Hadron spectroscopy and interactions from lattice QCD

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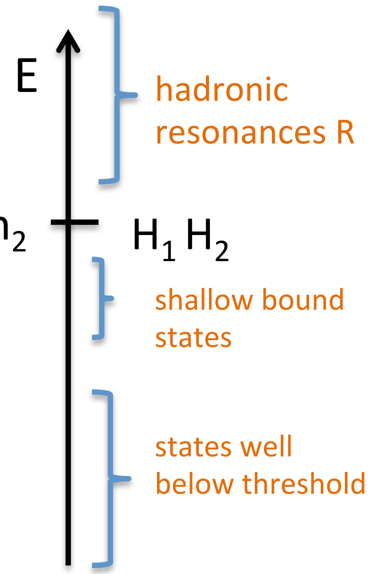
in collaboration with C. Lang, D. Mohler, L. Leskovec, R. Woloshyn

Outline with challenges

strong decay threshold: m_1+m_2



When one aims to study a hadron with given J^{PC}:
the location of the threshold in that channel pays a major role.
If the hadron mass is above threshold, it can strongly decay to H_1H_2
as long as quantum numbers allow decay.

- <u>Hadrons well bellow threshold:</u> *"easy"* all reproduced by lattice
  Unfortunately none of exotic experimental candidates is found well below threshold.

- <u>Hadron resonances</u> above threshold and <u>states slightly below threshold</u> : *challenging*
  simulate scattering of two hadrons H_1 H_2 on lattice and determine the scattering matrix
  a number of pioneering results in past few years

- <u>hadrons above several thresholds</u>: *very challenging* very few results available

✓ I will report on some examples for each case
conventional (ρ,...) and exotic hadrons (X(3872), Z_c^+,...) be given along the way

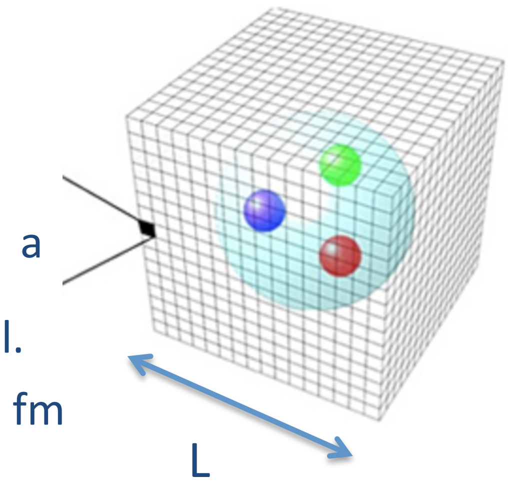
Non-perturbative method: QCD on lattice

$$L_{QCD} = -\tfrac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_{\mu} (\partial^{\mu} + i g_s G^{\mu}_{a} T^{a}) q - m_q \bar{q} q$$

input: g_s , m_q

output: hadron properties

hadron interactions



precision cal.

a~0.05-0.15 fm

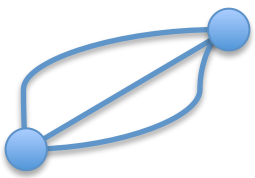
L~ 2-8 fm

Evaluation of Feynman path integrals in discretized space-time

quantum mechanics

$$\int Dx \, e^{i S/\hbar}$$

$$S = \int dt \, L[x(t)]$$



quantum field theory in Euclidian space-time

$$\int DG \, Dq \, D\bar{q} \, e^{-S_{QCD}/\hbar}$$

$$S_{QCD} = \int d^4x \, L_{QCD}[G(x), q(x), \bar{q}(x)]$$

x,t (Minkovsky) → x, i t (Euclidean)

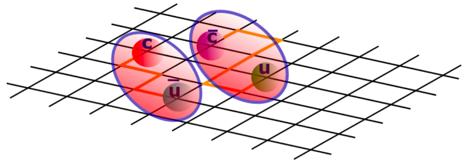
Discrete energies of eigenstates: E_n

mesons

$$J^{PC} \qquad \mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)_{\vec{p}_1}(\bar{q}\Gamma_2 q)_{\vec{p}_2}, \quad [\bar{q}\Gamma_3\bar{q}][q\Gamma_4 q], \ldots$$

$$\rho(770),\ 1^{--}: \qquad \bar{d}u, \quad (\bar{d}d)(\bar{d}u) = \pi\pi$$

$$X(3872),\ 1^{++}: \quad \bar{c}c, \quad (\bar{c}u)(\bar{u}c) = D\bar{D}^*, \qquad [\overline{cu}][cu]$$



baryon

$$p \qquad 1/2^+: \quad uud, \quad (\mathrm{uud})(\bar{u}c) = p\pi$$

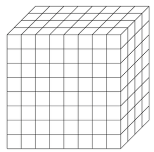
$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\ \mathcal{O}_j^+(0)|0\rangle = \sum_n \langle 0|\mathcal{O}_i|n\rangle\ e^{-E_n t}\ \langle n|\mathcal{O}_j^+|0\rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

All eigenstates with given J^{PC} appear in principle (example: proton channel ½⁺)

- single hadron states **χ_{c1}(1P)** $m=E_1$ for P=0 (after extrapolation)
- two-hadron states DD* E_n rigorously render two=hadron scattering matrix (for example DD* scattering matrix)

States well below threshold: “easy” precision spectrum

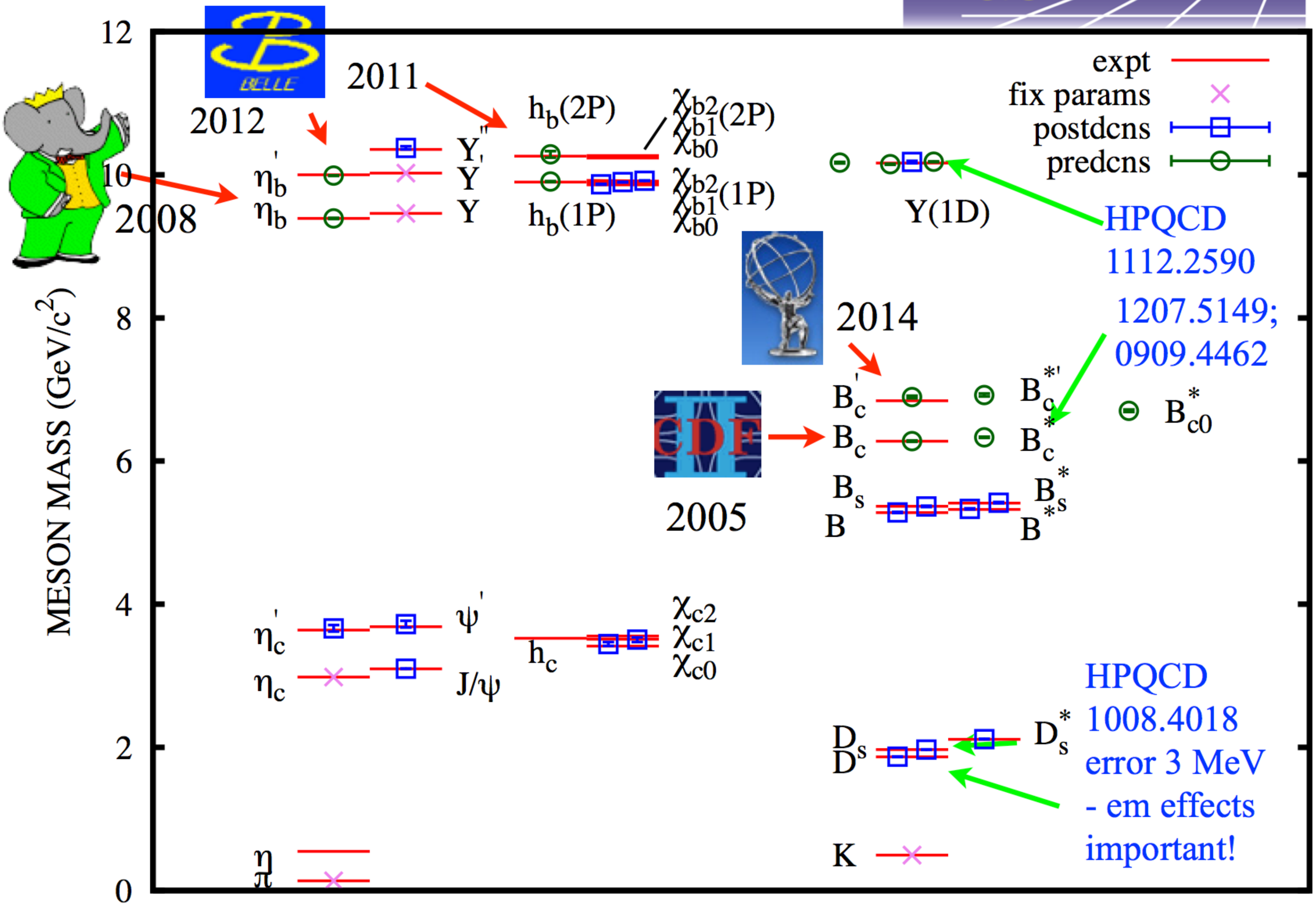
- $m=E_n$ for $P=0$ $a \to 0$, $L \to \infty$, $m_q \to m_q^{phy}$



- Available from a number of lattice QCD collaborations for a number of years

High Precision QCD

$m=E_n$ The gold-plated meson spectrum



Glance at PDG: almost all hadrons are hadronic resonances (decay strongly)

❑ stable on strong decay

In particular:

all exotic candidates

(tetraquarks, pentaquarks)

strongly decay

$\bar{u}u$	$\bar{s}u$	$\bar{c}u$	uud
$\pi^\pm$	$K^\pm$	$D^\pm$	p
π^0	K^0	D^0	n
η	K_S^0	$D^*(2007)^0$	$N(1440)\ 1/2^+$
$f_0(500)$ or σ was $f_0(60$	K_L^0	$D^*(2010)^\pm$	$N(1520)\ 3/2^-$
$\rho(770)$	$K_0^*(800)$ or κ	$D_0^*(2400)^0$	$N(1535)\ 1/2^-$
$\omega(782)$	$K^*(892)$	$D_0^*(2400)^\pm$	$N(1650)\ 1/2^-$
$\eta'(958)$	$K_1(1270)$	$D_1(2420)^0$	$N(1675)\ 5/2^-$
$f_0(980)$	$K_1(1400)$	$D_1(2420)^\pm$	$N(1680)\ 5/2^+$
$a_0(980)$	$K^*(1410)$	$D_1(2430)^0$	$N(1685)\ ??$
$\phi(1020)$	$K_0^*(1430)$	$D_2^*(2460)^0$	$N(1700)\ 3/2^-$
$h_1(1170)$	$K_2^*(1430)$	$D_2^*(2460)^\pm$	$N(1710)\ 1/2^+$
$b_1(1235)$	$K(1460)$	$D(2550)^0$	$N(1720)\ 3/2^+$
$a_1(1260)$	$K_2(1580)$	$D(2600)$	$N(1860)\ 5/2^+$
$f_2(1270)$	$K(1630)$	$D^*(2640)^\pm$	$N(1875)\ 3/2^-$
$f_1(1285)$	$K_1(1650)$	$D(2750)$	$N(1880)\ 1/2^+$
$\eta(1295)$	$K^*(1680)$		$N(1895)\ 1/2^-$
$\pi(1300)$	$K_2(1770)$		$N(1900)\ 3/2^+$
$a_2(1320)$	$K_3^*(1780)$		$N(1990)\ 7/2^+$
$f_0(1370)$	$K_2(1820)$		$N(2000)\ 5/2^+$
$h_1(1380)$	$K(1830)$		$N(2040)\ 3/2^+$
$\pi_1(1400)$			$N(2060)\ 5/2^-$
$\eta(1405)$			$N(2100)\ 1/2^+$
$f_1(1420)$			
$\omega(1420)$			
$f_2(1430)$			
$a_0(1450)$			
$\rho(1450)$			

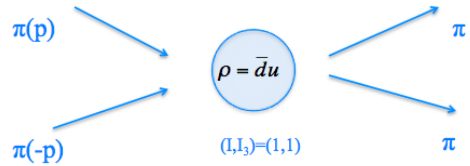
Hadrons near or above strong decay threshold

Rigorous approach

Scattering in one channel

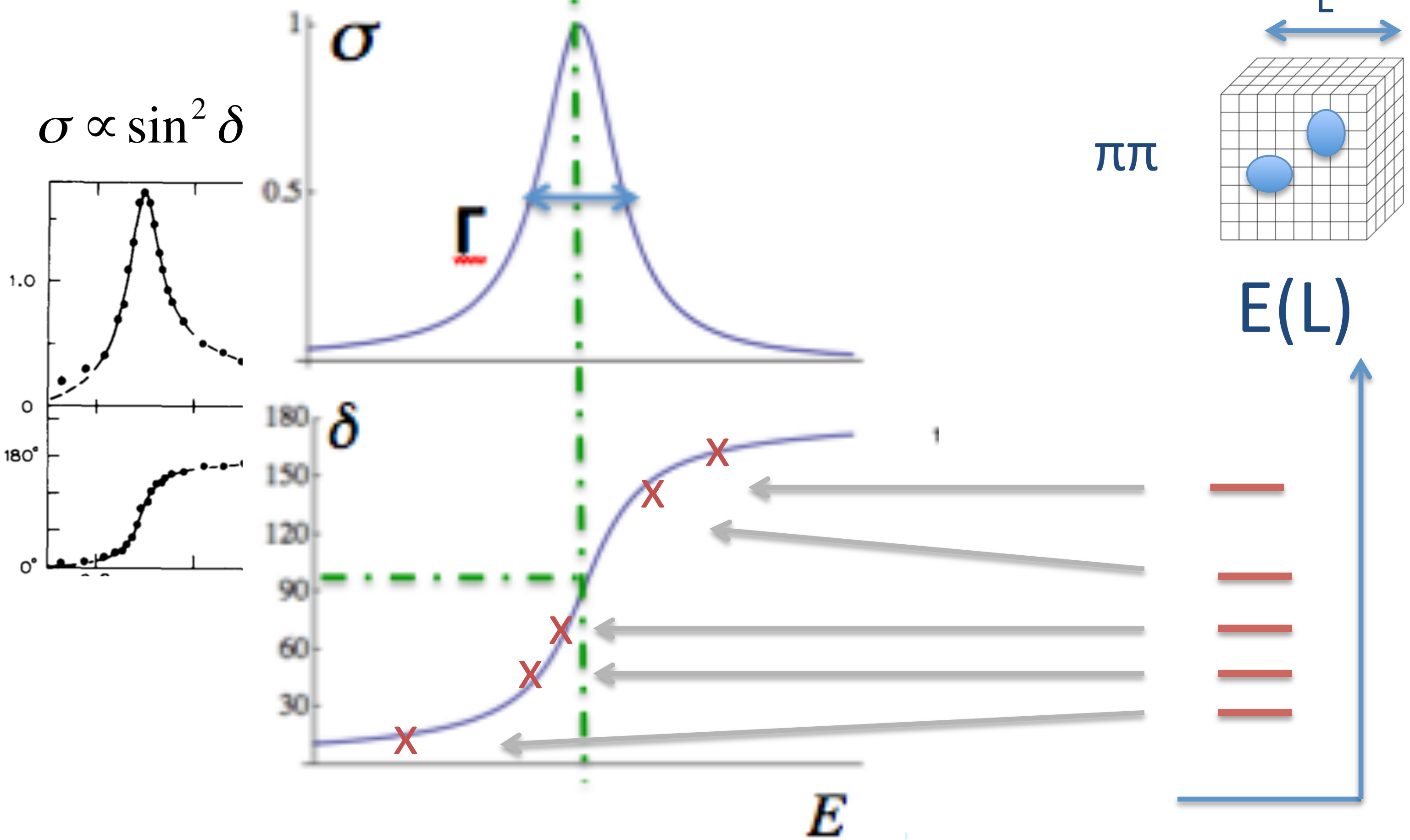
”challenging”

Extracting scattering phase shift



example

analytic proposal: Luscher 1991



two mesons

in a lattice box

scattering phase shifts at infinite volume $\delta(E) \longleftarrow E(L)$ energies from lattice with spatial extent L

Relation between E and δ(E)

analytic proposal: Luscher 1991

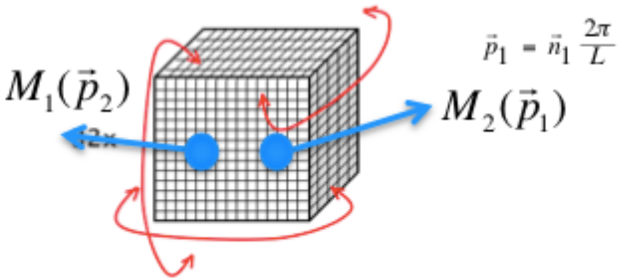
Energies of two hadrons in a box:

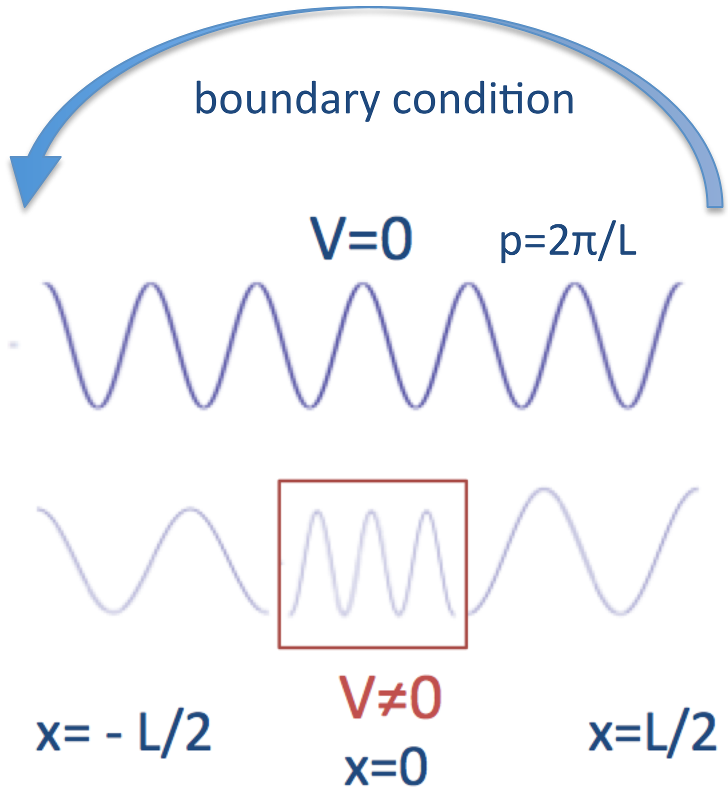
$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E$$

- due to strong interaction

- gives rigorous info on δ

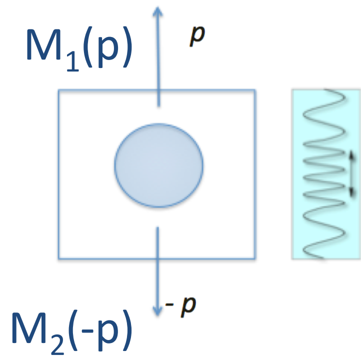
$$\vec{p}_{1,2} = \frac{2\pi}{L}\vec{n}$$





Scattering of two mesons

one-channel (elastic) scattering with total momentum P=0: $E=E_{cm}$



$$E_n(L) \xrightarrow{\text{Luscher's eq.}} \delta(E)$$

Scattering matrix for partial wave l

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot\delta(E) - i}$$

Bound state (B):

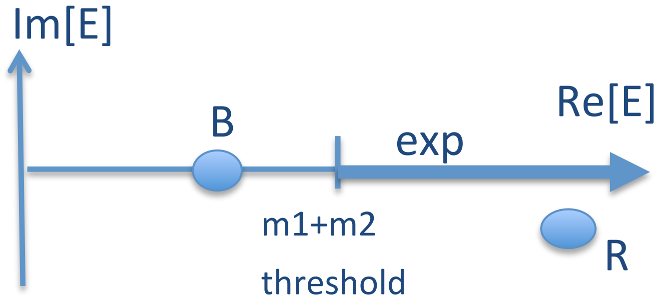
$\cot[\delta(E_B)] = i\,, \quad E_B < m_1+m_2$

Resonance (R) (of Breit-Wigner type):

$$T(E) = \frac{-E\,\Gamma}{E^2 - m_R^2 + i\,E\,\Gamma}, \quad \Gamma(E) = g^2\,\frac{p^{2l+1}}{E^2}$$

Locations of poles of T(E)

for res. and bound st.



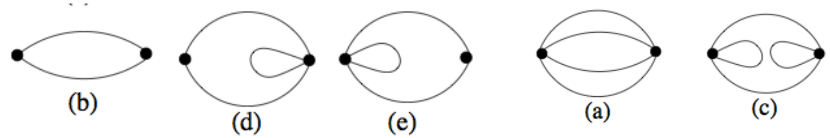
Two types of plots will be shown:

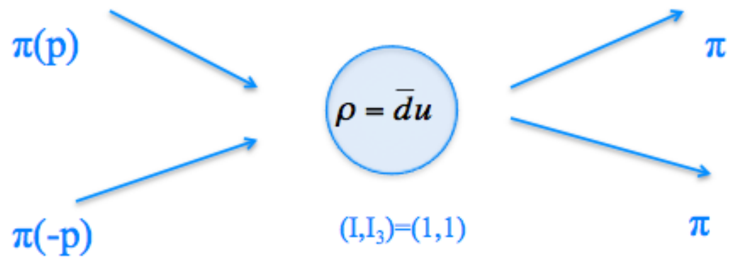
- E(L) energies of lat. eigenstates
- $m^{phy} = m_R$, m_B extracted from E(L)

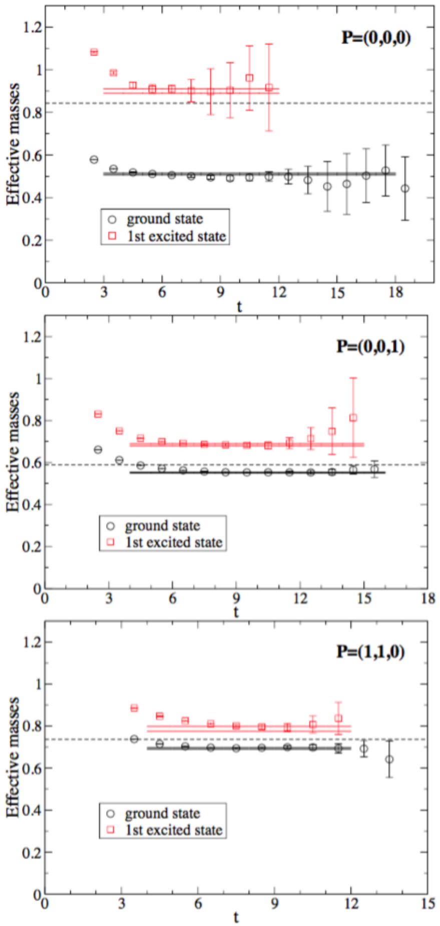
Verifying approach on conventional ρ

Lang, Mohler, S.P., Vidmar
PRD84 (2011) 054503

$$C_{jk}(t) = \langle 0 | O_j(t)\, O_k^{+}(0) | 0 \rangle, \quad O = \bar{q}q, (\bar{q}q)(\bar{q}q) \quad 1^{--}$$



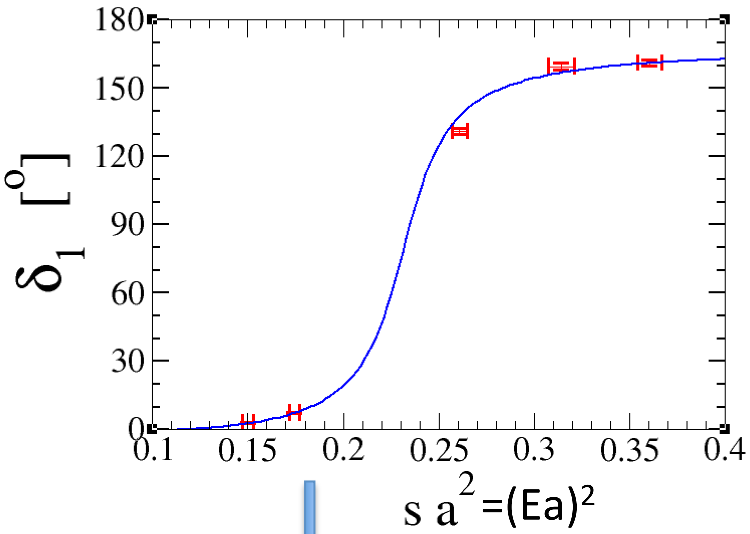




$E_{cm} \rightarrow \delta(E_{cm})$

generalized

Luscher's relation



$$T(E) = \frac{1}{\cot\delta(E) - i} = \frac{-E\,\Gamma}{E^2 - m_R^2 + i\,E\,\Gamma}$$

ρ meson	Mass [MeV]	$g_{\rho\pi\pi}$ (no unit)
Lat (m_π=266 MeV)	772 ± 6 ± 8	5.61 ± 0.12
Exp.	775	5.97

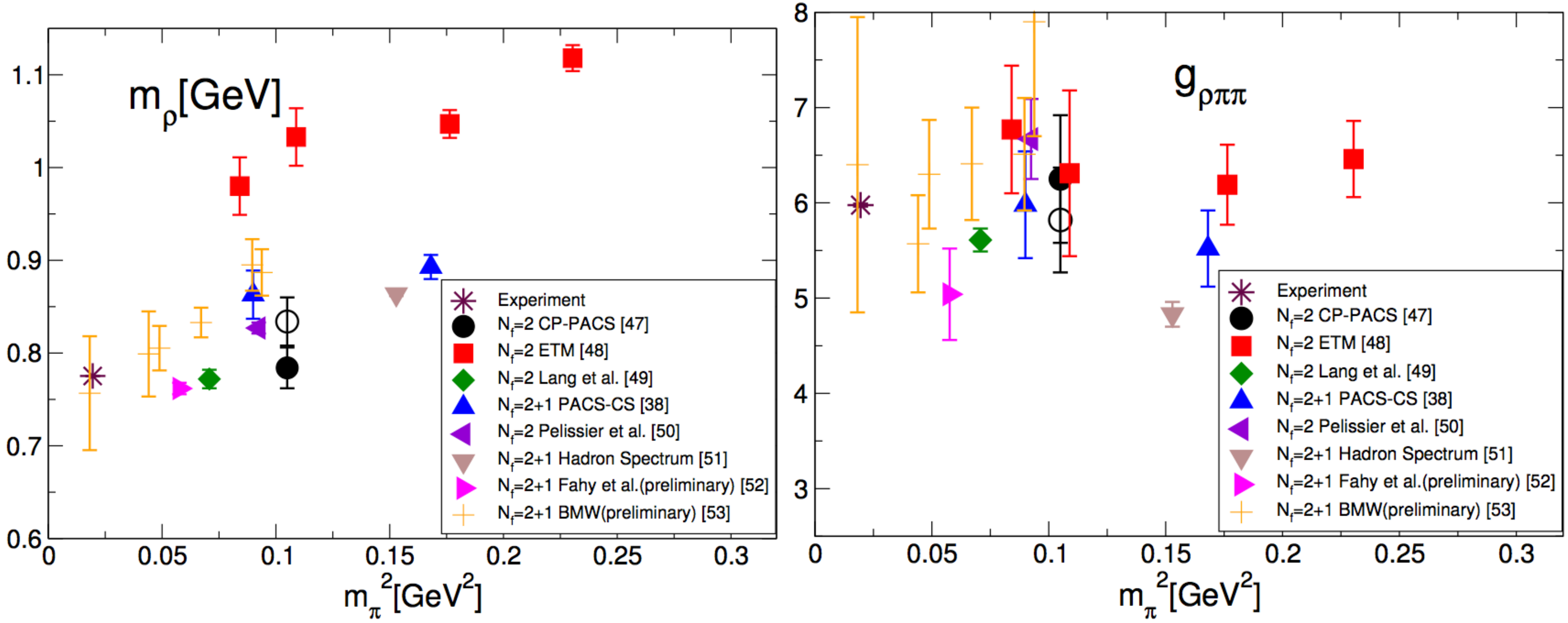
$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E^2}$$

ρ : review of lattice results

the only resonance studied by several collaborations

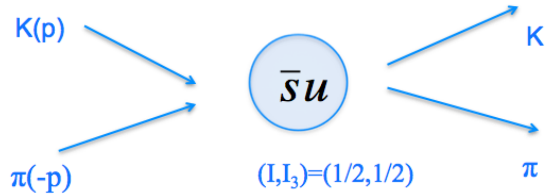
$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E^2}$$

from T. Yamazaki [1503.0867]

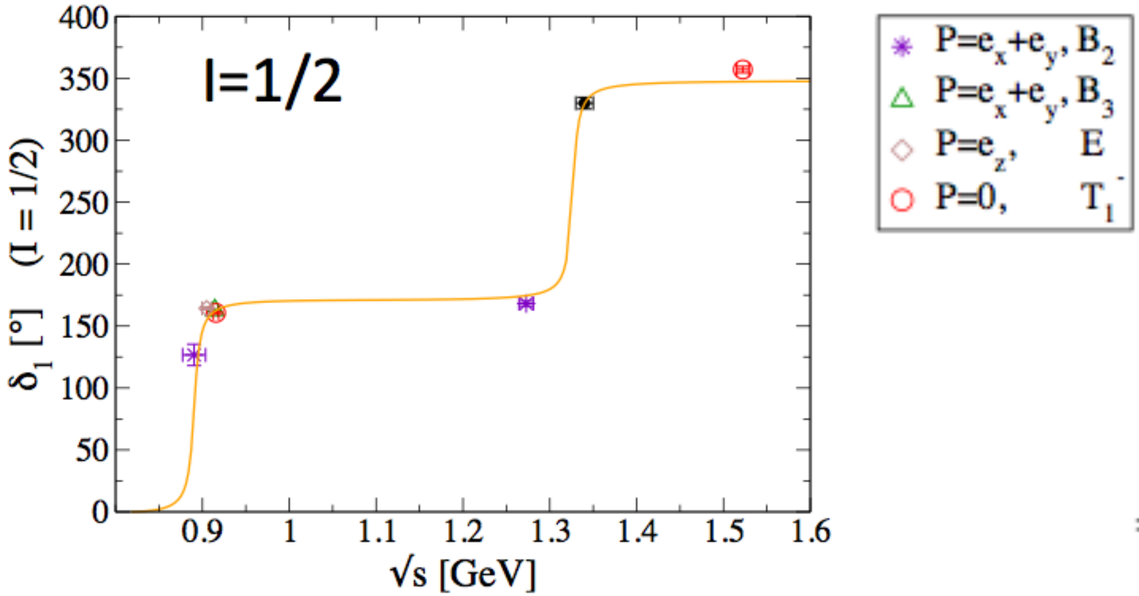


K*(892) resonance Kπ

reviewed by Takeshi Yamazaki, plenary talk



Kπ , I=1/2: p-wave phase shift



$$\text{BW}: \quad \delta = \text{acot}\,\frac{m_R^2 - E_{cms}^2}{m_R\,\Gamma}$$

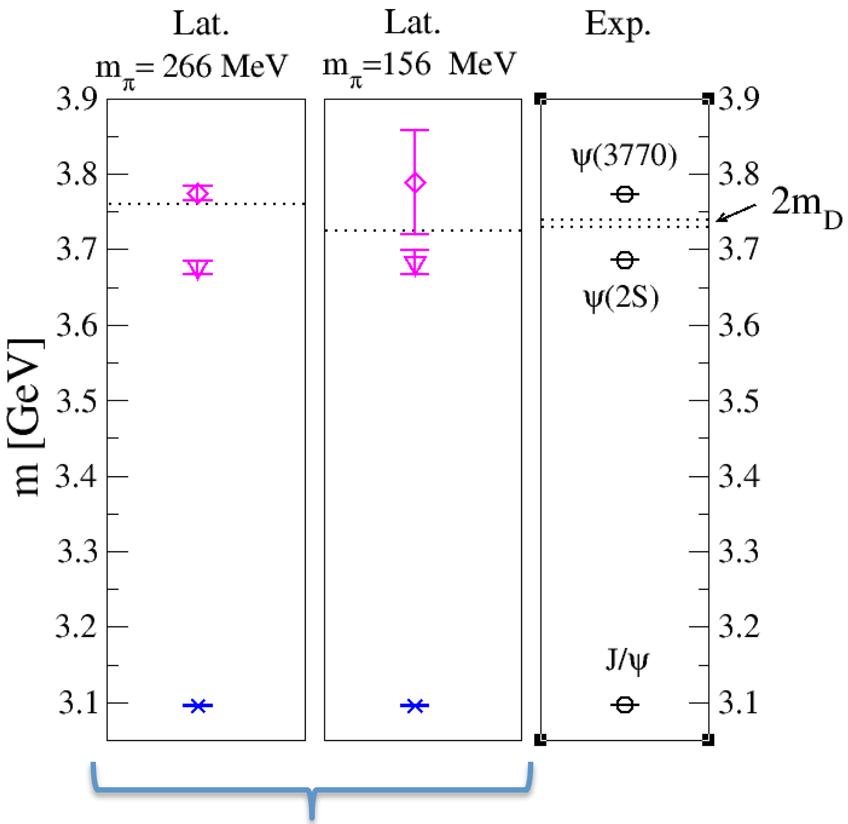
$$\Gamma[K^* \to K\pi] = \frac{g^2}{6\pi}\frac{p^{*3}}{s}$$

	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6
exp	891.66 ± 0.26	5.72 ± 0.06

[S.P. ,Lang, Leskovec, Mohler, 1307.0736, PRD 2013]

$m_\pi \approx 266$ MeV

Resonance ψ(3770) and bound st. ψ(2S) from D$\underline{\text{D}}$ scattering in p-wave



physical masses m^{phy}

$$\mathcal{O}: \bar{c}\, c, D\bar{D}, \quad J^{PC} = 1^{--}$$

D$\underline{\text{D}}$ scat. in p-wave is simulated

$$E_n \rightarrow \delta(E_n)$$

T-matrix is determined from E_n

and interpolated near threshold:

<u>Bound state</u> ψ(2S) from pole in T:

$\cot\delta(p_B) = i$

m_B (triangles)

$$T \propto \frac{1}{\cot\delta - i}$$

<u>Resonance</u> ψ(3770): $\delta(m_R)=90^o$

m_R (diamonds), Γ (given below)

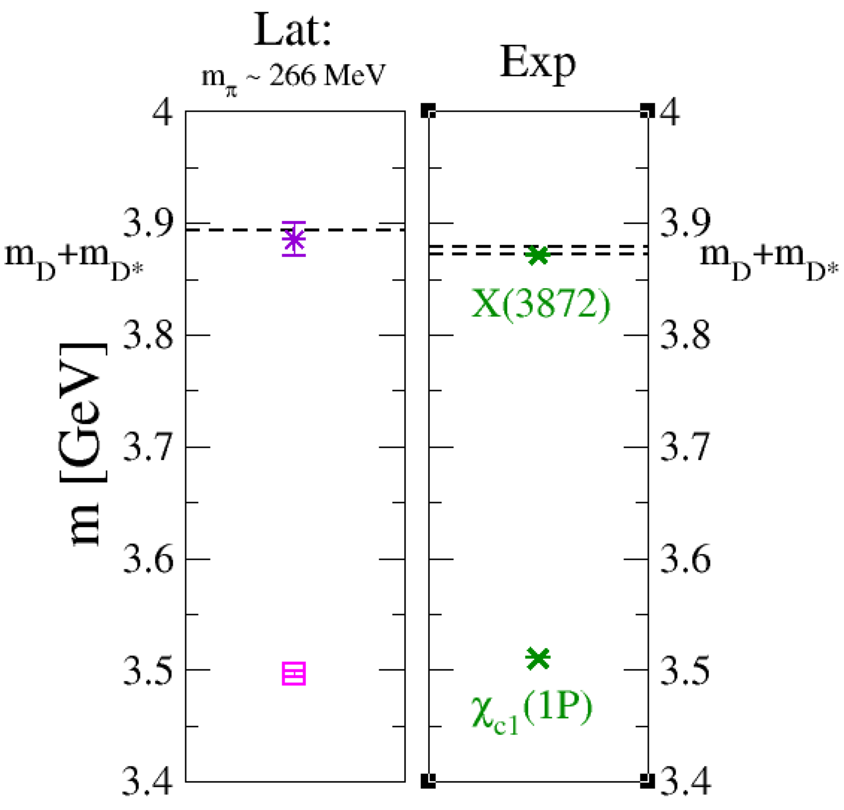
$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$h_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$
$2m_D$
$\psi(3770)$
$X(3872)$
$\chi_{c0}(2P)$wa
$\chi_{c2}(2P)$
$X(3940)$
$\psi(4040)$
$X(4050)^\pm$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^\pm$

ψ(3770)	Mass [MeV]	g (no unit)
Lat (m_π=266 MeV)	3774 ±6±10	19.7 ±1.4
Lat (m_π=156 MeV)	3789 ±68±10	28 ± 21
Exp.	3773.15± 0.33	18.7 ± 1.4

$$\Gamma = \frac{g^2}{6\pi}\frac{p^3}{s}$$

Lang, Leskovec, Mohler, S.P.,
1503.05363, JHEP 2015]

X(3872) as bound state from D$\underline{\text{D}}$* scattering, $J^{PC}=1^{++}$, I=0



physical masses m^{phy}

$$\mathcal{O}: \quad \bar{c}\,c, \quad D\bar{D}^*, \; [\overline{cu}]_{3c}[cu]_{3c}, \; [\overline{cu}]_{6c}[cu]_{6c}$$

- ground state: χ_{c1}(1P)
- D$\underline{\text{D}}$* scattering matrix near th. determined

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0^{DD^*} = -1.7 \pm 0.4 \text{ fm}, \quad r_0^{DD^*} = 0.5 \pm 0.1 \text{ fm}$$

$$T \propto \frac{1}{\cot\delta - i} = \infty$$

- A pole of found just below th. (violet star)
- The pole attributed to X(3872), which is a shallow bound state

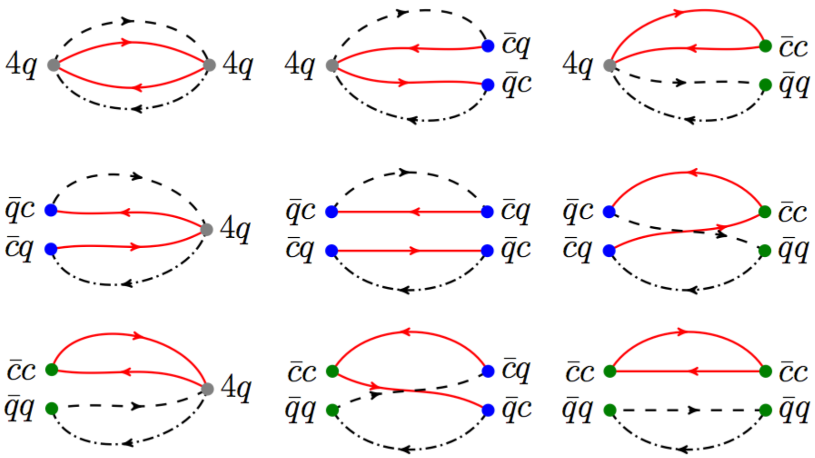
[M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]

$m_\pi \approx 266$ MeV, a=0.124 fm, L= 2 fm

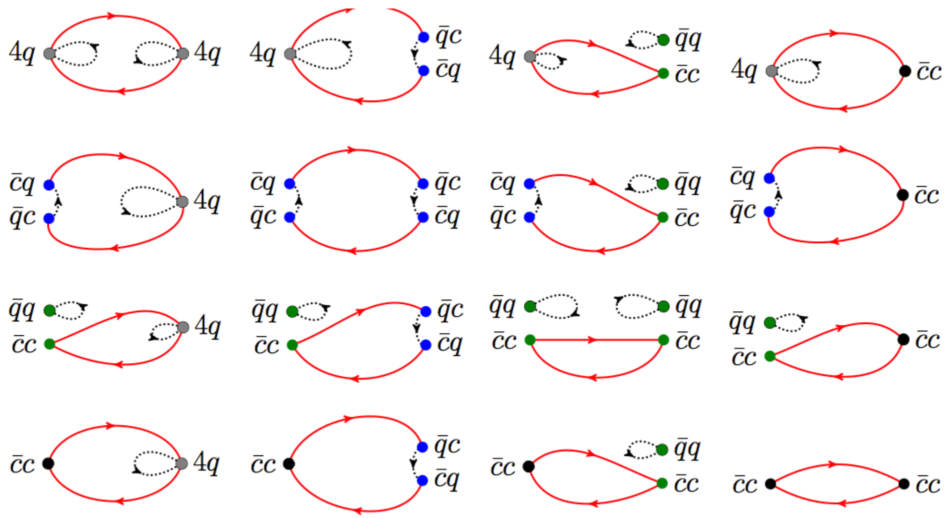
First evidence for X(3872) from lattice
S.P. and Leskovec: 1307.5172, PRL 2013

Which Fock components are essential for X(3872) with I=0 ?

$J^{PC}=1^{++}$ $\quad \mathcal{O}:\ \bar{c}\,c,\ \underbrace{D\bar{D}^{*},\ J/\psi\omega,\ \chi_{c1}\sigma,\ \eta_c\sigma,}_{(\bar{c}q)_{1_c}(c\bar{q})_{1_c},\ (\bar{c}c)_{1_c}(\bar{q}q)_{1_c}}\ [\overline{cu}]_{3c}[cu]_{3c},\ [\overline{cu}]_{6c}[cu]_{6c}$



(a)



(b)

[M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]

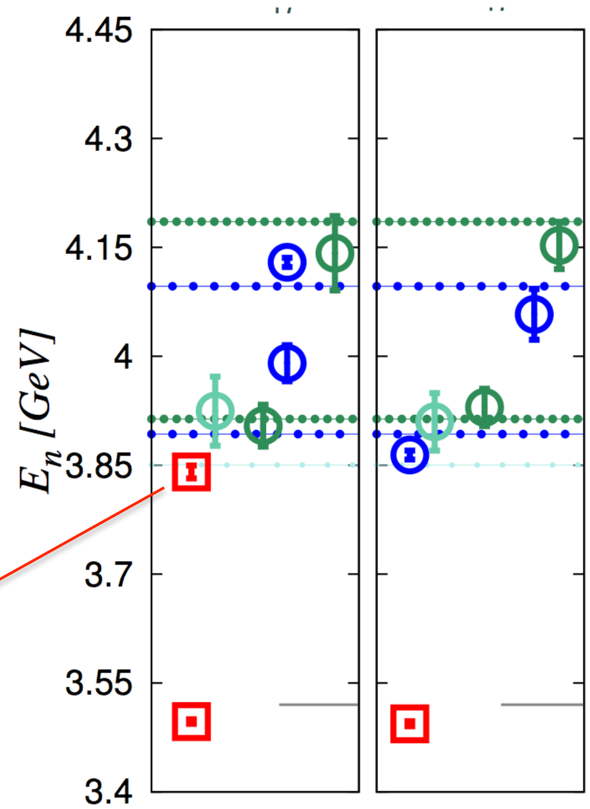
Which Fock components are essential for X(3872) with I=0 ?

localaized diquark antidiquark

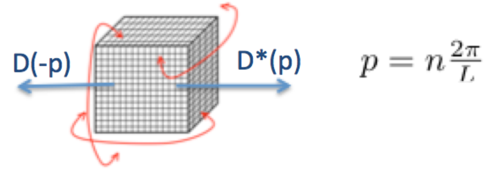
$\mathcal{O}$: $\underbrace{\bar{c}\,c,\quad D\bar{D}^*}_{\text{essential}}$, $J/\psi\omega$, $\chi_{c1}\sigma$, $\eta_c\sigma$, $\underbrace{[\overline{cu}]_{3c}[cu]_{3c},\ [\overline{cu}]_{6c}[cu]_{6c}}_{\text{do no seem not essential}}$

interpolating fields

O: all O: all without cc



energies of eigenstates on lattice E(L) (these are not m^{phy})



$m_\pi \approx 266$ MeV

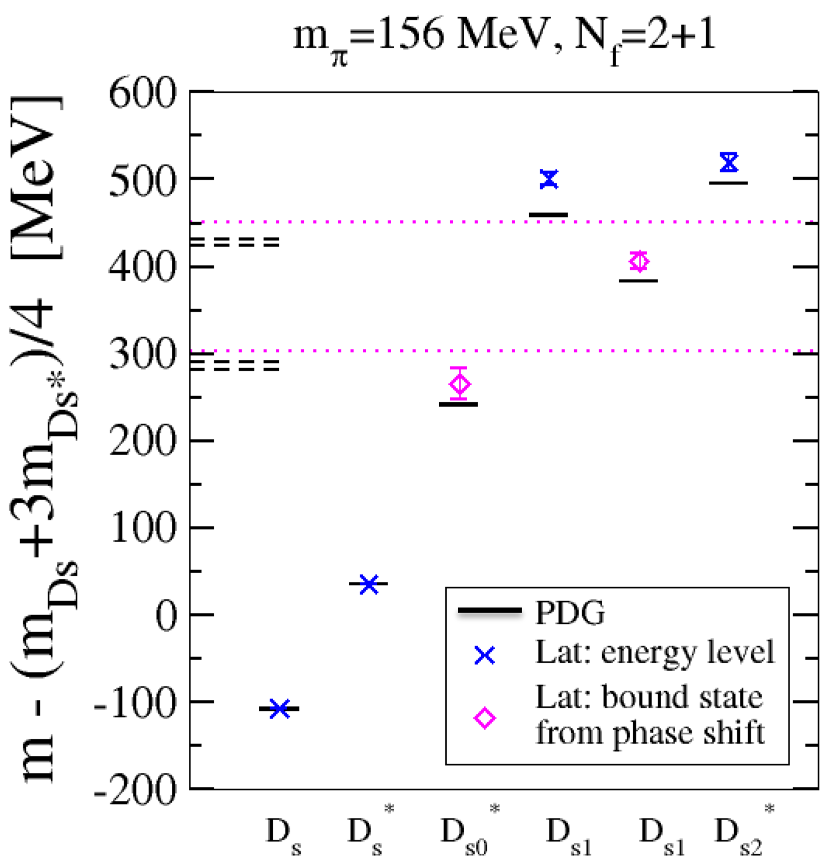
upper red square is candidate for X(3872): it is found only if cc in the basis

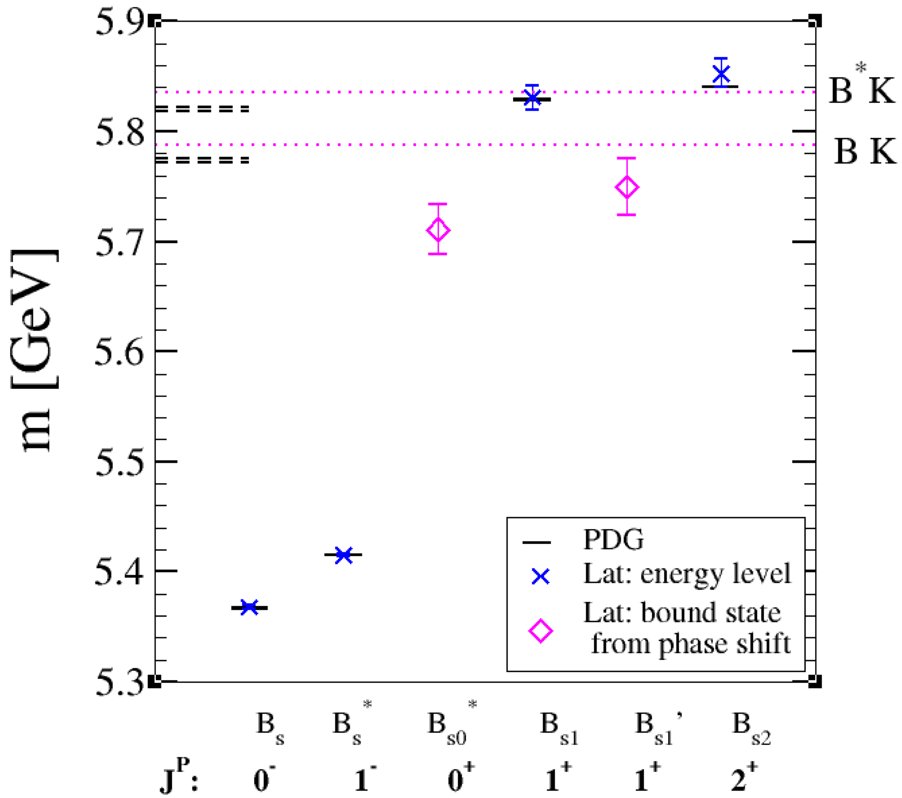
X(3872) not found if cc not in the basis, although [cu][cu] in the basis

[M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]

$D_{s0}(2317)$ and $D_{s1}(2460)$ below DK and D^*K thresholds

Mass prediction for missing B_{s0} and B_{s1}





[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn: 1308.3175, PRL 2013
1403.8103, PRD 2014]
[Martinez Torres, E. Oset, S.P., A. Ramos: 1412.1706, JHEP 2015]

[C. Lang, D. Mohler, S.P. ,
R. Woloshyn: 1501.0164, PLB2015]

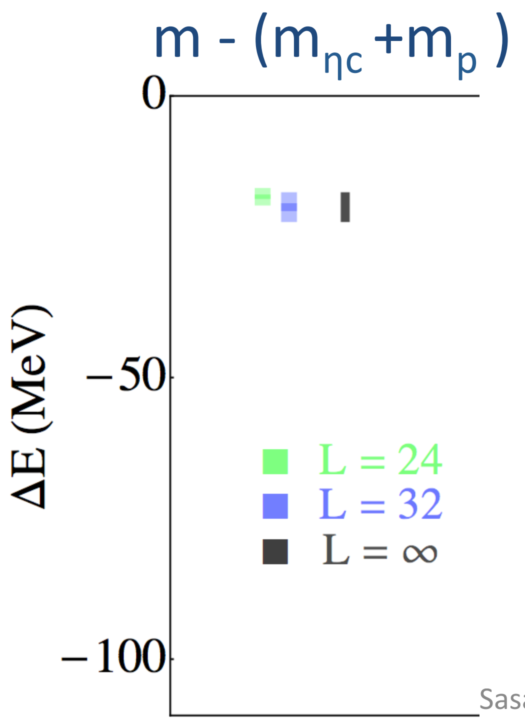
Bound state of a η_c and p from lattice

[NPLQCD, 1410.7069, PRD, m_π~800 MeV]

$\eta_c\ p$

$\bar{c}c\ uud$

~ 20 MeV below th. $m_{\eta c} + m_p$



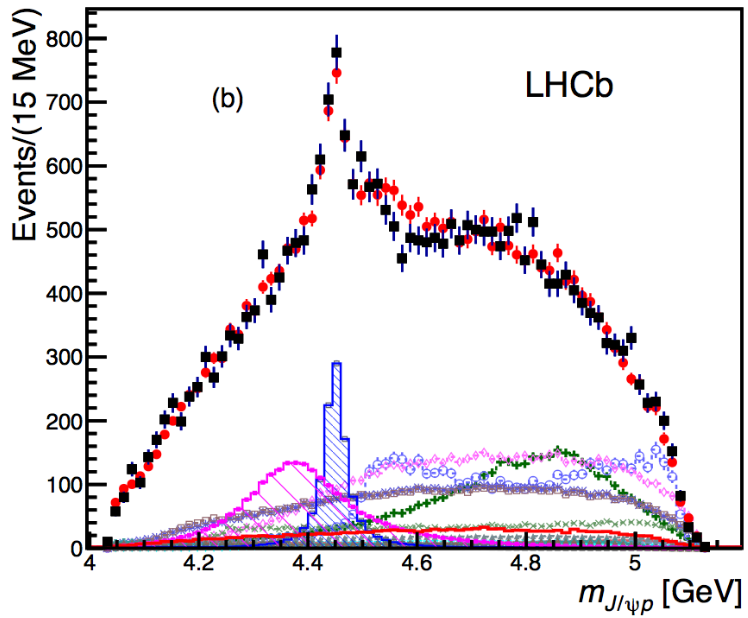
Two pentaquark resonances in J/ψ and p from exp

LHCb: 1507.03414

$J/\psi\ p$

$\bar{c}c\ uud$

~ 400 MeV above th. $m_{J/\psi} + m_p$



no lattice results for these Pc yet:

challenging as it can in principle decay to several decay ch.

Hadrons that decay to several final states

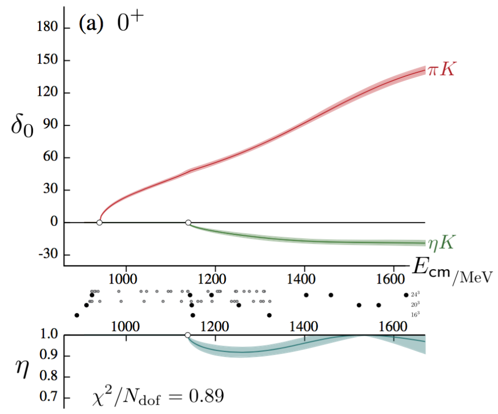
Scattering in two or more channels

”very challenging”

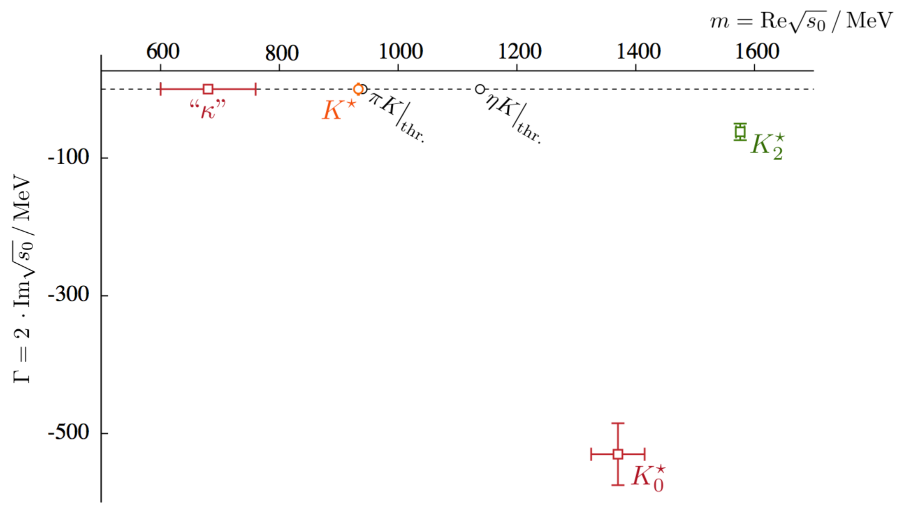
Resonances in Kπ, Kη coupled channels

- $\underline{q}q$, Kπ, Kη interpolators
- a number of different 0<P≤2
- for each E_n: one determinant equation for many unknowns
- T-matrix parametrized to get around this problem
- the location of poles of T-matrix in complex plain is given below
- K*(892) and κ are below threshold for this m_π
- $K_0^*(1400)$, K_2^* are resonances
- m_π=391 MeV, N_L=16, 20, 24

[Dudek, Edwards, Thomas, Wilson, HSC, 1406.4158, PRL; 1411.2004]



$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, \; t_{ij} = \frac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}}$$

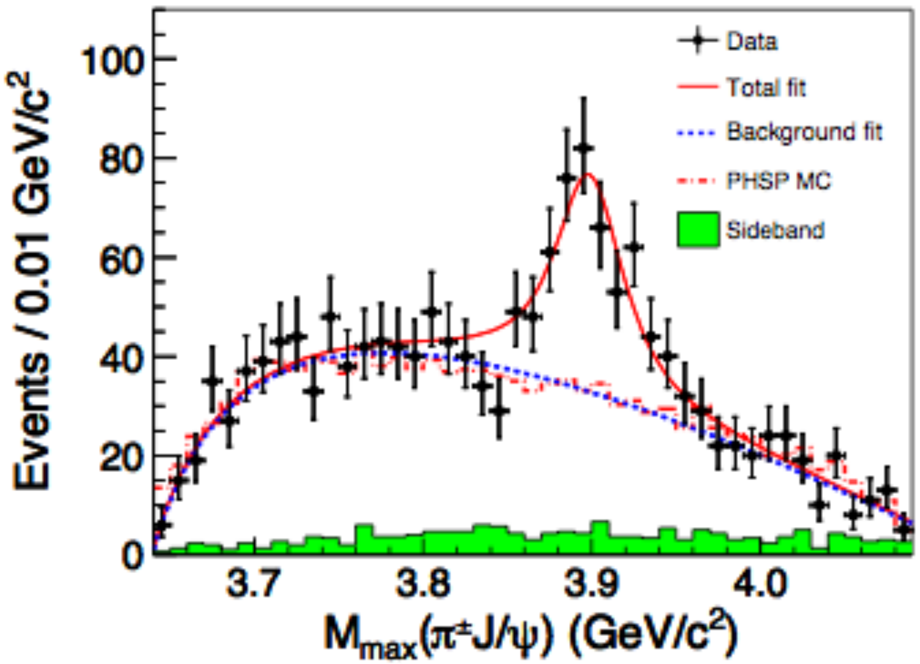


$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i\, t_{ij}^{(J)}(E_{\rm cm})\left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_iL)\right)\right] = 0,$$

location of poles in T matrix in complex plane

Charged charmonium Z_c^+ : manifestly exotic

$I^G=1^+$, $J^{PC}=1^{+-}$ (C is for neutral partner)



[BESIII, 2013, 1303.5949, PRL]

$Z_c^+(3900) \rightarrow \quad J/\Psi \ \ \pi^+$

cc du

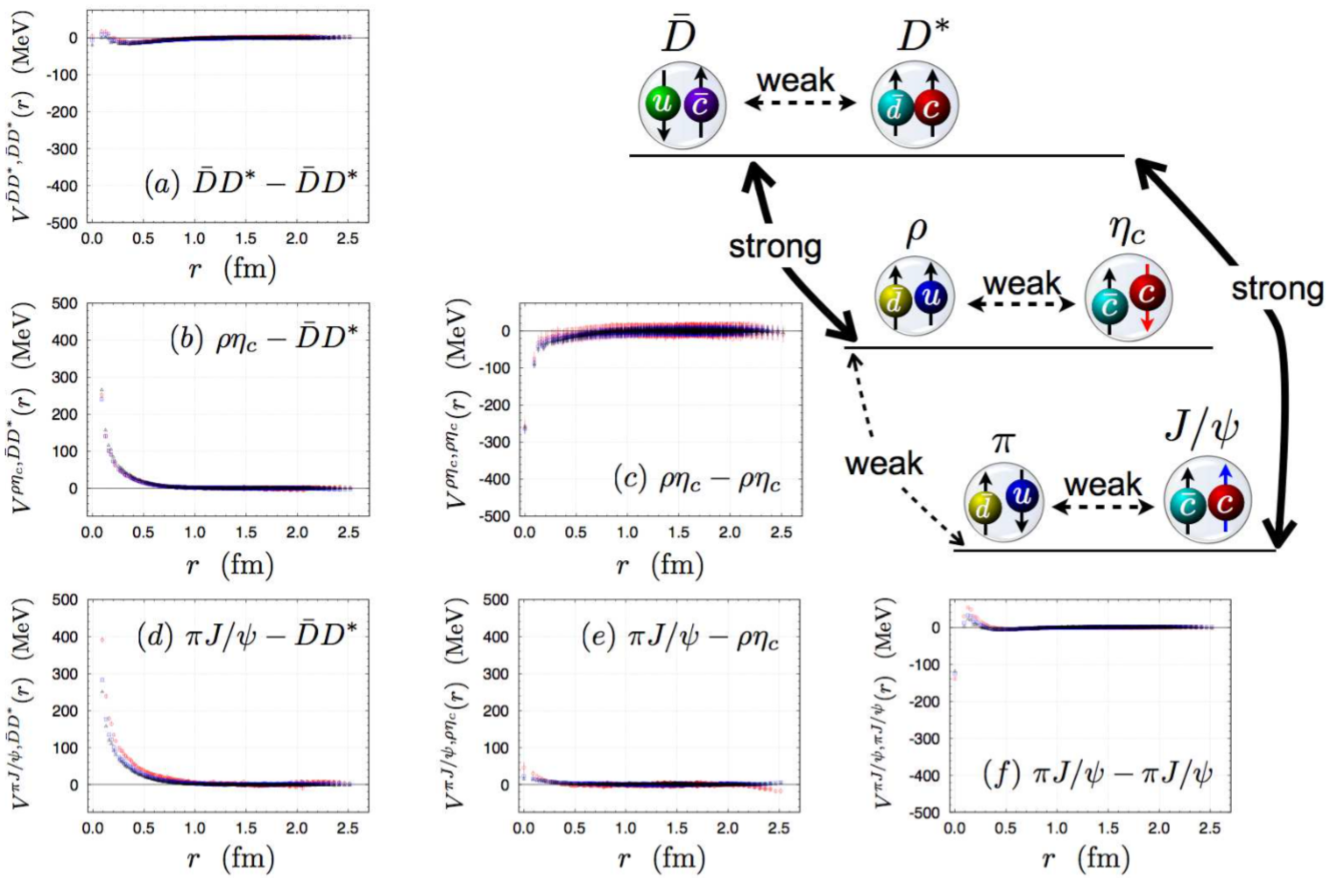
$\bar{c}\ u$
$c\ \bar{d}$

[HALQCD, Ikeda et al, 1602.03465]

$Z_c^+(3900)$ channel : $I^G=1^+$, $J^{PC}=1^{+-}$ HALQCD method V(r)

HALQCD is another method to extract scattering matrix from lattice

(considered to be less rigorous than the Luscher's method for coupled channels)

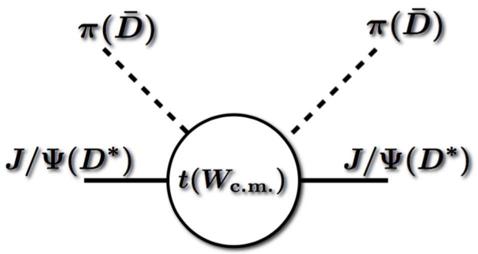


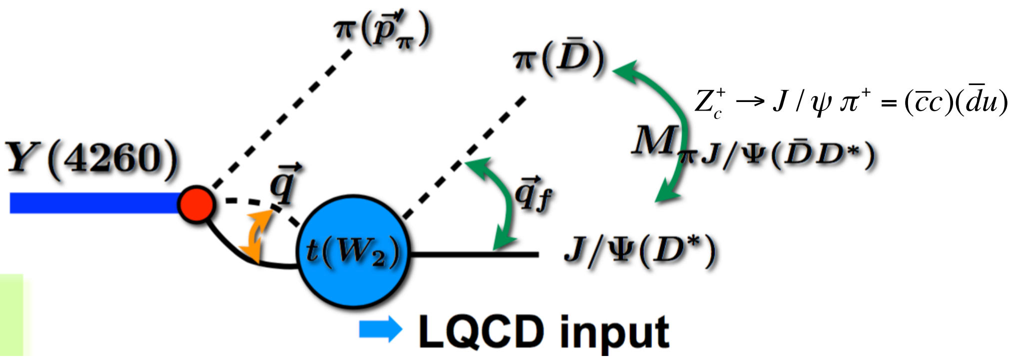
$$C^{\alpha\beta}(\vec{r},t) \equiv \sum_{\vec{x}} \langle 0|\, \phi_1^\alpha(\vec{x}+\vec{r},t)\phi_2^\alpha(\vec{x},t)\overline{\mathcal{J}}^\beta\, |0\rangle / \sqrt{Z_1^\alpha Z_2^\alpha},$$

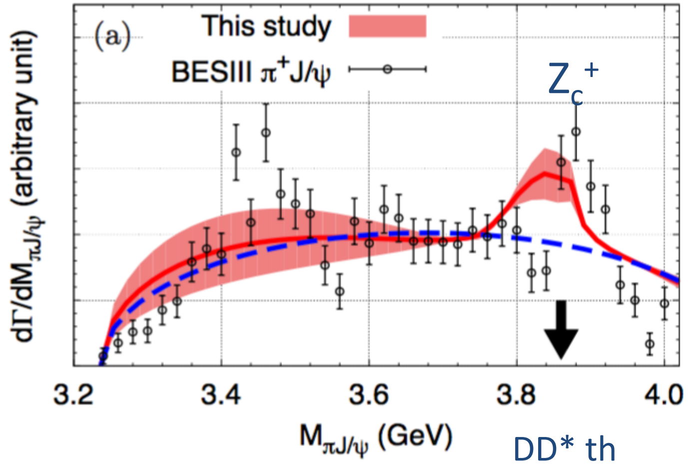
$$R^{\alpha\beta}(\vec{r},t) \equiv C^{\alpha\beta}(\vec{r},t)e^{(m_1^\alpha+m_2^\alpha)t}$$

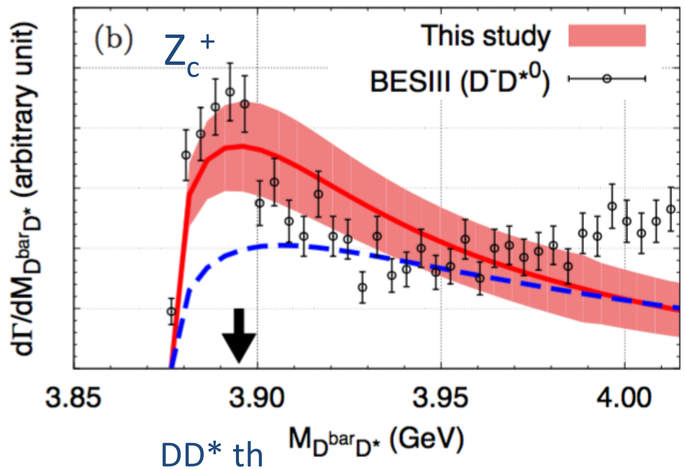
$$\left(-\frac{\partial}{\partial t} - H_0^\alpha\right)R^{\alpha\beta}(\vec{r},t) \simeq \sum_\gamma \Delta^{\alpha\gamma}\int d\vec{r'}U^{\alpha\gamma}(\vec{r},\vec{r'})R^{\gamma\beta}(\vec{r'},t)$$

Z_c^+ channel , three-body Y(4260) decay: lattice & exp



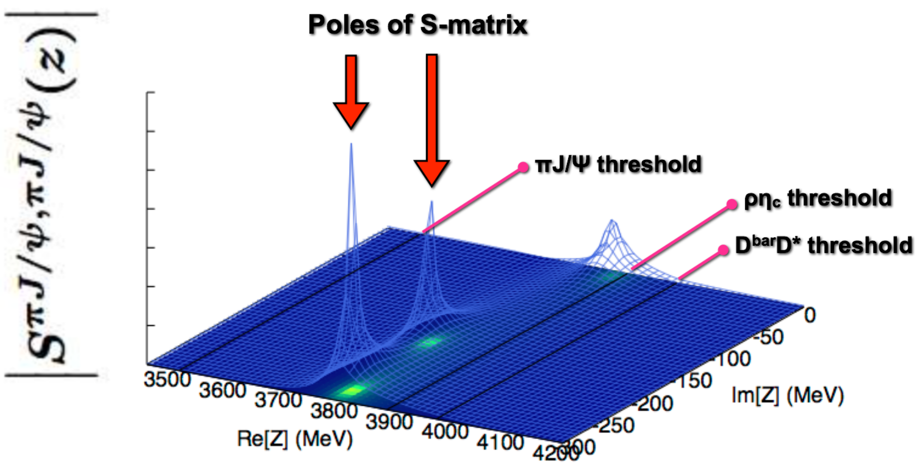




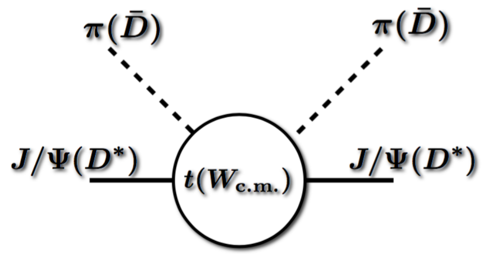


[HALQCD, Ikeda et al, 1602.03465]

Z_c^+ channel : HALQCD method , poles of S in complex plane



Second Riemann sheet for all three channels shown.



[HALQCD, Ikeda et al, 1602.03465]

Remarks:

- HALQCD method not considered as rigorous as the Luscher's method for coupled channels
- 3x3 matrix S in Zc channel has not been determined by Luscher method yet
- HALQCD method has not verified any of the conventional resonances yet (to my knowledge)
- Luscher's method has been verified on conventional res. like ρ, K*, ψ(3770), $D_0(2400)$ ...

Conclusions for the time being

Hadron spectroscopy from lattice (in brief):

- **hadrons well below threshold precisely obtained**: D, K ... and all the others: ✔✔✔
- Lattice QCD can extract scattering matrix and cross-section for scattering of two hadrons ✔✔
  (most easily if there is only one channel)
- **Hadron resonances are extracted from peaks in the cross-sections** via BW-type fits
  resonances verified : ρ, K^*, $K_0^*(1430)$, $K_{2,}$ D_0^*, D_1, a_1, b_1 , $\Psi(3770)$,.. ✔✔
- **Shallow bound states appear as poles in scattering matrix**
  shallow bound states verified : D_{s0}, D_{s1}, B_{s0}, B_{s1} , X(3872) with I=0,.. ✔✔
- First steps to extract scattering matrix for scattering with two or more **coupled channels** ✔
  indication that $\mathbf{Z_c^+}$ is not a usual resonance (with pole on the second Riemann sheet)

Lattice setup

PACS-CS

	Ensemble (1)	Ensemble (2)
$N_L^3 \times N_T$	$16^3 \times 32$	$32^3 \times 64$
N_f	2	2+1
a [fm]	0.1239(13)	0.0907(13)
L [fm]	1.98(2)	2.90(4)
m_π [MeV]	266(3)(3)	156(7)(2)

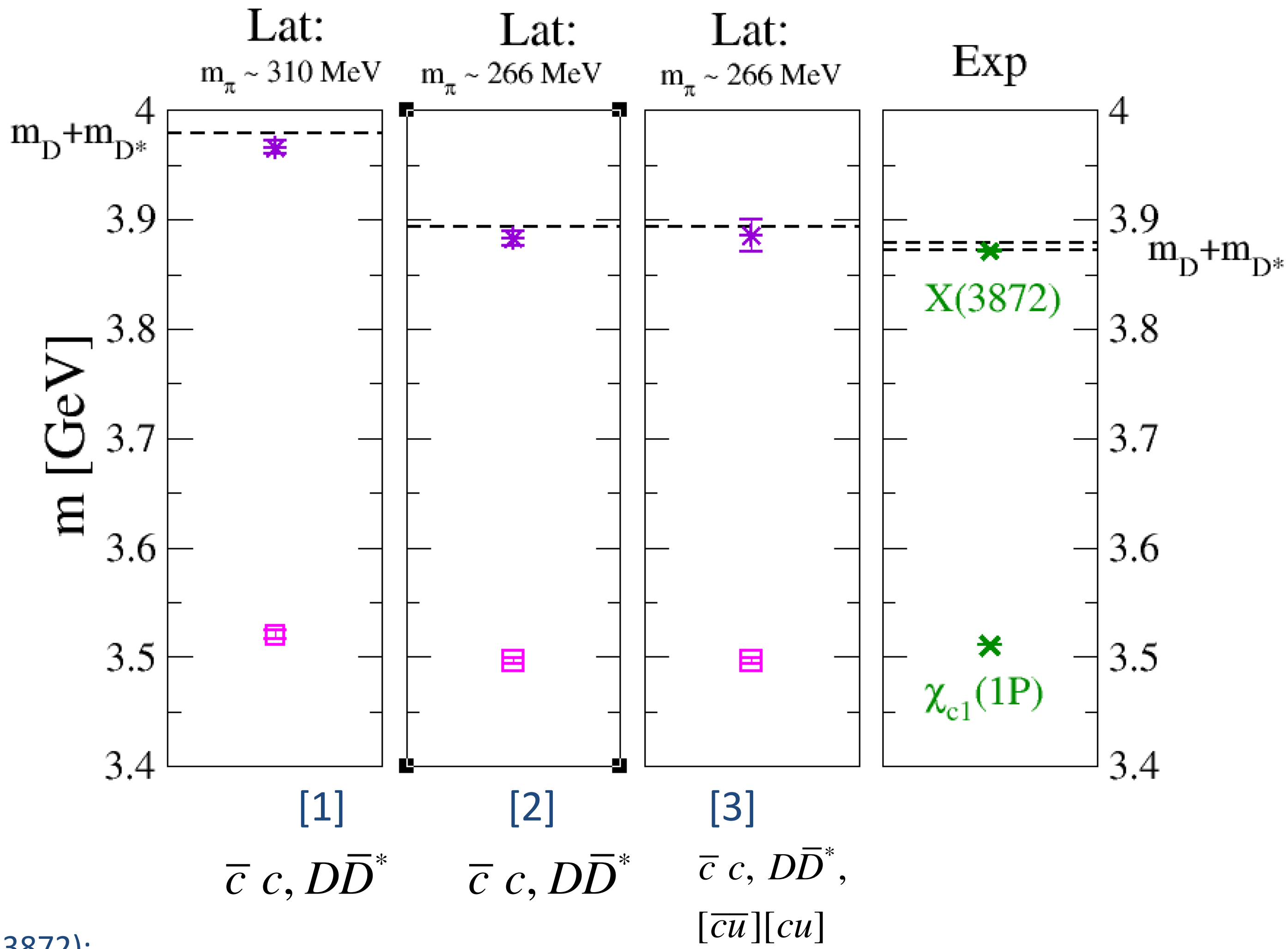
- Wilon-clover quarks
- Fermilab method for c and b : [El Khadra, Kronfeld et al, 1997]

  Rest hadron energies have sizable discretization errors but these largely cancel in splittings.

  Only splittings with respect to a chosen reference mass are compared to experiment.

- evaluating Wick contractions to simulate scattering on the lattice is challenging and computationally intensive – that is part of the reason why a small number of studies have been made. We apply two methods
  - distillation (Ensemble 1) [Peardon et. al., HSC, 2009]
  - stochastic distillation (Ensemble 2) [Morningstar et al., 2011]

Comparing lattice results for X(3872), $J^{PC}=1^{++}$, I=0



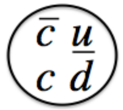
Lattice evidence for X(3872):

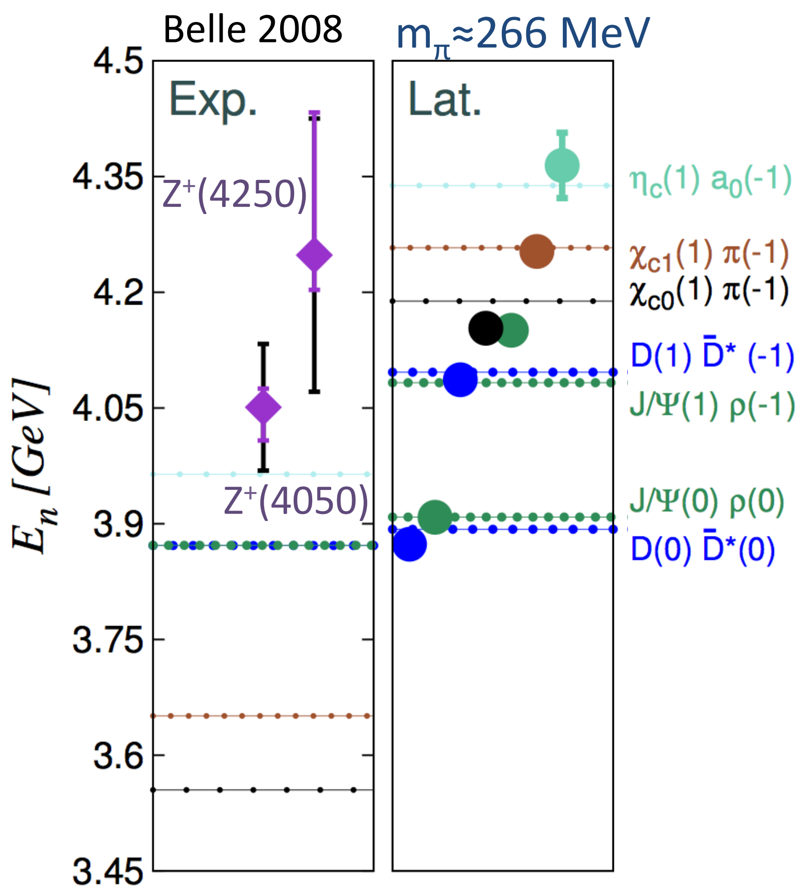
- [1] [Lee, DeTar, Na, Mohler , update of proc 1411.1389] $m_\pi \approx 310$ MeV, a=0.15 fm, L=2.4 fm , HISQ
- [2] [S.P. and Leskovec: 1307.5172, PRL 2013]
- [3] [M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD 2015]

$\Big\}$ $m_\pi \approx 266$ MeV, a=0.124 fm, L= 2 fm

- Position of DD* threshold depends on $m_{u/d}$, and may be affected by discretization effects related to charm quark

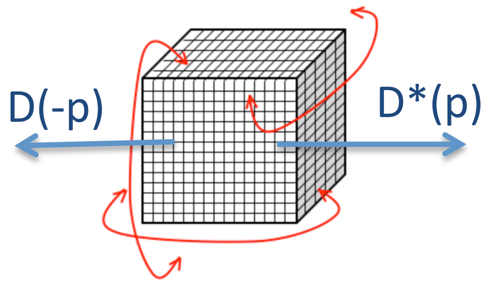
Search for charged partner of X(3872); channel $I^G=1^-$, $J^{PC}=1^{++}$, $\underline{c}\bar{c}\underline{d}u$





eigen-energies E(L)

(not m^{phy})



$$p = n\frac{2\pi}{L}$$

$$\mathcal{O}:\ (\bar{c}u)(\bar{d}c),\ \ (\bar{c}c)(\bar{d}u),\ \ [\bar{c}\bar{d}][cu]$$

- Horizontal lines: energies of expected two-meson states in limit of no interaction: $E = E[\ M_1(p_1)\ ] + E[\ M_2(p_2)\ ]$
- Circles: energies of eigenstates from latt
- Only expected two-meson states observed.
- **No lattice candidate for charged X(3872)**. In agreement with absence of such state in exp.
- **No lattice candidate for other charged state below 4.3 GeV**.
- Two Belle 2008 states are exp. unconfirmed.

[M. Padmanath, C.B. Lang, S.P., PRD 2015, 1503.03257]

Y(4140), $J^{PC}=?^{?+}$, $\underline{c}c\underline{s}s$

$$Y(4140) \to J/\psi\ \phi$$
$$\bar{c}c \quad \bar{s}s$$

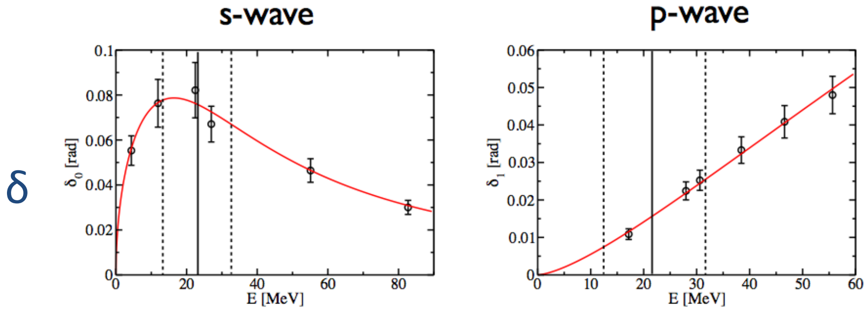
Experiment:

peak in J/ψ Φ just above J/ψ Φ threshold

found: CDF 2009, CMS 2012, D0 2013, Babar 2015

not found: Belle 2010, LHCb 2012

J/Ψ Φ scattering phase shift [rad]



Lattice:

- S. Ozaki and S. Sasaki, 1211.5512, PRD

  caveat: strange quark annihilation neglected

  no resonant Y(4140) structure found

- M. Padmanath, C.B. Lang, S.P., 1503.03257, PRD

$$\mathcal{O}:\ \bar{c}c,\ (\bar{c}s)(\bar{s}c),\ (\bar{c}c)(\bar{s}s),\ [\bar{c}\bar{s}][cs]$$

channel $J^P=1^+$ considered only: expected two-particle

eigenstates found and χ_{c1} , X(3872) but **not Y(4140)**

$m_\pi \approx 266$ MeV

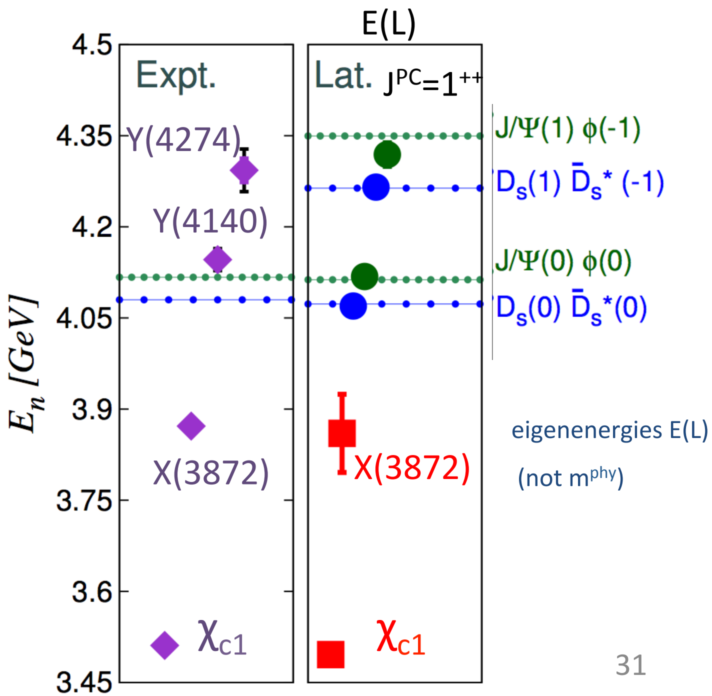
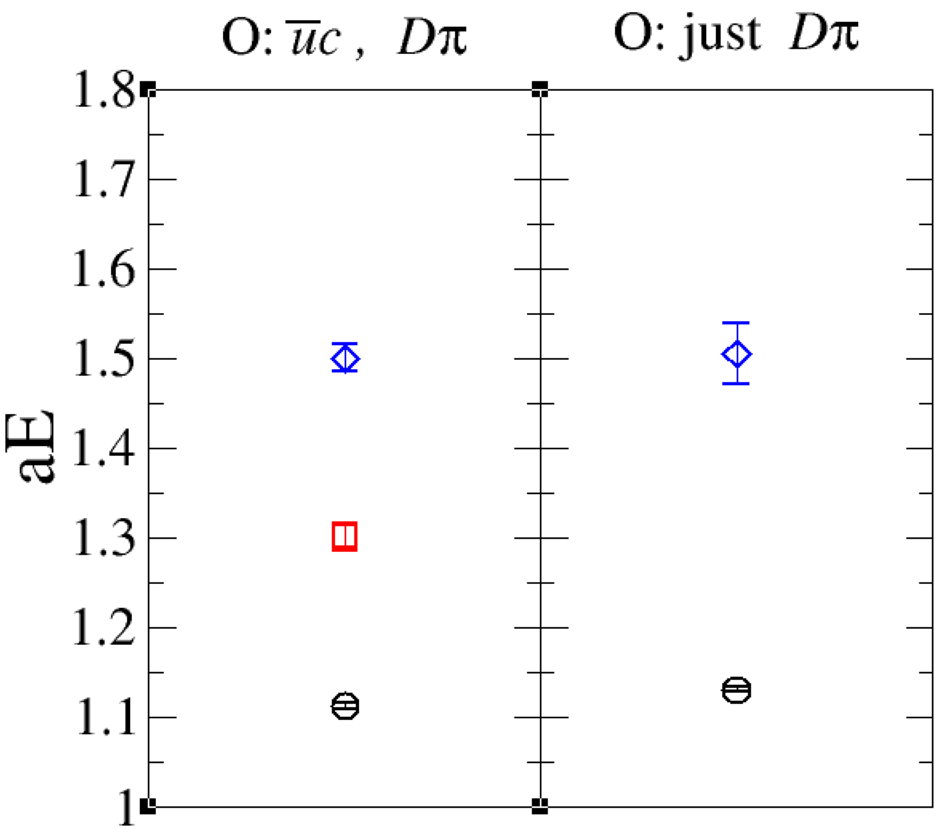


Illustration how eigenstate $D_0^*(2400)$ dominated by "$\underline{q}q$" dissapears when $\underline{q}q$ interpolators omitted



D(1)π(-1)

$D_0^*(2400)$ $\quad \overline{u}c \quad J^{PC}=0^{++}$

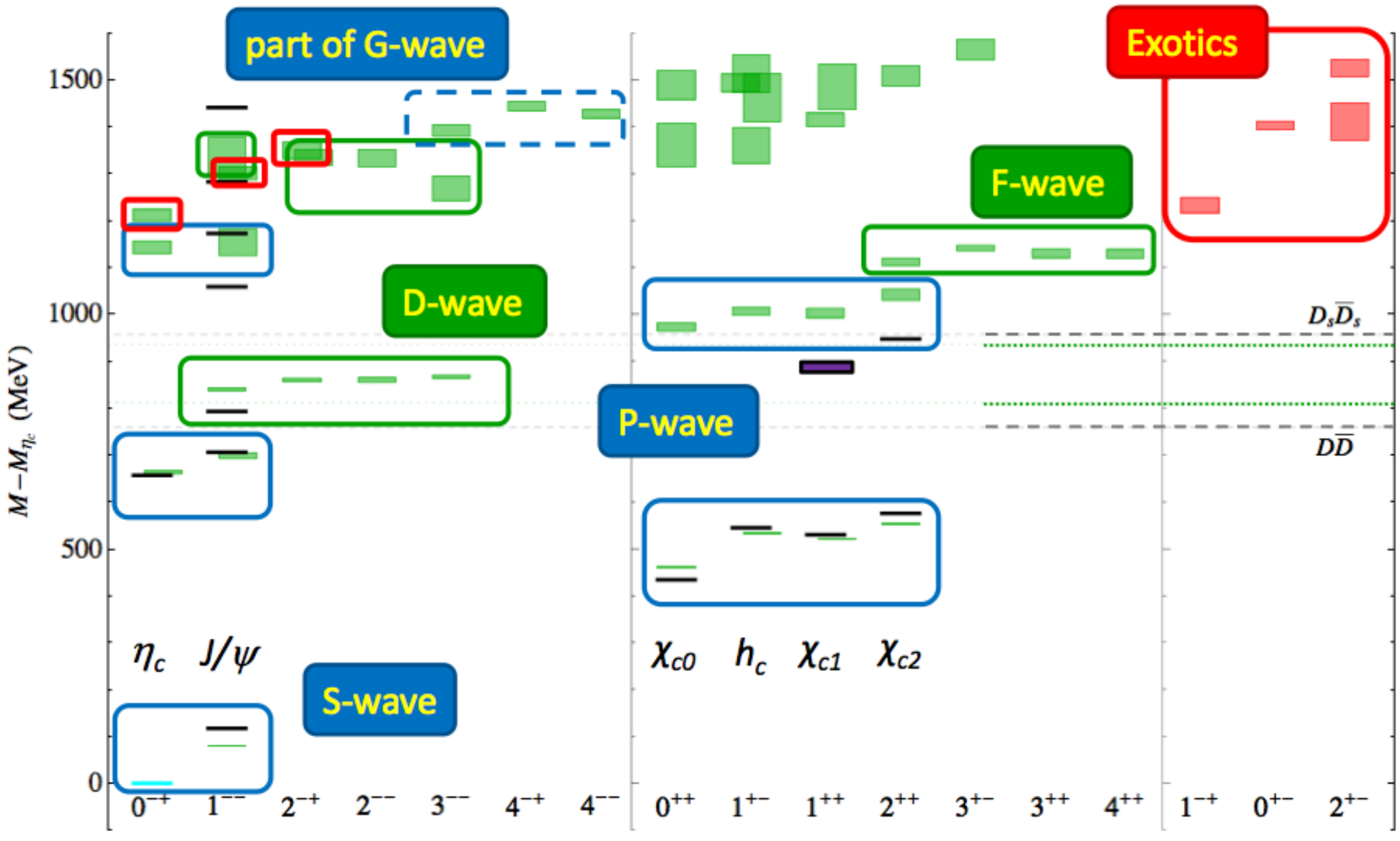
D(0)π(0)

Effective energies at t=8 plotted.

[Mohler, S.P., Woloshyn, PRD 87 (2013) 034501]

Charmonia: single-meson approximation

$\mathcal{O} : \bar{c}c$



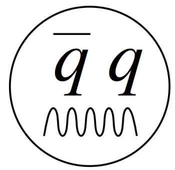
Lattice

Experiment

single-meson approx:

- used until few years ago:
- ignores strong decays of resonances
- ignores effects of thresholds

J^{PC}



[HSC , L. Liu et al: 1204.5425, JHEP]

- $m_\pi \approx 400$ MeV, L≈2.9 fm, Nf=2+1
- identification with n $^{2S+1}L_J$ multiplets using <O|n>
- green: lat, black: exp

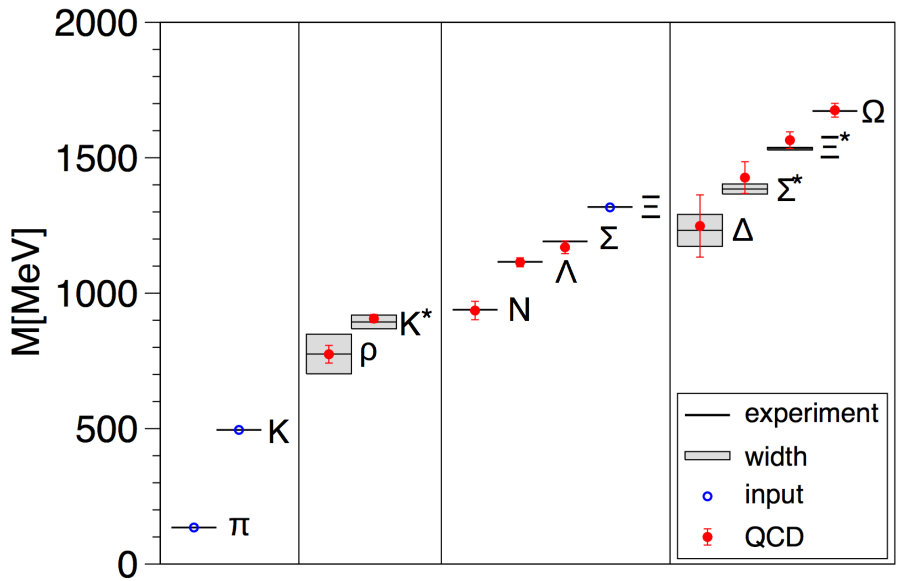
Hybrids:

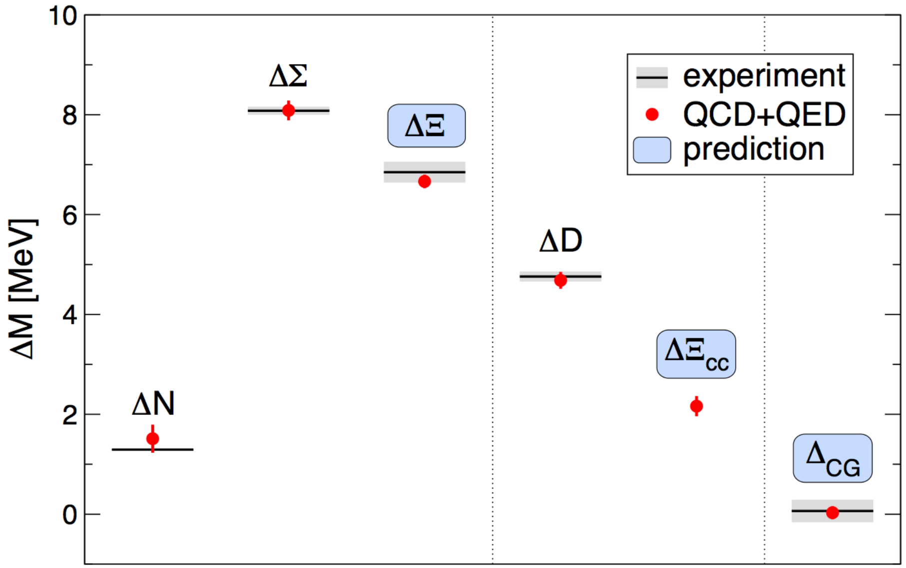
some of them have exotic J^{PC}

large overlap with $O = \underline{q}\, F_{ij}\, q$

Proton and neutron

constitute more than 99% of the bright side of universe





BMW collaboration
Science 322,2008

QCD

$m_u = m_d$

m_n , m_p

m_n - m_p

BMW collaboration
Science 347,2015

QCD + QED

$m_u \neq m_d$