

L. Liu, Orginos, Guo, Hanhart, Meissner, 1208.4535, PRD, $m_\pi \approx 300-620$ MeV, Nf=2+1

Sasa Prelovsek, Munich 2013
Conclusions & outlook

Present status of lattice results for $D$, $D_s$, $cc$ spectra:

• states well below strong decay threshold determined reliably and with good precision

• excited states: single-meson approximation

spectra with a number of full $qq$ multiplets and hybrids calculated during 2012, 2013

• excited states: rigorous treatment: first simulations during 2012, 2013

★ $D_0^*(2400)$, $D_1(2430)$, $D_{s0}^*(2317)$, $X(3872)$ identified

★ $Z_c^+(3900)$, $Y(4140)$, $ccud$ not (yet) found

Precision simulations of these channels will have to be performed in the future.
Outlook for lattice simulations of $D$, $D_s$, $cc$ spectra:

Which excited states can one treat rigorously in the near future?

- states not to far above strong decay threshold that have one (dominant) decay mode
  
  example: $Z_c^+(3900)$ is less challenging than $Z^+(4430)$

- states that are *not* accompanied by many lower states of the same quantum number
  
  example: higher lying $1^-$ charmonium states would be very challenging for rigorous treatment

Lots of exciting experimental results prompt for lots of exciting lattice simulations in the near future, encouraged by the pioneering exploratory steps made during the last year!
Backup slides
Two ensembles:

<table>
<thead>
<tr>
<th>ID</th>
<th>$N_L^3 \times N_T$</th>
<th>$N_f$</th>
<th>$a[\text{fm}]$</th>
<th>$L[\text{fm}]$</th>
<th>#configs</th>
<th>$m_{\pi}[\text{MeV}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$16^3 \times 32$</td>
<td>2</td>
<td>0.1239(13)</td>
<td>1.98</td>
<td>279</td>
<td>266(3)(3)</td>
</tr>
<tr>
<td>(2)</td>
<td>$32^3 \times 64$</td>
<td>2+1</td>
<td>0.0907(13)</td>
<td>2.90</td>
<td>196</td>
<td>156(7)(2)</td>
</tr>
</tbody>
</table>

On both ensembles:

- dynamical $u, d, (s)$, valence $u,d,s$: Improved Wilson Clover

- dispersion relation for mesons containing charm

$$E(p) = M_1 + \frac{p^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(p^2)^2}{8M_4^3} + \ldots$$

- $m_s$ set using $\phi$
- $m_c$ set using

$$\frac{1}{4} [M_2(\eta_c) + 3M_2(J/\psi)]_{\text{lat}} = \frac{1}{4} [M(\eta_c) + 3M(J/\psi)]_{\text{exp}}$$

- distillation method:
  1. conventional distillation method [Peardon et al. (2009)]
  2. stochastic version of distillation method [Morningstar et al. (2012)]
Identification of shallow bound state and Levinson's theorem

- example: non-rel. QM scattering with square-well (3D) potential radius R; $V_0$ is such that it contains N=1 bound state

- Levinson's theorem: $\delta(0) = N \pi$
  $N =$ number of bound states

- applications to case of DK scattering:
  one DK bound state $D_{s0}(2317) \Rightarrow \delta(0) = \pi$ and falls at small $p \Rightarrow$ negative $a_0$

- on lattice: negative $a_0 \Rightarrow$ positive $E$ shift

- up-shifted scattering state was observed also in the deuterium channel (pn)
  [NPLQCD:1301.5790, PACS-CS PRD84 (2011) 054506]