Doubly charm tetraquark from lattice QCD

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in collaboration with

Outline:

Doubly charm tetraquark $cc\bar{d}\bar{u}$ — M. Padmanath

Charmonium(like) states $\bar{c}c$, $\bar{c}q\bar{q}c$ — S. Collins, D. Mohler, M. Padmanath, S. Piemonte

Outline and lattice QCD results

CLS ensembles: u,d,s dynamical quarks

$m_u = m_d > m_{u,d}^{phy}$, $\mathbf{m_\pi \approx 280\ MeV}$

$a \approx 0.086$ fm, L = 2.1 fm, 2.7 fm

↓

Padmanath, S.P.: 2202.101101, PRL

Doubly charm tetraquark (T_{cc})

$c c \bar{d} \bar{u}$

DD^* scattering

I=0

$J^P=1^+$

- T_{cc} found as a virtual bound state ≈10 MeV below DD* threshold

- likely related to T_{cc} discovered by LHCb

Charmonium(like) states

$\bar{c}c$, $\bar{c}q\bar{q}c$

$D\bar{D} - D_s\bar{D}_s$ scattering

I=0

$J^{PC}=0^{++},1^{--},2^{++},3^{--}$

q=u,d,s

S.P. , Collins, Padmanath, Mohler, Piemonte 2011.02541 JHEP, 1905.03506 PRD 2111.02934

- masses and decay widths of conventional charmonia confirmed : ground states (bound states)

  first excitations (resonances)

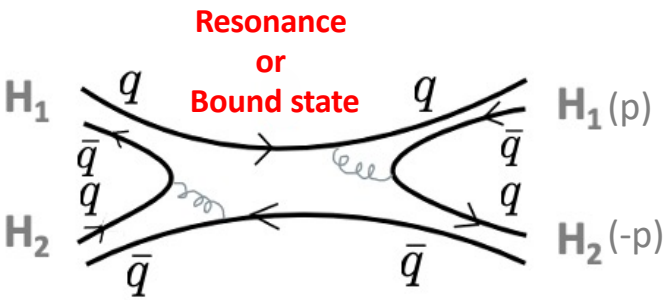
- two additional exotic charmonium-like states with $J^{PC}=0^{++}$ found just below thresholds

$\bar{D}D$ seen in dispersive re-analysis of exp. [Danilkin et al 2111.15033]

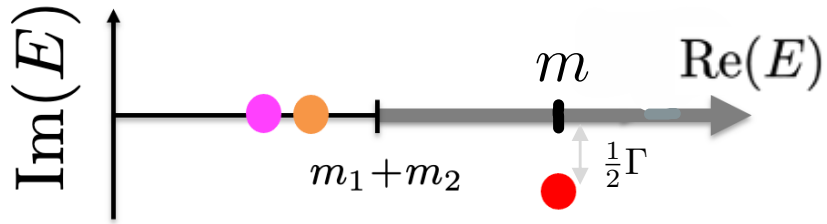
$\bar{D}_s D_s$ likely related to X(3915) / χ_{c0}(3930) / X(3960)

LHCb2020 LHCb2022

Extract resonances and (virtual) bound states from H_1 H_2 scattering



scattering matrix T(E)



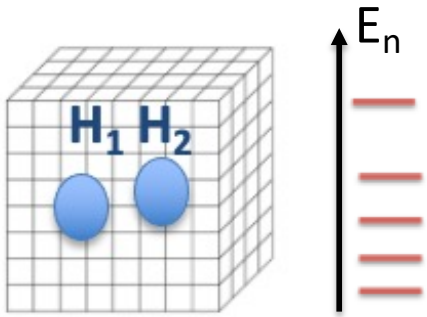
Virtual bound st. | Bound st. | Resonance

p = - i |p| | p = i |p|

$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$

Scattering matrix T(E) from lattice QCD



periodic b.c.

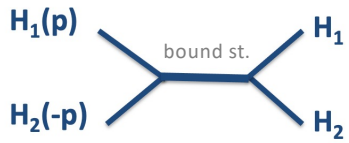
real E | for real E | for complex E

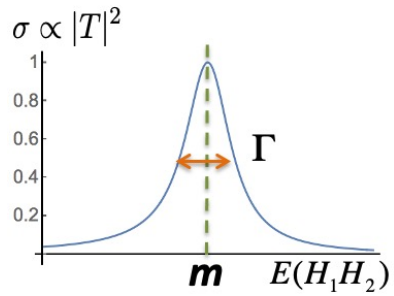
$$E \rightarrow T(E) \rightarrow T(E^c)$$

analytic relation:
Luscher 1991

analytic contin.
to complex E

simple argument: next slide





one-channel scattering

$$S = 1 + i\frac{4p}{E}T = e^{2i\delta} \qquad T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

Relation between E and δ(E), T(E):
1D nonrelativistic quantum mechanics

V= 0: outside the region of potential

$$\psi(x) = A\cos(p|x|+\gamma) = \begin{cases} A\cos(px+\gamma) & x>R \\ A\cos(-px+\gamma) & x<-\frac{R}{2} \end{cases}$$

- this form already ensures $\psi(L/2) = \psi(-L/2)$
- the other BC: $\psi'(L/2) = \psi'(-L/2)$

this requires

$$Ap\sin\left(p\left(\tfrac{L}{2}\right)+\gamma\right) = -Ap\sin\left(-p\left(-\tfrac{L}{2}\right)+\gamma\right)$$

$$\rightarrow \psi'(L/2) = 0, \quad \sin\left(p\tfrac{L}{2}+\gamma\right) = 0$$

$$p\tfrac{L}{2} + \delta = n\pi \qquad \boxed{p = n\frac{2\pi}{L} - \frac{2}{L}\gamma}$$

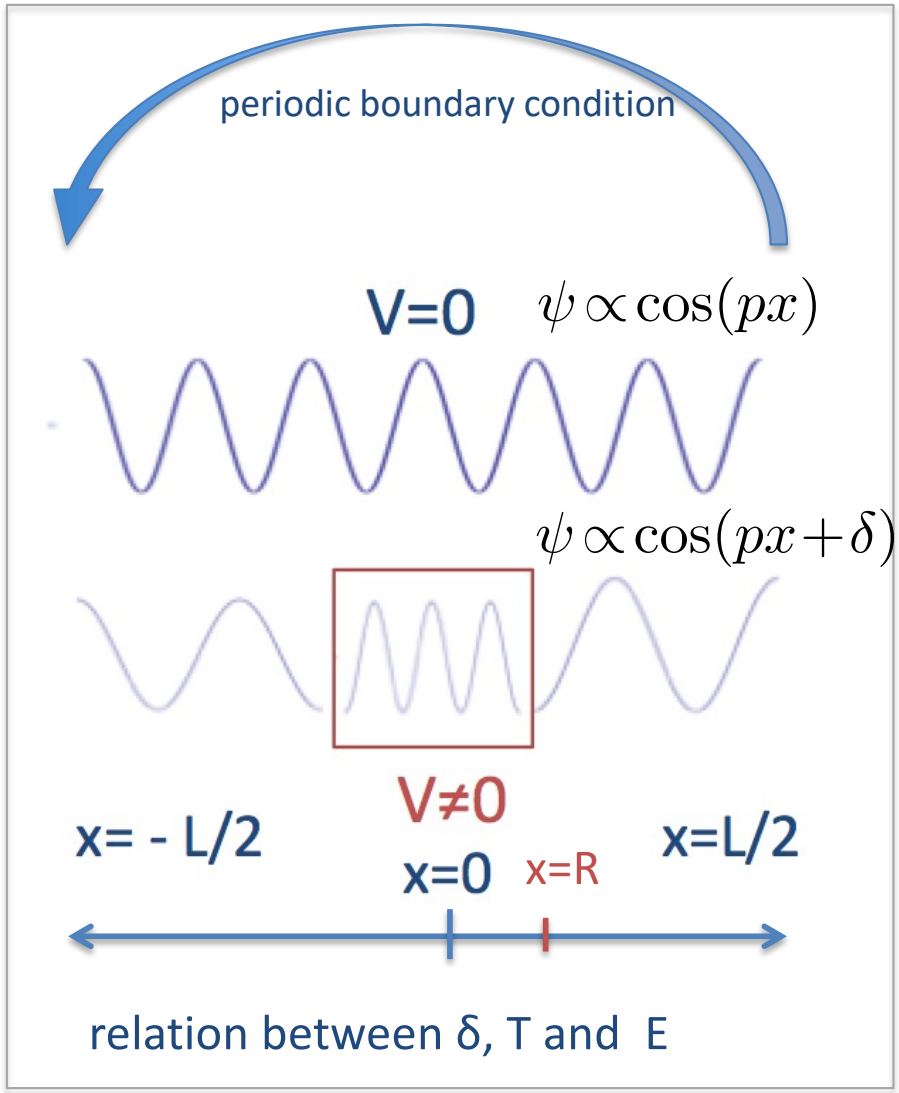
relation between δ, L

$$p = \frac{2\pi}{L}n$$

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta$$

$E=p^2/2m$

in both cases



$$cc\bar{d}\bar{u} = T_{cc}$$

Padmanath, S.P.: 2202.101101,
Phys.Rev.Lett. 129 (2022) 3, 032002
&
subsequent studies with S. Collins

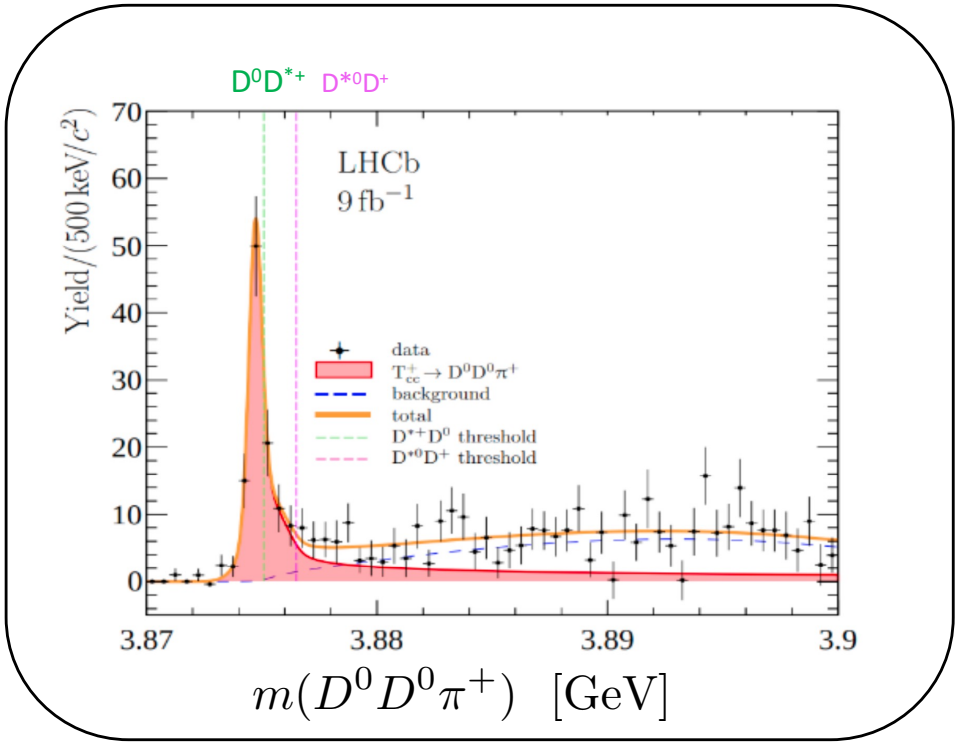
LHCb discovery of T_{cc}

$c c \bar{u} \bar{d}$

The longest lived exotic hadron ever discovered

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

I=0, $J^P=1^+$

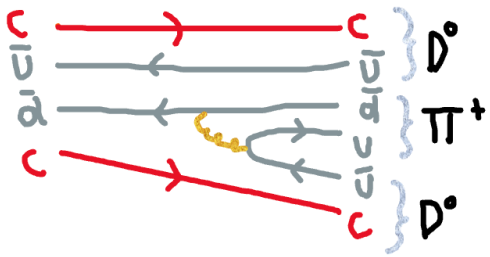


LHCb July 2021, 2109.01038, 2109.01056, Nature Physics

The doubly charmed tetraquark T_{cc}^+, $I = 0$ and favours $J^P = 1^+$.

No states observed in $D^0 D^+ \pi^+$: eliminates possibility of $I = 1$.

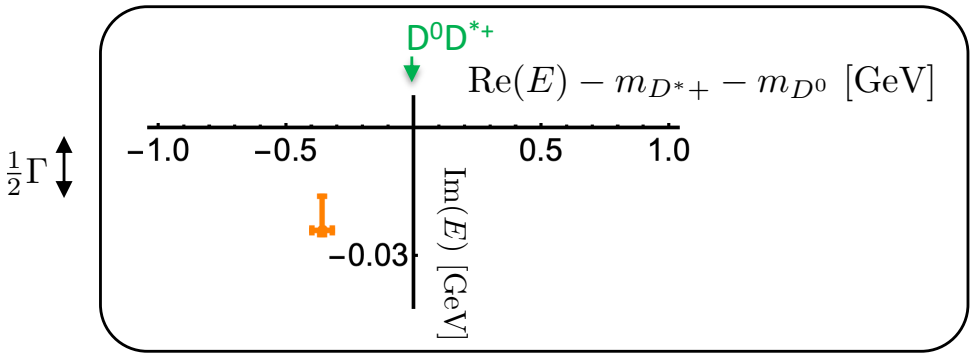
Near-threshold state: Demands pole identification to confirm existence.



Omitting $D^* \to D\pi$, $T_{cc} \to DD\pi$

T_{cc} would be a bound state

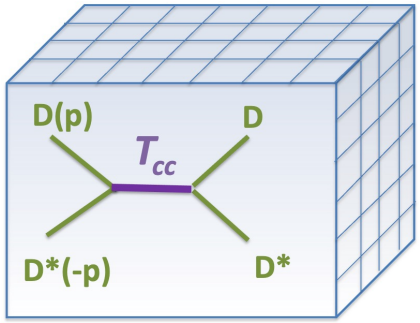
Pole in T(E) $\delta m = -0.36$ MeV



$$\delta m_{\rm pole} = -360 \pm 40^{+4}_{-0}\ {\rm keV}/c^2$$
$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14}\ {\rm keV}\,,$$

Lattice study

$$C \to E \to T(E)$$



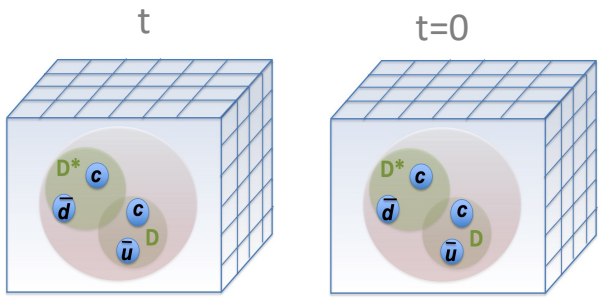
$m_\pi \simeq 280$ MeV :
$D^* \not\to D\pi,\ T_{cc} \not\to DD\pi$
$DD\pi$ above analyzed region

$\sum_n |n\rangle\langle n|$

Eucledian time

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\,\mathcal{O}_j^+(0)|0\rangle = \sum_n \langle 0|\mathcal{O}_i|n\rangle\, e^{-E_n t}\, \langle n|\mathcal{O}_j^+|0\rangle$$

$$\langle C\rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$

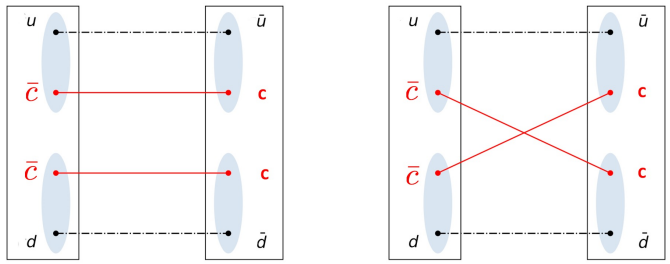


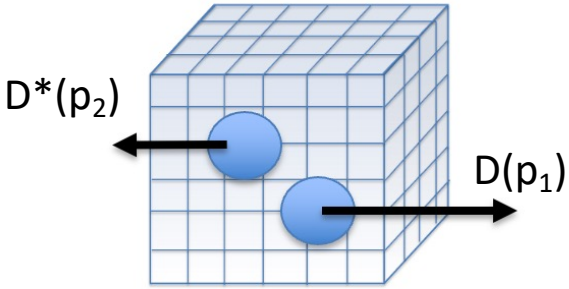
D D*

$$\mathcal{O} = (\bar{u}\gamma_5 c)_{\vec{p}_1}\ (\bar{d}\gamma_i c)_{\vec{p}_2} - (\vec{p}_1 \leftrightarrow \vec{p}_2)$$

$$\vec{p}_{1,2} = \vec{n}_{1,2}\ \frac{2\pi}{L}$$

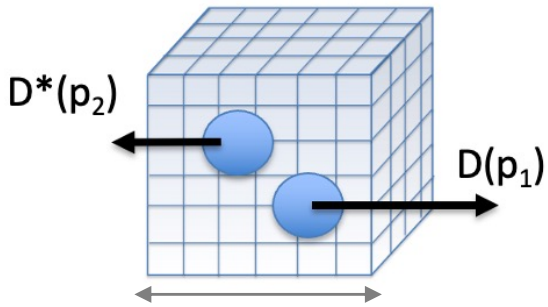
$$(\bar{u}\gamma_5\gamma_t c)_{\vec{p}_1}\ (\bar{d}\gamma_i\gamma_t c)_{\vec{p}_2}$$





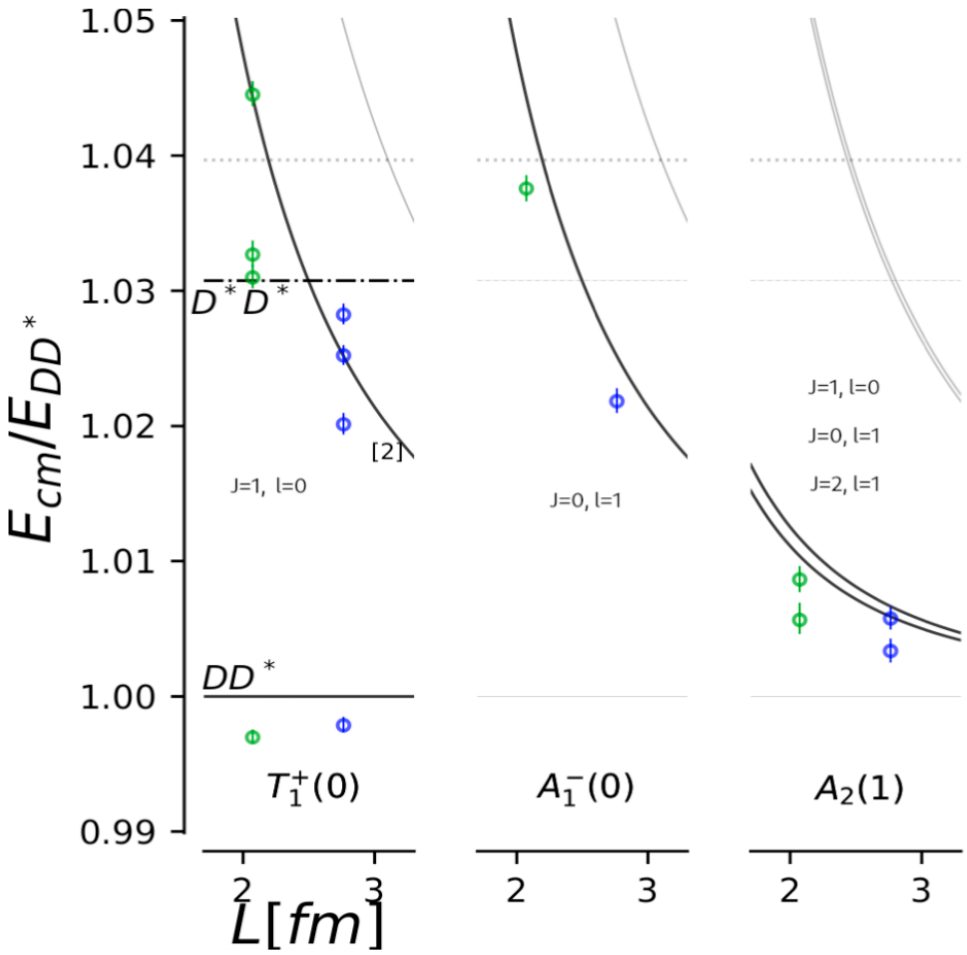
Eigen-energies on the lattice

at $m_\pi \approx 280\ MeV$



L = 2.1 fm, 2.7 fm

$\vec{P} = \vec{0}$ $\quad$ $\vec{P} = \frac{2\pi}{L}\ (0,0,1)$ $\quad$ $\vec{P} = \vec{p}_1 + \vec{p}_2$



lines

non-interacting energies

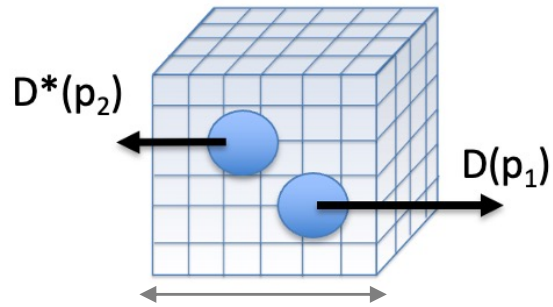
$$E^{n.i.} = \sqrt{m_D^2 + \vec{p}_1^{\,2}} + \sqrt{m_{D^*}^2 + \vec{p}_2^{\,2}}$$

$$\vec{p}_i = \vec{n}_i \frac{2\pi}{L}$$

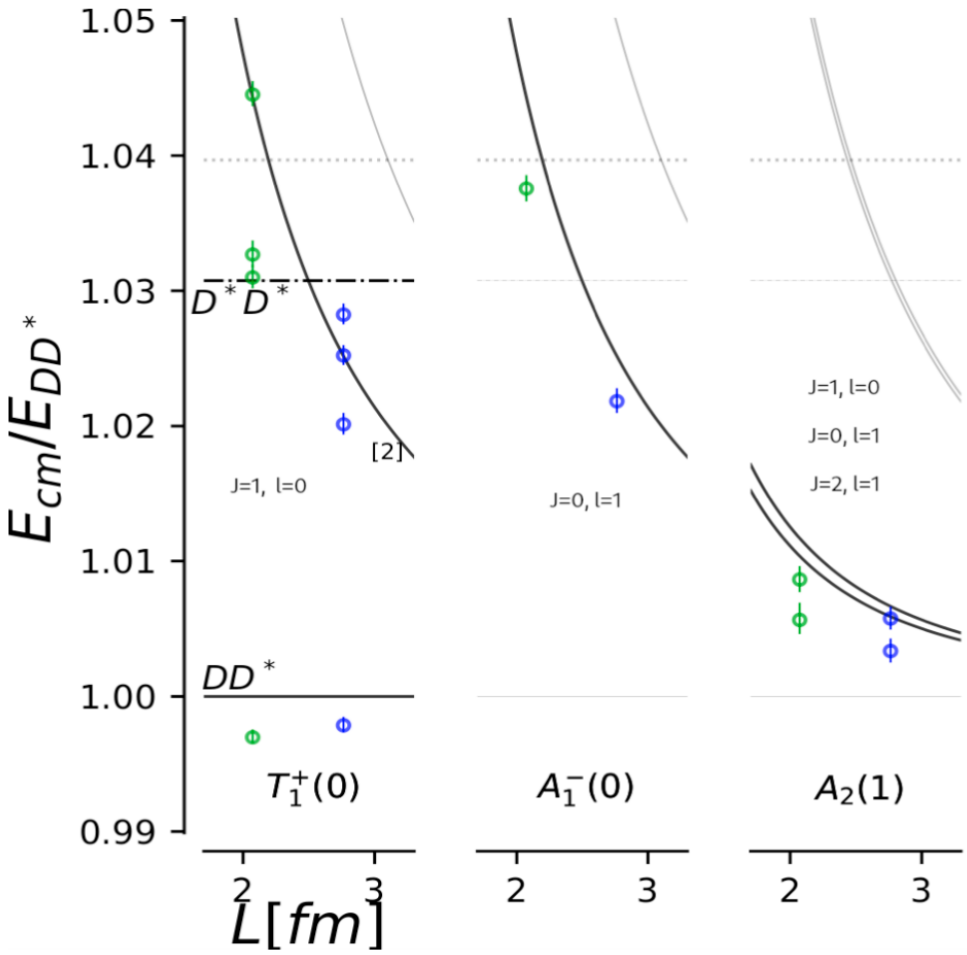
$$E_{DD^*} \equiv m_D + m_{D^*}$$

Eigen-energies and scattering amplitude

at $m_\pi \approx 280\ MeV$



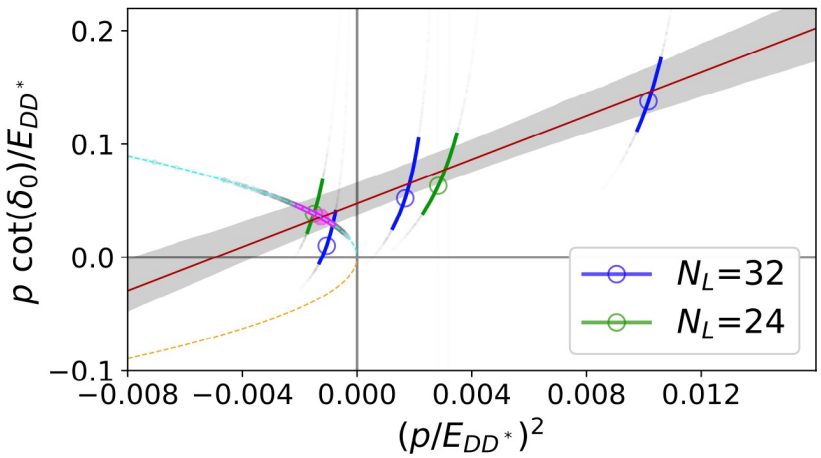
L = 2.1 fm, 2.7 fm



Luscher's relation

E -> T(E), $\delta(E)$

$p \cot \delta$ (for partial wave l=0)



$$E_{DD^*} \equiv m_D + m_{D^*}$$

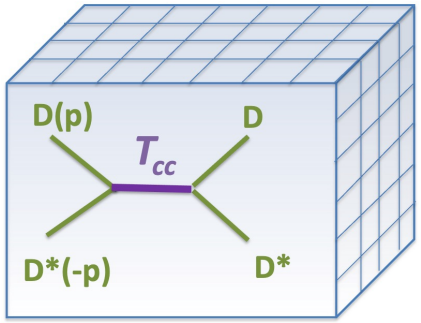
$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$

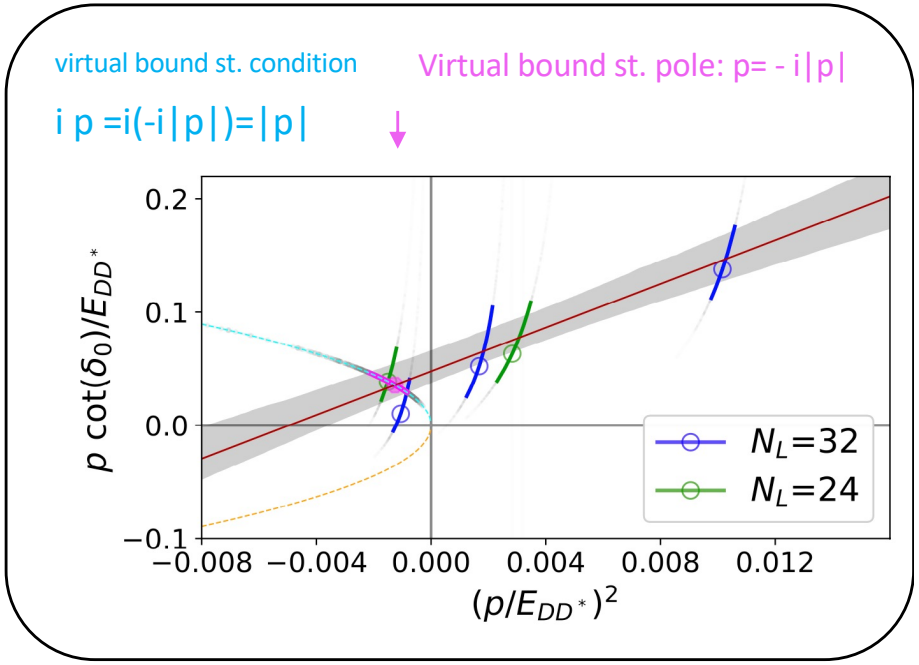
$a_0 = 1.04(0.29)$ fm & $r_0 = 0.96(^{+0.18}_{-0.20})$ fm

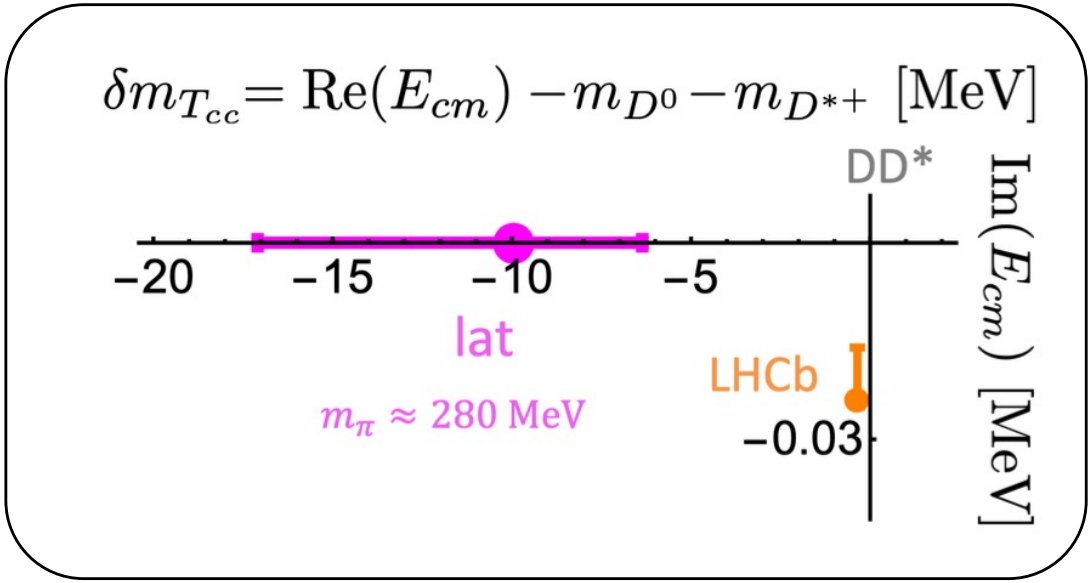
Scattering amplitude for l=0

at $m_\pi \approx 280\ MeV$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$







Lattice: virtual bound st. pole

Binding energy:

$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV

Nature (LHCb): bound st. pole

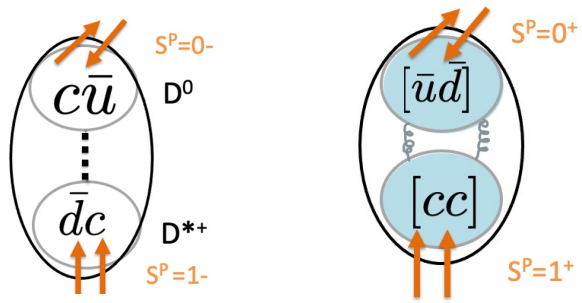
omitting $D^* \to D\pi,\ T_{cc} \to DD\pi$

Possible binding mechanisms of T_{cc}

molecular

likely dominant

[e.g. Janc, Rosina 2003]



"molecular"

Molecular component: dependence on $m_{u/d}$

exchanged particles:

light mesons $\pi, \rho, ..$

increasing $m_{u/d}$

increasing m_{ex}

decreasing R or

decreasing attraction |V|

Yukava-like potential

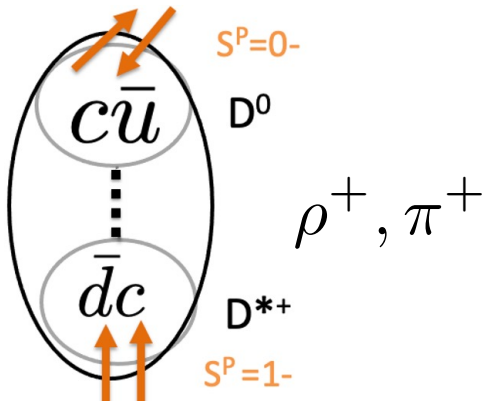
$$V(r) \propto -\frac{e^{-m_{ex} r}}{r}$$

analogous conclusion for any

fully attractive

$$V(r) = -V_0 \; f(r/R)$$

$$f = e^{-r/R}, e^{-r^2/R^2}, \theta(R-r), ...$$



subsequent lattice study:

CLQCD, Chen et al. 2206.06185

comparison of I=0,1 :

attraction in I=0 channel arises

mainly from ϱ exchange

Simplest Example: scattering in square-well potential in QM

$\delta = \arctan[\tan(qR)\frac{p}{q}] - pR$

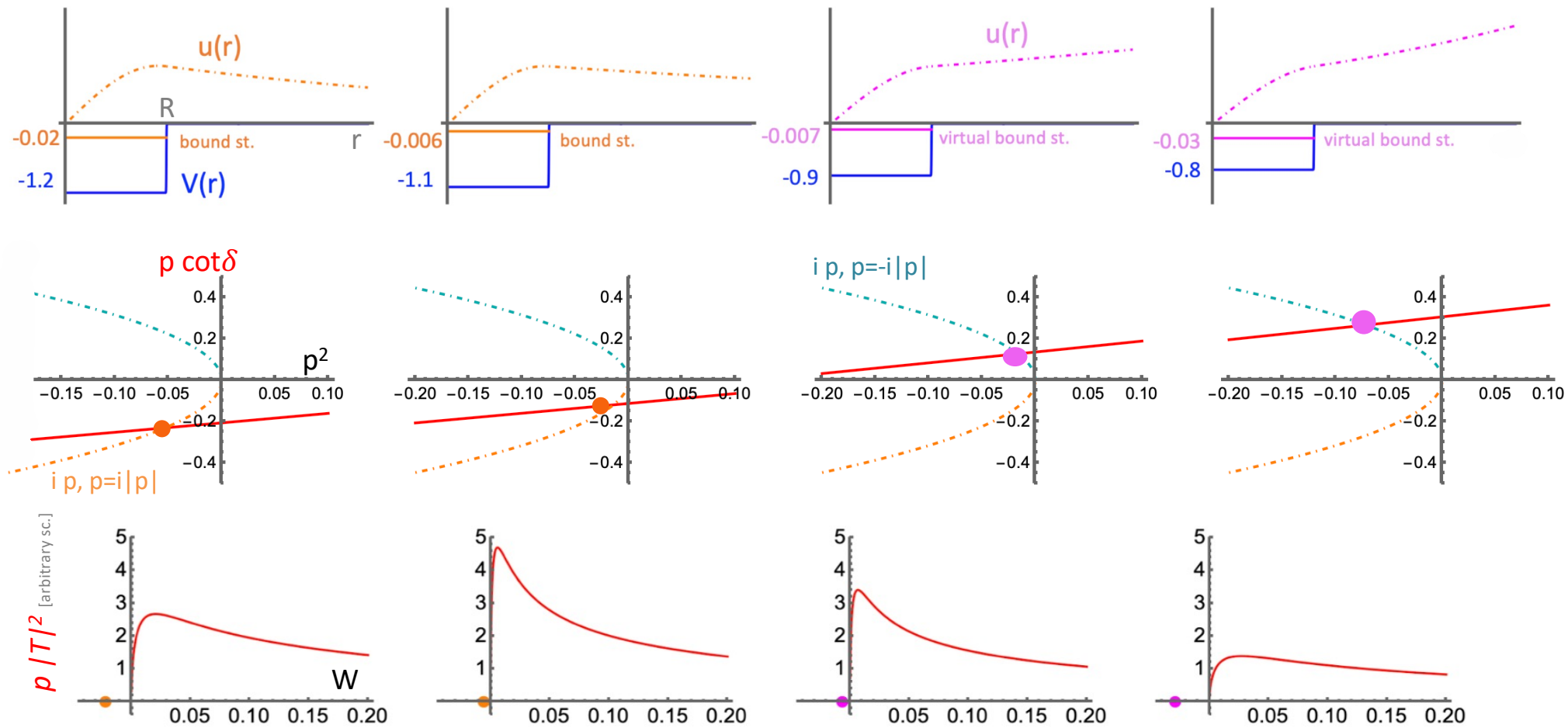
partial wave l=0

$T \propto (p\cot\delta - ip)^{-1}$

$u(r) = A\sin(qr)$ $\quad u(r) = B\sin(pr+\delta)$

p=i|p| $\quad e^{ipr} = e^{-|p|r}$

p=-i|p| $\quad e^{ipr} = e^{|p|r}$

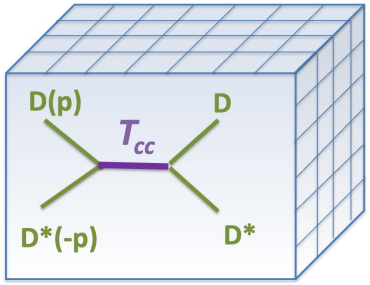


increasing $m_{u/d}$, decreasing attraction V_0 (or decreasing R)

Conclusions on T_{cc}

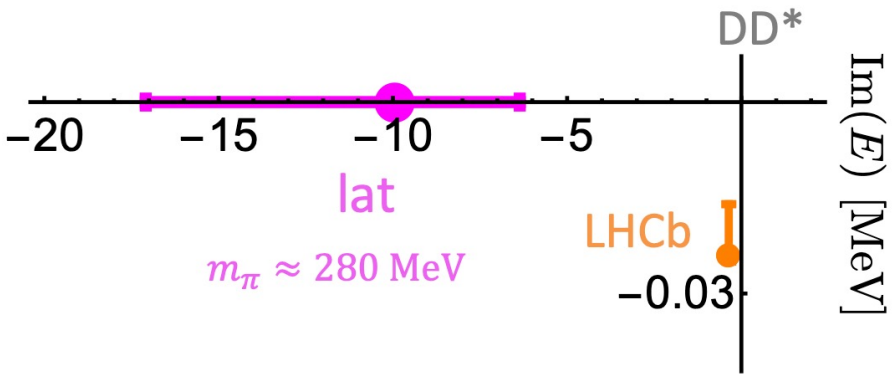
$cc\bar{d}\bar{u}$

The longest lived exotic hadron ever discovered



Pole of T(E) at $m_c^{(h)}$

$$\delta m_{T_{cc}} = \mathrm{Re}(E) - m_{D^0} - m_{D^{*+}} \ [\mathrm{MeV}]$$



	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	−0.36(4)	bound st.

closer-to physical m_c

$T_{cc} \to DD\pi$

$D^* \to D\pi$ omitting

Simple arguments within molecular picture:

$$m_{u/d} \text{ increases :}$$
$$m_{u/d}^{phy} \to m_{u/d}^{lat}$$

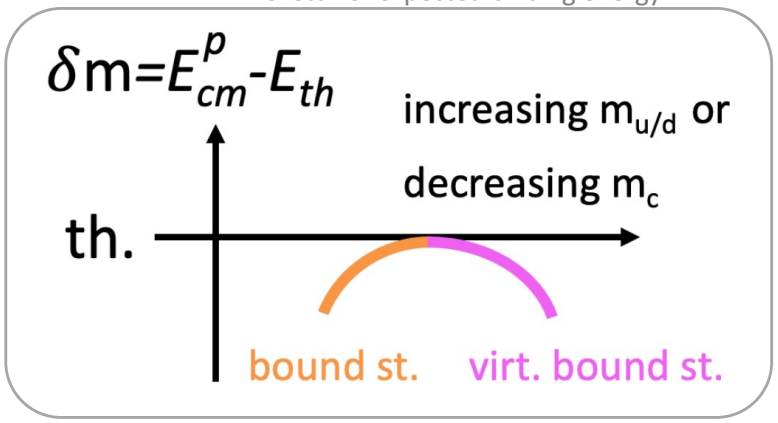
(LHCb) would-be bound st. → virtual bound st.

$$m_c \text{ decreases}$$
$$|\delta m_{T_{cc}}| \text{ increases for virtual bound st.}$$

(see backup slides)

Both in agreement with the lattice result

sketch of expected binding energy



Hypothesis to be verified by future simulations

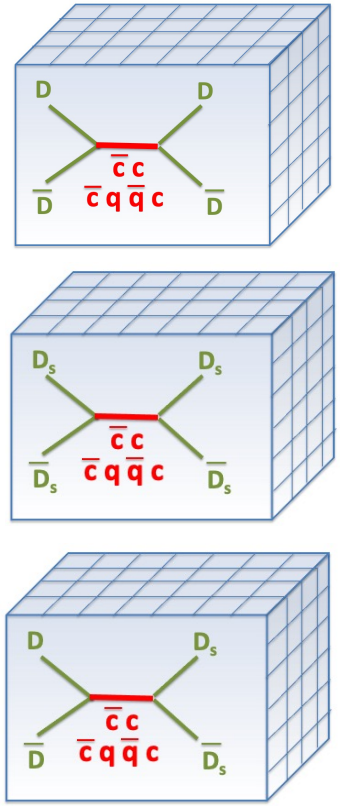
$\bar{c}c$, $\bar{c}q\bar{q}c$ I=0

S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP, 1905.03506 PRD, 2111.02934

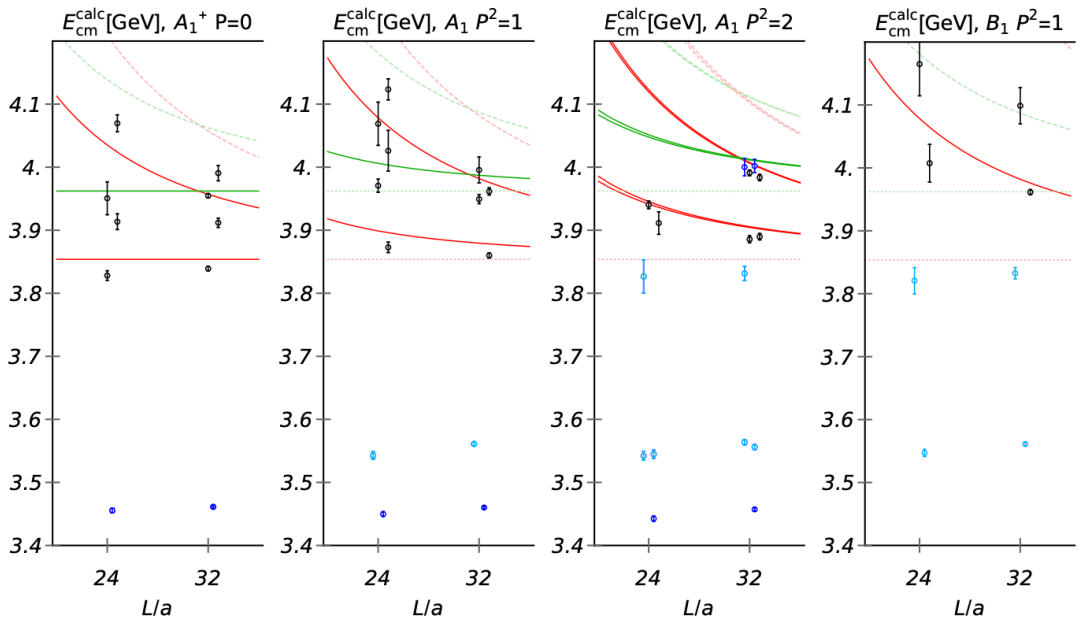
Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$

$D\bar{D} - D_s\bar{D}_s$

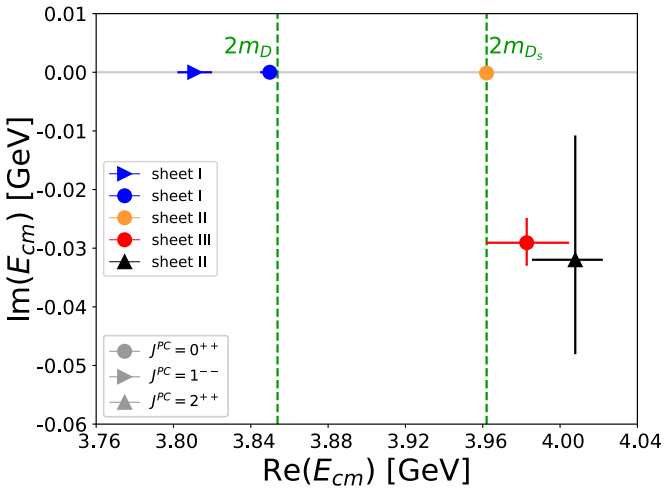


Eigen-energies



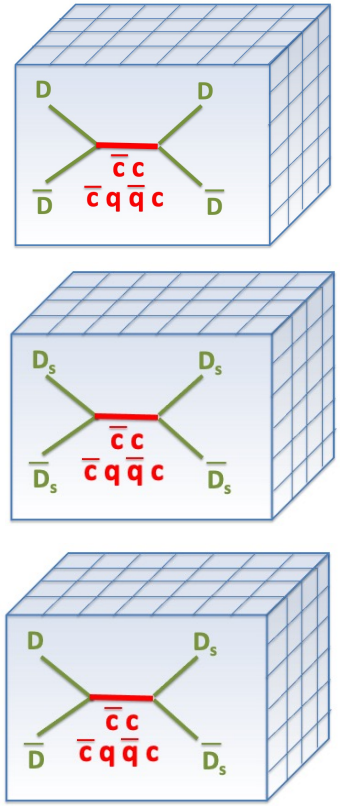
Luscher formalism

$T_{ij}(E)$



Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$



$\bar{D}_s D_s$ $J^P=0^+$ state

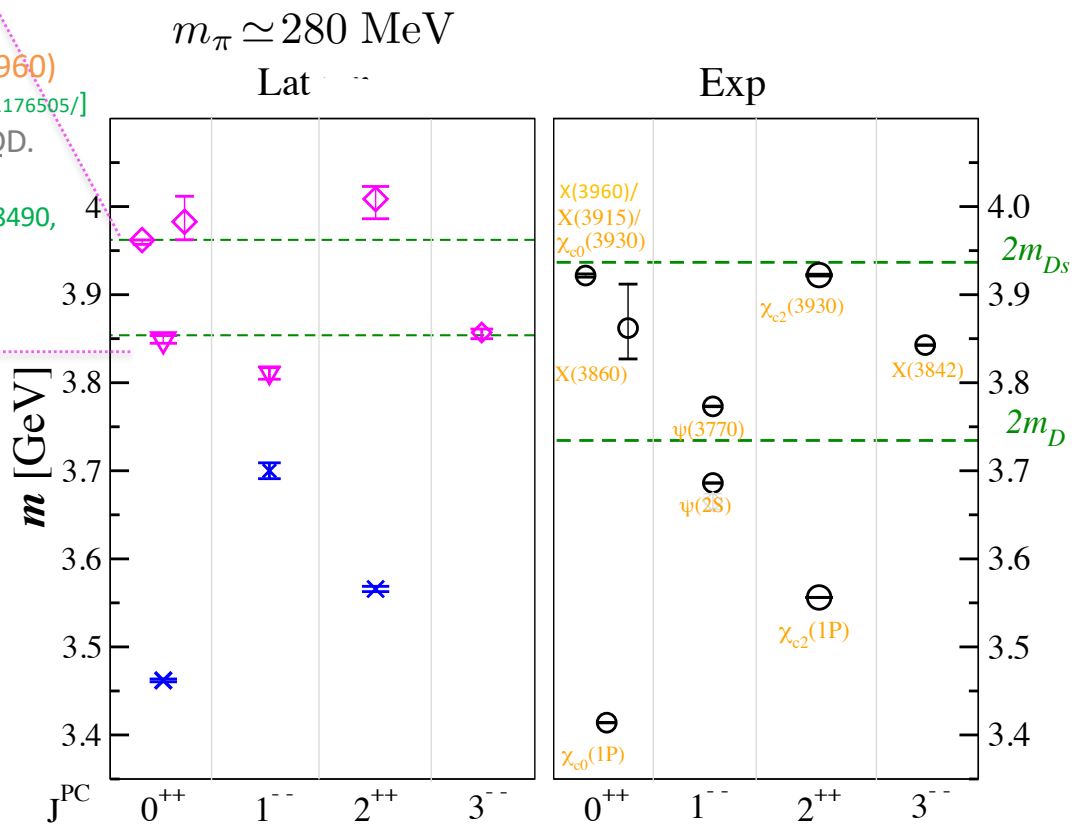
likely related to X(3915) / χ_{c0}(3930) / X(3960)
[BaBar, LHCb 2009.00026, LHCb 2022 indico..../1176505/]
explaining why it has narrow width to $\bar{D}D$.
Supported by some pheno studies:
Lebed, Polosa 1602.08421, Oset et al . 2207.08490,
Guo et al, 2101.01021, ...

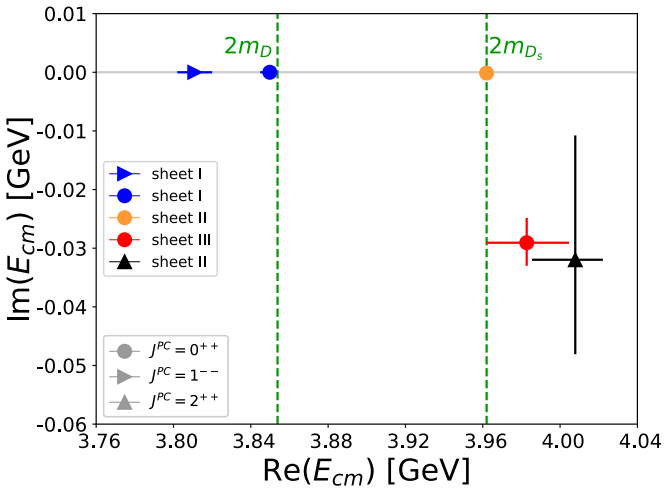
$\bar{D}D$ $J^P=0^+$ state

predicted in models [Oset et al, 0612179 PRD, Hildago Duque et al 1305.4487, Baru et al 1605.09649 PLB]

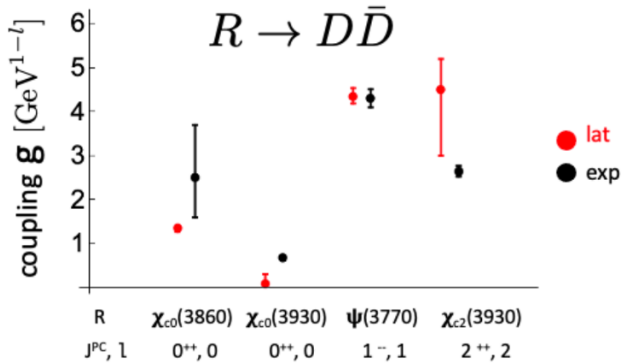
seen in dispersive re-analysis of exp. [Danilkin et al 2111.15033]

+ expected conventional charmonia





$$\Gamma \equiv g^2 \frac{p_D^{2l+1}}{m^2}$$



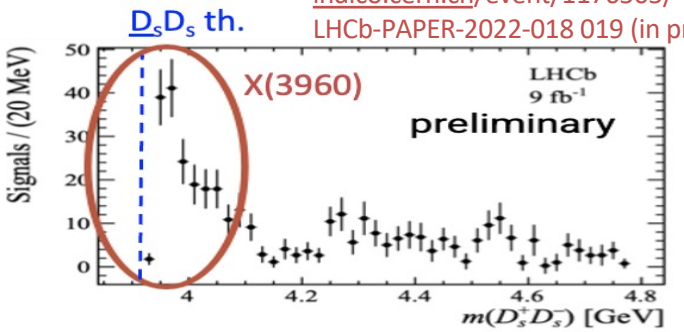
$\bar{D}_s D_s$

likely related to X(3915) / χ_{c0}(3930) / X(3960)

all three likely the same state
currently named χ_{c0}(3914) in PDG

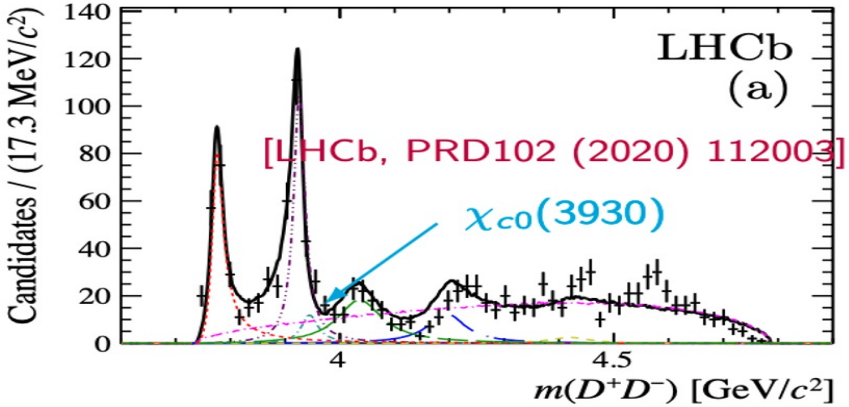
$J^{PC}=0^{++}$

indico.cern.ch/event/1176505/ july 2022
LHCb-PAPER-2022-018 019 (in preparation)

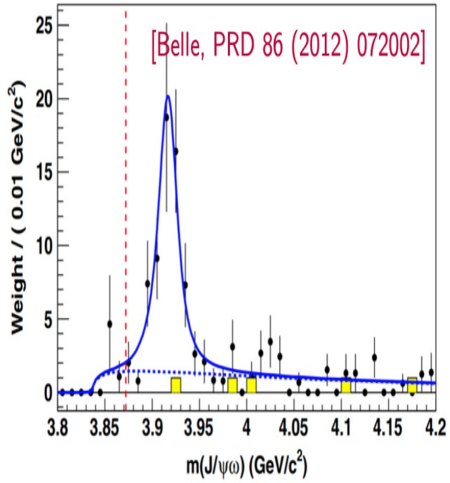


$$X(3960) \to D_s\bar{D}_s$$

$$\frac{Br(D\bar{D})}{Br(D_s\bar{D}_s)} \simeq 0.3$$



$$\chi_{c0}(3930) \to D\bar{D}$$



$$X(3915) \to J/\psi\ \omega$$

Conclusions on charmonium(like) states

$\bar{c}c$, $\bar{c}q\bar{q}c$

$D\bar{D} - D_s\bar{D}_s$ scattering

I=0
$J^{PC}=0^{++},1^{--},2^{++},3^{--}$
q=u,d,s

- masses and decay widths of conventional charmonia confirmed : ground states (bound states)
  first excitations (resonances)
- two additional exotic charmonium-like states with $J^{PC}=0^{++}$ found just below thresholds

$\bar{D}D$ seen in dispersive re-analysis of exp. [Danilkin et al 2111.15033]

$\bar{D}_sD_s$ likely related to X(3915) / χ_{c0}(3930) / X(3960)
LHCb2020 LHCb2022

Backup

Lattice details

CLS ensembles with u/d, s dynamical quarks

a≃0.086 fm

N_L=24, 32

lat exp

$m_{u/d} > m_{u/d}^{exp}$

$m_s < m_s^{exp}$

$m_u+m_d+m_s = m_u^{exp} + m_d^{exp} + m_s^{exp}$

$m_c \gtrsim m_c^{exp}$

m [MeV]	lat	exp
m_π	280(3)	137
m_D	1927(2)	1867
m_{Ds}	1981(1)	1968
M_{av}	3103(3)	3068

$M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$

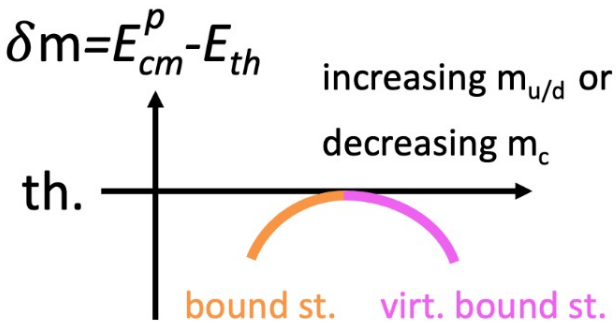
separation between DD and DsDs threshols smaller than in exp

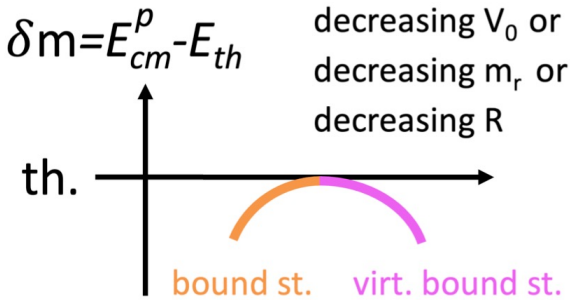
Wick contractions evaluated with distillation or stochastic distillation method.

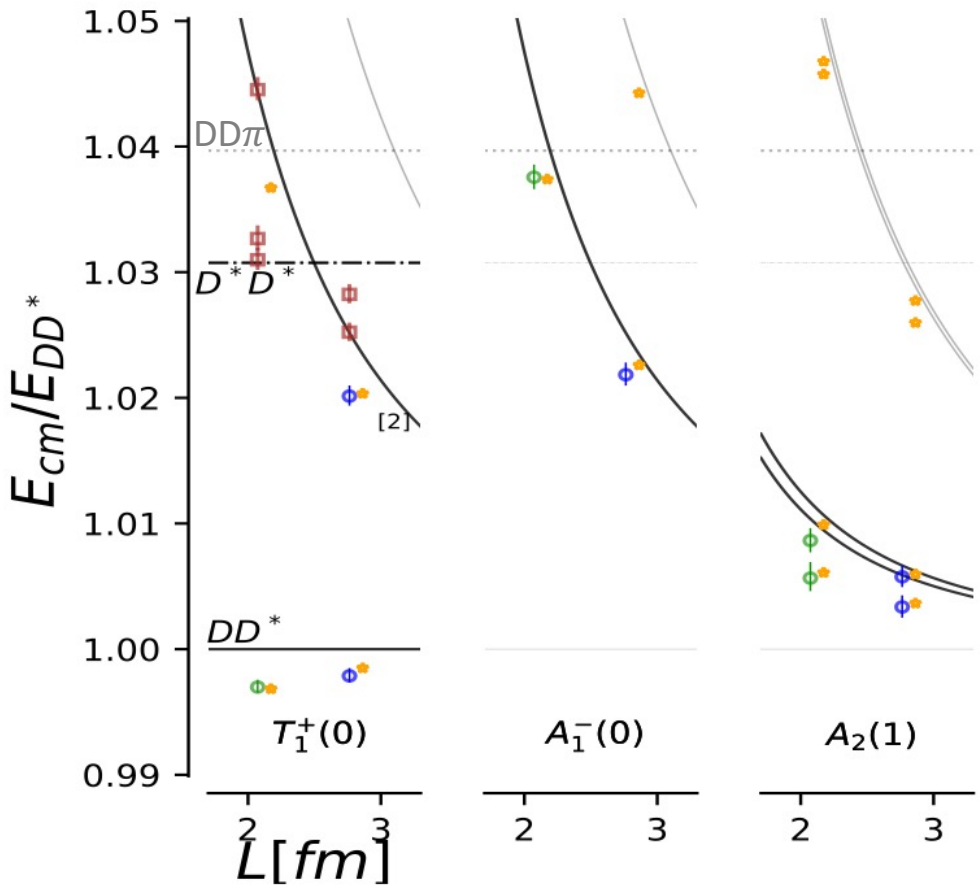
Lattice results on Tcc

	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$r_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp. [2, 37]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	[-11.9(16.9),0]	$-0.36(4)$	bound st.

$$V(r) = -V_0 \; f(r/R)$$







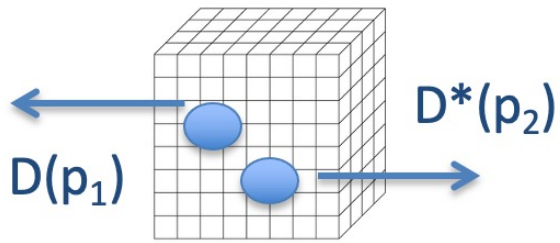
Interpolators for Tcc

Example: P=0

$J^P=1^+$ -> cubic irrep T_1^+

P=D, V=D*

$$
\begin{aligned}
O^{l=0} =& P(\{0,0,0\})V_z(\{0,0,0\}) \\
O^{l=0} =& P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\
&+P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\
&+P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \\
O^{l=2} =& P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\
&+P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\
&-2[P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \\
O^{l=0} =& V_{1x}[0,0,0]V_{2_y}[0,0,0] - V_{1y}[0,0,0]V_{2x}[0,0,0]
\end{aligned}
$$



$$\chi^2(\{a\}) = \sum_L \sum_{\vec{P}\Lambda n} \sum_{\vec{P}'\Lambda' n'} dE_{cm}(L, \vec{P}\Lambda n; \{a\}) \qquad (1)$$
$$\mathcal{C}^{-1}(L; \vec{P}\Lambda n; \vec{P}'\Lambda' n') dE_{cm}(L, \vec{P}'\Lambda' n'; \{a\}) \ .$$

Here

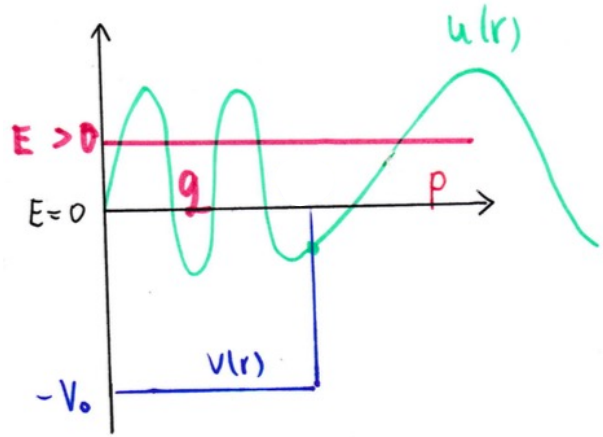
$$dE_{cm}(L, \vec{P}\Lambda n; \{a\}) = E_{cm}(L, \vec{P}\Lambda n) - E_{cm}^{an.}(L, \vec{P}\Lambda n; \{a\})$$

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm} p^{2l}} - i\frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1} \cot \delta_l^{(J)} \qquad (5)$$

We parametrize it with the effective range expansion

$$\tilde{K}^{-1} = \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)} p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)} p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix} . \qquad (6)$$

s-wave scattering on spherical potential well



$$A \sin qr \qquad B \sin(pr + \delta_0)$$

$$\left.\begin{aligned} u(R) &= A \sin qR = B \sin(pR + \delta) \\ u'(R) &= q\, A \cos qR = p\, B \cos(pR + \delta) \end{aligned}\right\}$$

dividing both eqs $\quad \frac{1}{q} \tan qR = \frac{1}{p} \tan(pR + \delta)$

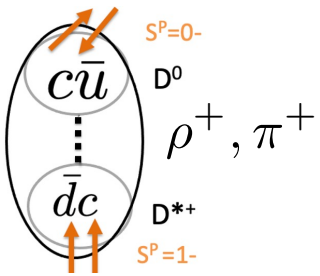
$$\boxed{\delta_0(p) = \arctan\left(\frac{p}{q}\tan(qR)\right) - pR + n\pi}$$

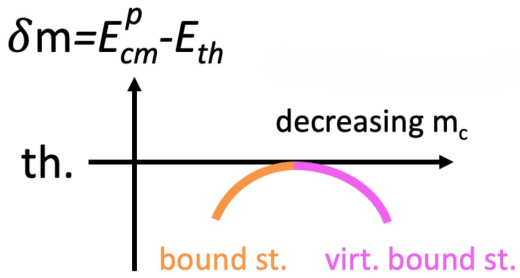
$$q = \sqrt{2\mu(V_0 + E)} = \sqrt{2\mu V_0 + p^2}$$

Molecular component: dependence on m_c

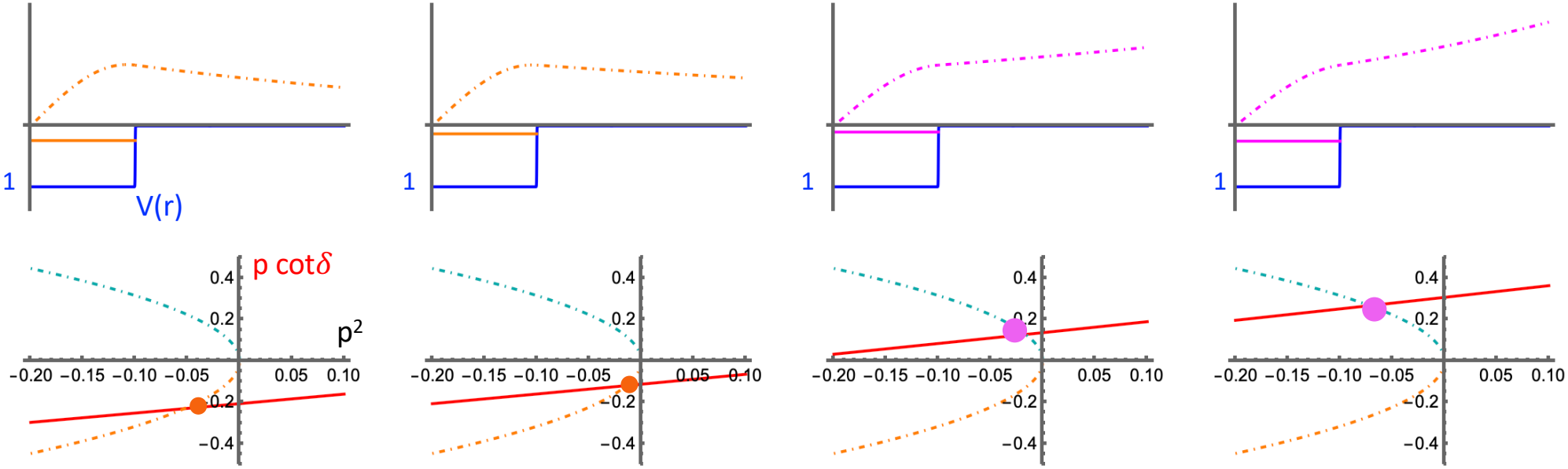
V(r) independent on m_c ,

m_c decreases : reduced mass m_r of D,D* system decreases



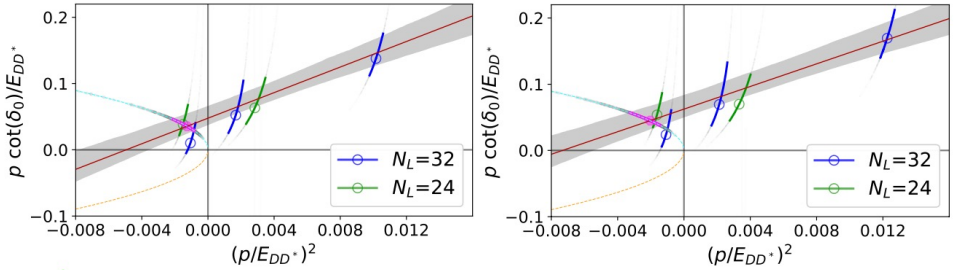


Square well potential (analogous conclusion for other shapes)



decreasing m_c and m_r

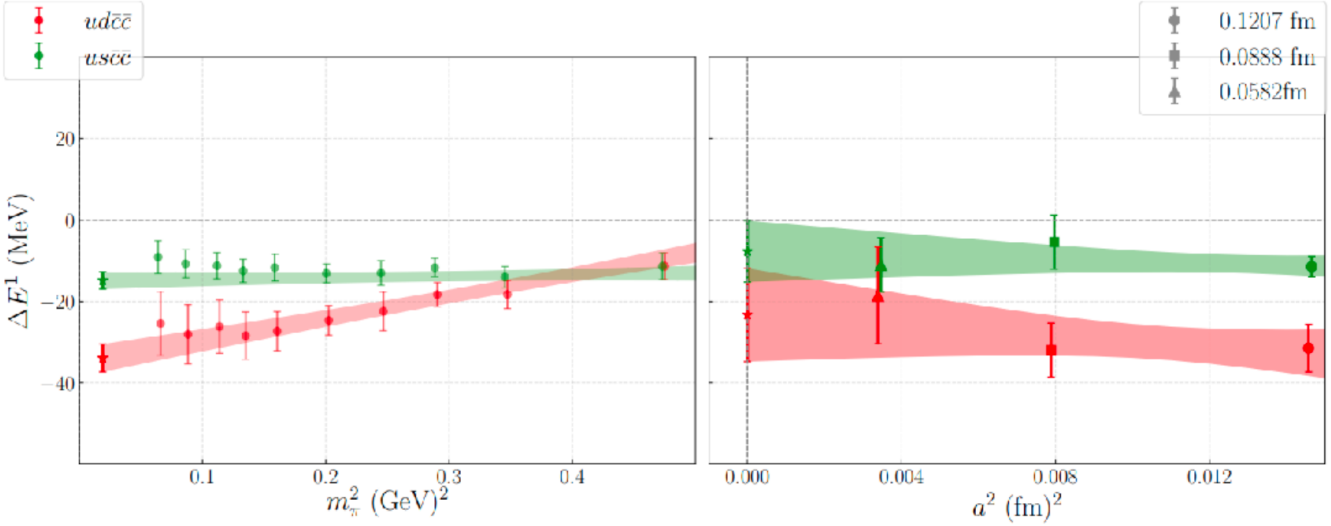
	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(h)}$	1927(1)	1.04(29)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
$m_c^{(l)}$	1762(1)	0.86(0.22)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.



lattice results

Previous lattice QCD study of T_{cc} channel

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285



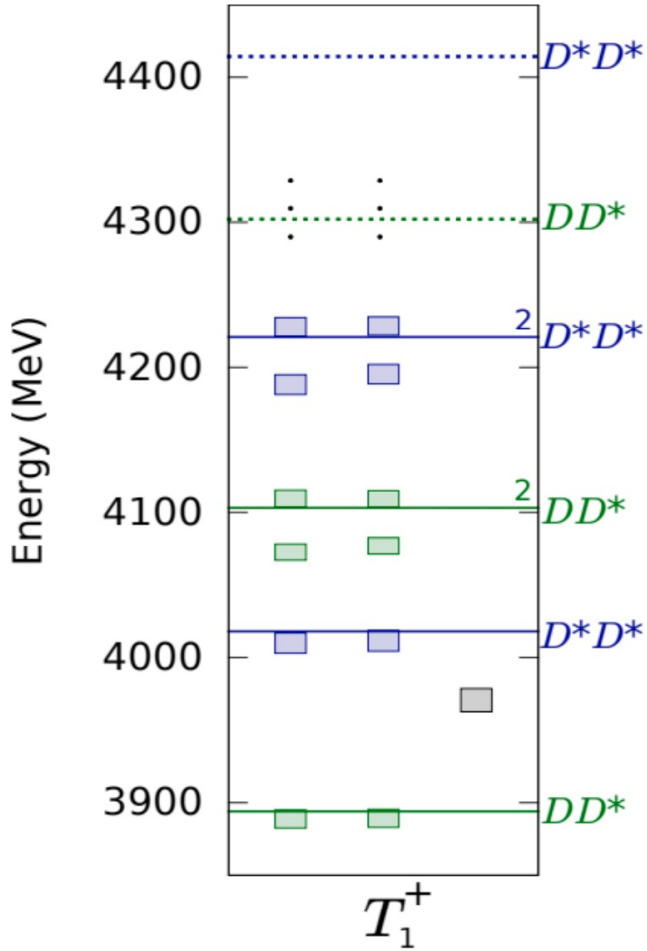
lowest finite-volume eigen-energy for P=0, $J^P=1^+$, I=0

- Study performed on LQCD ensembles with different lattice spacings.
  Single volume and only rest frame finite-volume irreps considered.
- Including a meson-meson and diquark-antidiquark interpolator.
  Diquark-antidiquark interpolators do not influence the low energy spectrum.
- The ground state energy subjected to chiral and continuum extrapolations.
- A finite-volume energy level 23(11) MeV below DD^* threshold.
  No rigorous scattering analysis and no pole structure determined.

Previous lattice QCD study of T_{cc} channel

Hadron Spectrum, JHEP 11, 033 (2017), 1709.01417

finite-volume eigen-energies for P=0, $J^P=1^+$, I=0

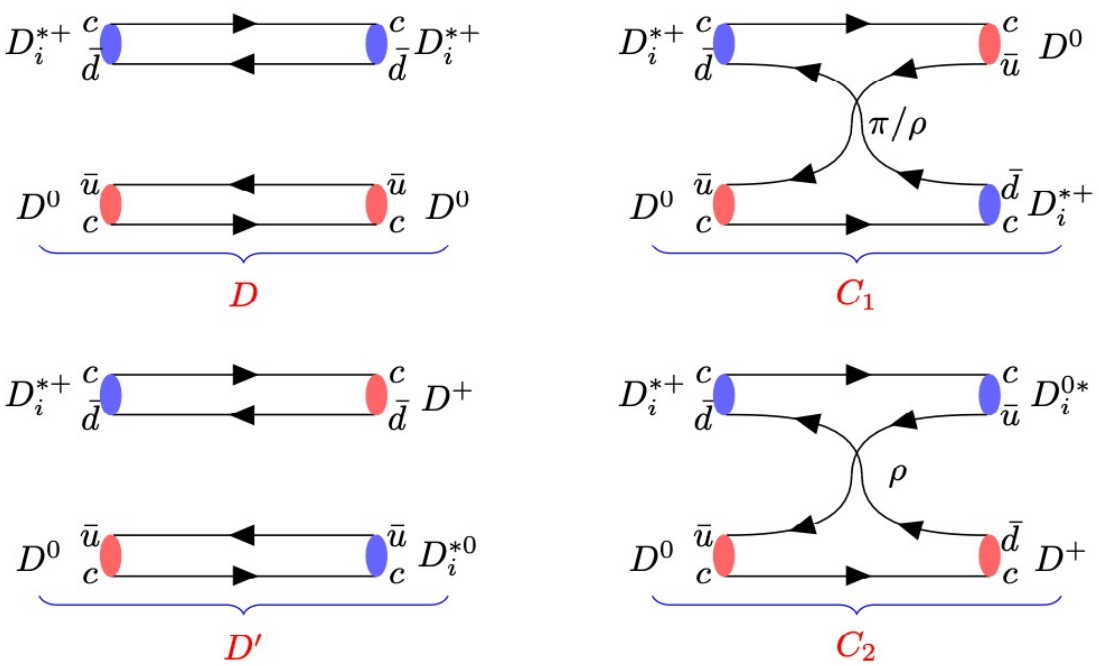


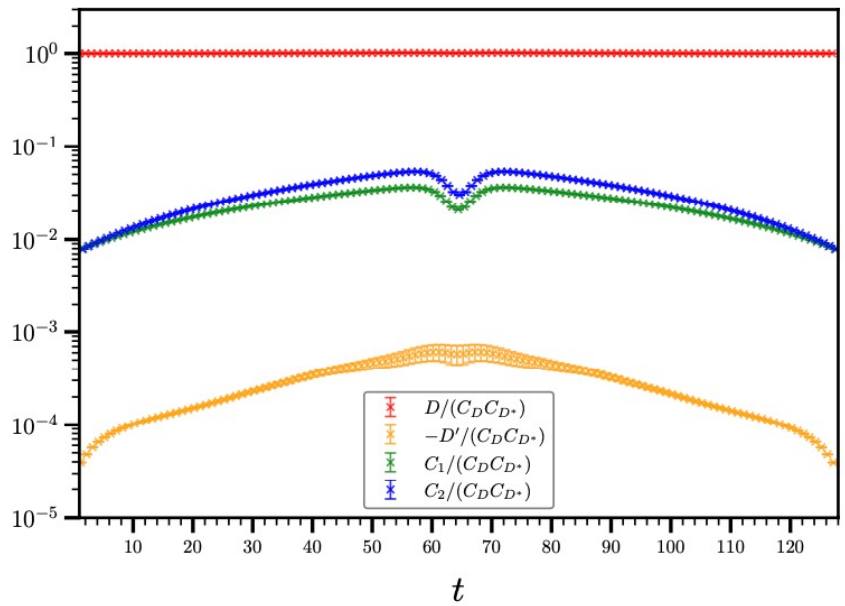
- Single volume rest frame study on a relatively coarse lattice ($a_s \sim 0.12$ fm).
- Large basis of meson-meson and diquark-antidiquark interpolators.
- Diquark-antidiquark interpolators do not influence the low energy spectrum.
- No statistically significant energy shifts observed near DD^* threshold.
  $\Rightarrow$ No scattering amplitude extraction.

Subsequent lattice QCD study of T_{cc} channel

CLQCD, Chen et al. 2206.06185

comparison of I=0,1 :
attraction in I=0 channel arises
mainly from ϱ exchange





$$C^{(I)}(p,t) = D - C_1(\pi/\rho) + (-)^{I+1}\left(D' - C_2(\rho)\right)$$