Advances in hadron interactions and spectroscopy from lattice QCD

mesons: this talk

baryons: next talk by M. Padmanath

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Oberwoltz 2018, Quantum Chromodynamics and its symmetries

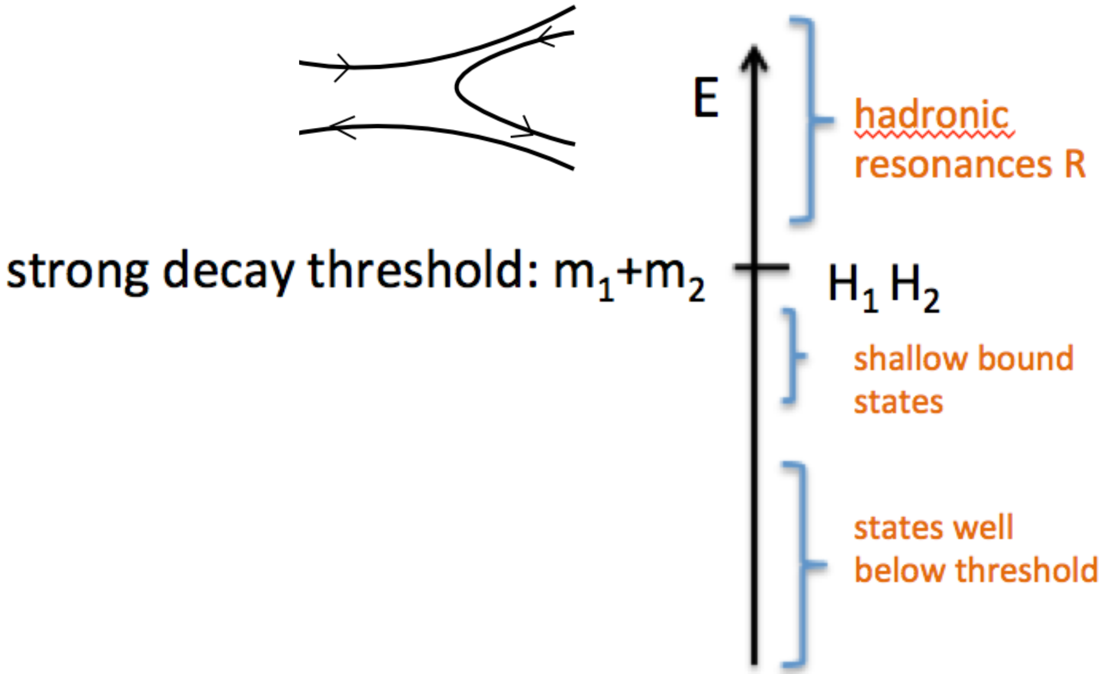
679 Heraeus Seminar, 9-15 September 2018

review & work in collaboration with

QCD: Lang, Padmanath, Mohler, Leskovec, Woloshyn, Collins, Piemonte, Bali, Skerbis

BSM: Drach, Janowski, Pica

Classification of hadron states



decay strongly

can not decay strongly

Meson classification with respect to threshold

- ❑ states well below threshold
- ❑ strongly decay: resonances
- ❑ candidates for shallow bound st.

$\bar{u}u$

- $\pi^{\pm}$
- π^{0}
- η
- $f_0(500)$ or σ was $f_0(60$
- $\rho(770)$
- $\omega(782)$
- $\eta'(958)$
- $f_0(980)$
- $a_0(980)$
- $\phi(1020)$
- $h_1(1170)$
- $b_1(1235)$
- $a_1(1260)$
- $f_2(1270)$
- $f_1(1285)$
- $\eta(1295)$
- $\pi(1300)$
- $a_2(1320)$
- $f_0(1370)$
- $h_1(1380)$
- $\pi_1(1400)$
- $\eta(1405)$
- $f_1(1420)$
- $\omega(1420)$
- $f_2(1430)$
- $a_0(1450)$
- $\rho(1450)$

$\bar{s}u$

- $K^{\pm}$
- K^{0}
- $K_S^{\,0}$
- $K_L^{\,0}$
- $K_0^*(800)$ or κ
- $K^*(892)$
- $K_1(1270)$
- $K_1(1400)$
- $K^*(1410)$
- $K_0^*(1430)$
- $K_2^*(1430)$
- $K(1460)$
- $K_2(1580)$
- $K(1630)$
- $K_1(1650)$
- $K^*(1680)$
- $K_2(1770)$
- $K_3^*(1780)$
- $K_2(1820)$
- $K(1830)$

$\bar{c}u$

- $D^{\pm}$
- D^{0}
- $D^*(2007)^0$
- $D^*(2010)^{\pm}$
- $D_0^*(2400)^0$
- $D_0^*(2400)^{\pm}$
- $D_1(2420)^0$
- $D_1(2420)^{\pm}$
- $D_1(2430)^0$
- $D_2^*(2460)^0$
- $D_2^*(2460)^{\pm}$
- $D(2550)^0$
- $D(2600)$
- $D^*(2640)^{\pm}$
- $D(2750)$

$\bar{c}s$

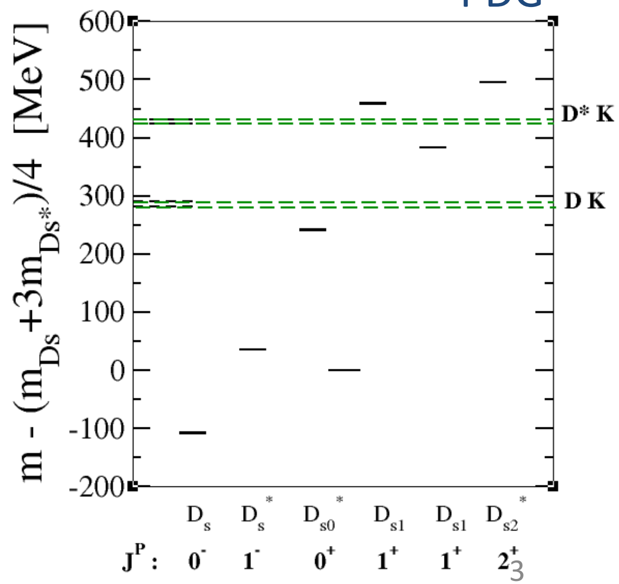
- $D_s^{\pm}$
- $D_s^{*\pm}$
- $D_{s0}^*(2317)^{\pm}$
- $D_{s1}(2460)^{\pm}$
- $D_{s1}(2536)^{\pm}$
- $D_{s2}^*(2573)$
- $D_{s1}^*(2700)^{\pm}$
- $D_{s1}^*(2860)^{\pm}$
- $D_{s3}^*(2860)^{\pm}$
- $D_{sJ}(3040)^{\pm}$

…

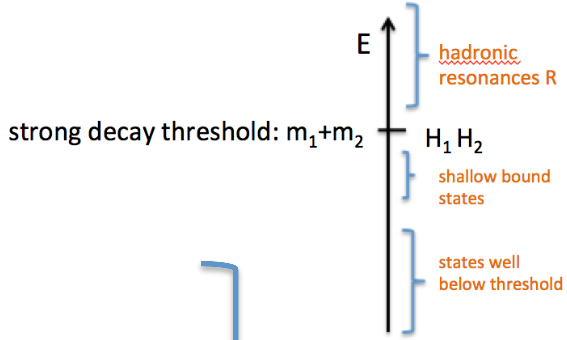
exotic mesons

all observed ones are near or above threshold

PDG



- T=0
- QCD: spontaneously broken chiral s., approx. $SU(3)_{F,}$ heavy quark symmetry for large m_Q
- no new symmetries ...



Outline

smulation of **scattering**

- states well below threshold (briefly)
- hadronic resonances : R -> H_1H_2
- shallow bound states
- resonances above several thresholds: R -> H_1H_2 , $H'_1H'_{2,\ldots}$

QCD

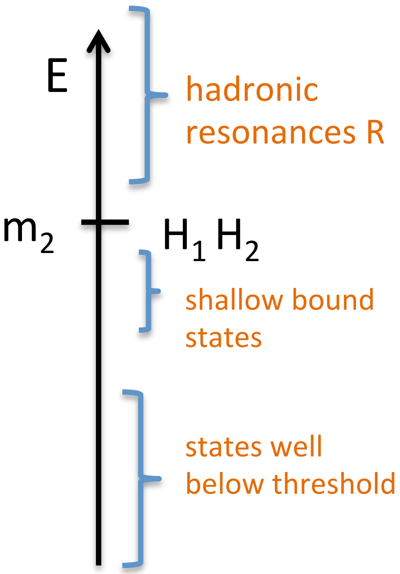
- resonance in Beyond SM strongly coupled scenario

BSM

Disclaimer: for clarity - “simpler” systems and pioneering (“simpler”) simulations are chosen for presentation

Status within Lattice QCD

strong decay threshold: m_1+m_2



- Hadrons well bellow threshold: easy, accomplished
  Unfortunately none of exotic experimental candidates
  is found well below threshold.

- Hadron resonances above one threshold and bound states slightly below threshold :
  - until about 2010: treated ignoring decay or threshold effect
  - now: rigorous treatment by determining scattering matrix for two hadrons $H_1\, H_2$

  more challenging, partly accomplished

- Resonances above several two-hadron thresholds:
  - requires simulation of coupled channel scattering
  - most exotic candidates belongs to this class

  very challenging, only few simulations exist

- Resonances in 3-hadron channels (e.g. $\omega \to \pi\pi\pi$)
  - (non-trivial!) progress on formal developments
  - no lattice QCD simulations yet

Lattice QFT: Discrete energies of eigenstates: E_n

mesons

meson$_1$ meson$_2$

$$J^{PC} \qquad \mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 q)_{\vec{p}_2}, \quad [\bar{q}\Gamma_3 \bar{q}][q\Gamma_4 q], \ldots$$

$$\text{charmonium}: \quad \bar{c}c\,, \quad (\bar{c}u)(\bar{u}c) = D\bar{D}\,, \qquad [\overline{cu}][cu]$$

intended to simulate meson-meson scattering

$$\sum |n\rangle\langle n|$$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle \; e^{-E_n t} \; \langle n | \mathcal{O}_j^+ | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

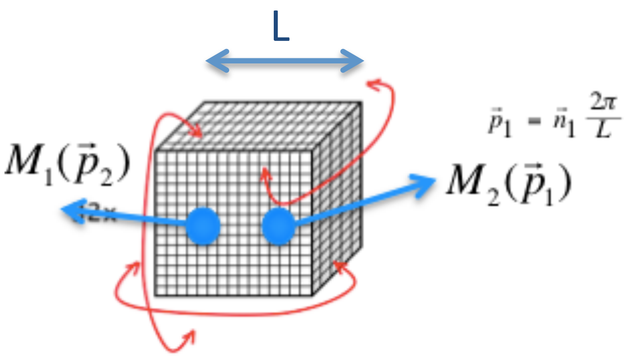
All eigenstates with given J^{PC} appear in principle

example: 1-- charmonium vector channel

- single hadron states $\quad \bar{c}c = J/\psi \quad$ m=E_1 for P=0
- two-hadron states $\quad D\,\bar{D} \quad$ E_n render $D\bar{D}$ scat. m.

Two-hadron energies in continuum and on the lattice

Lattice: periodic b.c. in space



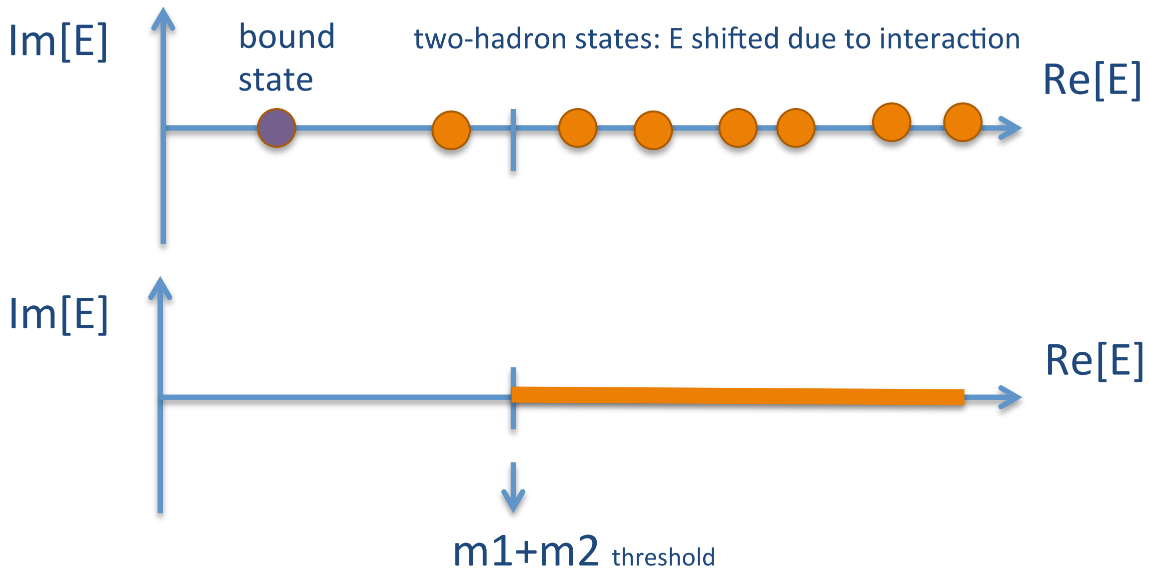
in non-interacting limit

$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}}, \qquad \vec{p}_1 = \tfrac{2\pi}{L}\vec{n}_1 \qquad \vec{p}_2 = \tfrac{2\pi}{L}\vec{n}_2$$

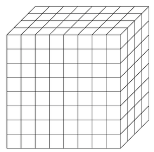
finite L (lat)

continuum (exp)

L=∞



Hadrons that do not decay strongly: "easy"

- $m=E_n$ for $P=0$ $a\to 0$, $L\to\infty$, $m_q\to m_q^{phy}$ 

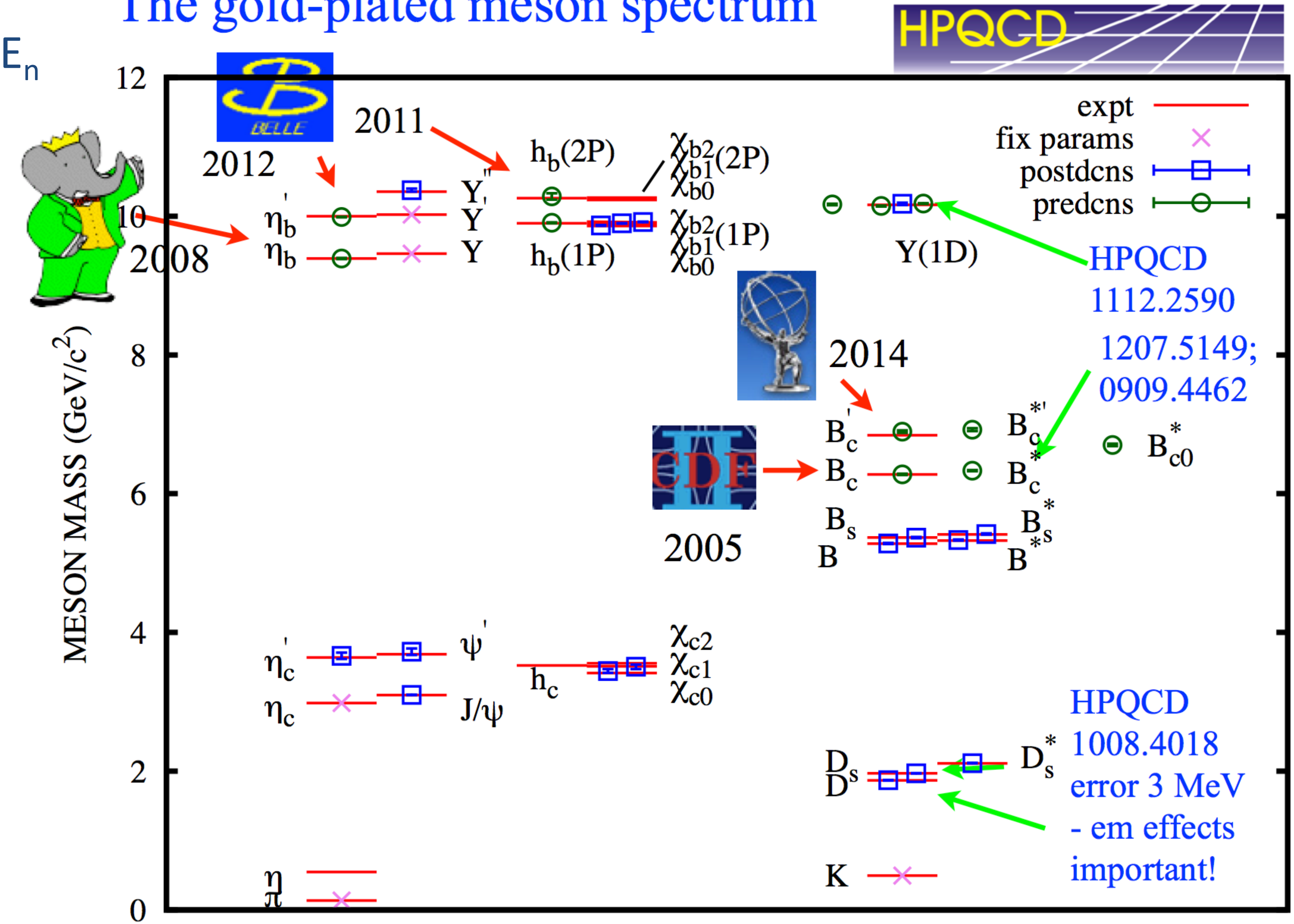
- Available from a number of lattice QCD collaborations for a number of years

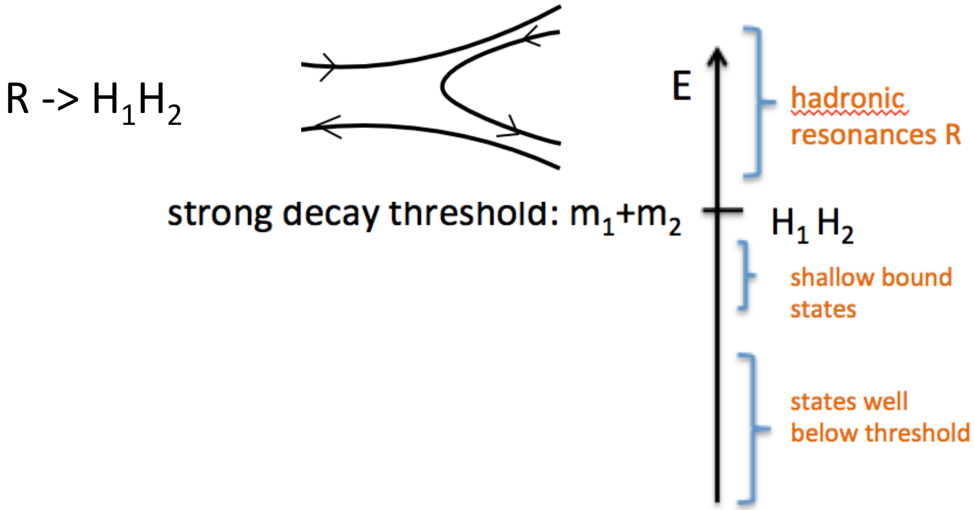
Solved for a while ...

High Precision QCD collab.

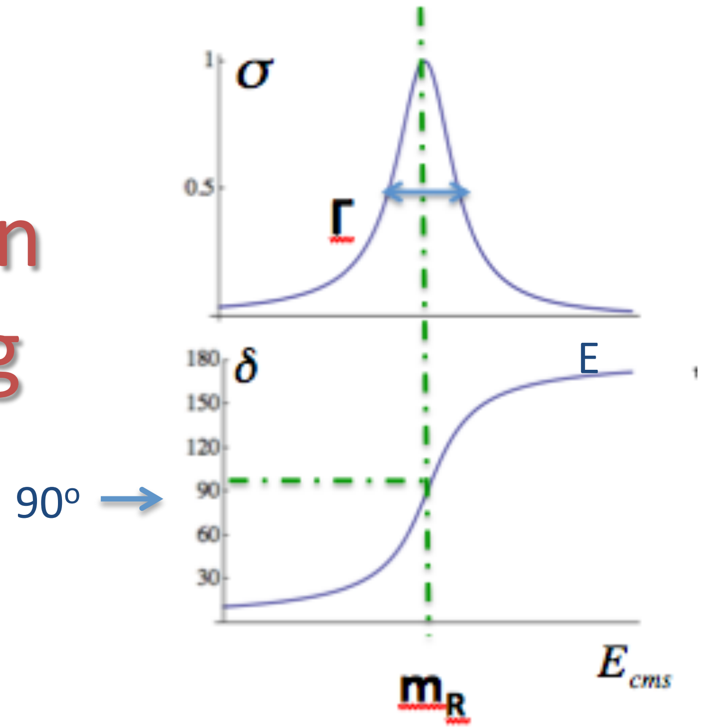
The gold-plated meson spectrum

$m = E_n$

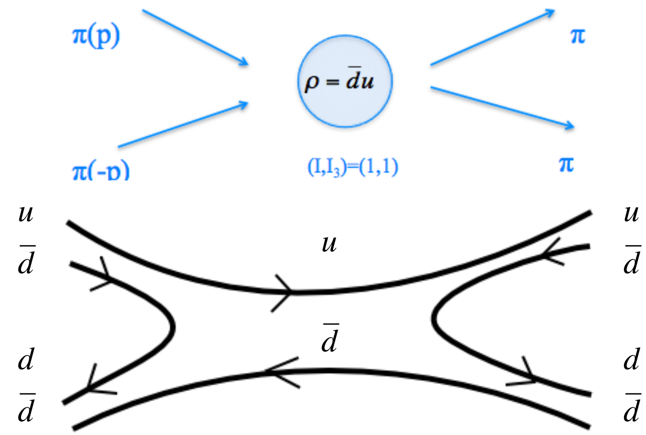




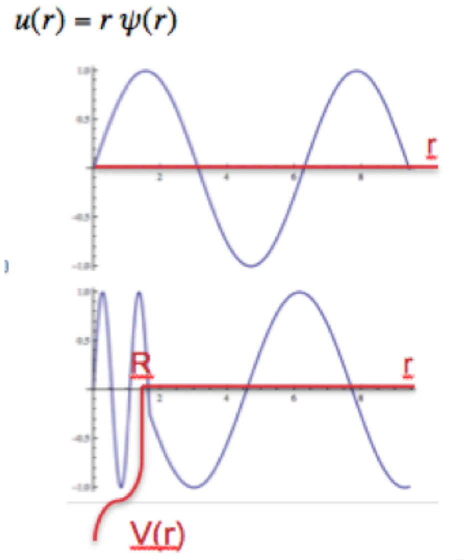
Hadronic resonances in one-channel scattering



Resonances in two-hadron scattering



Scattering phase shift δ

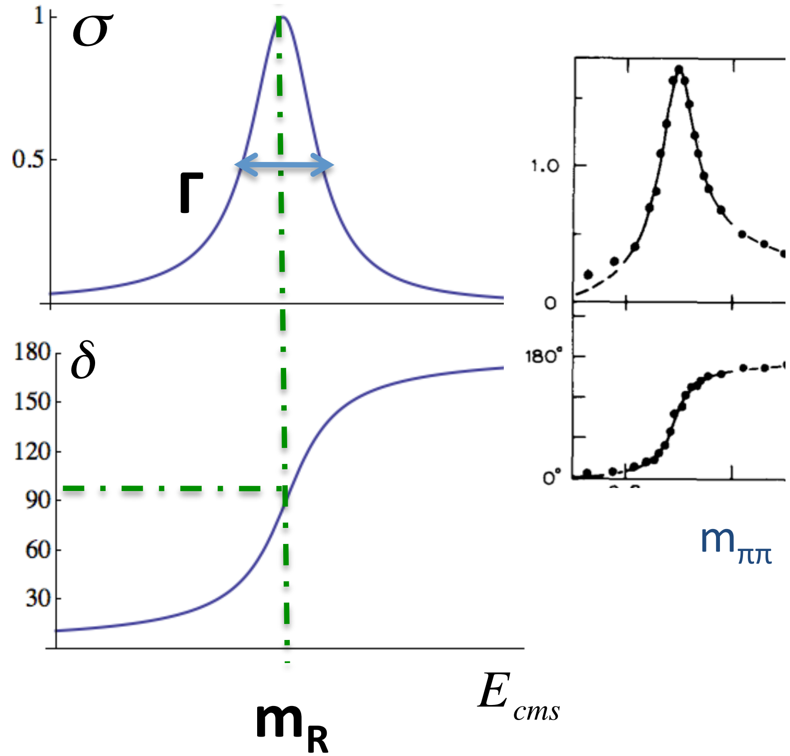


QM interpretation:

$$\psi(r) \propto \frac{\sin(pr)}{r}$$

$$\psi(r) \propto \frac{\sin(pr+\delta)}{r} \qquad r > R$$

$$\text{unknown} \qquad r < R$$



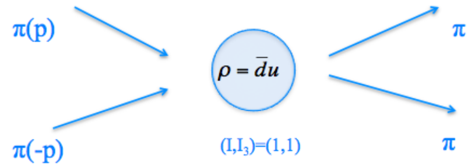
scattering matrix T(E) from δ(E) for elastic scat:

$$S(E) = 1 + 2\,i\,T(E) = e^{2\,i\,\delta(E)}$$

$$T = \frac{e^{2i\delta}-1}{2i} = \sin\delta\; e^{i\delta} = \frac{1}{\cot\delta - i}$$

$$\sigma \propto |T|^2 = \sin^2\delta$$

Extracting scattering phase shift δ



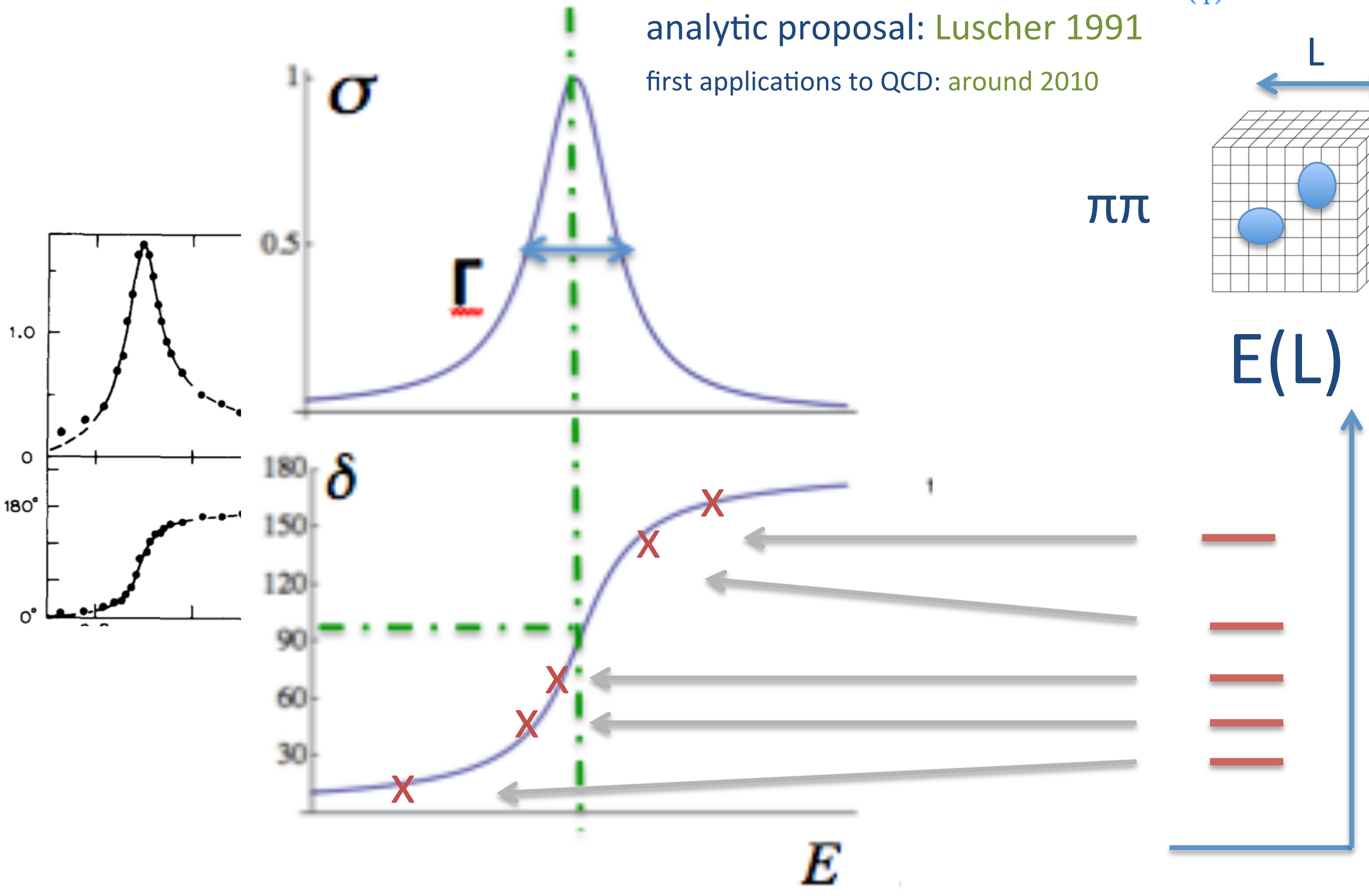
example

analytic proposal: Luscher 1991

first applications to QCD: around 2010

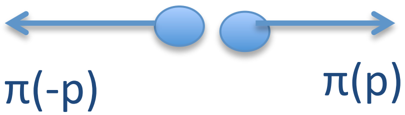
two mesons

in a lattice box



scattering phase shifts at infinite volume — δ(E) ← E(L) — energies from lattice with spatial extent L

Luscher's relation between E or p and δ in 1D



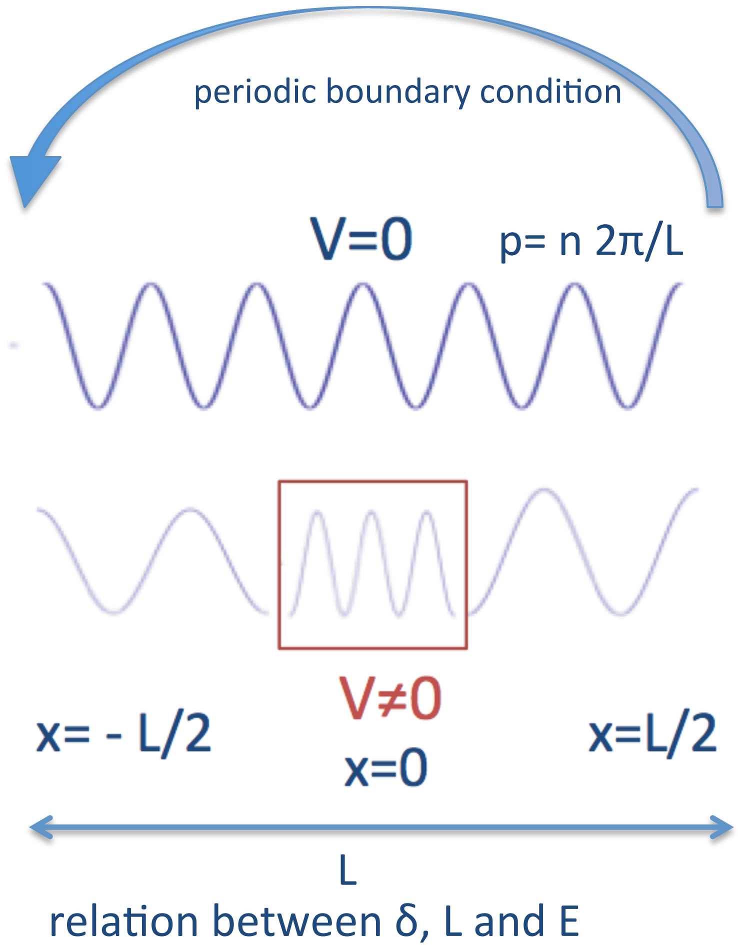
$$\psi(x) = A\cos(p\,|x|+\delta)$$ in region outside potential

$$\psi(\tfrac{L}{2}) = \psi(-\tfrac{L}{2})$$

$$\psi'(\tfrac{L}{2}) = \psi'(-\tfrac{L}{2}): \quad \sin(p\tfrac{L}{2}+\delta) = 0$$

$$\boxed{p = n\frac{2\pi}{L} - \delta\frac{2}{L}}$$

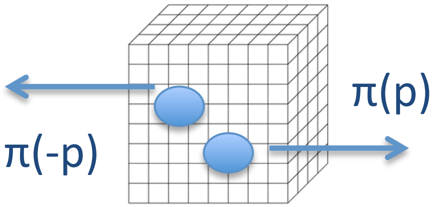
$$E = \sqrt{m_1^2+p^2} + \sqrt{m_2^2+p^2}$$



Relation between E and δ(E) in 1D and 3D (P=0)

analytic proposal: Luscher 1991

Relations if a single partial wave dominates

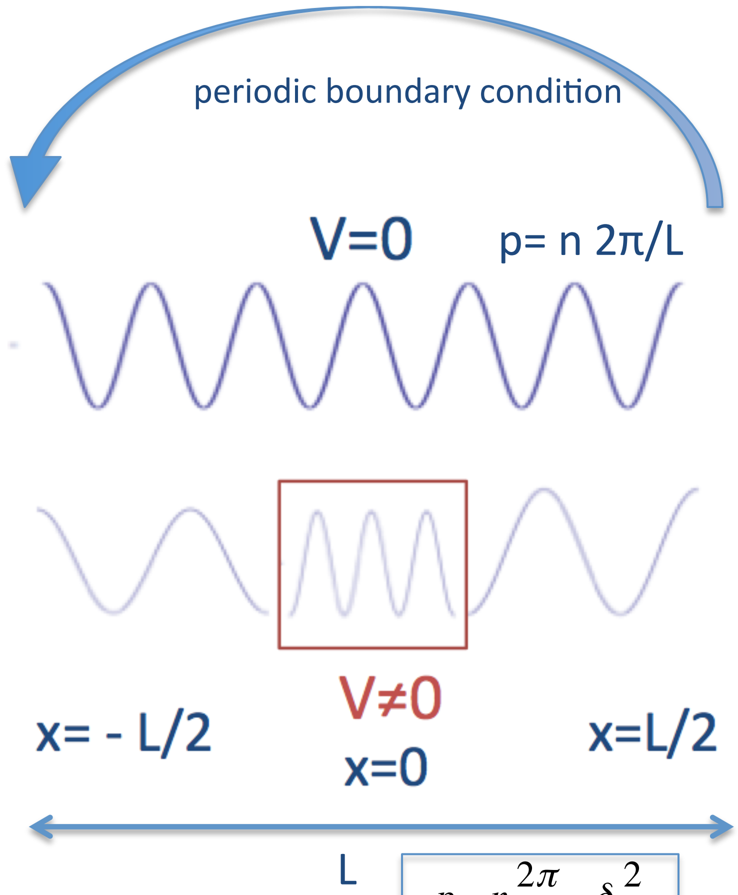


$$\cot\delta(p) = \frac{1}{pL\pi}\sum_{\vec{n}\in N^3}\frac{1}{\vec{n}^2 - (\frac{pL}{2\pi})^2}$$

$$E(p) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2}$$

relation between δ, L and E

for P_{tot}=0

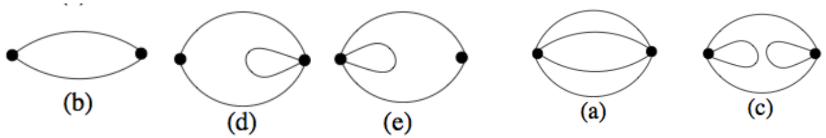


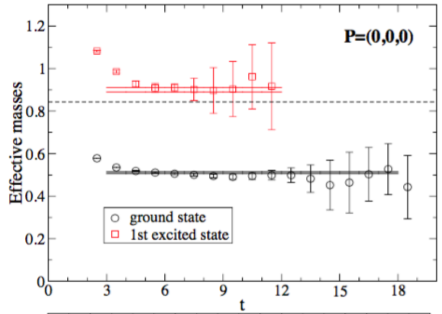
$$p = n\frac{2\pi}{L} - \delta\frac{2}{L}$$

ρ resonance: a bit of "history" from personal perspective

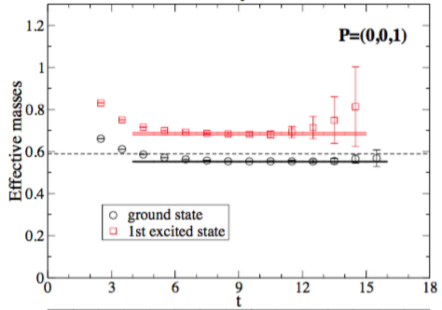
Lang, Mohler, S.P., Vidmar , PRD 2011

$$C_{jk}(t) = \langle 0 | O_j(t)\, O_k^{+}(0) | 0 \rangle, \quad O = \bar{q}q,\ (\bar{q}q)(\bar{q}q) = \pi\pi, \quad 1^{--}$$

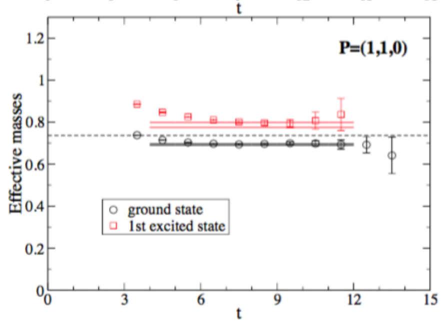




π(1)π(-1)



π(0)π(1)

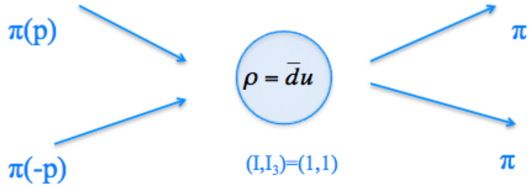


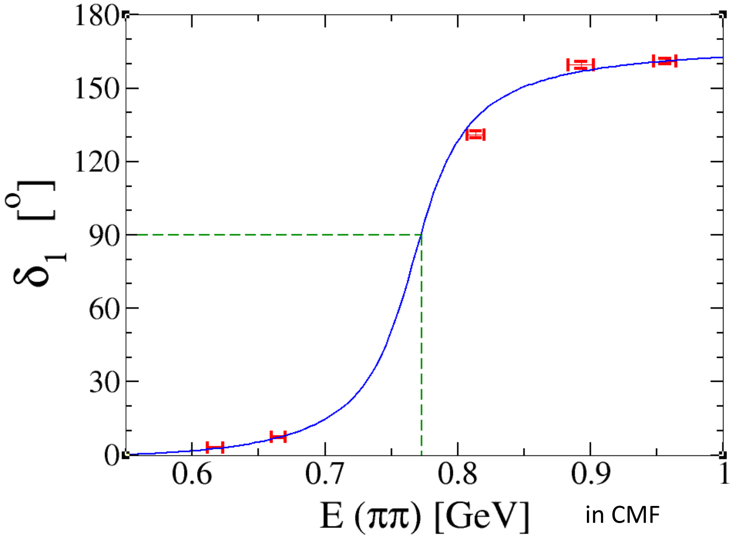
π(0)π(1,1,0)

$E_{cm} \rightarrow \delta(E_{cm})$

generalized

Luscher's relation





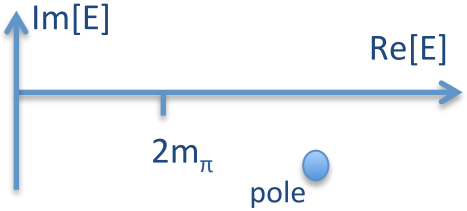
in CMF

simplest Breit-Wigner

$$\boxed{\frac{1}{\cot\delta(E) - i} = T(E) = \frac{-E\,\Gamma}{E^2 - m_R^2 + i\,E\,\Gamma}}$$

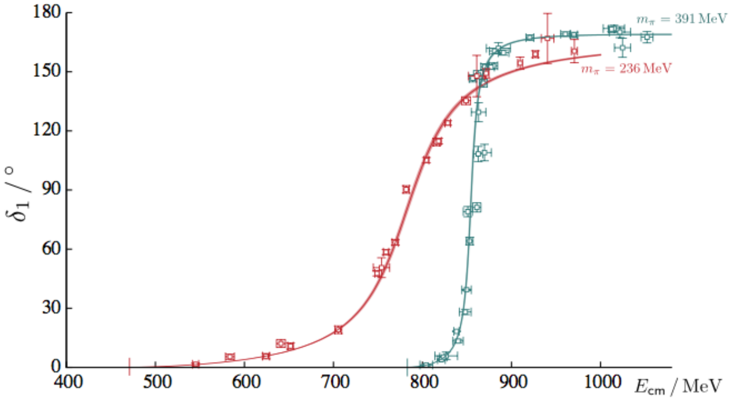
$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E^2}$$

ρ meson	Mass [MeV]	$g_{\rho\pi\pi}$
Lat (m_π=266 MeV)	772 ± 6 ± 8	5.61 ± 0.12
Exp.	775	5.97



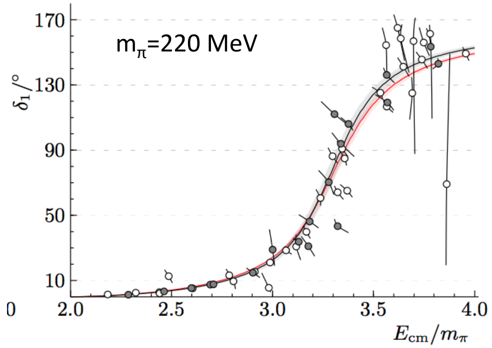
ρ resonance: impressive progress by LQCD community

HAD. spec (2015), $m_\pi \approx 230, 390$ MeV



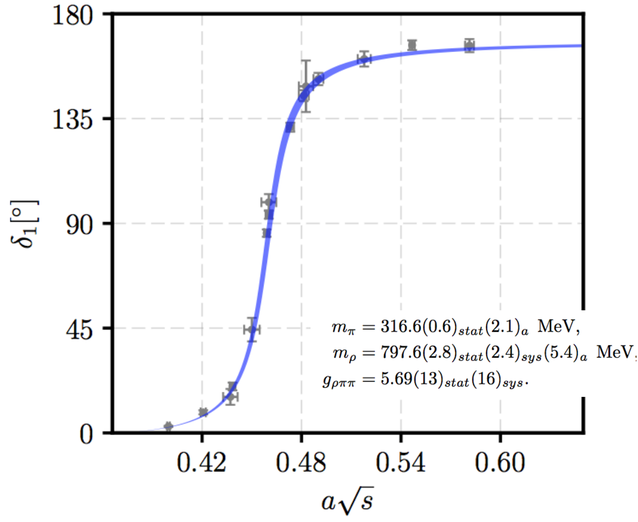
Andersen, Bulava, Horz, Morningstar

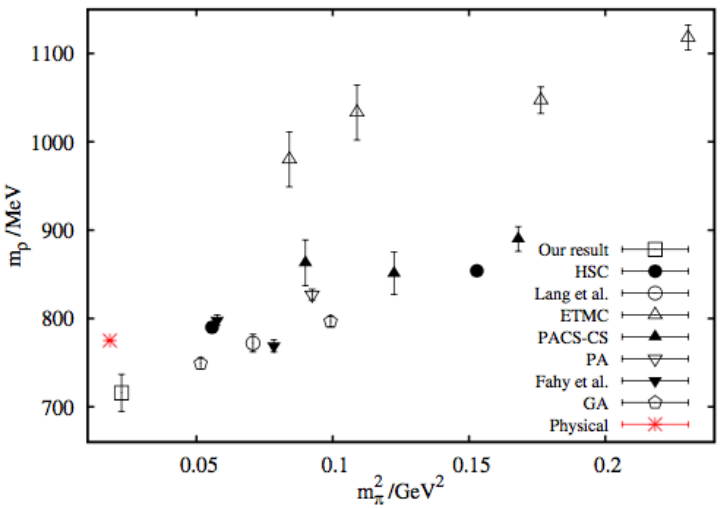
1808.05007, $m_\pi \approx 220$ MeV

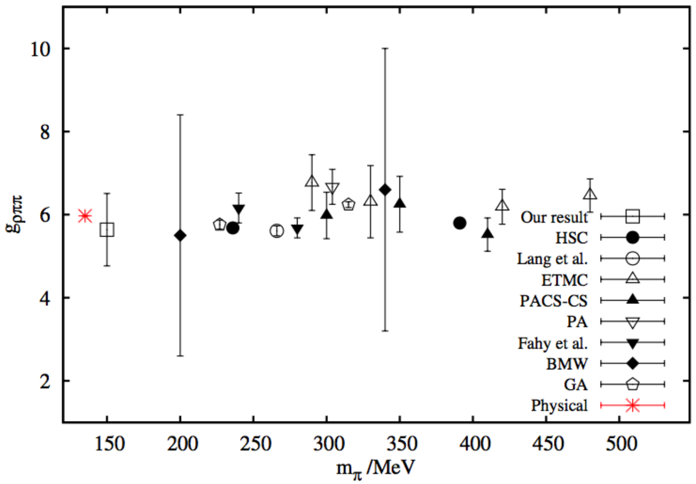


Alexandrou, Leskovec,et al PRD (2017),

$m_\pi \approx 320$ MeV







taken from:

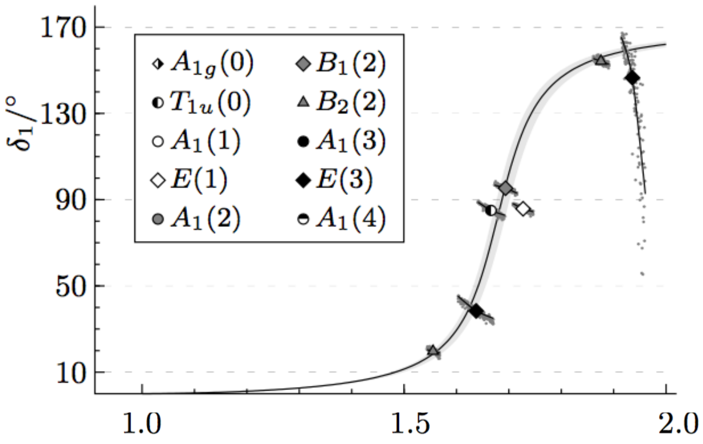
Bali, Collins, Lang et al

PRD (2015)

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{p^3}{E^2}$$

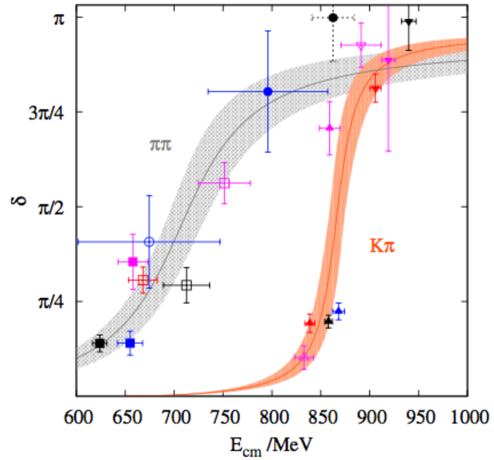
K*(892) resonance in p-wave Kπ

Brett et al. NPB 2018, m_π ≈230 MeV

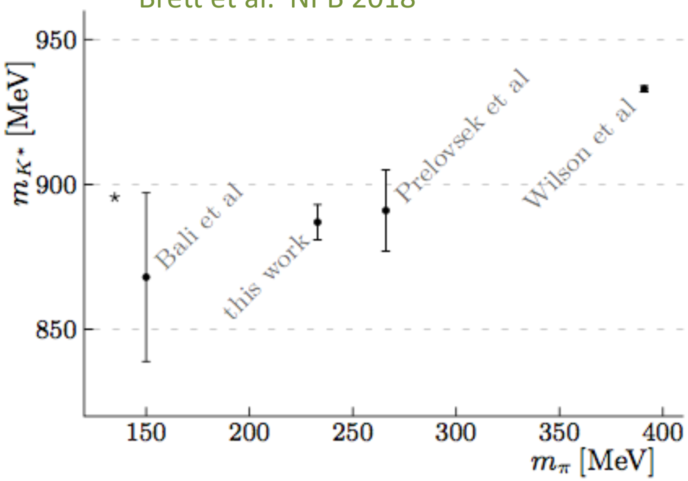


Bali, Collins, Lang et al

PRD (2015), m_π ≈150 MeV



Brett et al. NPB 2018

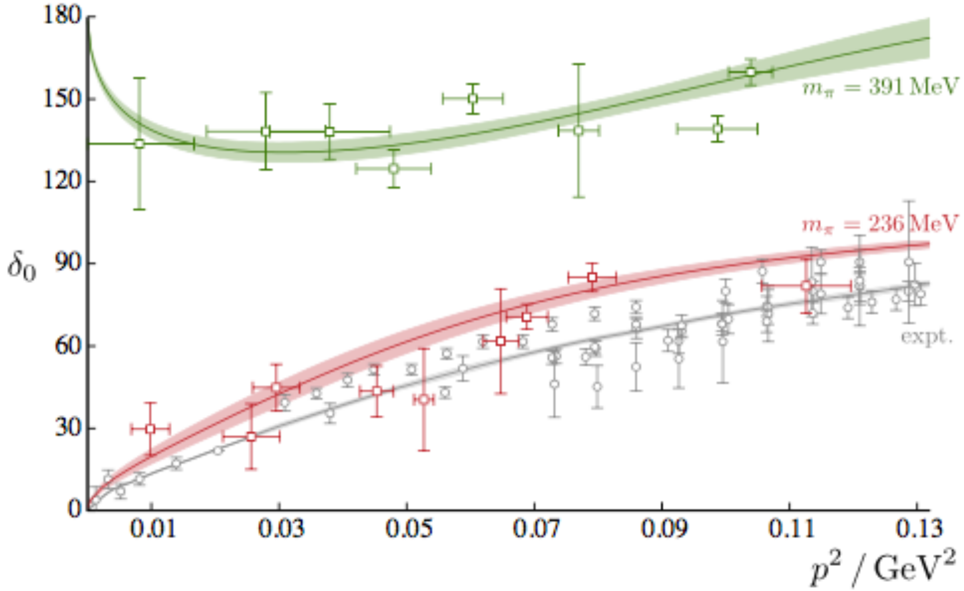


σ in ππ scattering

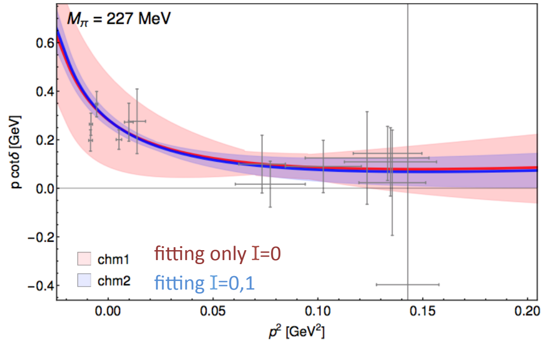
Guo, Alexandru, Molina, Mai, Doring, GWU

1803.02897, PRD (2018)

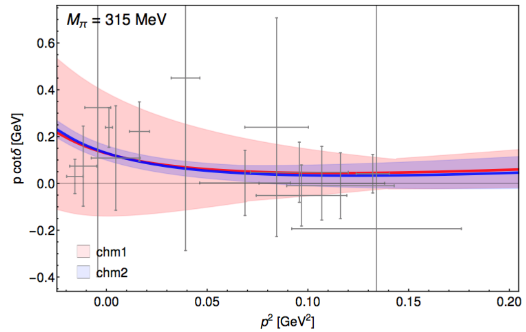
HAD. spec (PRL 2017), $m_\pi \approx 230$, 390 MeV

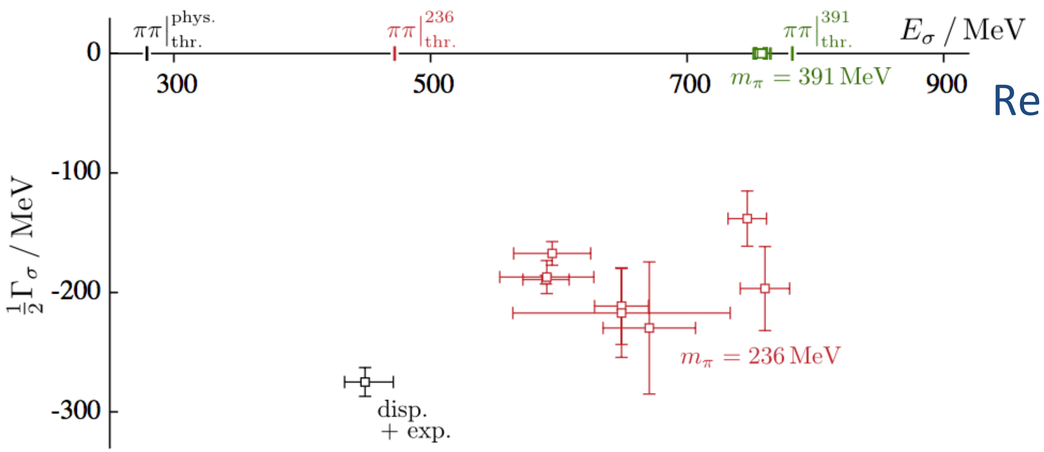


p cotδ



p cotδ





Re E [MeV]

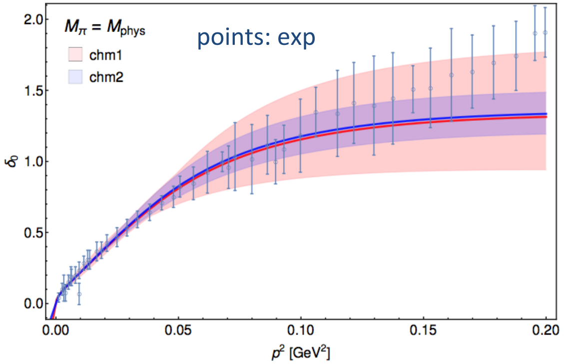
Im E [MeV]

Roy eq.

chiral extrapolation

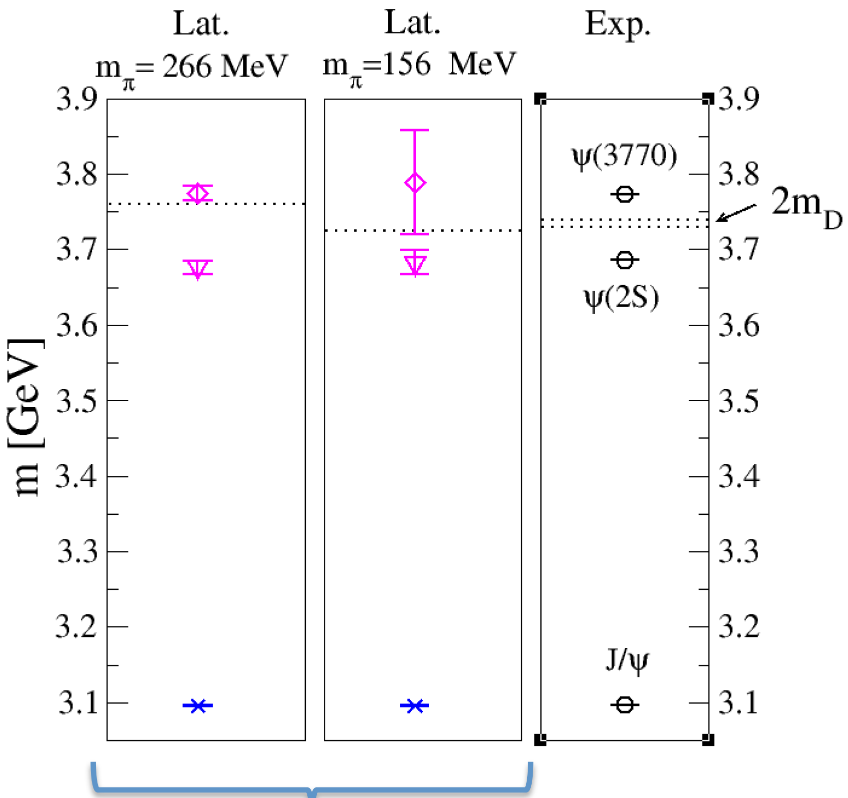
using chiral unitary approach

δ



p^2 [GeV^2]

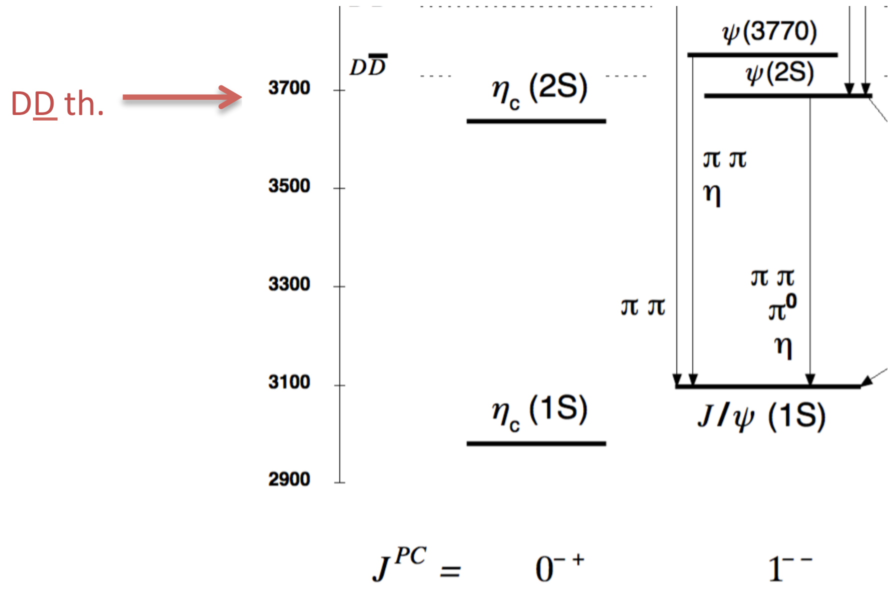
Resonance ψ(3770) from DD scattering in p-wave



physical masses m^{phy}

$\mathcal{O}: \bar{c}\, c, D\bar{D}, \quad J^{PC} = 1^{--}$

PDG:



$$\Gamma = \frac{g^2}{6\pi}\frac{p^3}{s}$$

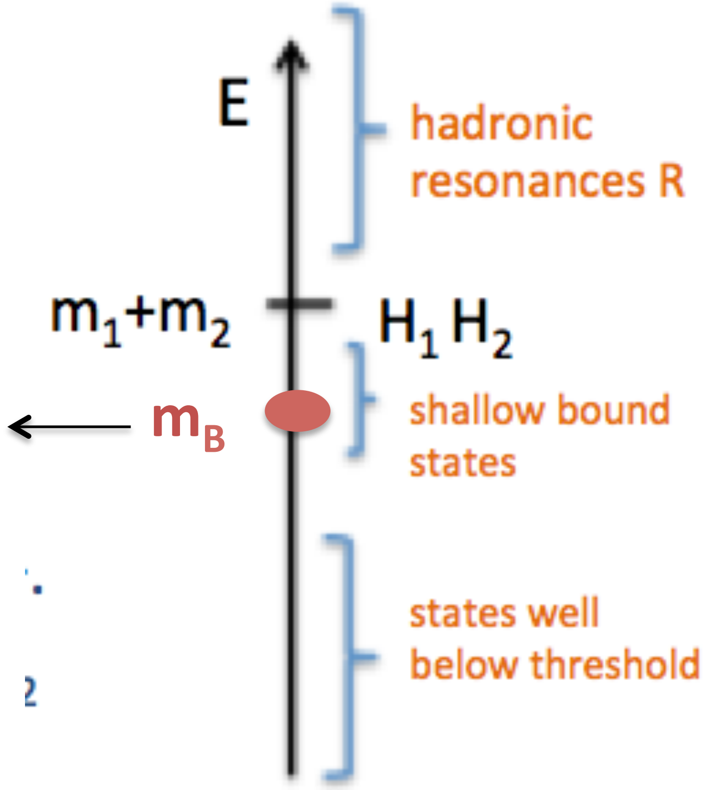
ψ(3770)	Mass [MeV]	g (no unit)
Lat (m_π=266 MeV)	3774 ±6±10	19.7 ±1.4
Lat (m_π=156 MeV)	3789 ±68±10	28 ± 21
Exp.	3773.15± 0.33	18.7 ± 1.4

Lang, Leskovec, Mohler, S.P.,
[1503.05363, JHEP 2015]

First determination of Γ
for charmonium resonance.

deuterium-like systems

Two-hadron bound states slightly below threshold



Bound states vs. resonances

for one channel scattering dominated by single partial wave L

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot\delta(E) - i}$$

simplest E-dependence expected in a region
near a relatively narrow reson.

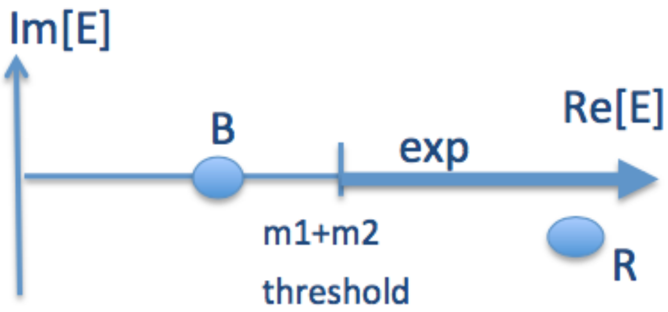
Bound state (B):

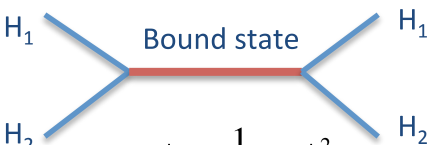
$$\cot(\delta(m_B)) = i, \quad m_B < m_1 + m_2$$

Resonance (R) (of Breit-Wigner type):

$$T(E) = \frac{E\,\Gamma(E)}{m_R^2 - E^2 - i\,E\,\Gamma(E)} \qquad \Gamma(E) = g^2\,\frac{p^{2l+1}}{E^2}$$

Locations of poles of T(E)
for res. and bound st.





$$T \propto \left| \frac{1}{s - m_B^2} \right|^2$$

$$T(E = m_B) = \infty$$

Mesonic bound states in s-wave ? (analogues of deuterium)

- states well below threshold
- strongly decay: resonances
- candidates for shallow bound st.

$\bar{u}u$

States well below threshold:
- $\pi^{\pm}$
- π^0
- η

Resonances:
- $f_0(500)$ or σ was $f_0(60$
- $\rho(770)$
- $\omega(782)$
- $\eta'(958)$
- $f_0(980)$
- $a_0(980)$
- $\phi(1020)$
- $h_1(1170)$
- $b_1(1235)$
- $a_1(1260)$
- $f_2(1270)$
- $f_1(1285)$
- $\eta(1295)$
- $\pi(1300)$
- $a_2(1320)$
- $f_0(1370)$
- $h_1(1380)$
- $\pi_1(1400)$
- $\eta(1405)$
- $f_1(1420)$
- $\omega(1420)$
- $f_2(1430)$
- $a_0(1450)$
- $\rho(1450)$

$\bar{s}u$

States well below threshold:
- $K^{\pm}$
- K^0
- K_S^0
- K_L^0

Resonances:
- $K_0^*(800)$ or
- $K^*(892)$
- $K_1(1270)$
- $K_1(1400)$
- $K^*(1410)$
- $K_0^*(1430)$
- $K_2^*(1430)$
- $K(1460)$
- $K_2(1580)$
- $K(1630)$
- $K_1(1650)$
- $K^*(1680)$
- $K_2(1770)$
- $K_3^*(1780)$
- $K_2(1820)$
- $K(1830)$

$\bar{c}u$

States well below threshold:
- $D^{\pm}$
- D^0

Resonances:
- $D^*(2007)^0$
- $D^*(2010)^{\pm}$
- $D_0^*(2400)^0$
- $D_0^*(2400)^{\pm}$
- $D_1(2420)^0$
- $D_1(2420)^{\pm}$
- $D_1(2430)^0$
- $D_2^*(2460)^0$
- $D_2^*(2460)^{\pm}$
- $D(2550)^0$
- $D(2600)$
- $D^*(2640)^{\pm}$
- $D(2750)$

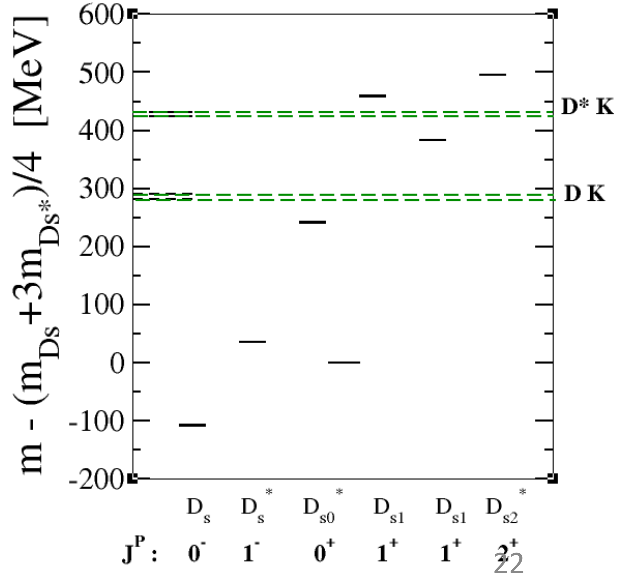
$\bar{c}s$

States well below threshold:
- $D_s^{\pm}$
- $D_s^{*\pm}$

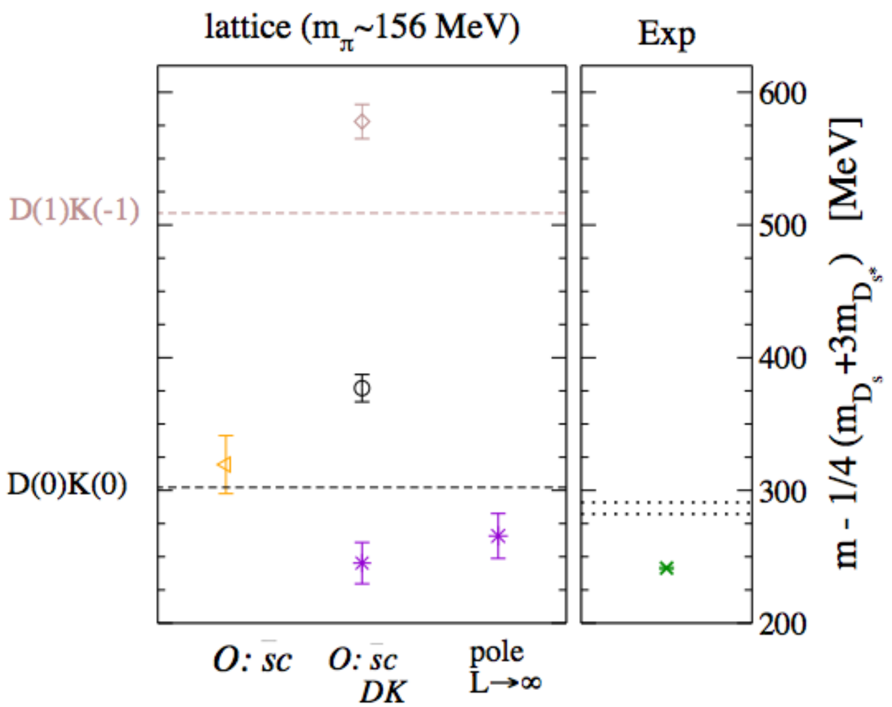
Candidates for shallow bound states:
- $D_{s0}^*(2317)^{\pm}$
- $D_{s1}(2460)^{\pm}$

Resonances:
- $D_{s1}(2536)^{\pm}$
- $D_{s2}^*(2573)$
- $D_{s1}^*(2700)^{\pm}$
- $D_{s1}^*(2860)^{\pm}$
- $D_{s3}^*(2860)^{\pm}$
- $D_{sJ}(3040)^{\pm}$

PDG



D_{s0}^*, pole in DK scat.

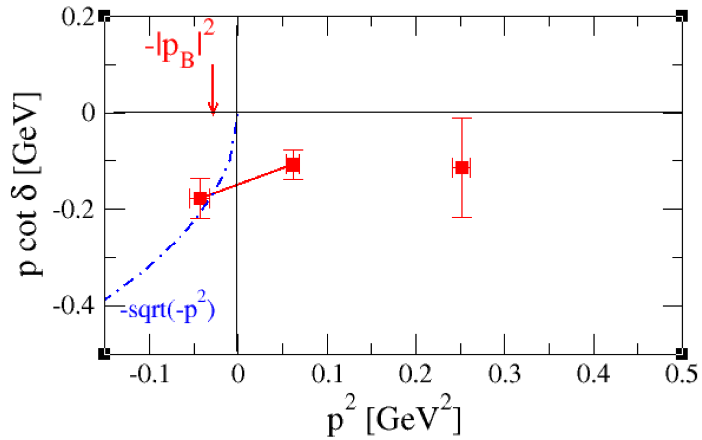


D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn:

Phys. Rev. Lett. 2013: $m_\pi \approx 156$ MeV, L≈2.9 fm, Nf=2+1, PACSCS

mesonic bound st. established on lattice for the first time

$D_{s0}^*(2317)$	m - ¼ (m_{Ds}+3m_{Ds^*})	mD+mK-m
lat	266 ± 16±4 MeV	36 ± 17 MeV
exp	241.45 ± 0.6 MeV	45 MeV

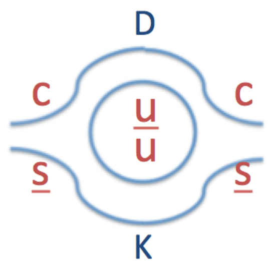


$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = -1.33 \pm 0.20 \text{ fm} \quad r_0 = 0.27 \pm 0.17 \; fm$$

$$T \propto \frac{1}{\cot\delta - i} = \infty, \quad \cot\delta(p_B) = i, \quad p_B = i\,|\,p_B\,|$$

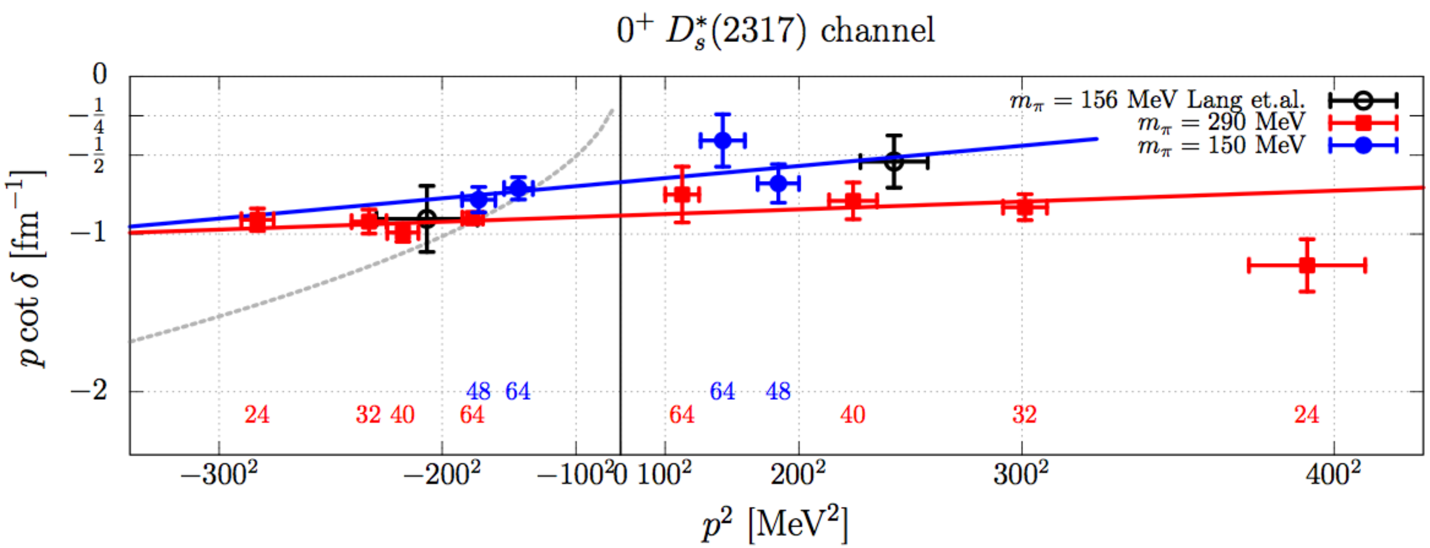
$$m_{D_{s0}}^{lat,\,L\to\infty} = \sqrt{m_D^2 - |\,p_B\,|^2} + \sqrt{m_K^2 - |\,p_B\,|^2}$$



D_{s0}^*

Bali, Collins, Cox, Schafer (RQCD):

PRD (2017) 074501



$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

0^+ channel

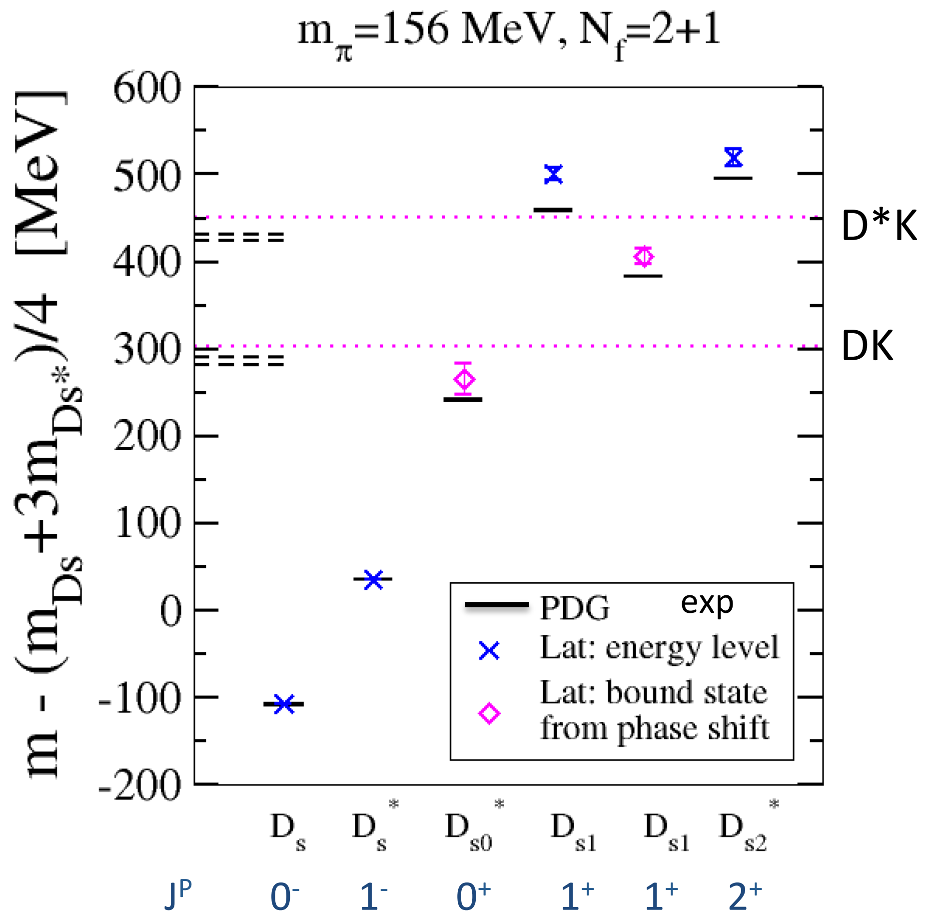
	$m_\pi = 290$ MeV	$m_\pi = 150$ MeV	Expt.	m_π=156 MeV
a_0 [fm]	$-1.13(0.04)(+0.05)$	$-1.49(0.13)(-0.30)$		-1.33(20)
r_0 [fm]	$0.08(0.03)(+0.08)$	$0.20(0.09)(+0.31)$		0.27(17)
$\lvert p_B \rvert$ [MeV]	$180(6)(0)$	$142(11)(-9)$		
Δm [MeV]	$40(3)(0)$	$26(4)(-3)$	$42.6(0.7)(2.0)$	36 (17)
m_{D_s} [MeV]	$2384(2)(-1)$	$2348(4)(+6)$	$2317.7(0.6)(2.0)$	

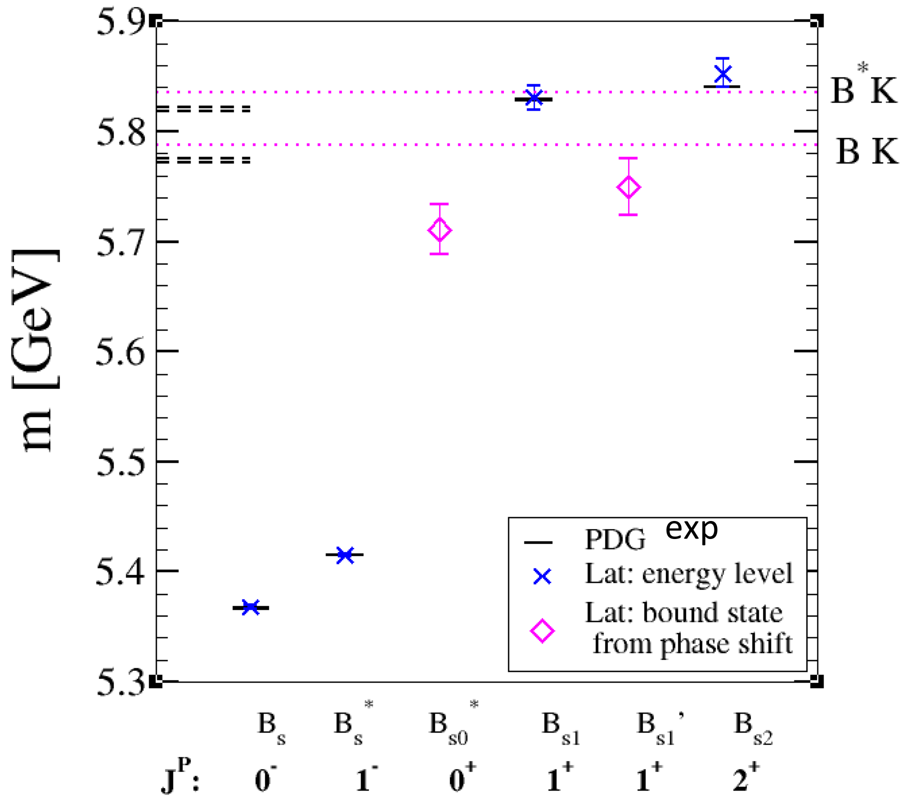
RQCD

Lang et al.

$D_{s0}(2317)$ and $D_{s1}(2460)$ below DK and D^*K thresholds $\bar{s}c$

Mass prediction for missing B_{s0} and B_{s1} $\bar{s}b$





[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn: 1308.3175, PRL 2013 1403.8103, PRD 2014]
[Martinez Torres, E. Oset, S.P., A. Ramos: 1412.1706, JHEP 2015]

[C. Lang, D. Mohler, S.P. , R. Woloshyn: 1501.0164, PLB2015]

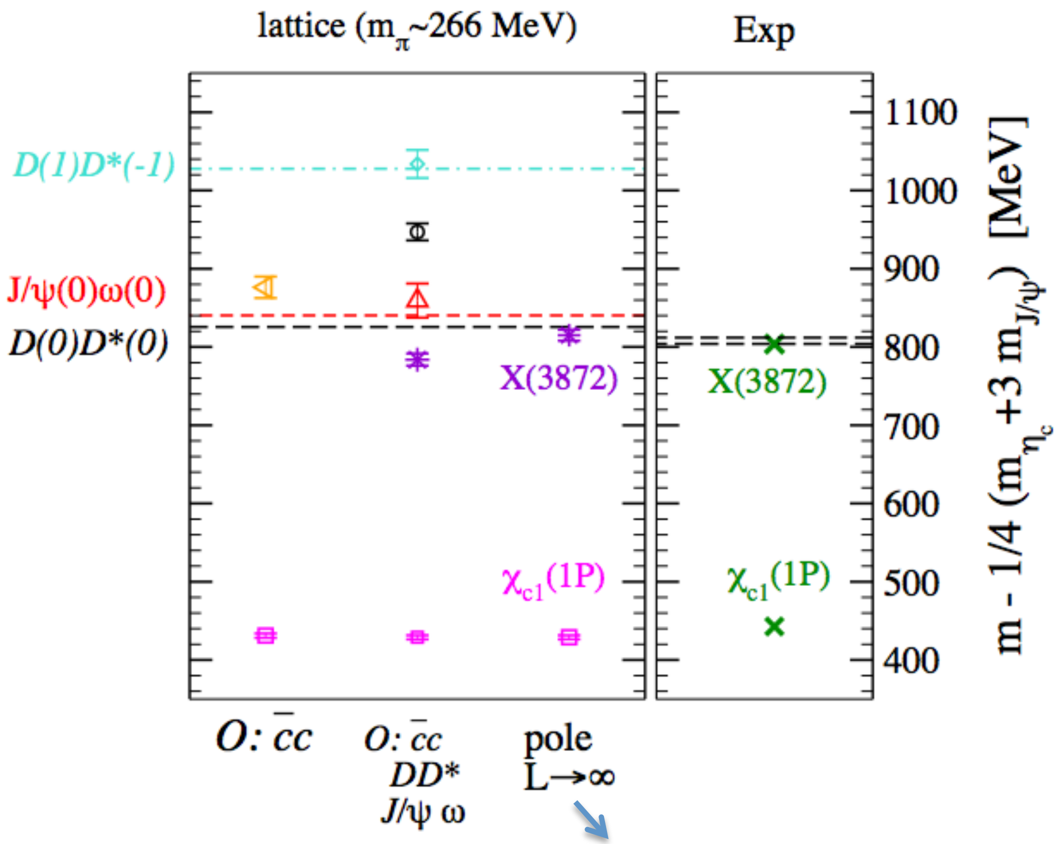
waiting for exp discovery by LHCb

D_{s0} discovered by [BaBar, 2003]

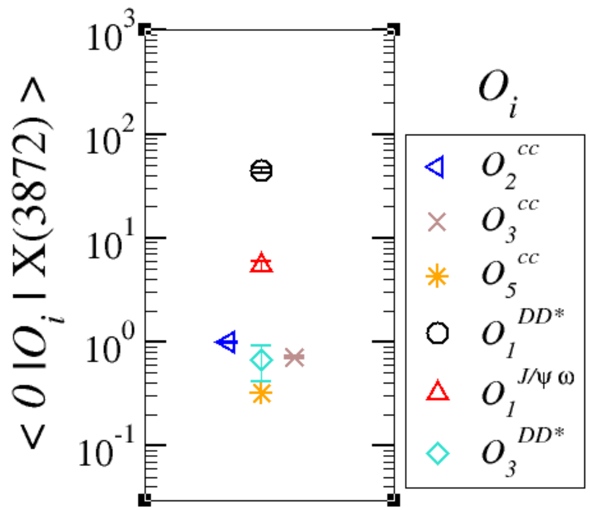
first time bound states were searched as poles in T for mesonic systems

X(3872): pole in DD* $J^{PC}=1^{++}, I=0$

$\mathcal{O}: \bar{c}\,c,\quad DD^*,\quad J/\psi\omega$

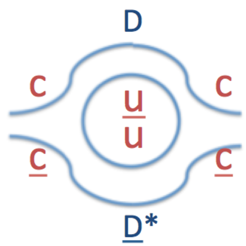


$$\langle 0|O_j|X(3872)\rangle$$



Overlaps normalized to $\langle 0|O_1^{cc}|X(3872)\rangle$

X(3872)	m - $(m_{D0}+m_{D0^*})$
lat	- 11 ± 7 MeV
exp	- 0.14 ± 0.22 MeV



- δ_0 for DD* extracted using Luscher's rel. and interpolated near threshold
- pole in T-matrix $T \propto [\cot\delta - i]^{-1} = \infty$ found just below DD* threshold.

[S.P. and L. Leskovec : 1307.5172, Phys. Rev. Lett.]

$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

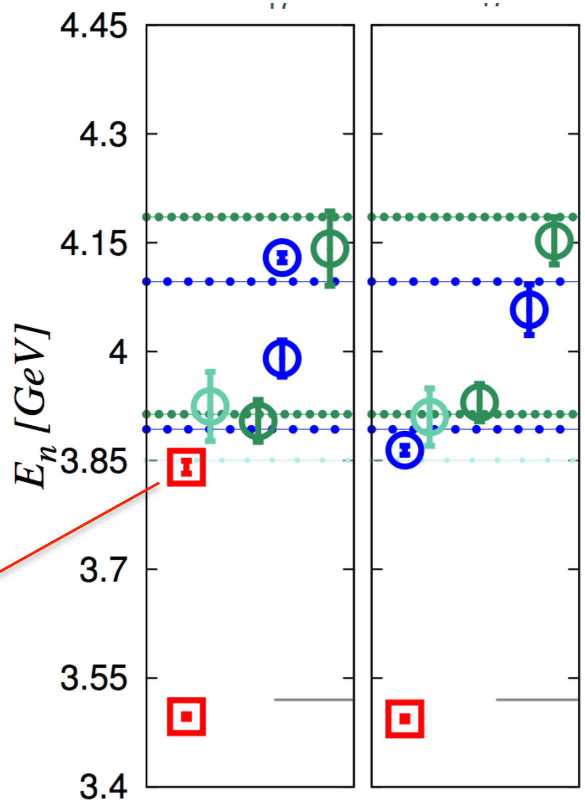
Which Fock components are essential for X(3872) with I=0 ?

localaized diquark antidiquark

$$\mathcal{O}:\ \underbrace{\bar{c}\,c,\quad D\bar{D}^*}_{\text{essential}},\ J/\psi\omega,\ \chi_{c1}\sigma,\ \eta_c\sigma,\ \underbrace{[\overline{cu}]_{3c}[cu]_{3c},\ [\overline{cu}]_{6c}[cu]_{6c}}_{\text{do no seem not essential}}$$

interpolating fields

O: all O: all without $\bar{c}c$



energies of eigenstates on lattice E(L)
(these are not m^{phy})

$m_\pi \approx 266$ MeV

upper red square is candidate for X(3872): it is found only if $\bar{c}c$ in the basis

C(t) provides E_n as well as overlaps

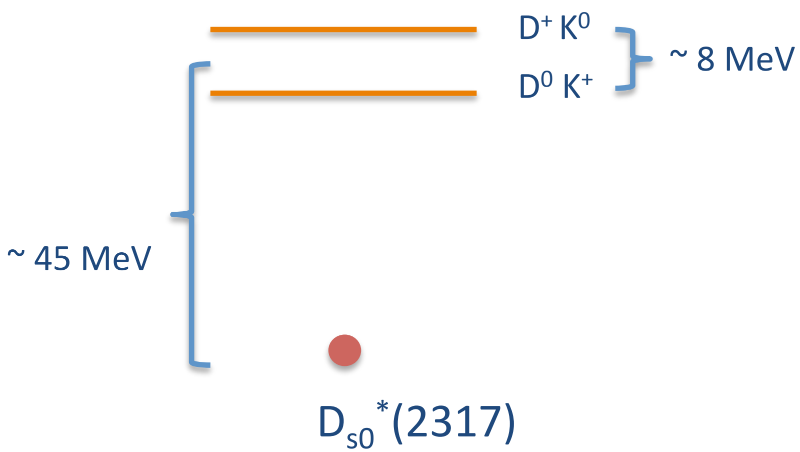
$$\langle n|\mathcal{O}_i|0\rangle$$

X(3872) not found if $\bar{c}c$ not in the basis, although $[\bar{c}\bar{u}][cu]$ in the basis

[M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]

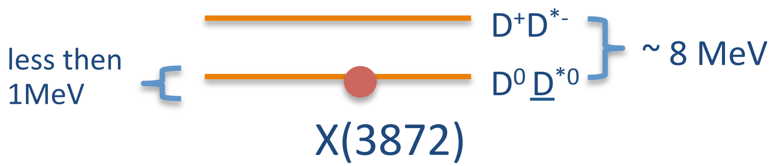
D_{s0}^* in DK vs. X(3872) in $D\underline{D}^*$

- very narrow: width not measured
- theoretically cleaner
- no Wick contraction omitted
- no other nearby threshold
- isospin breaking less relevant
- only s-wave contributes to $J^P=0^+$



- same
- currently theoretically less clean -> more to be done
- charm annihilation omitted
- I=0 state in isospin limit:
  J/Ψ ω (I=0) threshold at 3879 MeV , J/Ψ πππ
  J/Ψ ππ (I=0) threshold formally below X(3872)
- isospin breaking more relevant (not considered on lat)

  another threshold for broken I
  J/Ψ ρ (I=1) threshold 3873 MeV , J/Ψ ππ
- s and d-wave in $J^P=1^+$ (d-wave not considered on lat)
- nevertheless: the lattice result obtained is believed to be rather solid (X has width much less than MeV)
- more work to be done



Prediction of strongly stable doubly-bottom tetraquarks

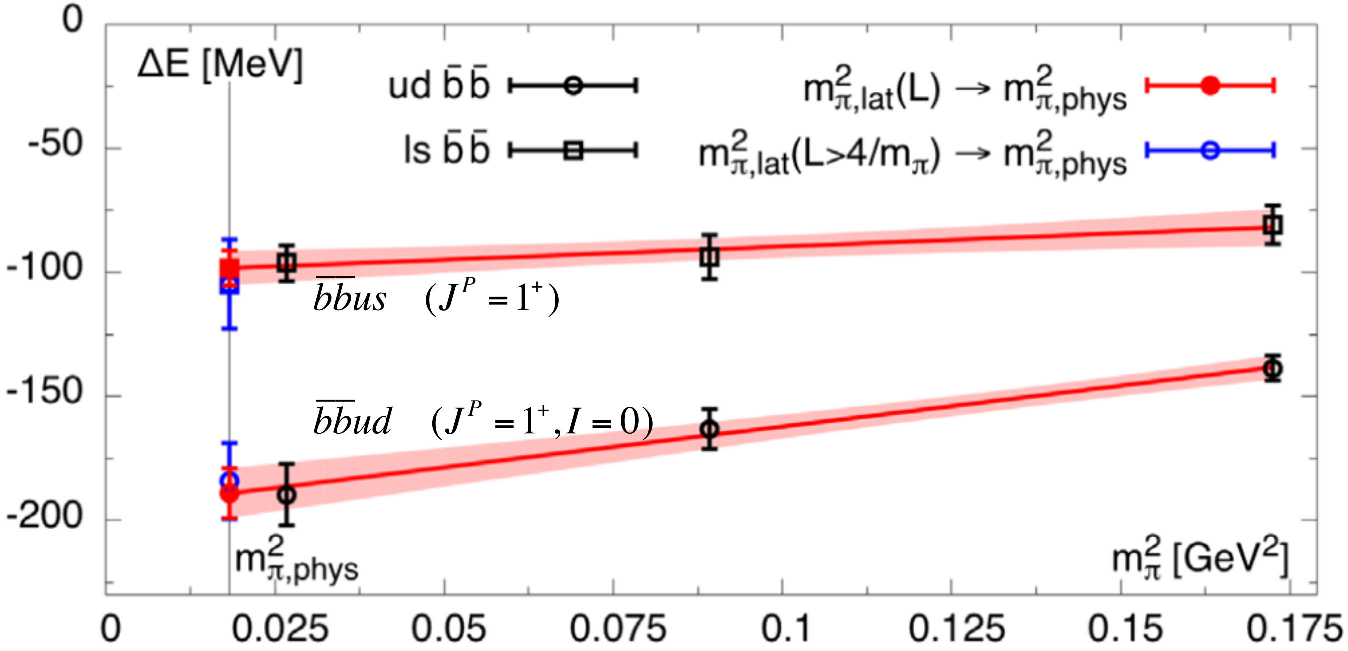
$\bar{b}\bar{b}ud \quad (J^P = 1^+, I = 0)$

$\bar{b}\bar{b}us \quad (J^P = 1^+)$

NRQCD for b-quarks; no phase shift analysis. Determining E_1 and assuming $m_B = E_1$

$$\Delta E = m_{tetra} - m_B - m_{B^*_{(s)}}$$

[Francis, Hudspith, Lewis, Maltman, PRL 2017]



B B* threshold

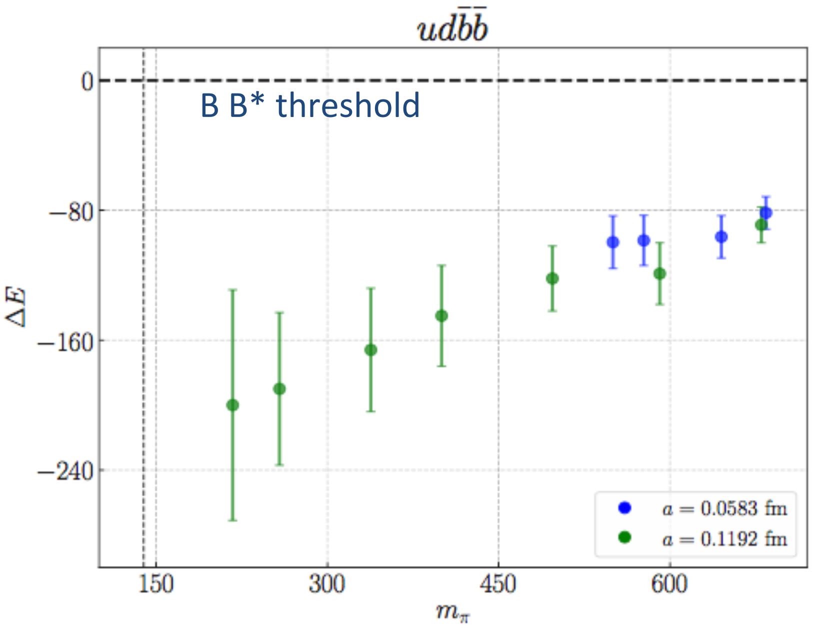
$(u_a^T C\gamma_5 d_b)(\bar{b}_a C\gamma_i \bar{b}_b^T)$ and $(\bar{b}_a\gamma_5 u_a)(\bar{b}_b\gamma_i d_b) - (\bar{b}_a\gamma_5 d_a)(\bar{b}_b\gamma_i u_b)$

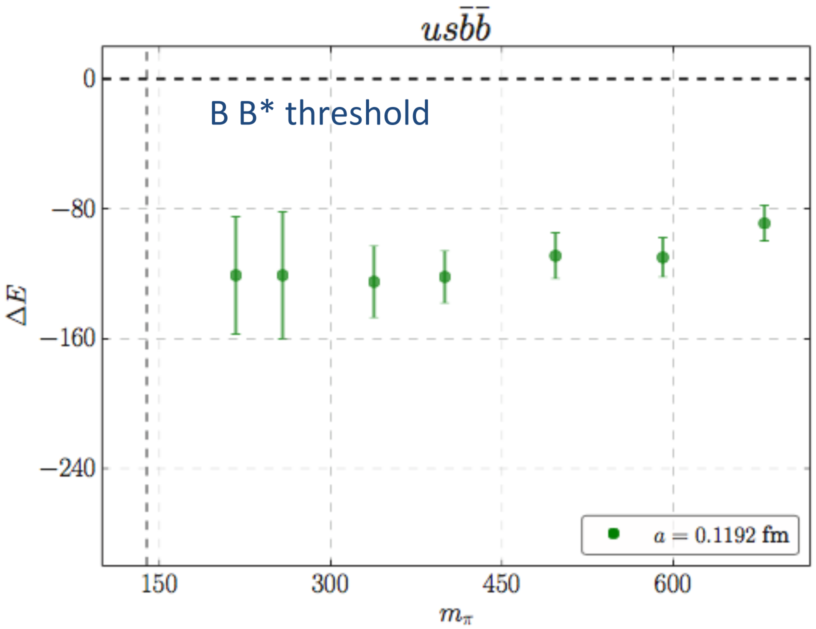
Confirmation of prediction based on similar methods

$\bar{b}\bar{b}ud \quad (J^P = 1^+, I = 0)$

$\bar{b}\bar{b}us \quad (J^P = 1^+)$

$$\Delta E = m_{tetra} - m_B - m_{B^*_{(s)}}$$

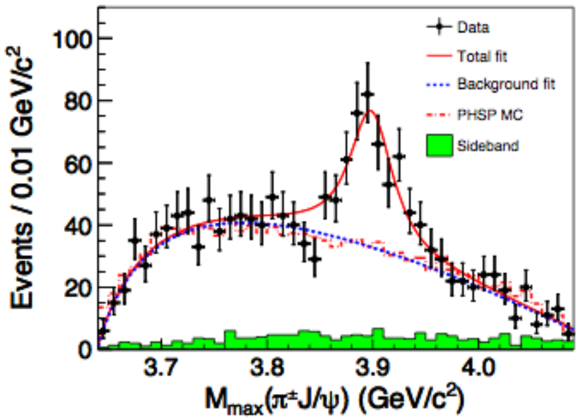




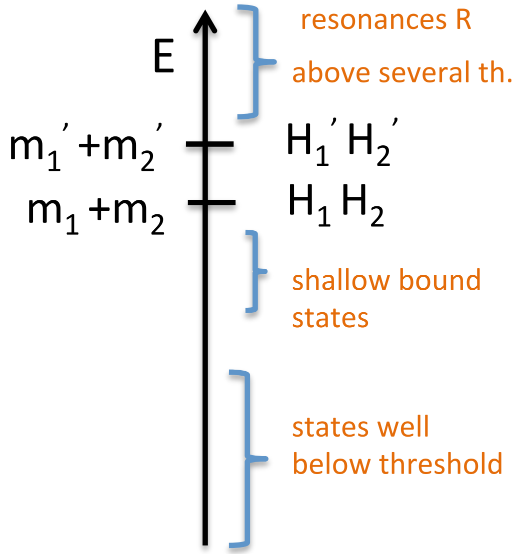
[Junnarkar, Padmanath, Mathur, EPJWebConf175(2018)05014, preliminary]

Similar conclusion using static b quarks and extracting BB* potential V(r):

Bicudo, Wagner, Peters (2015-2017)



$R \to H_1 H_2\ ,\ H'_1 H'_2, \ldots$



Hadrons that decay to several final states

Scattering in two or more channels

"very challenging"

most of interesting exotic hadrons lie above several thresholds

$$\bar{c}c\bar{d}u: \quad Z_c(3900) \to J/\psi\,\pi,\ \eta_c\,\rho,\ D\bar{D}^*$$

$$\bar{b}b\bar{d}u: \quad Z_b \to \Upsilon(nS)\pi,\ h_b(nP)\pi,\ B\bar{B}^*$$

$$\bar{c}cuud: \quad P_c \to J/\psi\, p, \ldots$$

simplified case when only one partial-wave L contributes

One channel scattering

$$S(E) = e^{2i\delta(E)} = I + 2iT$$

Luscher's equation: $$f[E_n, \delta(E_n)] = 0: \quad E_n \to \delta(E_n)$$

Two coupled channel scattering

a: O=H_1 H_2

b: O=H'_1 H'_2

↓

E_n

$$\begin{array}{cc} \text{a -> a} & \text{a -> b} \\ \end{array}$$

$$S(E) = \begin{vmatrix} \eta(E)\, e^{2i\delta_a(E)} & i\sqrt{1-\eta^2(E)}e^{i(\delta_a(E)+\delta_b(E))} \\ i\sqrt{1-\eta^2(E)}e^{i(\delta_a(E)+\delta_b(E))} & \eta(E)\, e^{2i\delta_b(E)} \end{vmatrix} = I + 2iT(E)$$

$$\begin{array}{cc} \text{b->a} & \text{b -> b} \\ \end{array}$$

generalized Luscher's (det) eq.:

1 equation with three unknowns

$$f[E_n, \delta_1(E_n), \delta_2(E_n), \eta(E_n)] = 0: \quad E_n \to ??$$

Parametrizing T matrix and

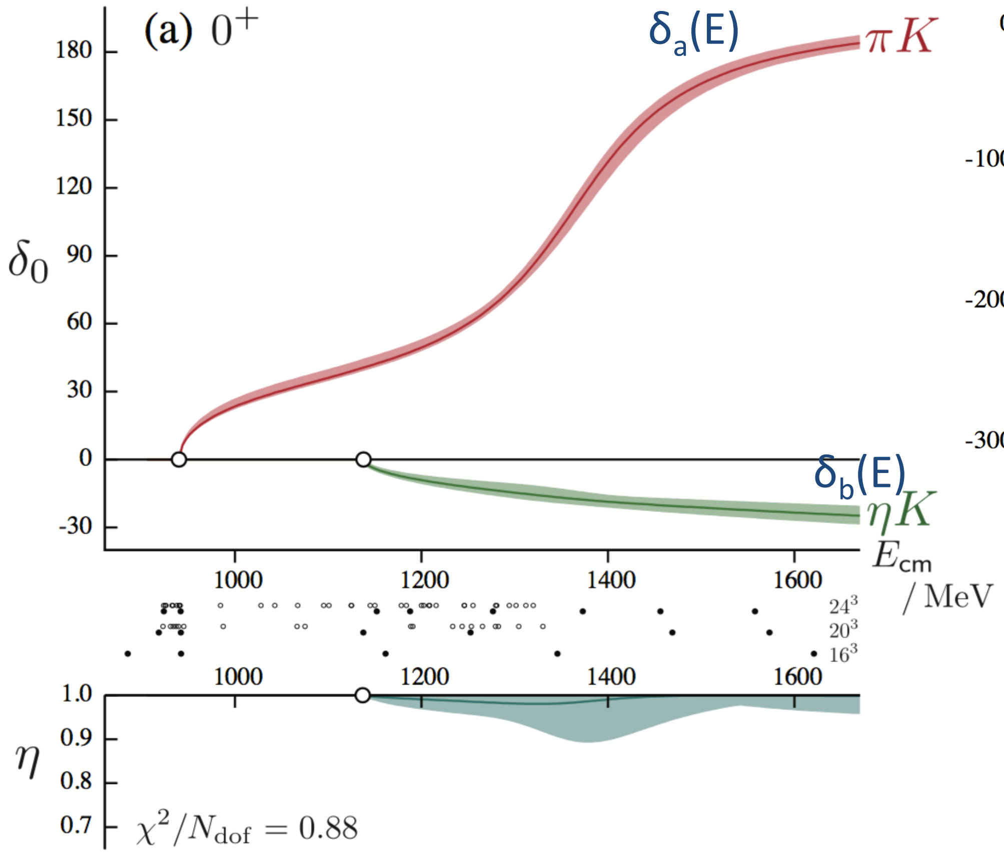
determine parameters from the fit to all E_n

$$\mathrm{Re}[T_{ij}^{-1}(E)] = a_{ij} + b_{ij}E^2 + c_{ij}E^4 + \ldots$$

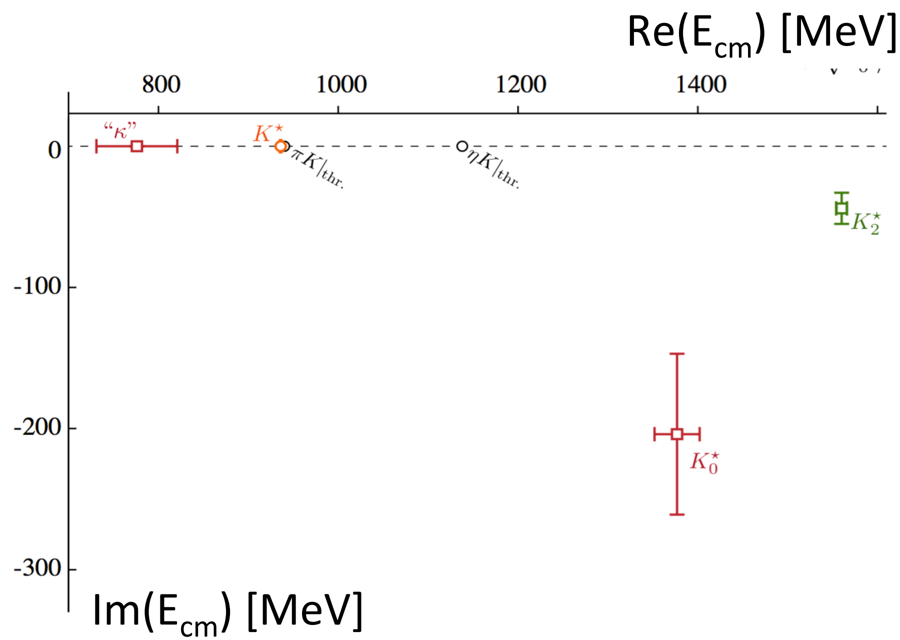
$$\text{fit to all } \mathrm{E}_n: \quad \text{values } \mathrm{a}_{ij}, b_{ij}, c_{ij}$$

First determination of coupled-channel S: Kπ - Kη

[Hadron Spectrum coll. (PRL 2014), m_π=390 MeV]



Location of poles (from all 4 sheets)



$$C_{ij}(t) = \langle O_i(t) O_j^+(0) \rangle$$

$$O \approx K\pi = (\bar{s}q)(\bar{q}u)$$

$$K\eta = (\bar{s}q)(\bar{u}u + \bar{d}d - C\bar{s}s)$$

Scalar charmonium channel: D^+D^- - $D_s^+D_s^-$

[Regensburg QCD: Collins, Padmanath, Pimonte, Mohler, S.P., Bali]

- not settled yet which is first excited scalar charmonia
- testing group before attacking more exotic channels
- CLS ensembles, two volumes, three different total momenta

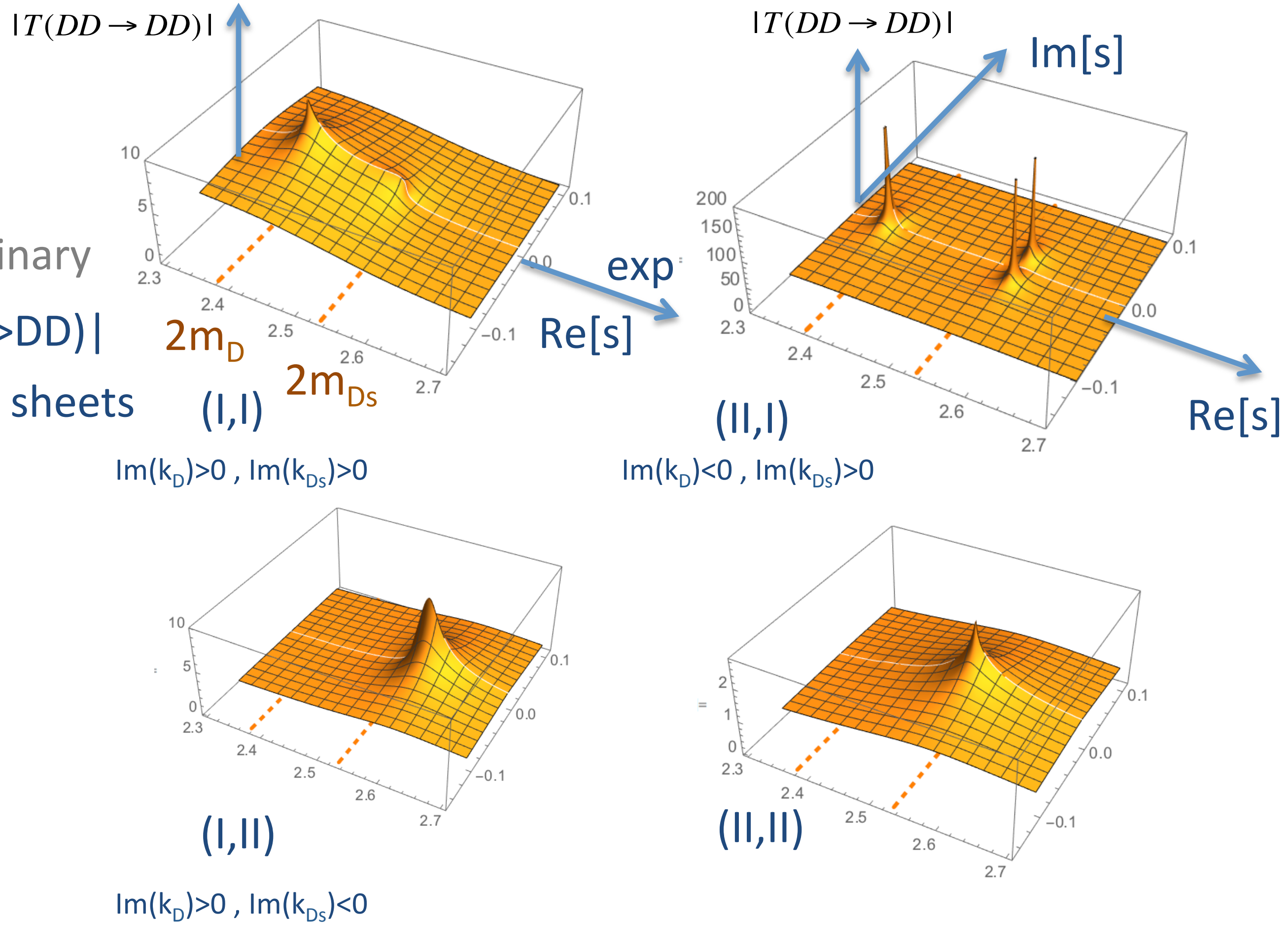
$$O \approx DD = (\bar{c}u)(\bar{u}c) + (\bar{c}d)(\bar{d}c)$$

$$D_s D_s = (\bar{c}s)(\bar{s}c)$$

preliminary

|T(DD->DD)|

on four sheets



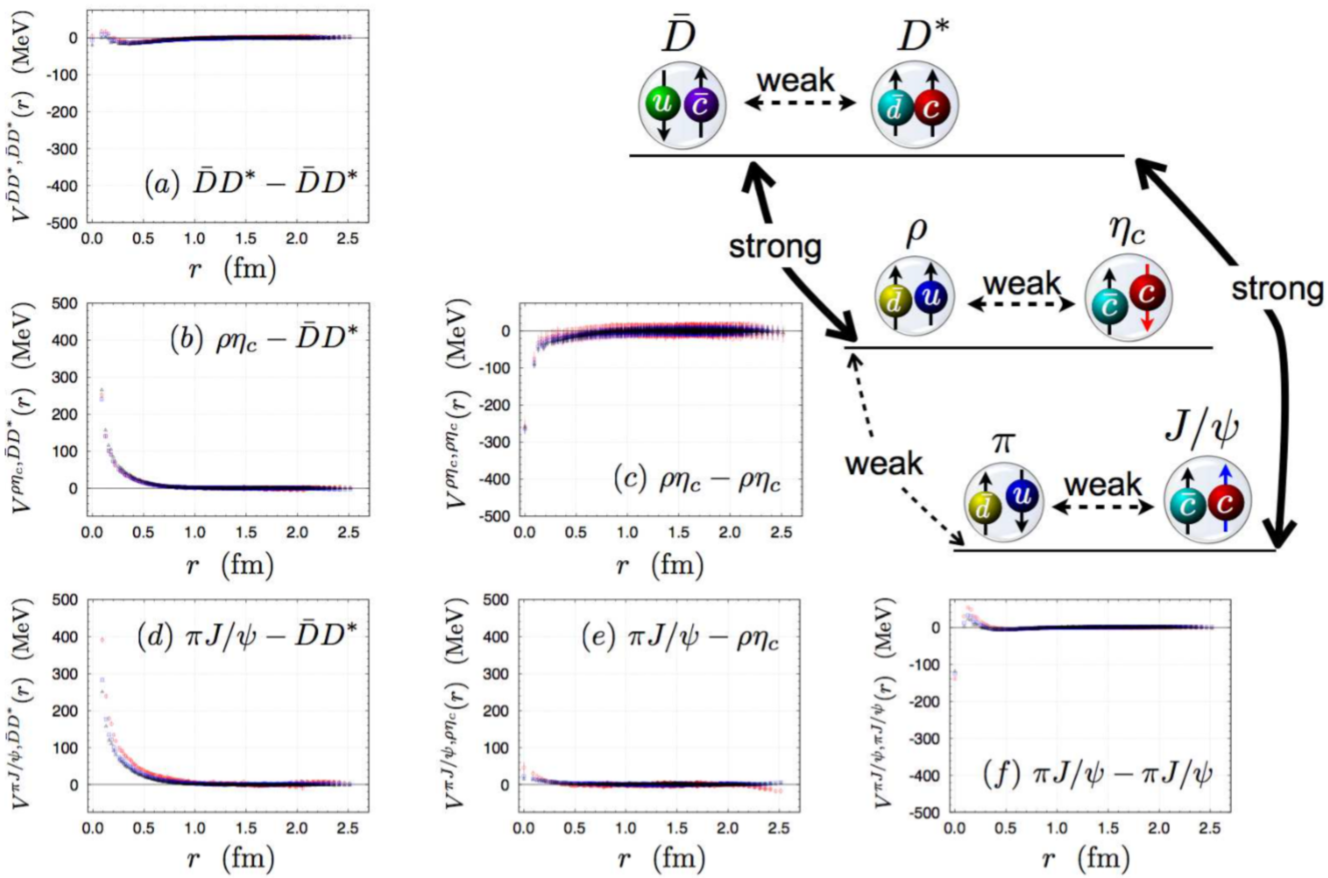
$s=E^2_{cm}$

[HALQCD, Ikeda et al, 1602.03465, PRL]

$Z_c^+(3900)$ channel : $I^G=1^+$, $J^{PC}=1^{+-}$ HALQCD method V(r)

HALQCD is another method to extract scattering matrix from lattice

(considered to be less rigorous than the Luscher's method for coupled channels)

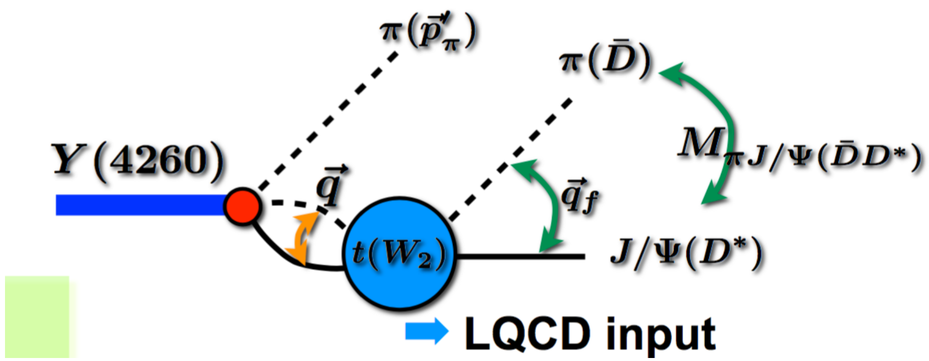


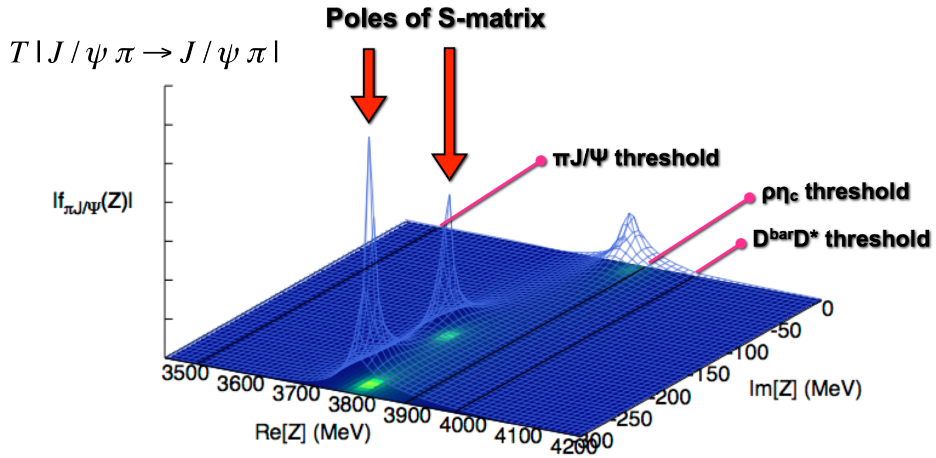
$$C^{\alpha\beta}(\vec{r},t) \equiv \sum_{\vec{x}} \langle 0|\, \phi_1^\alpha(\vec{x}+\vec{r},t)\phi_2^\alpha(\vec{x},t)\overline{\mathcal{J}}^\beta\, |0\rangle \,/\sqrt{Z_1^\alpha Z_2^\alpha},$$

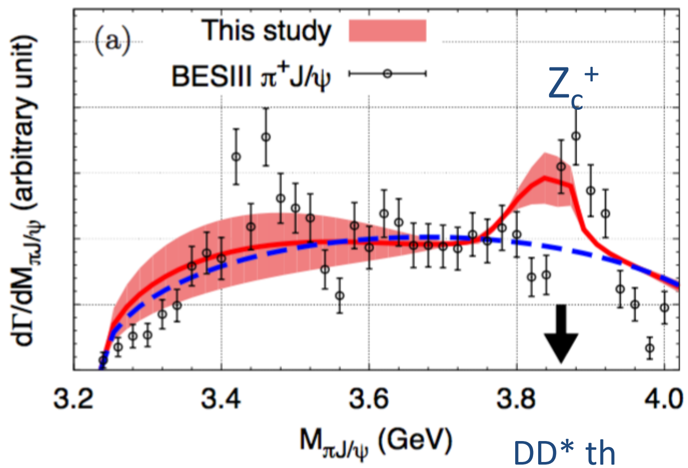
$$R^{\alpha\beta}(\vec{r},t) \equiv C^{\alpha\beta}(\vec{r},t)e^{(m_1^\alpha+m_2^\alpha)t}$$

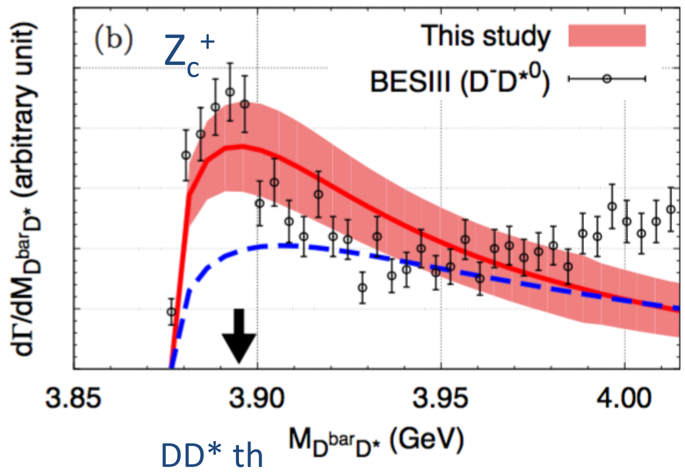
$$\left(-\frac{\partial}{\partial t} - H_0^\alpha\right)R^{\alpha\beta}(\vec{r},t) \simeq \sum_\gamma \Delta^{\alpha\gamma}\int d\vec{r'}U^{\alpha\gamma}(\vec{r},\vec{r'})R^{\gamma\beta}(\vec{r'},t)$$

Z_c^+ channel , three-body Y(4260) decay: lattice & exp







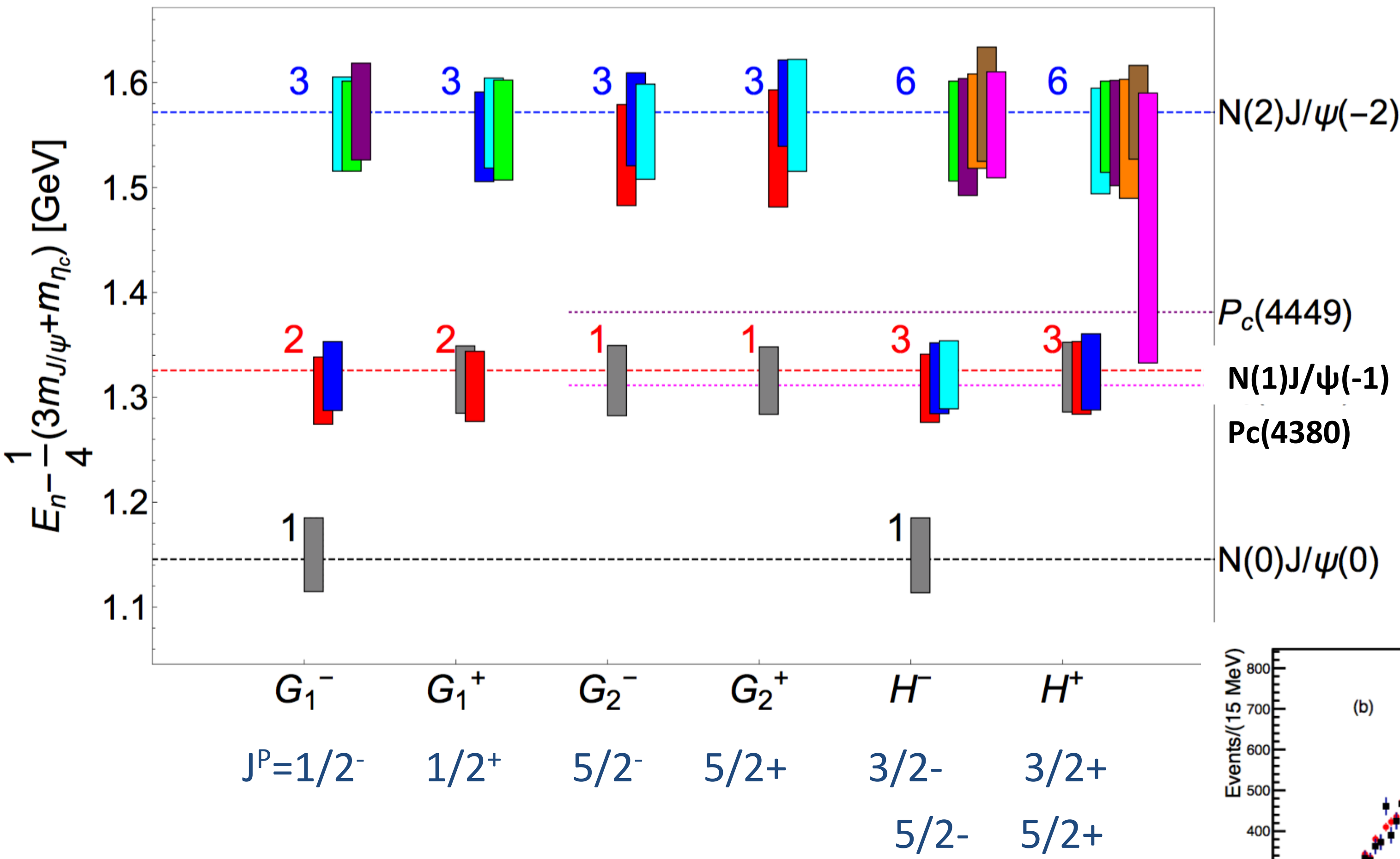


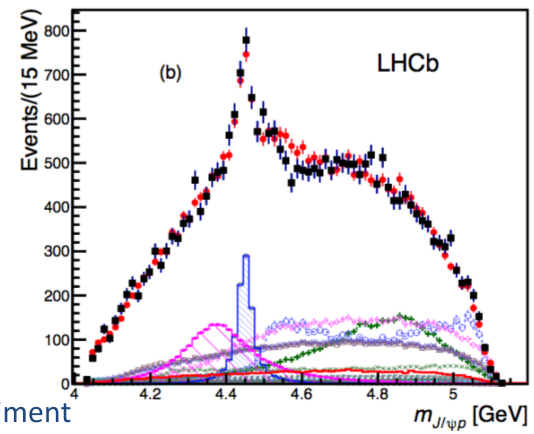
[HALQCD, Ikeda et al, 1602.03465, PRL]

conclusion: Zc peak is consequence of strong coupling between DD* and J/ψ π channels

J/Ψ p scattering in P_c channels (one-channel approx.)

U. Skerbis, S. P., 18010.xxxx, first lat. simulation that reaches energies where Pc were found

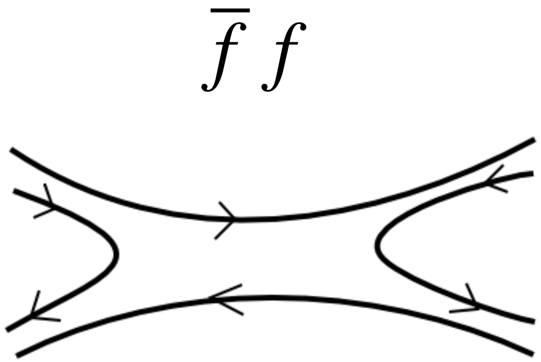




- No indication for Pc in one-channel approximation.
- This suggests that coupling of J/Ψ p with other channels may be responsible for Pc in experiment

Strongly coupled scenario
Beyond Standard Model

Scattering and resonance study in a composite Higgs model

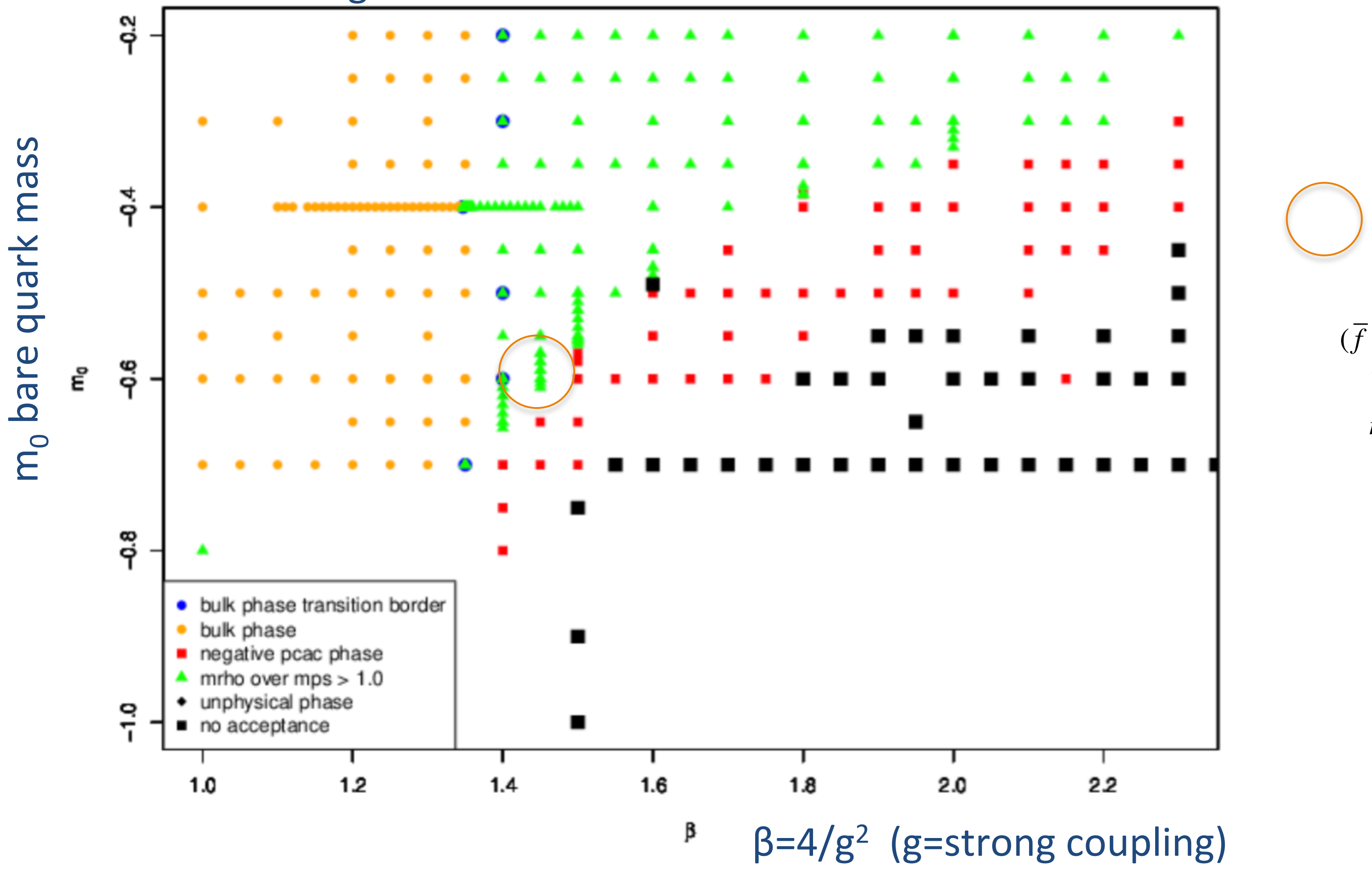


SU(2) gauge sym. with two fundamental in fundamental rep.

Phenomenology and Symanzik unimproved Lat: Sannino, Pica et al. (series of papers)

This lattice study (Symanzik improved Wilson fermions): Drach, Janowski, Pica, S.P., preliminary

Phase diagram



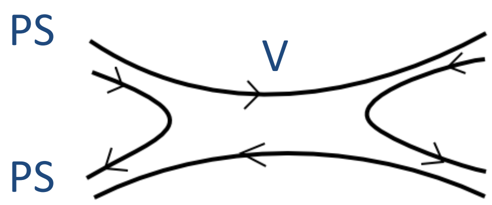
$$(\bar{f}\gamma_i f) \to (\bar{f}\gamma_5 f)(\bar{f}\gamma_5 f)$$

$$V \to PS \quad PS$$

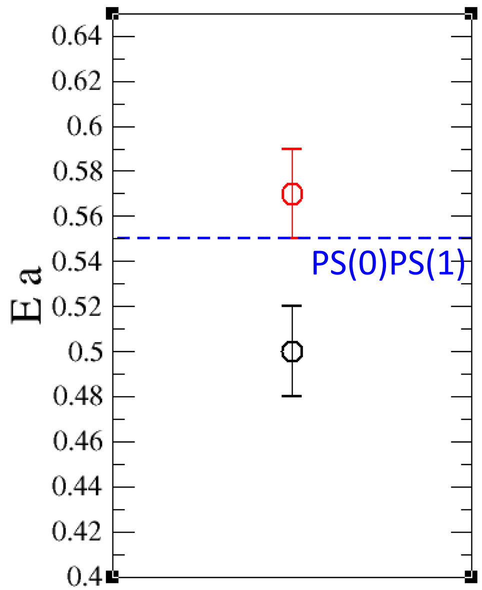
$$m_V > 2\, m_{PS}$$

Pseudoscalar-pseudoscalar scattering

Drach, Janowski, Pica, S.P., preliminary



energies for total momentum 1



Luscher's relation: $E_1 \to \delta(E_1), \quad E_2 \to \delta(E_2)$

$$\frac{1}{\cot\delta(E) - i} = T(E) = \frac{-E\,\Gamma}{E^2 - m_R^2 + i\,E\,\Gamma}$$

$$\Gamma = \frac{g^2}{6\pi}\frac{p^3}{E^2}$$

g=5.84(?) m_V a=0.437(?) > 2*m_{PS} m_V^{naive} a=0.444(9)

m_{PS} a=0.2114(8)

Preliminary: statistics is being increased and other total momenta analyzed

note: $g_{\rho\pi\pi}^{QCD} \approx 6$

Conclusions

Hadrons near and above threshold from lattice:

- progress, no doubt
- lots to be done
- many observed exotic hadrons (very/to) challenging, for now

Concerning QCD symmetries (spontaneously broken chiral s., approx. $SU(3)_{F,}$ heavy quark symmetry for large m_Q):

- those will be easier to consider once all these resonance masses are interpolated/extrapolated to m_q^{phy}
- presently, resonance simulations for one m_q, m_s, a
- systematics not under control (yet), unlike for strongly stable states
- unfortunately, more thresholds open as $m_u \to m_u^{phy}$

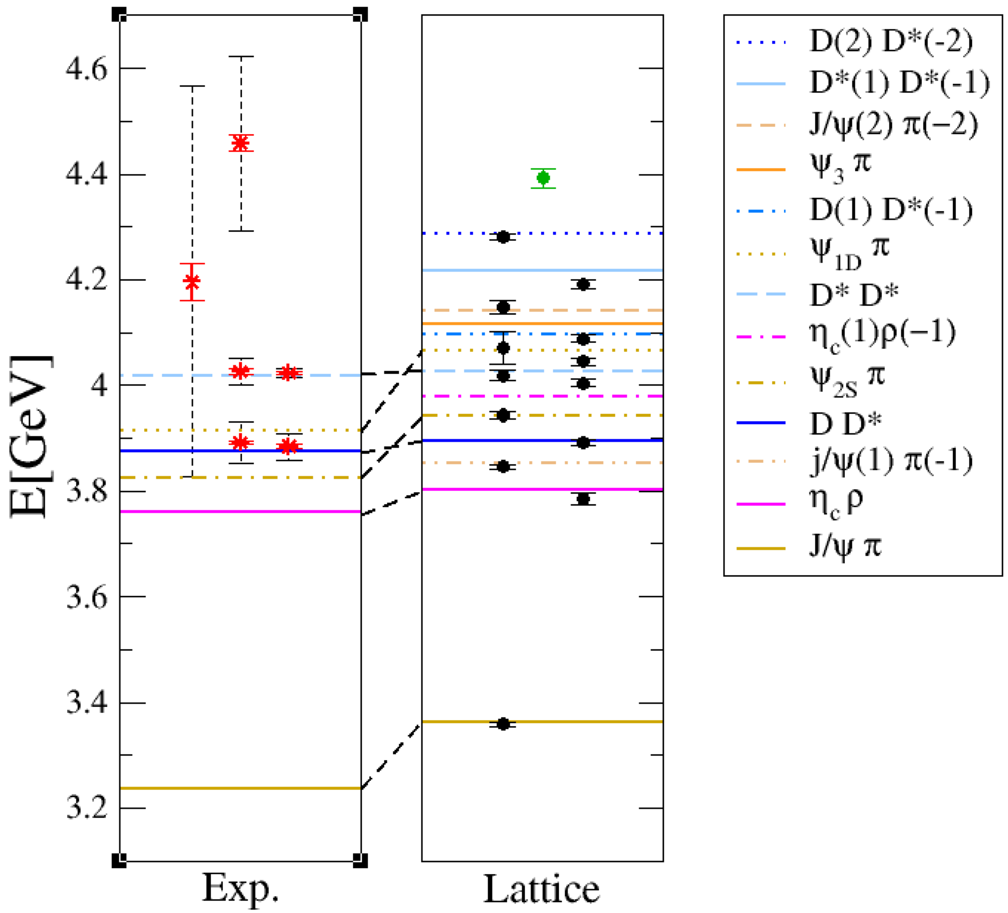
Scattering and resonance studies of strongly coupled scenarios Beyond SM are on the way

Backup slides

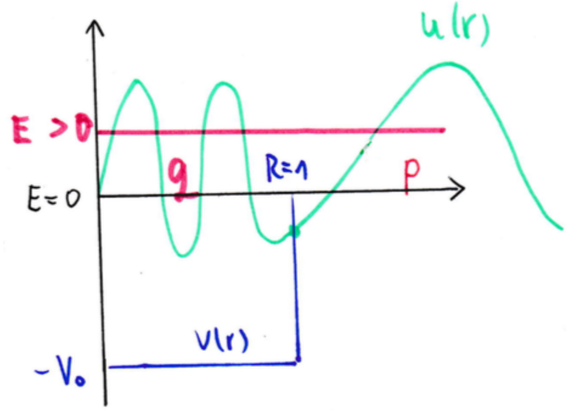
$$\mathcal{O} : (\bar{c}u)(\bar{d}c), \ (\bar{c}c)(\bar{d}u), \ [\bar{c}\bar{d}][cu]$$

Eigen-energies in Zc channel

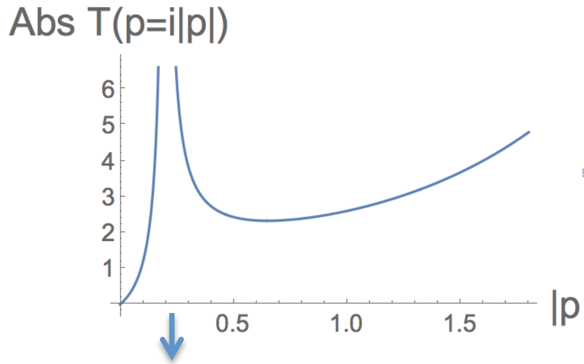
[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

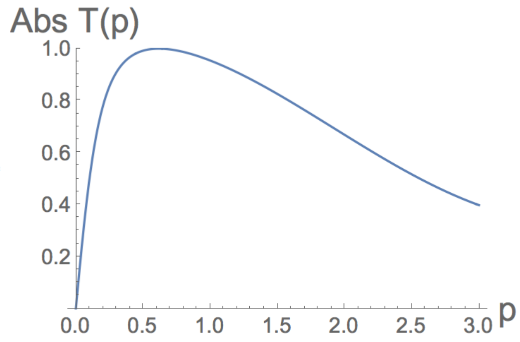


s-wave scattering on spherical well with shallow bound state in QM: T(p=i|p$_B$|)=∞



$A \sin qr \qquad B \sin(pr + \delta_0)$





$|p|=|p_B|=0.2$: pole of T at the position of the bound st.

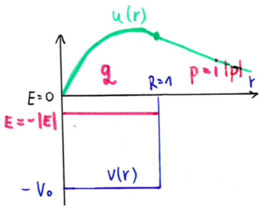
$$u(R) = A \sin qR = B \sin(pR + \delta_0)$$
$$u'(R) = qA \cos qR = kB \cos(pR + \delta_0)$$

$$\frac{1}{q}\tan qR = \frac{1}{k}\tan(pR + \delta_0)$$

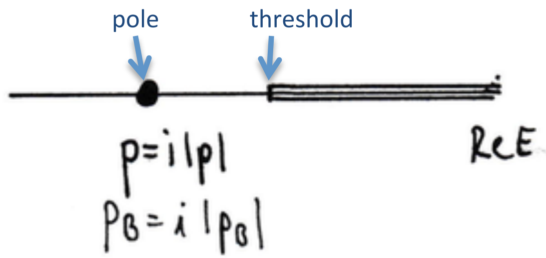
$$\delta_0(p) = \arctan\left(\frac{p}{q}\tan(pR)\right) - pR + n\pi$$

$$q = \sqrt{2\mu(V_0 + E)} = \sqrt{\underbrace{2\mu V_0}_{c^2} + p^2}$$

$$S = e^{2i\delta(p)} = 1 + 2iT(p) \qquad T(p) = \frac{1}{\cot\delta(p) - i}$$



$c^2 \approx 1.7^2$: V0 chosen such that there is one bound state solution in this potential with $|p_B|=0.2$

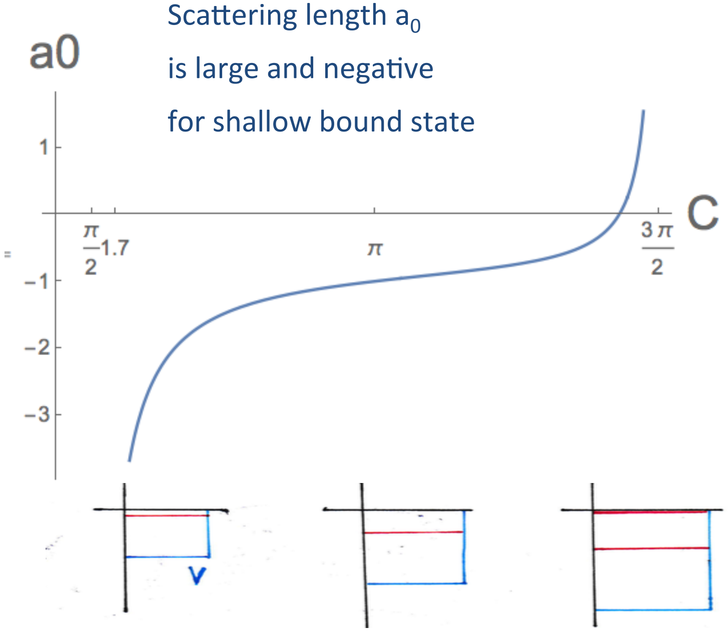


s-wave scattering on a spherical well with a shallow bound state in QM

$$\delta_0(p) = \arctan\left(\frac{p}{q}\tan(pR)\right) - pR + n\pi \qquad q = \sqrt{2\mu V_0 + p^2} = \sqrt{C^2 + p^2}$$

Taylor expanding

$$\boxed{p\cot\delta_0(p) =} \underbrace{\frac{C}{-C + \mathrm{Tan}[C]}}_{1/a_0} + \frac{1}{6}\left(3 - \frac{C^2}{(C - \mathrm{Tan}[C])^2} - \frac{3}{C^2 - C\,\mathrm{Tan}[C]}\right) p^2 + O[p]^4 \boxed{= \frac{1}{a_0} + \frac{1}{2} r_0\, p^2}$$



Signature of a shallow bound state:

a_0 negative and large