Resonances with heavy quarks from lattice

(Bound states will be discussed by Daniel Mohler)

Sasa Prelovsek

University of Ljubljana & Jozef Stefan Institute, Slovenia

SFB meeting

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in collaboration with: C.B. Lang, D. Mohler, L. Leskovec, U. Skerbis

Outline

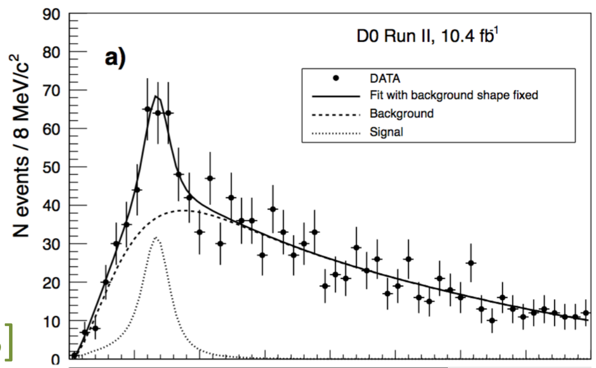
Lattice QCD studies considering **scattering of two hadrons** in the rest frame $H_1(p)H_2(-p)$

- ❑ charmonium resonances above open-charm threshold
  [Lang, Leskovec, Mohler, S.P.,1503.05363, JHEP 2015]

$$J^{PC} = 1^{--}: \quad D\bar{D} \to \psi(3770) \to D\bar{D} \quad \text{in p-wave}$$

- ❑ lattice search for resonance X(5568) in $B_s\,\pi^+$ scat.
  [Lang, Mohler, S.P., 1607.03185, PRD 2016]

D0 collaboration, 1602.07588



$B_s\,\pi^+$ invariant mass $\bar{b}s\bar{d}u$

- ❑ aimed resonances that appear in scat. of hadrons with spin
  construction of lattice operators [S.P., Lang, Skerbis, 1607.06738]

vector-pseudoscalar (e.g J/ψ π for Z_c resonance)
vector-nucleon (e.g. J/ψ p for P_c resonance)
pseudoscalar-nucleon (e.g. π p for Roper resonance)

↓

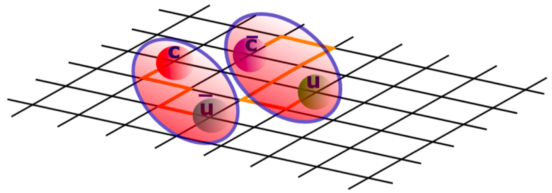
lattice simulation of π N in the Roper channel with $J^P=1/2^+$
[Lang, Leskovec, Padmanath, S.P., arxiv:1610.01422]

Lattice gives discrete energies of eigenstates: E_n

Meson(like) system with given J^{PC} is created by a number of interpolating fields

$$J^{PC} \qquad \mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 q)_{\vec{p}_2}, \quad [\bar{q}\Gamma_3 \bar{q}][q\Gamma_4 q], \ldots$$

$$\psi(3770),\ 1^{--}: \quad \bar{c}c\,, \quad (\bar{c}u)(\bar{u}c) = D\bar{D}\,, \qquad [\overline{cu}][cu]$$



$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}, \qquad Z_i^n = \langle 0 | \mathcal{O}_i | n \rangle$$

All physical states with given J^{PC} appear as E_n in principle (example: charmonium with 1^{++})

- "single-meson" states J/Ψ $\quad m_{J/\psi}=E_1 \quad$ for P=0 (after extrapolations)

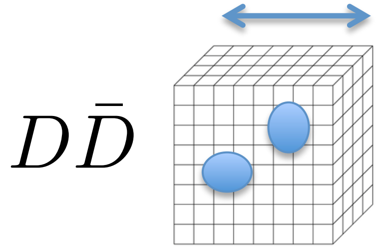
- "two-meson" states $D\bar{D},\ldots$ $\quad E_n$ rigorously render two-hadron scattering matrix (for example D$\underline{\text{D}}$ scattering matrix)

Rigorous treatment of hadrons near or above threshold

scattering of two spin-less mesons in rest frame

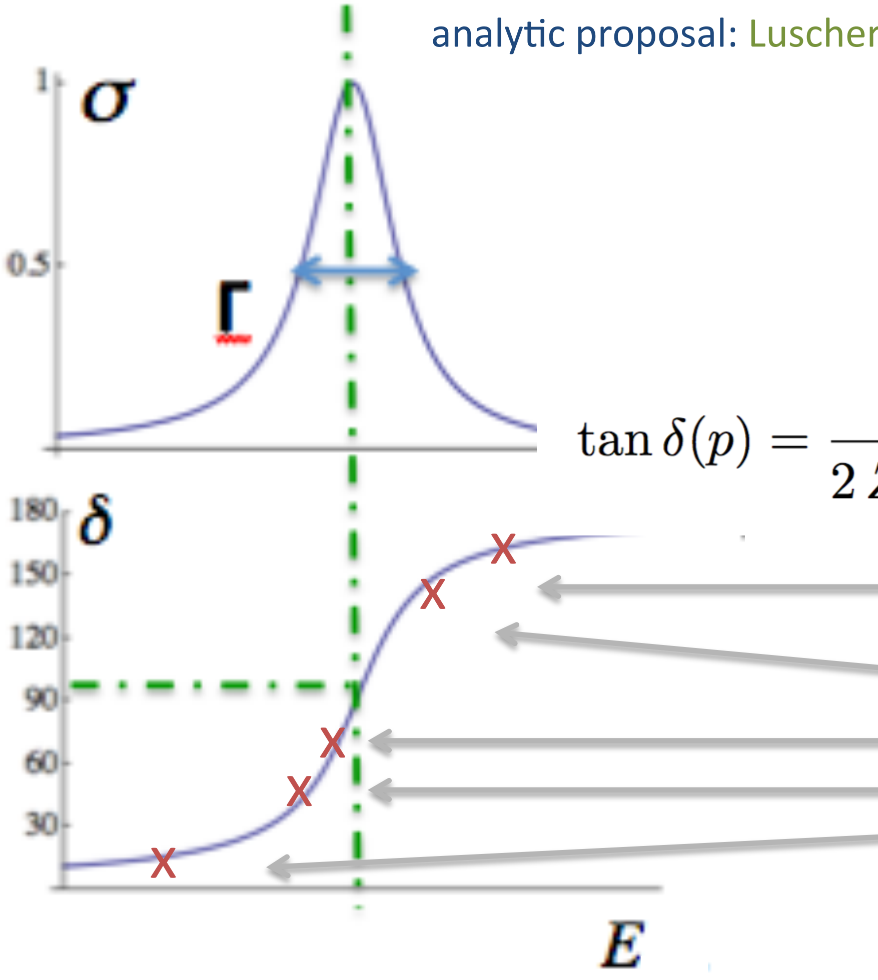
analytic proposal: Luscher 1991

$D\bar{D} \to \psi(3770) \to D\bar{D}$ example



two mesons

in a lattice box



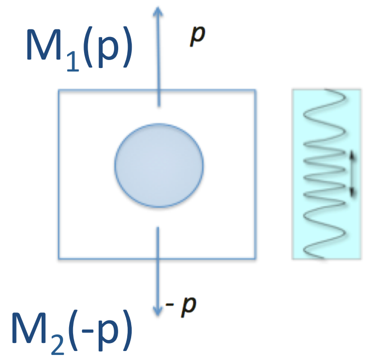
$$\tan\delta(p) = \frac{\sqrt{\pi} p L}{2\, Z_{00}(1; (\frac{pL}{2\pi})^2)}$$

$$E(p) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2}$$

scattering phase shifts at infinite volume	δ(E)	←	E(L)	energies from lattice with spatial extent L

Scattering of two mesons

one-channel (elastic) scattering with total momentum P=0: $E=E_{cm}$



$E_n(L) \xrightarrow{\text{Luscher's eq.}} \delta(E)$

Scattering matrix for partial wave l

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot\delta(E) - i}$$

Bound state (B):

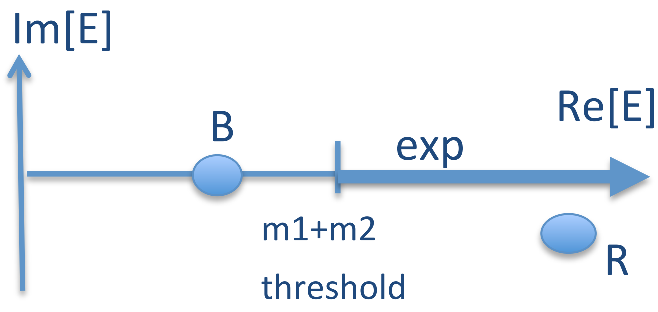
$\cot[\delta(E_B)] = i\,,\quad E_B < m_1 + m_2$

Resonance (R) (of Breit-Wigner type):

$$T(E) = \frac{-E\,\Gamma}{E^2 - m_R^2 + i\,E\,\Gamma}, \quad \Gamma(E) = g^2\,\frac{p^{2l+1}}{E^2}$$

Locations of poles of T(E)

for res. and bound st.



Two types of plots will be shown:

- E(L) energies of lat. eigenstates
- $m^{phy} = m_R$, m_B extracted from E(L)

Charmonium

in particular

charmonium resonances

Charmonia well below D$\underline{D}$ precisely known

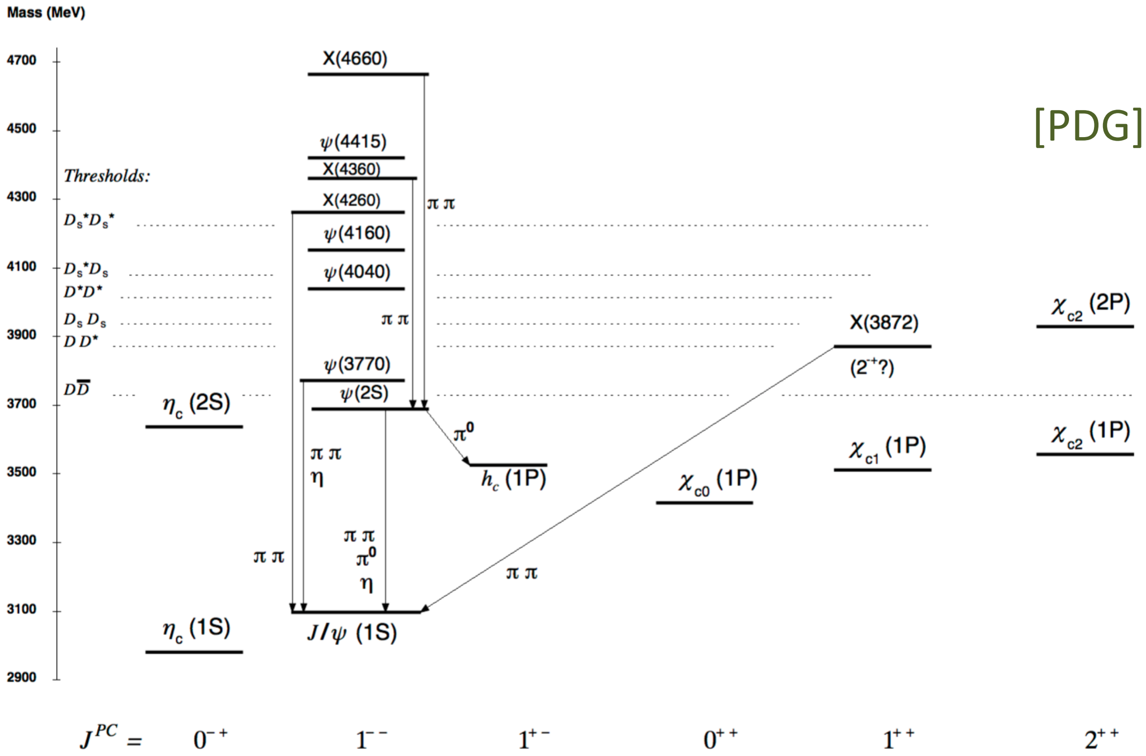
m=E (P=0):

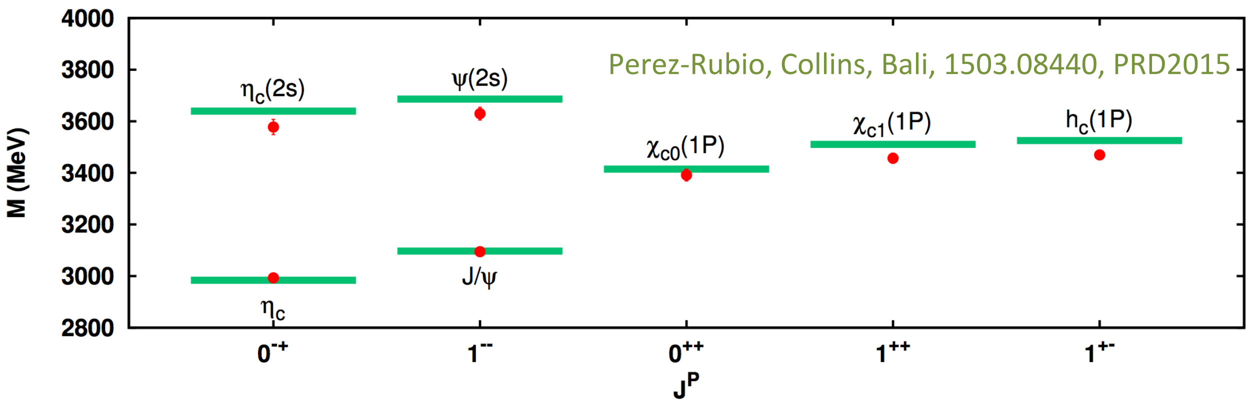
a→0

L→∞

$m_q \rightarrow m_q^{phy}$

[PDG]





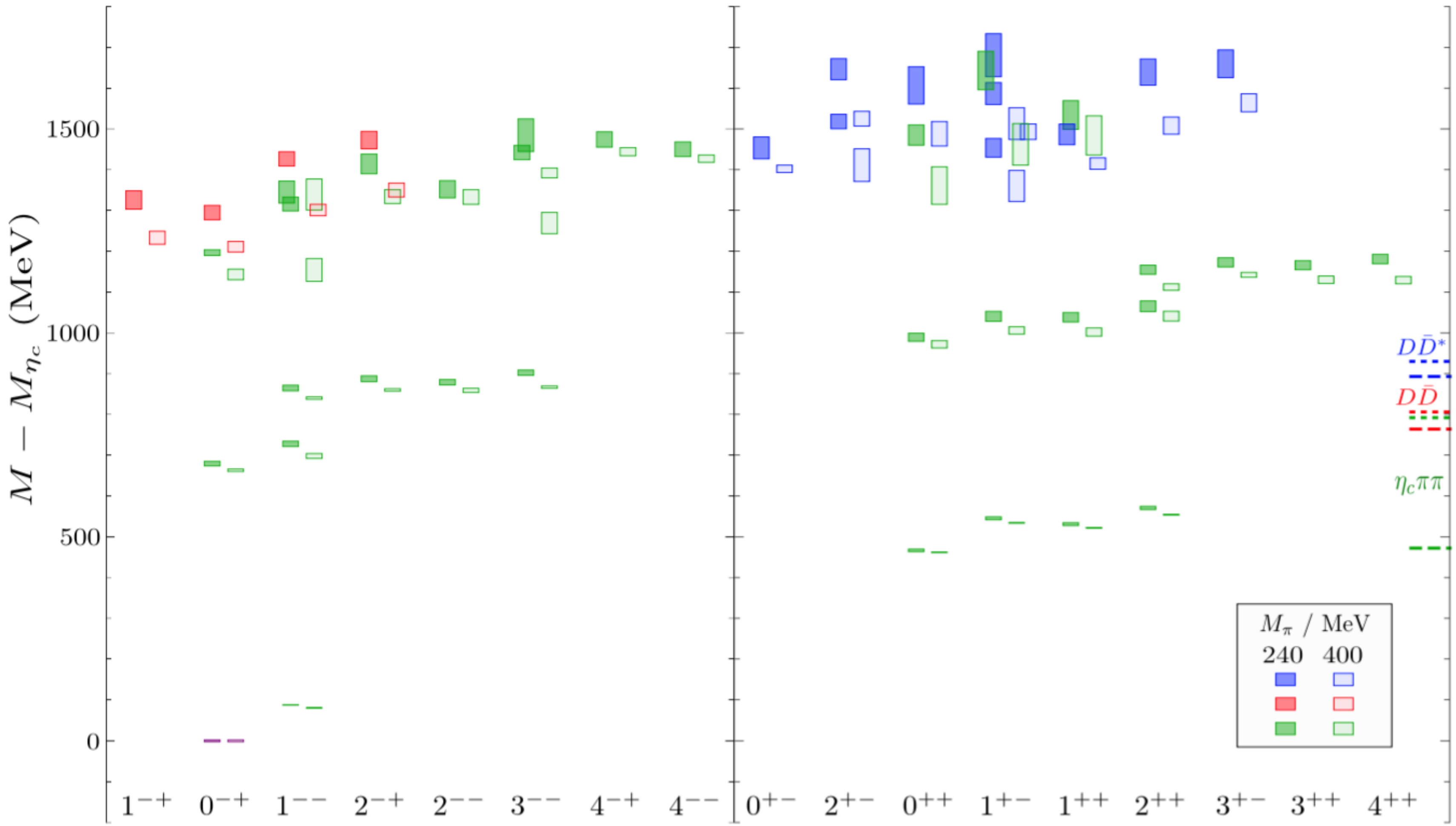
see also:

FNAL/MILC, 1412.1057

The omission of charm annihilation is the main remaining uncertainty

Excited charmonia within single-hadron approach

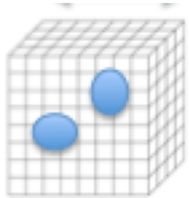
just $\underline{c}c$ interpolators used, strong decay of resonances above DD not taken into account



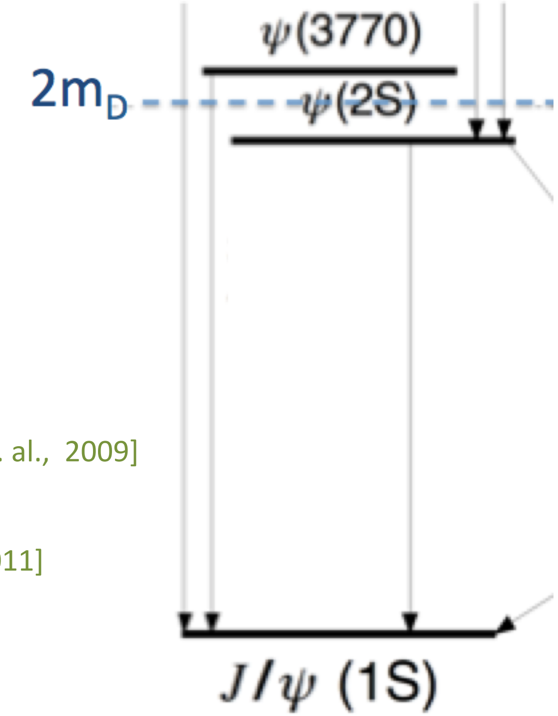
Had. Spec. Coll, 1610.01073

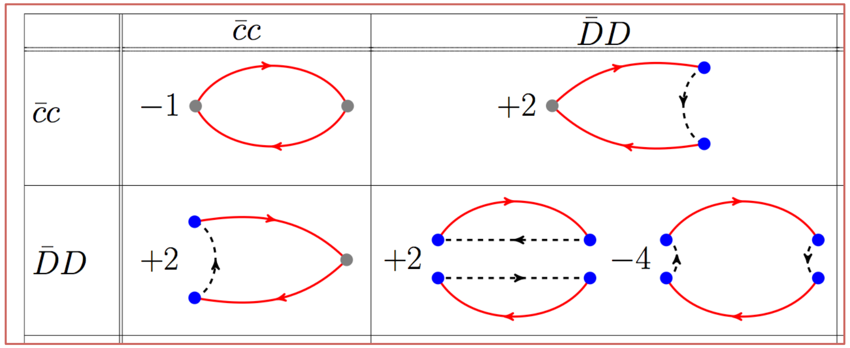
Resonance ψ(3770) and bound st. ψ(2S) from DD scattering in p-wave

$D\bar{D}$



$$O^{\bar{c}c}_{1,..,16} = \bar{c}\Gamma c \quad (16 \text{ operators})$$
$$O^{DD}_1 = [\bar{c}\gamma_5 u(e_i)\ \bar{u}\gamma_5 c(-e_i) - \bar{c}\gamma_5 u(-e_i)\ \bar{u}\gamma_5 c(e_i)] + \{u \to d\}$$
$$O^{DD}_2 = [\bar{c}\gamma_5\gamma_t u(e_i)\ \bar{u}\gamma_5\gamma_t c(-e_i) - \bar{c}\gamma_5\gamma_t u(-e_i)\ \bar{u}\gamma_5\gamma_t c(e_i)] + \{u \to d\}$$





c quark

u,d quarks

Wick contractions evaluated using

- distillation [Peardon et. al., 2009]
- stochastic distillation [Morningstar et al., 2011]

First (exploratory) lattice simulation of a charmonium resonance above open-charm threshold taking into account its strong decay $D\bar{D} \to \psi(3770) \to D\bar{D}$

Lang, Leskovec, Mohler, S.P., 1503.05363, JHEP 2015

Eigen-energies E_n in finite volume

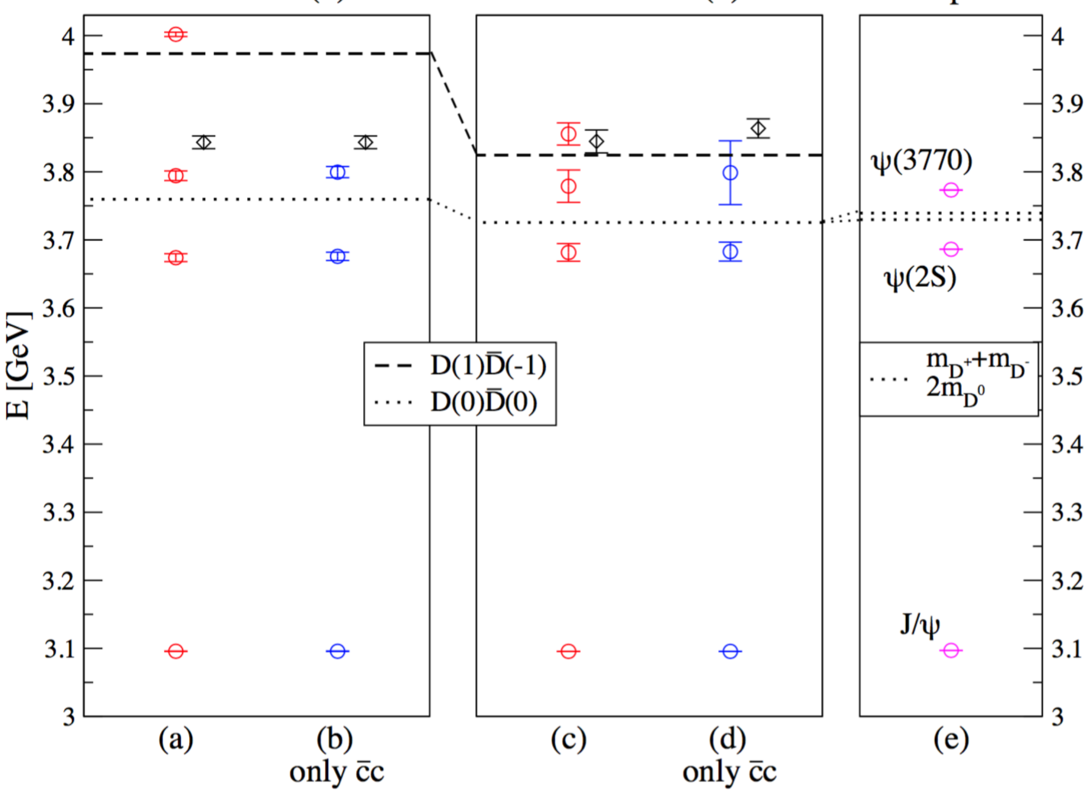
PACS-CS

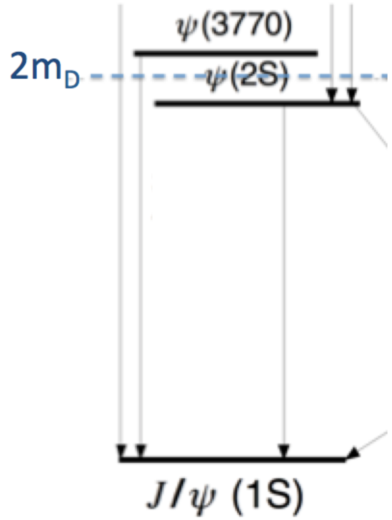
m_π=266 MeV

L=2 fm

m_π=156 MeV

L=2.9 fm





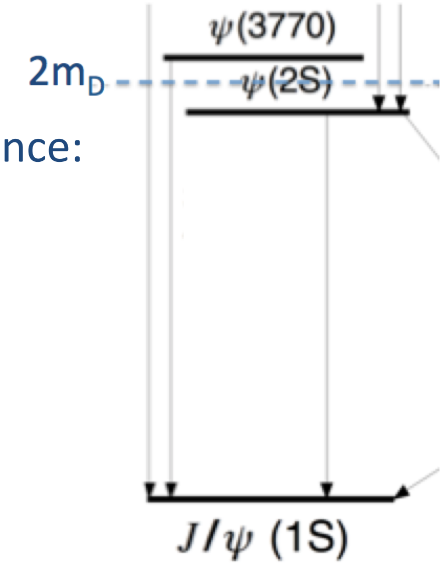
$$E_n \rightarrow \delta(E_n)$$

$$\tan\delta(p) = \frac{\sqrt{\pi}pL}{2\, Z_{00}(1;(\frac{pL}{2\pi})^2)}$$

Lang, Leskovec, Mohler, S.P.,
1503.05363, JHEP 2015

Resonance ψ(3770) and bound st. ψ(2S) from D$\underline{\text{D}}$ scattering in p-wave

$$E_n \;\rightarrow\; \delta(E_n) \;\rightarrow\; p^3 \cot\delta(p)/\sqrt{s}$$



This quantity presented as it is approx. linear near simple BW resonance:

- BW fit (i):

$$T_l(s) = \frac{\sqrt{s}\,\Gamma(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma(s)} = \frac{1}{\cot\delta_l(s) - i}$$

$$\Gamma(s) = \frac{g^2}{6\pi}\frac{p^3}{s} \qquad \frac{p^3\cot\delta(s)}{\sqrt{s}} = \frac{6\pi}{g^2}4(p_R^2 - p^2)$$

- fit (ii): captures also bound st.:

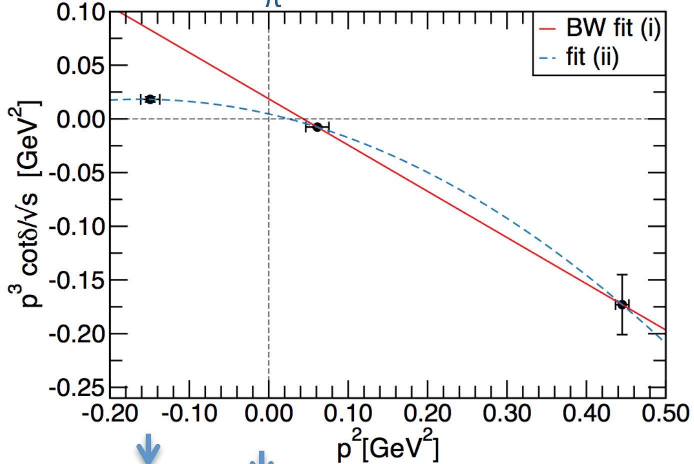
$$\frac{p^3}{\sqrt{s}}\cot\delta_1(s) = A + Bp^2 + Cp^4$$

R : m_R: zero, Γ_R : slope near zero

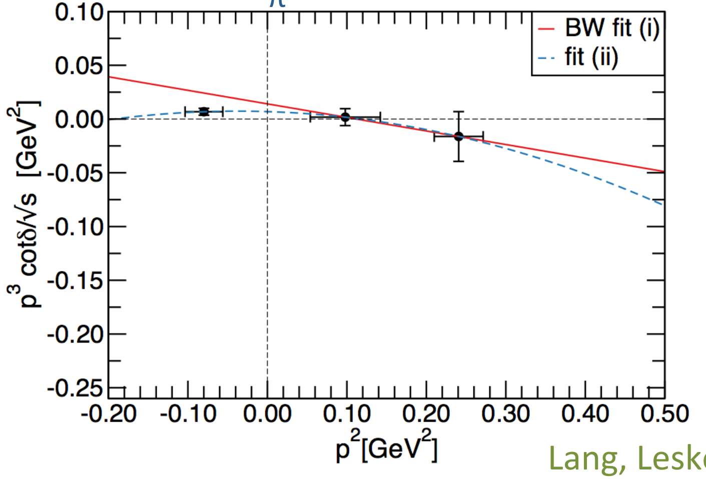
B: cot δ = i

$$p = i|p| \;\rightarrow\; p^3 cot\delta = (i|p|)^3 i = |p|^3 > 0$$

m_π=266 MeV



m_π=156 MeV

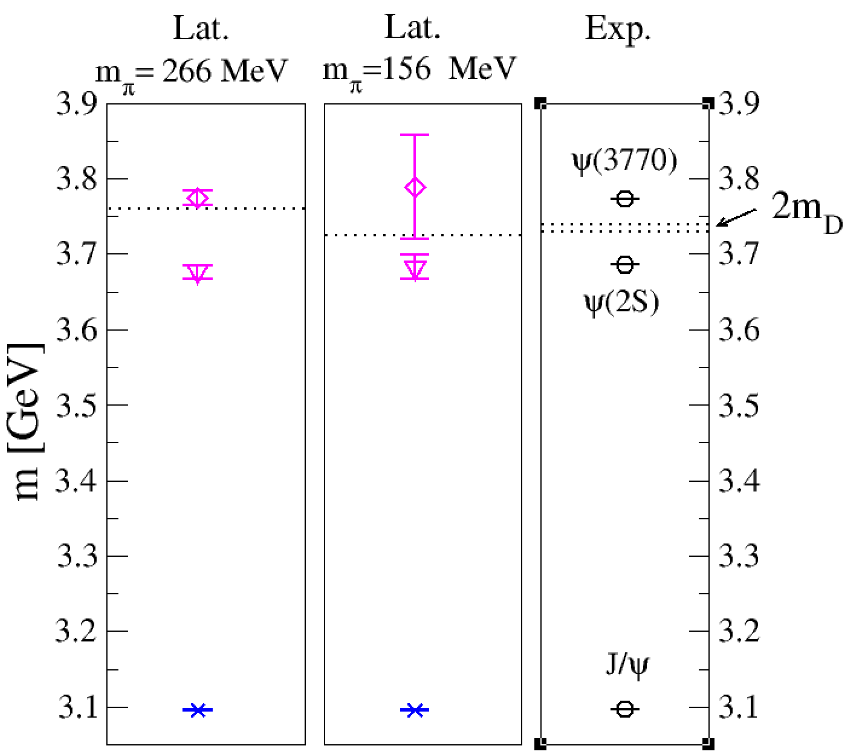


$$\frac{p^3\cot\delta}{\sqrt{s}}$$

$\psi(2S)$ DD threshold

Lang, Leskovec, Mohler, S.P., 1503.05363, JHEP 2015

Resonance ψ(3770) and bound st. ψ(2S) from $D\bar{D}$ scattering in p-wave



extracted masses of hadrons

$$\mathcal{O}: \bar{c}\,c, D\bar{D}, \quad J^{PC}=1^{--}$$

$D\bar{D}$ scat. in p-wave is simulated

T-matrix is determined from E_n

Fit of T-matrix gives:

BW resonance ψ(3770):

m_R (magenta diamonds): $\delta=\pi/2$

Γ (given below): from slope near $\delta=\pi/2$

Bound state ψ(2S) from pole in T:

m_B (magetna triangles): $\cot\delta=i$

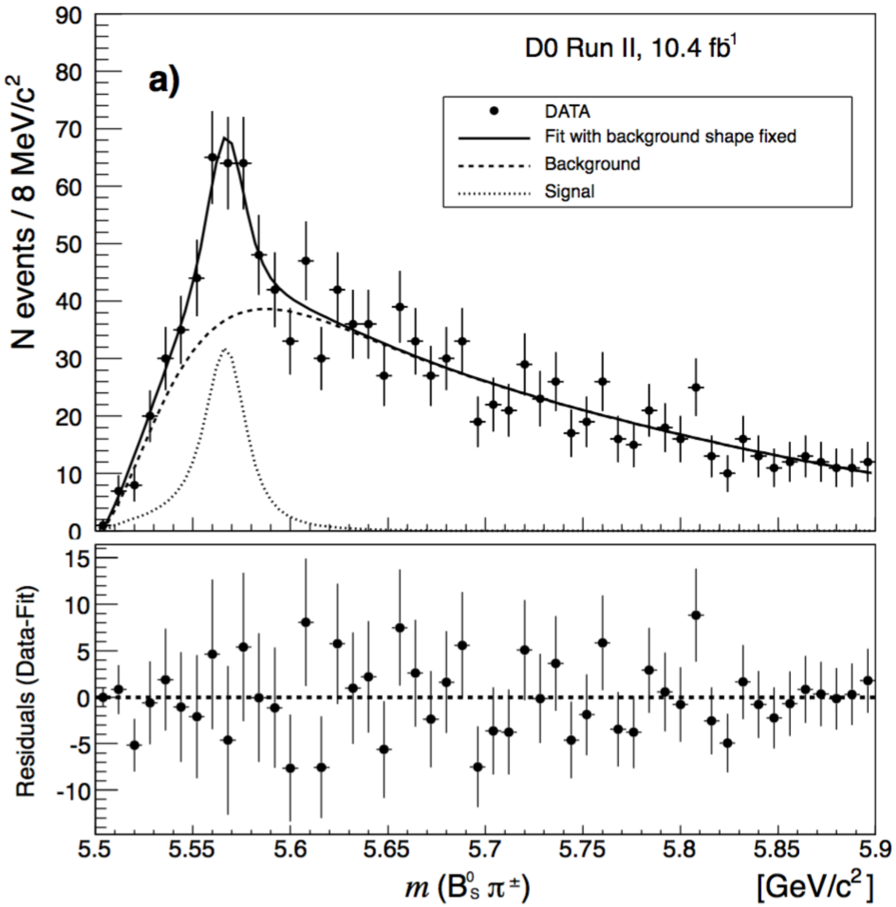
ψ(3770) , fit (ii)	Mass [MeV]	g (no unit)
Lat (m_π=266 MeV)	3774 ±6±10	19.7 ±1.4
Lat (m_π=156 MeV)	3789 ±68±10	28 ± 21
Exp.	3773.15± 0.33	18.7 ± 1.4

$$\Gamma = \frac{g^2}{6\pi}\frac{p^3}{s}$$

Lang, Leskovec, Mohler, S.P.,
1503.05363, JHEP 2015]

Search for resonance X(5568) in $B_s\ \pi^+$ scattering

D0 coll. found resonance X(5568) in $B_s\ \pi^+$ scattering



$B_s\ \pi^+$ invariant mass

D0 collaboration

february 2016

1602.07588

- Γ=22 MeV
- J^P not measured
- The only hadron with 4 different flavors ?!

$$B_s\ \pi^+$$
$$\bar{b}s\ \bar{d}u$$

Search for X(5568) in $B_s\ \pi^+$ scattering

$$B_s\ \pi^+$$
$$\bar{b}s\ \bar{d}u$$

- soon after D0 result, several phenomenological studies appeared
- those who could find support, suggested it has $J^P=0^+$
- if X(5568) is $J^P=0^+$:

  - the only strong decay channel is $B_s\ \pi^+$

  - the next threshold is BK and it is 210 MeV above X(5568) !

  - exotic resonance in elastic channel !? This is something lattice QCD can do!

[Lang, Mohler, S.P., 1607.03185, PRD 2016]

- note: all other exotic candidates (Z_c, Z_b, ...) are resonances but lie

  - next to a higher threshold
  - above several thresholds

this is much more challenging

that is why it is difficult to establish

those exotic states on lattice

Employed operators: $B_s\,\pi^+$ and BK

$$O_{1,2}^{B_s(0)\pi(0)} = \left[\bar{b}\Gamma_{1,2}s\right](\mathbf{p}=0)\left[\bar{d}\Gamma_{1,2}u\right](\mathbf{p}=0)$$

$$O_{1,2}^{B_s(1)\pi(-1)} = \sum_{\mathbf{p}=\pm\mathbf{e_{x,y,z}}\ 2\pi/L} \left[\bar{b}\Gamma_{1,2}s\right](\mathbf{p})\left[\bar{d}\Gamma_{1,2}u\right](-\mathbf{p})$$

$$O_{1,2}^{B(0)K(0)} = \left[\bar{b}\Gamma_{1,2}u\right](\mathbf{p}=0)\left[\bar{d}\Gamma_{1,2}s\right](\mathbf{p}=0)$$

$$\Gamma_1 = \gamma_5 \text{ and } \Gamma_2 = \gamma_5\gamma_t$$

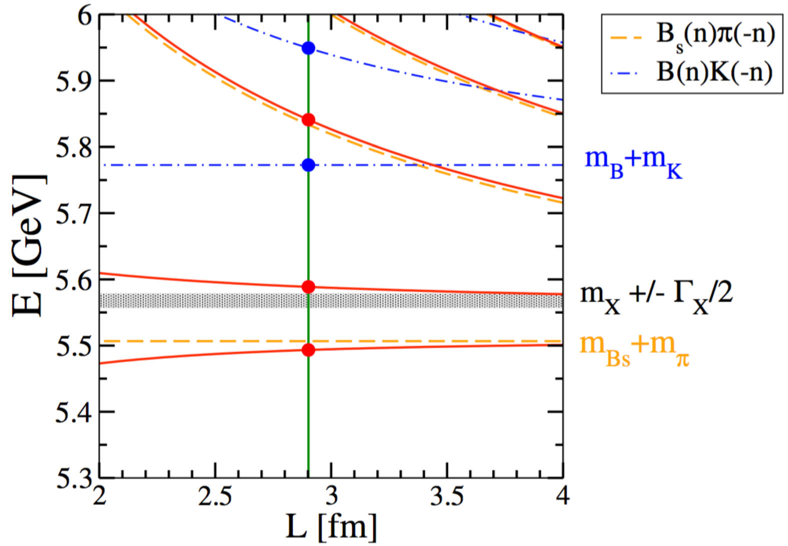
BK threshold 210 MeV higher;

the interpolators are employed just for completeness

[Lang, Mohler, S.P., 1607.03185, PRD 2016]

Analytic expectation for E_n if X(5568) exists

based on m_X and Γ_X as measured by D0 exp



$$\delta_{B_s\pi}(p) = \text{atan}\left[\frac{E\,\Gamma(E)}{m_X^2 - E^2}\right], \quad \Gamma(E) = \Gamma_X \frac{p(E)m_X^2}{p(m_X)E^2}$$

$$\delta_{B_s\pi}(p) = \text{atan}\left[\frac{\sqrt{\pi}pL}{2\,Z_{00}(1;(pL/2\pi)^2)}\right]$$

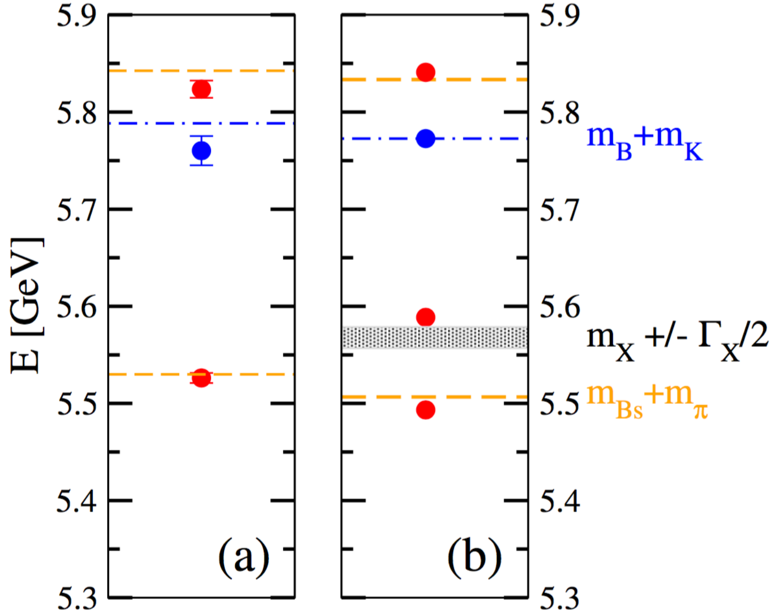
Results of E_n from actual lattice simulation

Analytic expectation for E_n if X(5568) exists

L=2.9 fm

Lattice

Analytic (if X exists)



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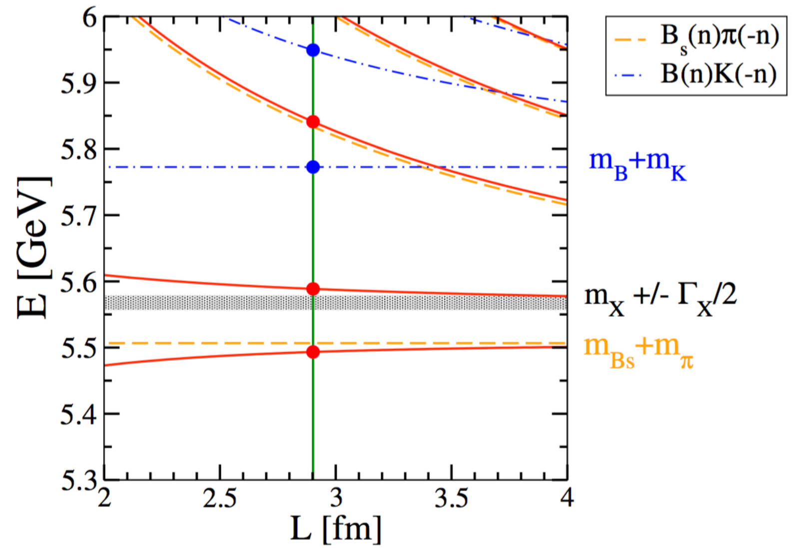

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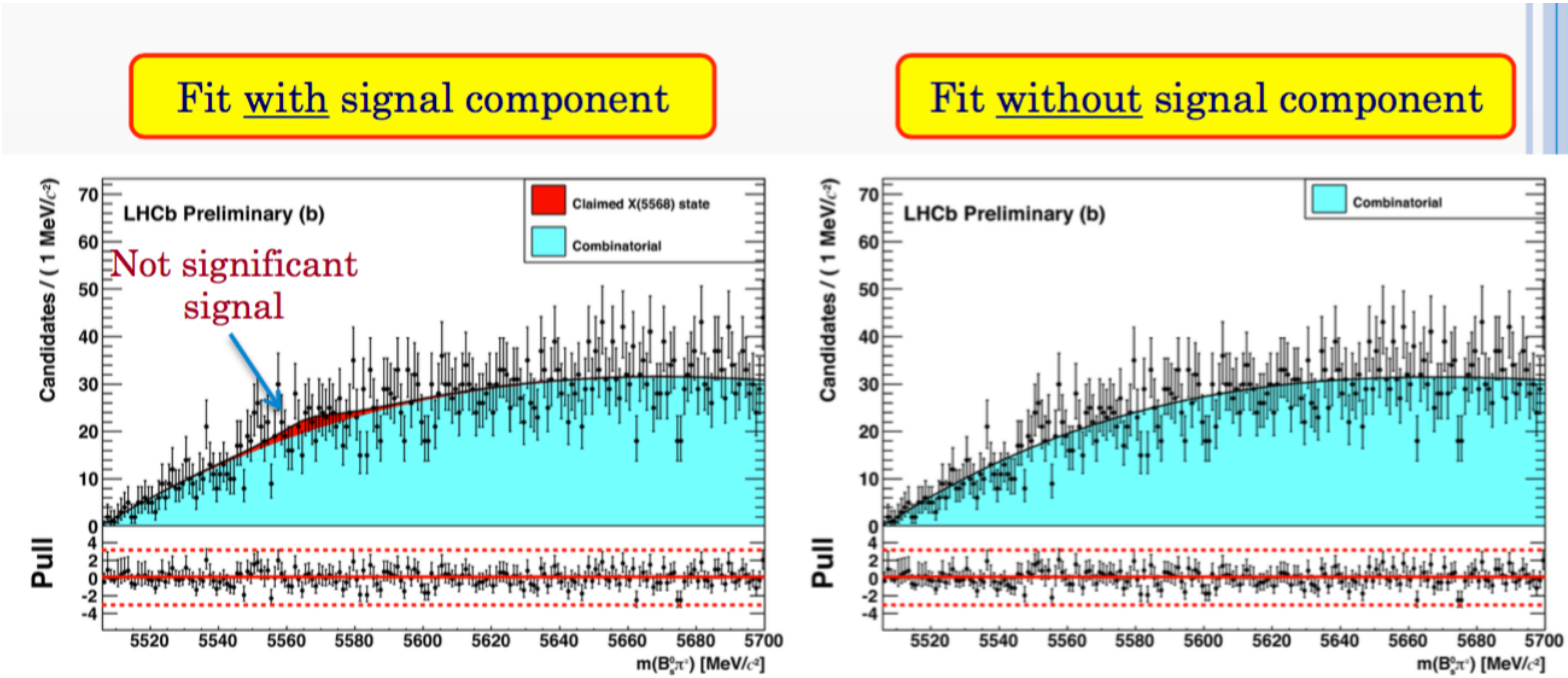
$$\delta_{B_s\pi}(p) = \mathrm{atan}\left[\frac{\sqrt{\pi} p L}{2\, Z_{00}(1; (pL/2\pi)^2)}\right]$$

Lattice result (obtained before march 22nd 2016 – see next slide):

- no indication of X(5568)
- interactions in $B_s\,\pi^+$ and BK system are small

[Lang, Mohler, S.P., 1607.03185, PRD 2016]

LHCb data published in 1608.00435: PRL 2016



$B_s\ \pi^+$ invariant mass

$B_s\ \pi^+$ invariant mass

Previous two examples concerned scattering of two spinless hadrons:
pseudoscalar-pseudoscalar (DD and $B_s\, \pi^+$)

Lattice operators for scattering of particles with spin

[S.P., Lang, Skerbis, 1607.06738]

Motivation

- Mainly PP scattering was simulated on lattice up to now → scattering phase shift extracted (P has no spin)

- $H^{(1)}\,H^{(2)}$: where one or both H carry spin was explored mostly only for L=0
  many interesting channels still unexplored, particularly for L>0

I will consider construction of $H^{(1)}\,H^{(2)}$ interpolators
where H is one of P,V,N hadrons, which is (almost) stable with respect to strong decay:

P=psuedoscalar ($J^P=0^{\wedge -}$) $= \pi$, K, D, B, η_c , ...
V=vector ($J^P=1^{\wedge -}$) = D*, B*, J/ψ, Υ_b , B_c^*,... (but not directly applicable to ρ as is unstable...)
N=nucleon ($J^P=1/2^{\wedge +}$) = p, n, Λ, Λ_c, Σ, ... (but not directly applicable to $N^-(1535)$ as is unstable...)

I will consider interpolators for channels :

PV: meson resonances and <u>Q</u>Q-like exotics (e.g. π J/ψ, D <u>D</u>* ..)

PN: baryon resonances (e.g. π N, K N ...) and pentaquarks

NV: baryon resonances and pentaquarks

NN: nucleon-nucleon and deuterium, baryon-baryon

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$$\langle O_i(t)|O_j^\dagger(0)\rangle \ \rightarrow\ E_n\ \rightarrow\ \delta(E)$$

- O=HH needed to create/annihilate HH system
- E_n related to phase shifts for HH scattering
  - two spinless particles Luscher (1991):
  - two particles with arbitrary spin

    Briceno, PRD89, 074507 (2014)

    (other authors: some specific cases)

Some previous related work on lattice HH operators for hadrons with spin and L≠0

Partial-wave method for HH:
Berkowitz, Kurth, Nicolson, Joo, Rinaldi, Strother, Walker-Loud, 1508.00886
Wallace, Phys. Rev. D92, 034520 (2015), [arXiv:1506.05492]

Projection method for HH:
M. Göckeler et al., Phys.Rev. D86, 094513 (2012), [arXiv:1206.4141].

Helicity operators for single-H:
Thomas, Edwards and Dudek, Phys. Rev. D85, 014507 (2012), [arXiv:1107.1930]

Some aspects of helicity operators for HH:
Wallace, Phys. Rev. D92, 034520 (2015), [arXiv:1506.05492].
Dudek, Edwards and Thomas, Phys. Rev. D86, 034031 (2012), [arXiv:1203.6041].

Which CG of H_1 and H_2 to H_1H_2 irreps are nonzero; values of CG not published:
Moore and Fleming, Phys. Rev. D 74, 054504 (2006), [arXiv:hep- lat/0607004].

etc ...

However: for a lattice practitioner who was interested in a certain channel, for example
(PV scattering in L=2 or VN scattering with λ_V=1 and λ_N=1/2)
there were still lots of puzzles to beat before constructing a reliable interpolator ..

We restrict to total momentum zero

$H^{(1)}(p)\, H^{(2)}(-p)$, $P_{tot}=0$

Advantage of $P_{tot}=0$:

- parity P is a good number
- channels with even and odd L do not mix in the same irrep

} not true for $P_{tot}\neq 0$

Building blocks H: required transformation properties of H

to prove correct transformation properties of HH

rotations R

$$R H_{m_s}(p) R^{-1} = \sum_{m'_s} \underbrace{D^s_{m'_s m_s}(R)^*}_{\text{Wigner D matrix}} H_{m'_s}(Rp) \;,$$

inversion I

$$I H_{m_s}(p) I = (-1)^P H_{m_s}(-p)$$

annihilation field

m_s is a good quantum number at p=0: $S_z H_{m_s}(0) S_z^{-1} = m_s H_{m_s}(0)$

m_s is not good quantum number in general for p≠0: in this case it denotes m_s of corresponding $H_{ms}(p=0)$

Non-practical choice of H: canonical fields $H^{(c)}$

with correct transformation properties under R and I

$$H^{(c)}_{m_s}(p) \equiv L(p) H_{m_s}(0)$$

L(p) is boost from 0 to p; drawback: $H^{(c)}(p)$ depend on m, E,..

$$V_{m_s=1}(0) = \tfrac{1}{\sqrt{2}}[-V_x(0) + iV_y(0)] \;\rightarrow\; V^{(c)}_{m_s=1}(p_x) = \tfrac{1}{\sqrt{2}}[-\gamma V_x(p_x) + iV_y(p_x)]$$

$$\begin{pmatrix}-1\\ i\\ 0\end{pmatrix} \xrightarrow{\Lambda^1(p_x)} \begin{pmatrix}\gamma & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}-1\\ i\\ 0\end{pmatrix} = \begin{pmatrix}-\gamma\\ i\\ 0\end{pmatrix}$$

$$N_{m_s=1/2}(0) = \mathcal{N}_1(0) \;\rightarrow\; N^{(c)}_{m_s=1/2}(p_x) \propto \mathcal{N}_1(p_x) + \tfrac{p_x}{E+m}\mathcal{N}_4(p_x)$$

$$\begin{pmatrix}1\\ 0\\ 0\\ 0\end{pmatrix} \xrightarrow{\Lambda^{1/2}(p_x)} \begin{pmatrix}1\\ 0\\ 0\\ \frac{p_x}{E+m}\end{pmatrix}$$

$\mathcal{N}_{\mu=1,..,4}$ are Dirac components in Dirac basis

Non-practical choice of H: canonical fields $H^{(c)}$

with correct transformation properties under R and I

$$H^{(c)}_{m_s}(p) \equiv L(p) H_{m_s}(0)$$

L(p) is boost from 0 to p; drawback: $H^{(c)}(p)$ depend on m, E,..

$$V_{m_s=1}(0) = \tfrac{1}{\sqrt{2}}[-V_x(0) + iV_y(0)] \;\rightarrow\; V^{(c)}_{m_s=1}(p_x) = \tfrac{1}{\sqrt{2}}[-\gamma V_x(p_x) + iV_y(p_x)]$$

$$\begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix} \xrightarrow{\Lambda^1(p_x)} \begin{pmatrix} \gamma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix} = \begin{pmatrix} -\gamma \\ i \\ 0 \end{pmatrix}$$

$$N_{m_s=1/2}(0) = \mathcal{N}_1(0) \;\rightarrow\; N^{(c)}_{m_s=1/2}(p_x) \propto \mathcal{N}_1(p_x) + \tfrac{p_x}{E+m}\mathcal{N}_4(p_x)$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\Lambda^{1/2}(p_x)} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \frac{p_x}{E+m} \end{pmatrix}$$

$\mathcal{N}_{\mu=1,..,4}$ are Dirac components in Dirac basis

Practical choice of H

with correct transformation properties under R and I

$$V_{m_s=\pm 1}(p) = \tfrac{1}{\sqrt{2}}[\mp V_x(p) + iV_y(p)], \quad V_{m_s=0}(p) = V_z(p)$$

$$N_{m_s=1/2}(p) = \mathcal{N}_{\mu=1}(p)\;, \quad N_{m_s=-1/2}(p) = \mathcal{N}_{\mu=2}(p)$$

$$P(p) = \sum_x \bar{q}(x)\gamma_5 q(x) e^{ipx}$$

$$V_i(p) = \sum_x \bar{q}(x)\gamma_i q(x) e^{ipx}, \quad i = x, y, z$$

$$\mathcal{N}_\mu(p) = \sum_x \epsilon_{abc}[q^{aT}(x) C\gamma_5 q^b(x)]\; q^c_\mu(x)\; e^{ipx}, \quad \mu = 1, .., 4$$

These H are employed as building block in our HH operators

simple examples

Required transformation properties of O=HH

$$RO^{J,m_J}(P_{tot}\!=\!0)R^{-1} = \sum_{m'_J} D^J_{m_J m'_J}(R^{-1}) O^{J,m'_J}(0) \qquad IO^{J,m_J}(0)I = (-1)^P O^{J,m_J}(0)$$

continuum R

good parity since P_{tot}=0 !

relevant rotations: $R \in O^{(2)}$ O with 24 el. for J=integer ; O^2 with 48 elements for J=half-integer

The group including inversion I: O_h with 48 el. for J=integer ; O^2_h with 96 elements for J=half-integer

The representation O^J reducible under $O^{(2)}$. Irreducible representations (irreps) are denoted by Γ and rows r

$$RO_{\Gamma,r}R^{-1} = \sum_{r'} \underbrace{T^{\Gamma}_{r,r'}(R^{-1})}_{} O_{\Gamma,r'} \quad R \in O^{(2)}, \qquad IO_{\Gamma,r}I = (-1)^P O_{\Gamma,r}$$

discrete R

T(R) given for all irreps in
Bernard, Lage, Meißner, Rusetsky,
JHEP 2008, 0806.4495
We use same conventions for rows.

J	Γ (dim$_\Gamma$)
0	$A_1(1)$
$\frac{1}{2}$	$G_1(2)$
1	$T_1(3)$
$\frac{3}{2}$	$H(4)$
2	$E(2) \oplus T_2(3)$
$\frac{5}{2}$	$H(4) \oplus G_2(2)$
3	$A_2(1) \oplus T_1(3) \oplus T_2(3)$

Method I: Projection operators

$$O_{|p|,\Gamma,r,n} = \sum_{\tilde{R}\in O_h^{(2)}} T^{\Gamma}_{r,r}(\tilde{R})\ \tilde{R}\ H^{(1),a}(p)\ H^{(2),a}(-p)\ \tilde{R}^{-1}\ , \qquad n=1,..,n_{max}$$

T(R) given for all irreps in
Bernard, Lage, Meißner, Rusetsky, JHEP 2008, 0806.4495

"seed": Each H^a can have any polarization m_s and direction p with given |p|. Different choices lead to different linearly independent O_n

Some examples for |p|=1:

PV in T_1^+ , n_{max}=2:

$$O_{T_1^+,r=3,n=1} = \mathrm{P}(e_z)V_z(-e_z) + \mathrm{P}(-e_z)V_z(e_z)$$
$$O_{T_1^+,r=3,n=2} = \mathrm{P}(e_x)V_z(-e_x) + \mathrm{P}(-e_x)V_z(e_x) + \mathrm{P}(e_y)V_z(-e_y) + \mathrm{P}(-e_y)V_z(e_y)$$

PN in H^+ , n_{max}=1:

$$O_{H^+,r=1} = -iN_{\frac{1}{2}}(-e_x)\mathrm{P}(e_x) + iN_{\frac{1}{2}}(e_x)\mathrm{P}(-e_x) - N_{\frac{1}{2}}(-e_y)\mathrm{P}(e_y) + N_{\frac{1}{2}}(e_y)\mathrm{P}(-e_y)$$

VN in H^- , n_{max}=3:

$$O_{H^-,r=1,n=1} = iN_{\frac{1}{2}}(e_x)V_x(-e_x) + iN_{\frac{1}{2}}(-e_x)V_x(e_x) + N_{\frac{1}{2}}(e_y)V_y(-e_y) + N_{\frac{1}{2}}(-e_y)V_y(e_y)$$
$$O_{H^-,r=1,n=2} = ...$$
$$O_{H^-,r=1,n=3} = ...$$

<u>Disadvantage:</u>

not informative which continuum numbers (partial wave L or helicity) each O_n corresponds

This is remedied in next two method.s

Method II: Partial-wave operators

Starting annihilation operator
(before subduction to irreps)

$$O^{|p|,J,m_J,S,L} = \sum_{m_L,m_S,m_{s1},m_{s2}} \underbrace{C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1 m_{s1},s_2 m_{s2}}}_{\text{Clebsch-Gordans}} \sum_{R\in O} \underbrace{Y^*_{Lm_L}(\widehat{Rp})}_{\text{Spherical Harmonics}} \underbrace{H^{(1)}_{m_{s1}}(Rp)\ H^{(2)}_{m_{s2}}(-Rp)}_{\text{building blocks H mentioned on slide 6 below}}$$

Proposed for NN in [Berkowitz, Kurth, Nicolson, Joo, Rinaldi, Strother, Walker-Loud, CALLAT, 1508.00886] There Y_{lm}^* appears where we have Y_{lm}

Proof (in our paper and backup slides): the correct transformation properties

$$R_a O^{J,m_J,S,L} R_a^{-1} = \sum_{m'_J} D^J_{m_J m'_J}(R_a^{-1}) O^{J,m'_J,S,L}$$

follow from transformations of H (slide 4) and properties of C, Y_{lm} and D.

Example of PV operators

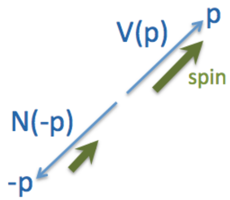
$$O^{|p|=1,J=1,m_J=0,L=0,S=1} = \sum_{p=\pm e_x,\pm e_y,\pm e_z} \mathrm{P}(p)V_z(-p)\ ,$$

$$O^{|p|=1,J=1,m_J=0,L=2,S=1} = \sum_{p=\pm e_x,\pm e_y} \mathrm{P}(p)V_z(-p) - 2\sum_{p=\pm e_z} \mathrm{P}(p)V_z(-p)$$

Subduction to irreps discussed later on.

Method III: helicity operators

[HH in continuum: Jacob, Wick (1959)]
[for single H on lattice: HSC, Thomas et al. (2012)]
[not widely used for HH on lattice yet]



- building blocks in partial-wave operators are $H_{ms}(p)$ and m_s is not good for p≠0:
- Helicity λ is projection of S to p. It is good also for particles in flight $$h \equiv S \cdot p \;/\; |p|$$
- Definition of single-hadron helicity operator

  rotation from p_z to p

  $$H^h_\lambda(p) \equiv R^p_0 \; H_{m_s=\lambda}(p_z) \; (R^p_0)^{-1}$$

  denoted by superscript h

  good m_s
- Helicity is not modified under R (p and S transform the same way) $$R H^h_\lambda(p) R^{-1} = e^{i\varphi(R)} H^h_\lambda(Rp)$$
- Two-hadron O: $$\boxed{O^{|p|,J,m_J,\lambda_1,\lambda_2,\lambda} = \sum_{R \in O^{(2)}} D^J_{m_J,\lambda}(R) \; R \; H^{(1),h}_{\lambda_1}(p) \; H^{(2),h}_{\lambda_2}(-p) \; R^{-1}}$$

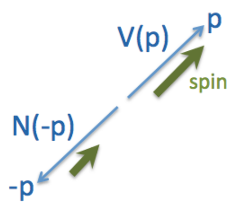
  p is arbitrary momentum in given shell |p|; R does not modify $\lambda_{1,2}$, so $H_{1,2}$ have chosen $\lambda_{1,2}$ in all terms
- Proof:

$$\begin{aligned}
R_a O^{J,m_J,\lambda_1,\lambda_2} R_a^{-1} &= \sum_{R \in O^{(2)}} D^J_{m_J,\lambda}(R) \; R_a R \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R^{-1} R_a^{-1} \\
&= \sum_{R \in O^{(2)}} D^J_{m_J,\lambda}(R_a^{-1} R') \; R' \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R'^{-1} \\
&= \sum_{R' \in O^{(2)}} \sum_{m'_J} D^J_{m_J,m'_J}(R_a^{-1}) D^J_{m'_J,\lambda}(R') \;\; R' \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R'^{-1} \\
&= \sum_{m'_J} D^J_{m_J,m'_J}(R_a^{-1}) \; O^{J,m'_J,\lambda_1,\lambda_2} \; ,
\end{aligned}$$

$R' = R_a R$

$$D(R_1 R_2) = D(R_1) D(R_2)$$

Method III: helicity operators (continued)



Using definitions of $H^h_\lambda(p) \equiv R^p_0 \; H_{m_s=\lambda}(p_z) \; (R^p_0)^{-1}$ and parity projection $\frac{1}{2}(\mathcal{O} + PI\mathcal{O}I)$

$$O^{|p|,J,m_J,P,\lambda_1,\lambda_2,\lambda} = \frac{1}{2}\sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R) \; RR^p_0 \; [H^{(1)}_{m_{s_1}=\lambda_1}(p_z) H^{(2)}_{m_{s_2}=-\lambda_2}(-p_z)$$

$$+ P \; I \; H^{(1)}_{m_{s_1}=\lambda_1}(p_z) H^{(2)}_{m_{s_2}=-\lambda_2}(-p_z) \; I] \; (R^p_0)^{-1} R^{-1}$$

$$\lambda = \lambda_1 - \lambda_2$$

- H are building blocks from slide 6 below: actions of R and I on $H_{ms}(p)$ are given in slide 4
- There are several choices of R_0^p which rotate from p_z to p:
  - these lead to different phases in definition of H_λ^h : inconvenience
  - but they lead to the same O above (modulo irrelevant overall factor): so no problem for such construction
- Simple choice for momentum shell |p|=1 : $p=p_z$ and R_0^p=Identity
- paper provides details how to use functions from Mathematica for construction, also since Mathematica uses non-conventional defnition of D

$$D^j_{m,m'}[R^\omega_{\alpha\beta\gamma}] = F \cdot \mathtt{WignerD}[\{j,m,m'\},-\alpha,-\beta,-\gamma], \qquad F = \begin{cases} 1 \;:\; j = \text{integer} \\ \pm 1 \;:\; j = \text{halfinteger}, \;\; \mathrm{F}(\omega+2\pi) = -\mathrm{F}(\omega) \end{cases}$$, choice of sign in our paper

$$\{\alpha,\beta,\gamma\} = \underbrace{\mathtt{EulerAngles}}_{\text{MATHEMATICA}}[T] \quad T = \exp(-i\vec{n}\vec{J}\omega) \text{ and } (J_k)_{ij} = -i\epsilon_{ijk}$$

one last step before reaching the results ...

Subduction of O^J to irreducible representations

subduction

continuum R

discrete R in discrete group $O^{(2)}$

Partial-wave operators $O^{J,\, mJ,\, L,\, S}$

$$O^{[J,S,L]}_{|p|,\Gamma,r} = \sum_{m_J} \mathcal{S}^{J,m_J}_{\Gamma,r} O^{|p|,J,m_J,S,L}$$

Helicity operators $O^{J,\, mJ,\, \lambda1,\, \lambda2}$

$$O^{[J,P,\lambda_1,\lambda_2,\lambda]}_{|p|,\Gamma,r} = \sum_{m_J} \underbrace{\mathcal{S}^{J,m_J}_{\Gamma,r}}_{\text{Subduction matrices S}} O^{|p|,J,m_J,P,\lambda_1,\lambda_2,\lambda}$$

The representation O^J is irreducible under continuum R.

But it is reducible under R in discrete group lattice $O^{(2)}$.

Operators that transform according to irrep Γ and row r obtained via subduction.

Subduction matrices S

[Dudek et al., PRD82, 034508 (2010)

Edwards et al, PRD84, 074508 (2011)]

Single-hadron operators H: experience by Hadron Spectrum collaboration Phys. Rev. D 82, 034508 (2010)

- subduced operators $O^{[J]}{}_{\Gamma}$ carry memory of continuum spin and dominantly couple to states with this J

Expectation for partial-wave and helicity operators HH obtained by subduction :

- $O^{[J,S,L]}_{|p|,\Gamma,r}$ would dominantly couple to eigen-states with continuum (J,L,S)
- $O^{[J,P,\lambda_1,\lambda_2,\lambda]}_{|p|,\Gamma,r}$ would dominantly couple to eigen-states with continuum (J,λ1,λ2)

valuable for simulations

give physics intuition on quant. num.

P(1)V(-1) operators, T_1^+, row=r=1

T_1^+ :

projection op.

$$\begin{cases} O_{T_1^+,r=1,n=1} = \mathrm{P}(e_x)V_x(-e_x) + \mathrm{P}(-e_x)V_x(e_x) \\ O_{T_1^+,r=1,n=2} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) + \mathrm{P}(e_z)V_x(-e_z) + \mathrm{P}(-e_z)V_x(e_z) \end{cases}$$

partial-wave op.

$$\begin{cases} O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2} \\ O^{[J=1,L=2,S=1]}_{T_1^+,r=1} = -2\ O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2} \end{cases}$$

$J^P=1^+$, S=1,L=0

$J^P=1^+$, S=1,L=2

helicity op.

$$\begin{cases} O^{[J=1,P=+,\lambda_V=0,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1} \\ O^{[J=1,P=+,\lambda_V=\pm 1,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=2} \end{cases}$$

$J^P=1^+$, $\lambda_V=0$

$J^P=1^+$, $\lambda_V=1$

Partial-wave and helicity operators expressed in terms of projection operators throughout.

N(1)P(-1) operators, G_1^+, row=r=1, p-wave

projection op.

$$O_{G_1^+,r=1} = N_{-\frac{1}{2}}(-e_x)\mathrm{P}(e_x) - N_{-\frac{1}{2}}(e_x)\mathrm{P}(-e_x) - iN_{-\frac{1}{2}}(-e_y)\mathrm{P}(e_y) + iN_{-\frac{1}{2}}(e_y)\mathrm{P}(-e_y) + N_{\frac{1}{2}}(-e_z)\mathrm{P}(e_z) - N_{\frac{1}{2}}(e_z)\mathrm{P}(-e_z)$$

partial-wave op.

$$O^{[J=\frac{1}{2},m_J=\frac{1}{2},L=1,S=\frac{1}{2}]}_{G_1^-,r=1} = O_{G_1^+,r=1}$$

$J^P=1/2^+$, $S=1/2$, $L=1$

helicity op.

$$O^{[J=\frac{1}{2},m_J=\frac{1}{2},P=+,\lambda_N=\pm\frac{1}{2},\lambda_P=0]}_{G_1^+,r=1} = O_{G_1^+,r=1}$$

$J^P=1/2^+$, $\lambda_N=1/2$

employed in lattice simulation of
π N scattering in p-wave
to study the Roper channel with $J^P=1/2^+$

[Lang, Leskovec, Padmanath, S.P., arxiv:1610.01422]

Explicit expressions for all $H^{(1)}(p)H^{(2)}(-p)$

PV, PN, VN, NN

all irreps, $|p|=0,1$

consistent results found in three methods

given in [S. Prelovsek, U. Skerbis, C.B. Lang, arXiv: 1607:06738]

Operators valuable to study resonances with heavy quarks

PV: meson resonances and $\underline{Q}Q$-like exotics (e.g. π J/ψ, D $\underline{D}$* for Z_c)

PN: baryon resonances (e.g. π N for N*, K N ...) and pentaquarks

NV: baryon resonances and pentaquarks (e.g. p J/ψ for P_c)

NN: nucleon-nucleon and deuterium, baryon-baryon

Conclusions

I have discussed

- first study of charmonium resonance that takes into account its strong decay
- search for an exotic state X(5568) that contains four different flavors in elastic scattering: we did not find evidence for it from lattice, in agreement with recent LHCb result
- construction of lattice operators for simulating scattering of two-hadrons with spin: neccessary to extract information on resonances that appear in PV, PN and VN, NN scattering

Backup

Lattice setup

PACS-CS

	Ensemble (1)	Ensemble (2)
$N_L^3 \times N_T$	$16^3 \times 32$	$32^3 \times 64$
N_f	2	2+1
a [fm]	0.1239(13)	0.0907(13)
L [fm]	1.98(2)	2.90(4)
m_π [MeV]	266(3)(3)	156(7)(2)

- Wilon-clover quarks
- Fermilab method for c and b : [El Khadra, Kronfeld et al, 1997]

  Rest hadron energies have sizable discretization errors but these largely cancel in splittings.

  Only splittings with respect to a chosen reference mass are compared to experiment.

- evaluating Wick contractions to simulate scattering on the lattice is challenging and computationally intensive – that is part of the reason why a small number of studies have been made. We apply two methods

  - distillation (Ensemble 1) [Peardon et. al., HSC, 2009]
  - stochastic distillation (Ensemble 2) [Morningstar et al., 2011]