Hadron spectroscopy and interactions from lattice QCD

Sasa Prelovsek
University of Ljubljana & Jozef Stefan Institute, Slovenia

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in collaboration with C. Lang, D. Mohler, L. Leskovec, R. Woloshyn
Outline with challenges

When one aims to study a hadron with given $J^{PC}$, the location of the threshold in that channel pays a major role. If the hadron mass is above threshold, it can strongly decay to $H_1 H_2$ as long as quantum numbers allow decay.

- **Hadrons well below threshold:** "easy" all reproduced by lattice. Unfortunately none of exotic experimental candidates is found well below threshold.

- **Hadron resonances above threshold and states slightly below threshold:** challenging. simulate scattering of two hadrons $H_1 H_2$ on lattice and determine the scattering matrix. a number of pioneering results in past few years.

- **Hadrons above several thresholds:** very challenging very few results available.

✓ I will report on some examples for each case. conventional ($\rho,...$) and exotic hadrons ($X(3872), Z_c^+,...$) be given along the way.

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**Non-perturbative method: QCD on lattice**

\[ L_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + ig_s G^a_{\mu} T^a) q - m_q \bar{q} q \]

input: \( g_s, m_q \)

output: hadron properties
hadron interactions

**Evaluation of Feynman path integrals in discretized space-time**

quantum mechanics

\[ \int Dx \ e^{i S/\hbar} \]

\[ S = \int dt \ L[x(t)] \]

quantum field theory in Euclidean space-time

\[ \int DG \ Dq \ D\bar{q} \ e^{-S_{QCD}/\hbar} \]

\[ S_{QCD} = \int d^4x \ L_{QCD}[G(x), q(x), \bar{q}(x)] \]

x, t (Minkovsky) \( \rightarrow \) x, i t (Euclidean)

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Discrete energies of eigenstates: $E_n$

\[ J^{PC} \quad \mathcal{O} = \bar{q} \Gamma q, \quad (\bar{q} \Gamma_1 q) \bar{p}_1 (\bar{q} \Gamma_2 q) \bar{p}_2, \quad [\bar{q} \Gamma_3 \bar{q}] [q \Gamma_4 q], \ldots \]

$\rho(770)$, $1^- : \quad \bar{d}u, \quad (\bar{d}d)(\bar{d}u) = \pi \pi$

$X(3872)$, $1^{++} : \quad \bar{c}c, \quad (\bar{c}u)(\bar{uc}) = D \bar{D}^*, \quad [\bar{c}u][cu]$  

\[ p \quad 1/2^+ : \quad uud, \quad (uud)(\bar{uc}) = p\pi \]

\[
C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^+ (0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle \ e^{-E_n t} \langle n | \mathcal{O}_j^+ | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t} \]

All eigenstates with given $J^{PC}$ appear in principle (example: proton channel $1/2^+$)

- **single hadron states** $\chi_c(1P)$  \hspace{1cm} $m=E_1$ for $P=0$ (after extrapolation)

- **two-hadron states** $DD^*$  \hspace{1cm} $E_n$ rigorously render two-hadron scattering matrix (for example $DD^*$ scattering matrix)

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States well below threshold: “easy” precision spectrum

- $m = E_n$ for $P=0$, $a \rightarrow 0$, $L \rightarrow \infty$, $m_q \rightarrow m_q^{\text{phy}}$

- Available from a number of lattice QCD collaborations for a number of years
m = E_n

The gold-plated meson spectrum

High Precision QCD

HPQCD
1112.2590
1207.5149;
0909.4462

1008.4018
error 3 MeV
- em effects important!
Glance at PDG: almost all hadrons are hadronic resonances (decay strongly)

In particular:
all exotic candidates
(tetraquarks, pentaquarks)
strongly decay

<table>
<thead>
<tr>
<th>$\bar{u}u$</th>
<th>$\bar{s}u$</th>
<th>$\bar{c}u$</th>
<th>$uud$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$</td>
<td>$K^+$</td>
<td>$D^*$</td>
<td>$p$</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>$K^0$</td>
<td>$D^0$</td>
<td>$n$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$K_S^0$</td>
<td>$D^*$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_L^0$</td>
<td>$D^*$</td>
<td></td>
</tr>
<tr>
<td>$\rho(770)$</td>
<td>$K_0(800)$ or $\kappa$</td>
<td>$D^*(2007)^+$</td>
<td>( N(1440) ) 1/2$^+ $</td>
</tr>
<tr>
<td>$\omega(782)$</td>
<td>$K'(892)$</td>
<td>$D^*(2010)^0$</td>
<td>( N(1520) ) 3/2$^- $</td>
</tr>
<tr>
<td>$\eta'(958)$</td>
<td>$K_1(1270)$</td>
<td>$D_0^*(2400)^0$</td>
<td>( N(1535) ) 1/2$^+ $</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>$K_1(1400)$</td>
<td>$D_0(2440)^0$</td>
<td>( N(1650) ) 1/2$^- $</td>
</tr>
<tr>
<td>$a_0(980)$</td>
<td>$K_{1}^{*}(1410)$</td>
<td>$D_1(2420)^0$</td>
<td>( N(1675) ) 5/2$^- $</td>
</tr>
<tr>
<td>$\phi(1020)$</td>
<td>$K_{1}^{*}(1430)$</td>
<td>$D_{1}(2420)^{+}$</td>
<td>( N(1680) ) 5/2$^- $</td>
</tr>
<tr>
<td>$h_1(1170)$</td>
<td>$D_{2}^{*}(2460)^0$</td>
<td>$D_{1}(2430)^0$</td>
<td>( N(1685) ) ?</td>
</tr>
<tr>
<td>$b_1(1235)$</td>
<td>$D_{2}^{*}(2460)^{+}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(1700) ) 3/2$^- $</td>
</tr>
<tr>
<td>$a_1(1260)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2430)^{0}$</td>
<td>( N(1710) ) 1/2$^+ $</td>
</tr>
<tr>
<td>$f_2(1270)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(1720) ) 3/2$^+ $</td>
</tr>
<tr>
<td>$f_1(1285)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2430)^{0}$</td>
<td>( N(1860) ) 5/2$^+$</td>
</tr>
<tr>
<td>$\eta(1295)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(1875) ) 3/2$^+$</td>
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<tr>
<td>$\eta(1300)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(1880) ) 1/2$^+$</td>
</tr>
<tr>
<td>$a_2(1320)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(1895) ) 1/2$^+$</td>
</tr>
<tr>
<td>$f_0(1370)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(1900) ) 3/2$^+$</td>
</tr>
<tr>
<td>$\eta(1405)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(1990) ) 7/2$^+$</td>
</tr>
<tr>
<td>$f_1(1420)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(2000) ) 5/2$^+$</td>
</tr>
<tr>
<td>$\omega(1420)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(2040) ) 3/2$^+$</td>
</tr>
<tr>
<td>$f_2(1430)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(2060) ) 5/2$^+$</td>
</tr>
<tr>
<td>$a_0(1450)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td>( N(2100) ) 1/2$^+$</td>
</tr>
<tr>
<td>$\rho(1450)$</td>
<td>$D_{2}^{*}(2460)^{0}$</td>
<td>$D_{1}(2440)^{0}$</td>
<td></td>
</tr>
</tbody>
</table>

\( \square \) stable on strong decay
Hadrons near or above strong decay threshold

Rigorous approach

Scattering in one channel

“challenging”
Extracting scattering phase shift

\[ \sigma \propto \sin^2 \delta \]

analytic proposal: Luscher 1991

two mesons in a lattice box

scattering phase shifts at infinite volume \[ \delta(E) \]

energies from lattice with spatial extent \( L \)

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Relation between $E$ and $\delta(E)$

analytic proposal: Luscher 1991

Energies of two hadrons in a box:

$$E(L) = \sqrt{m_1^2 + \vec{p}_1^2} + \sqrt{m_2^2 + \vec{p}_2^2} + \Delta E$$

boundary condition

- due to strong interaction
- gives rigorous info on $\delta$

$$\vec{p}_{1,2} = \frac{2\pi}{L} \hat{n}$$

V=0, $p=2\pi/L$

$V\neq 0$

x=0

x=L/2

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Scattering of two mesons

one-channel (elastic) scattering with total momentum $P=0$: $E=E_{cm}$

$E_n(L) \xrightarrow{\text{Luscher's eq.}} \delta(E)$

Scattering matrix for partial wave $l$

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot \delta(E) - i}$$

**Bound state (B):**

$$\cot[\delta(E_B)] = i, \quad E_B < m_1 + m_2$$

**Resonance (R) (of Breit-Wigner type):**

$$T(E') = \frac{-E \Gamma}{E^2 - m_R^2 + iE \Gamma}, \quad \Gamma(E) = g^2 \frac{p^{2l+1}}{E^2}$$

Locations of poles of $T(E)$ for res. and bound st.

Two types of plots will be shown:

- $E(L)$ energies of lat. eigenstates
- $m_{\text{phy}} = m_R$, $m_B$ extracted from $E(L)$
Verifying approach on conventional $\rho$

$$C_{jk}(t) = \langle 0 | O_j(t) O_k^+(0) | 0 \rangle, \quad O = \bar{q}q, (\bar{q}q)(\bar{q}q) \quad 1$$

$\rho$ meson

<table>
<thead>
<tr>
<th>Mass [MeV]</th>
<th>$g_{\rho\pi\pi}$ (no unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lat ($m_\rho=266$ MeV)</td>
<td>$772 \pm 6 \pm 8$</td>
</tr>
<tr>
<td>Exp.</td>
<td>$775$</td>
</tr>
</tbody>
</table>

$\Gamma = \frac{g_{\rho\pi\pi}^2 p^3}{6\pi E^2}$
ρ : review of lattice results
the only resonance studied by several collaborations

from T. Yamazaki [1503.0867]
**$K^*(892)$ resonance $K\pi$**

**$K\pi$, $I=1/2$: p-wave phase shift**

\[ \delta = \text{acot} \left( \frac{m_R^2 - E_{\text{cms}}^2}{m_R \Gamma} \right) \]

\[ \Gamma[K^* \to K\pi] = \frac{g^2 p^3}{6\pi s} \]

<table>
<thead>
<tr>
<th>$m_{K^*(892)}$ [MeV]</th>
<th>$g_{K^*(892)}$ [no unit]</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat 891 ± 14</td>
<td>5.7 ± 1.6</td>
</tr>
<tr>
<td>exp 891.66 ± 0.26</td>
<td>5.72 ± 0.06</td>
</tr>
</tbody>
</table>

[S.P. ,Lang, Leskovec, Mohler, 1307.0736, PRD 2013]

$m_\pi \approx 266$ MeV

Reviewed by Takeshi Yamazaki, plenary talk.
Resonance $\psi(3770)$ and bound st. $\psi(2S)$ from $D\bar{D}$ scattering in p-wave

$D\bar{D}$ scat. in p-wave is simulated

$E_n \rightarrow \delta(E_n)$

$T$-matrix is determined from $E_n$ and interpolated near threshold:

Bound state $\psi(2S)$ from pole in $T$:

$\cot \delta(p_B) = i$

$m_B$ (triangles)

Resonance $\psi(3770)$: $\delta(m_R) = 90^\circ$

$m_R$ (diamonds), $\Gamma$ (given below)

<table>
<thead>
<tr>
<th>$\psi(3770)$</th>
<th>Mass [MeV]</th>
<th>$g$ (no unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lat ($m_\pi=266$ MeV)</td>
<td>3774 ±6±10</td>
<td>19.7 ±1.4</td>
</tr>
<tr>
<td>Lat ($m_\pi=156$ MeV)</td>
<td>3789 ±68±10</td>
<td>28 ±21</td>
</tr>
<tr>
<td>Exp.</td>
<td>3773.15±0.33</td>
<td>18.7 ±1.4</td>
</tr>
</tbody>
</table>

$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$\eta_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$

$X(3872)$
$X(3940)$
$\psi(4040)$
$X(4050)^\pm$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^\pm$

$\Gamma = \frac{g^2}{6\pi} \frac{p^3}{s}$

Lang, Leskovec, Mohler, S.P.,
1503.05363, JHEP 2015]
$X(3872)$ as bound state from $D\bar{D}^*$ scattering, $J^{PC}=1^{++}$, $I=0$

- ground state: $\chi_{c1}(1P)$
- $D\bar{D}^*$ scattering matrix near th. determined

\[ p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 \]

\[ a_0^{D\bar{D}^*} = -1.7 \pm 0.4 \text{ fm} \quad r_0^{D\bar{D}^*} = 0.5 \pm 0.1 \text{ fm} \]

\[ T \propto \frac{1}{\cot \delta - i} = \infty \]

- A pole found just below th. (violet star)
- The pole attributed to $X(3872)$, which is a shallow bound state

First evidence for $X(3872)$ from lattice
S.P. and Leskovec: 1307.5172, PRL 2013

$M.\;\text{Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015}$

- physical masses $m^{\text{phy}}$
- $m_\pi \approx 266 \text{ MeV}$, $a=0.124 \text{ fm}$, $L=2 \text{ fm}$
Which Fock components are essential for $X(3872)$ with $I=0$?

$J^{PC}=1^{++}$

$\mathcal{O}: \bar{c}c, \ D\bar{D}^*, \ J/\psi \omega, \ \chi_{c1}\sigma, \ \eta_c \sigma, \ \{\bar{c}u\}_{3c}[cu]_{3c}, \ \{\bar{c}u\}_{6c}[cu]_{6c}$

$(\bar{c}q)_{1c}(c\bar{q})_{1c}$, $(\bar{c}c)_{1c}(\bar{q}q)_{1c}$

[M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]
Which Fock components are essential for $X(3872)$ with $I=0$?

\[ \Phi: \overline{c}c, \ D\overline{D}^{*}, \ J/\psi\omega, \ \chi_{c1}\sigma, \ \eta_{c}\sigma, \ \left[ \overline{c}u \right]_{3c} \left[ cu \right]_{3c}, \ \left[ \overline{c}u \right]_{6c} \left[ cu \right]_{6c} \]

Upper red square is candidate for $X(3872)$: it is found only if $cc$ in the basis.

$E_\eta = 3.85 \text{ GeV}$

$\approx 266 \text{ MeV}$

$[M. \text{ Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015}]$

Energies of eigenstates on lattice $E(L)$ (these are not $m^{\text{phy}}$)

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Localised diquark antidiquark

Do not seem not essential

If $cc$ not in the basis, although $[cu][cu]$ in the basis

$X(3872)$ not found
$D_{s0}(2317)$ and $D_{s1}(2460)$ below DK and $D^*K$ thresholds

Mass prediction for missing $B_{s0}$ and $B_{s1}$

$M_{\pi} = 156$ MeV, $N_f = 2 + 1$

[1403.8103, PRD 2014]

[C. Lang, D. Mohler, S.P., R. Woloshyn: 1501.0164, PLB2015]
Bound state of a $\eta_c$ and p from lattice

[NPLQCD, 1410.7069, PRD, $m_\pi \sim 800$ MeV]

$\eta_c$ p
$\bar{c}c$ uud

$\sim 20$ MeV below th. $m_{\eta_c} + m_p$

Two pentaquark resonances in $J/\psi$ and p from exp

LHCb: 1507.03414

$L/\psi$ p
$\bar{c}c$ uud

$\sim 400$ MeV above th. $m_{J/\psi} + m_p$

no lattice results for these Pc yet: challenging as it can in principle decay to several decay ch.
Hadrons that decay to several final states

Scattering in two or more channels

"very challenging"
Resonances in $K\pi$, $K\eta$ coupled channels

- $q\bar{q}$, $K\pi$, $K\eta$ interpolators
- a number of different $0<P\leq2$
- for each $E_n$: one determinant equation for many unknowns
- T-matrix parametrized to get around this problem
- the location of poles of T-matrix in complex plain is given below
- $K^*(892)$ and $\kappa$ are below threshold for this $m_\pi$
- $K_0^*(1400)$, $K_2^*$ are resonances
- $m_\pi=391$ MeV, $N_L=16, 20, 24$

[Dudek, Edwards, Thomas, Wilson, HSC, 1406.4158, PRL; 1411.2004]

\[
\text{det} \left[ \delta_{ij} \delta_{jj'} + i \rho_i t_{ij}^{(j)}(E_{cm}) \left( \delta_{jj'} + i M^{E\lambda}_{jj'}(p_i L) \right) \right] = 0.
\]

location of poles in T matrix in complex plane
Charged charmonium \( Z_c^+ \): manifestly exotic

\[ I^G = 1^+, \ J^{PC} = 1^{++} \quad (C \text{ is for neutral partner}) \]

\[ Z_c^+(3900) \rightarrow J/\Psi \ \pi^+ \quad \text{cc\, du} \]

[BESIII, 2013, 1303.5949, PRL]
HALQCD is another method to extract scattering matrix from lattice (considered to be less rigorous than the Luscher’s method for coupled channels)
**$Z_c^+$ channel, three-body $Y(4260)$ decay: lattice & exp**

\[ Z_c^+ \rightarrow J/\psi \pi^+ = (c\bar{c})(d\bar{u}) \]

[HALQCD, Ikeda et al, 1602.03465]
**Z_c^+ channel:** HALQCD method, poles of S in complex plane

Second Riemann sheet for all three channels shown.

**Remarks:**
- HALQCD method not considered as rigorous as the Luscher’s method for coupled channels
- 3x3 matrix S in Zc channel has not been determined by Luscher method yet
- HALQCD method has not verified any of the conventional resonances yet (to my knowledge)
- Luscher’s method has been verified on conventional res. like ρ, K*, ψ(3770), D_0(2400) ...

[HALQCD, Ikeda et al, 1602.03465]
Conclusions for the time being

Hadron spectroscopy from lattice (in brief):

- **hadrons well below threshold precisely obtained**: D, K ... and all the others: ✔✔✔

- **Lattice QCD can extract scattering matrix and cross-section for scattering of two hadrons ✔✔**
  (most easily if there is only one channel)

- **Hadron resonances are extracted from peaks in the cross-sections** via BW-type fits
  resonances verified: \( \rho, K^*, K_0^*(1430), K_2, D_0^*, D_1, a_1, b_1, \Psi(3770),.. \) ✔✔

- **Shallow bound states appear as poles in scattering matrix**
  shallow bound states verified: \( D_{s0}, D_{s1}, B_{s0}, B_{s1}, X(3872) \) with \( I=0,.. \) ✔✔

- **First steps to extract scattering matrix for scattering with two or more coupled channels ✔**
  indication that \( Z_c^+ \) is not a usual resonance (with pole on the second Riemann sheet)
**Lattice setup**

<table>
<thead>
<tr>
<th>PACS-CS</th>
<th>Ensemble (1)</th>
<th>Ensemble (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_L^3 \times N_T$</td>
<td>$16^3 \times 32$</td>
<td>$32^3 \times 64$</td>
</tr>
<tr>
<td>$N_f$</td>
<td>2</td>
<td>2+1</td>
</tr>
<tr>
<td>$a$ [fm]</td>
<td>0.1239(13)</td>
<td>0.0907(13)</td>
</tr>
<tr>
<td>$L$ [fm]</td>
<td>1.98(2)</td>
<td>2.90(4)</td>
</tr>
<tr>
<td>$m_\pi$ [MeV]</td>
<td>266(3)(3)</td>
<td>156(7)(2)</td>
</tr>
</tbody>
</table>

- Wilon-clover quarks

- **Fermilab method for c and b**: [El Khadra, Kronfeld et al, 1997]
  
  Rest hadron energies have sizable discretization errors but these largely cancel in splittings. Only splittings with respect to a chosen reference mass are compared to experiment.

- **evaluating Wick contractions** to simulate scattering on the lattice is challenging and computationally intensive – that is part of the reason why a small number of studies have been made. We apply two methods
  
  - distillation (Ensemble 1) [Peardon et. al., HSC, 2009]
  
  - stochastic distillation (Ensemble 2) [Morningstar et al., 2011]
Comparing lattice results for $X(3872)$, $J^{PC}=1^{++}$, $I=0$

Lattice evidence for $X(3872)$:

- [1] [Lee, DeTar, Na, Mohler, update of proc 1411.1389] $m_\pi \approx 310$ MeV, $a=0.15$ fm, $L=2.4$ fm, HISQ

- [2] [S.P. and Leskovec: 1307.5172, PRL 2013]

- [3] [M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]

- Position of $D\bar{D}^*$ threshold depends on $m_{u/d}$ and may be affected by discretization effects related to charm quark
Search for charged partner of $X(3872)$; channel $I^G=1^-$, $J^{PC}=1^{++}$, $c\bar{c}d\bar{u}$

Horizontal lines: energies of expected two-meson states in limit of no interaction:

$$E = E[ M_1(p_1) ] + E[ M_2(p_2) ]$$

- Circles: energies of eigenstates from lattice
- Only expected two-meson states observed.

- **No lattice candidate for charged $X(3872)$**.
  In agreement with absence of such state in exp.

- **No lattice candidate for other charged state below 4.3 GeV**.

- Two Belle 2008 states are exp. unconfirmed.

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- Two Belle 2008 states are exp. unconfirmed.

[M. Padmanath, C.B. Lang, S.P., PRD 2015, 1503.03257]
**Y(4140), J^{PC}=??^+, c\bar{c}s s**

**Experiment:**
peak in $J/\psi \Phi$ just above $J/\psi \Phi$ threshold

found: CDF 2009, CMS 2012, D0 2013, Babar 2015
not found: Belle 2010, LHCb 2012

**Lattice:**
- S. Ozaki and S. Sasaki, 1211.5512, PRD
  caveat: strange quark annihilation neglected
  no resonant $Y(4140)$ structure found

- M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD
  $\mathcal{O} : \bar{c}c, (\bar{c}s)(s\bar{c}), (\bar{c}c)(s\bar{s}), [\bar{c}s][c\bar{s}]$
channel $J^p=1^+$ considered only: expected two-particle
eigenstates found and $\chi_{c1}, X(3872)$ but not $Y(4140)$

$m_{\pi} = 266 \text{ MeV}$
Illustration how eigenstate $D_0^*(2400)$ dominated by “qq" dissapears when qq interpolators omitted

Effective energies at $t=8$ plotted.

[Mohler, S.P., Woloshyn, PRD 87 (2013) 034501]
Charmonia: single-meson approximation

\[ \mathcal{O} : \bar{c}c \]

- Lattice
- Experiment

single-meson approx:
- used until few years ago:
  - ignores strong decays of resonances
  - ignores effects of thresholds

Hybrids:
- some of them have exotic \( J^{PC} \)
- large overlap with \( O = q F_{ij} q \)

\[ [HSC, \ L. \ Liu \ et \ al: \ 1204.5425, \ JHEP] \]

- \( m_\pi \approx 400 \text{ MeV}, L \approx 2.9 \text{ fm}, N_f = 2 + 1 \)
- identification with \( n^{2S+1L_j} \) multiplets using \( \langle O | n \rangle \)
- green: lat, black: exp

Sasa Prelovsek, Hadron interactions from lattice
Proton and neutron constitute more than 99% of the bright side of universe

\[ m_u = m_d \]

Sasa Prelovsek, Hadron interactions from lattice