

Letter of interest for

Hadron Spectroscopy with Lattice QCD

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SNOWMASS21–RF7_RF0–TF5_TF4–052

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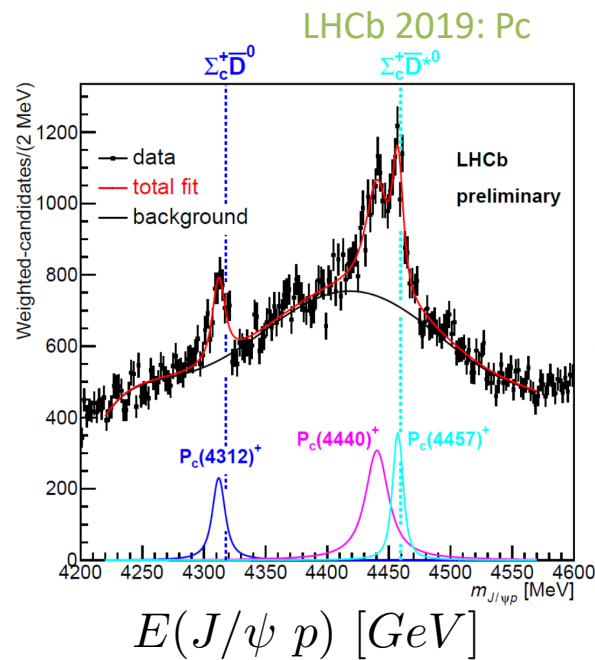
University of Regensburg

2th October 2020

townhall meeting

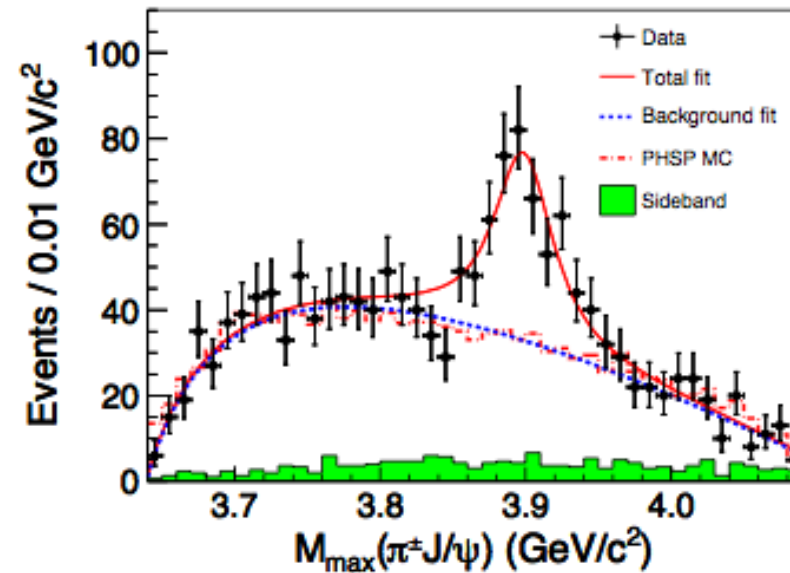
Snowmass, RF7

Experimentally discovered exotic hadrons decay strongly



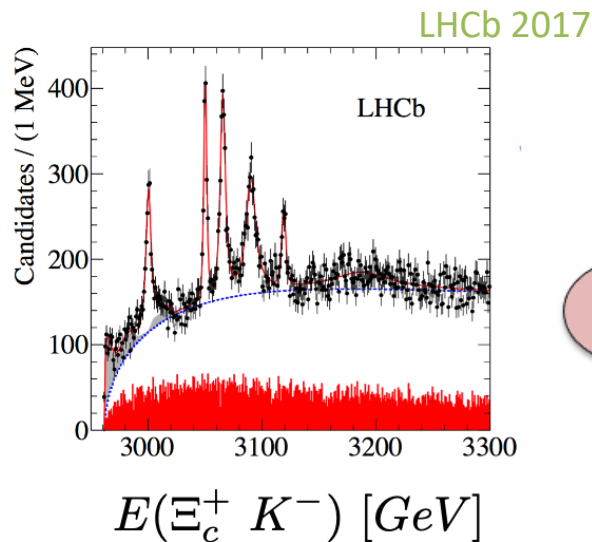
$\bar{c}c u u d$

Bess III, Belle 2013 Z_c



$\bar{c}c d u$

Most of conventional hadrons decay strongly



$\bar{c} s s$

many many more

Non-perturbative theory approach: QCD on lattice

x, t (Minkovsky) \rightarrow x, i, t (Euclidean)

$$L_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$$

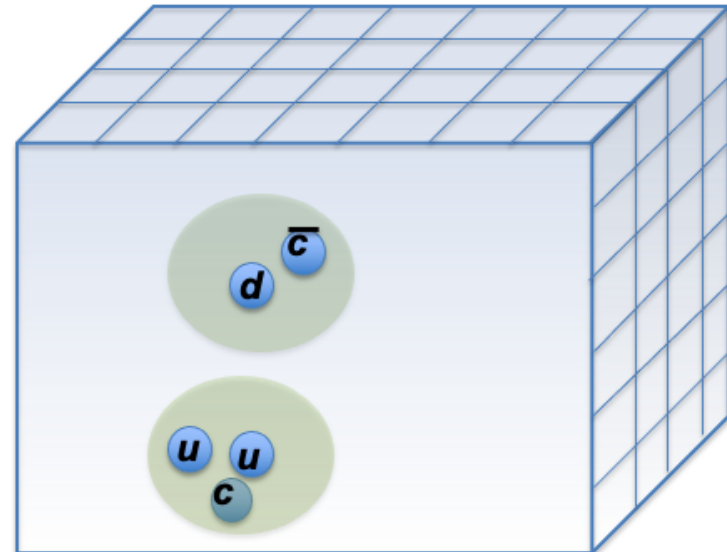
input: g_s , m_q

output: hadron properties

hadron interactions

Evaluation of Feynman path integrals
in discretized space-time

$$\int DG Dq D\bar{q} e^{-S_{QCD}/\hbar}$$



Conventional and exotic hadrons with lattice QCD

resonances:

- not QCD eigenstates
- inferred from decay products (like in exp)

$$H_1 H_2 \rightarrow R \rightarrow H_1 H_2$$

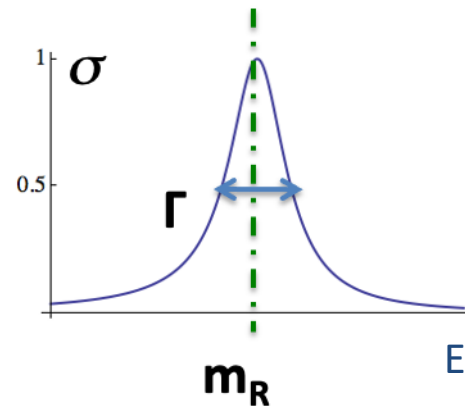
scattering amplitudes T



cross sections, poles of T



masses and widths of R



Lol:

- solved problems
- challenges to address

"Solved problems" in lattice QCD

more difficult ↑

Resonances that have only one strong decay channel

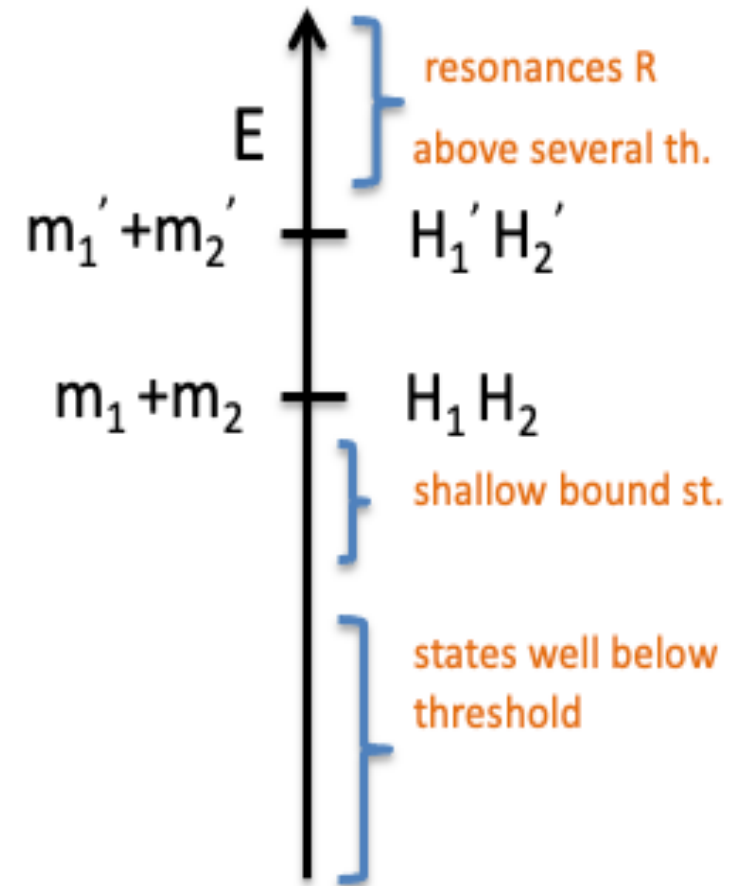
- masses
- widths $R \rightarrow H_1 H_2$

Hadrons that are slightly below threshold:

- masses

Hadrons that can not decay strongly (well below threshold):

- masses
- transition matrix elements $\langle H_f | J | H_i \rangle$



Status

$$R \rightarrow H_1 H_2, H_1' H_2'$$

"difficult": - decay via two or three channels:

T for coupled-channel scattering

[Hadron Spectrum collaboration

extracted coupled-channel T for several channels]

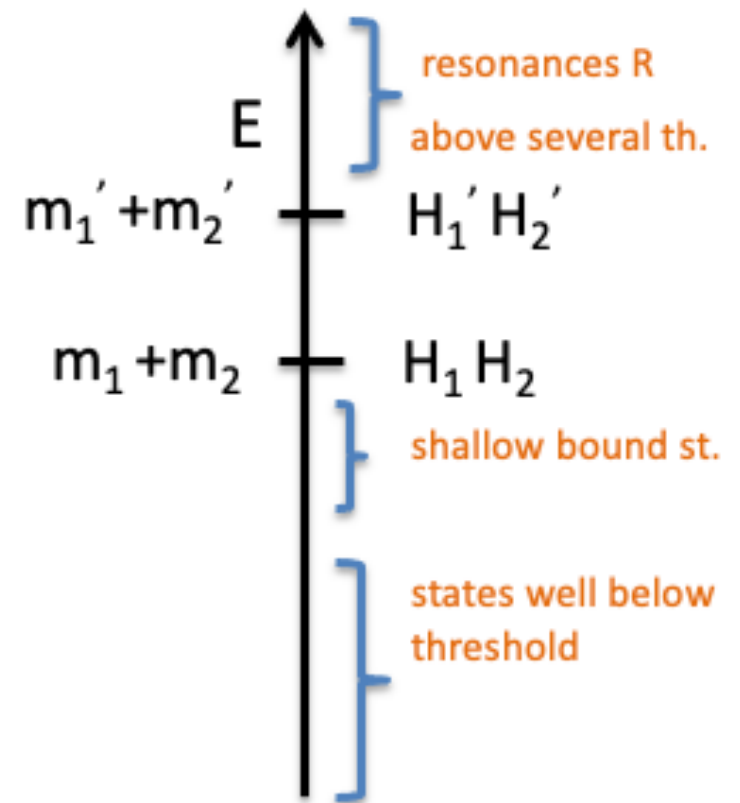
"too challenging" (for now):

- decay to more than three channels

(if none of them can be "neglected")

- if $H_1 H_2 H_3$ decays are important

in addition to two-hadron decays



Lol: challenges to focus on

- Identify certain exotic hadrons that could be studied reliably on lattice and in exp

exp. discovered exotic hadrons

P_c , Z_c , $X(6900)$, ...



difficult for lattice QCD

(several strong decay channels)

some exotic hadrons identified on lattice

$b\bar{b}\bar{u}d$, $b\bar{b}\bar{u}s$



(too) difficult for experiment

those two not discovered yet

- hadrons that can decay via two or three channels, including those with heavy quarks

$$R \rightarrow H_1 H_2, H_1' H_2'$$

- hadrons that can decay via more than three channels

- try to find simplifications

- determine which channels could be treated as decoupled

- hadrons that have two-particle and three-particle decay channels:

- try to simulate

- look for simplifications

$$R \rightarrow H_1 H_2, H_1' H_2' H_3'$$

Lol: challenges to focus on (continued)

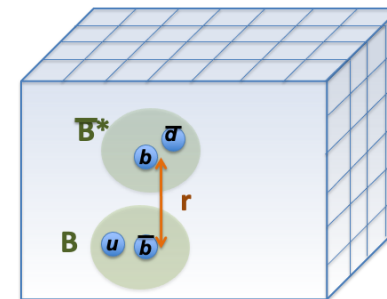
- identify reliable criterion on the importance of molecular and diquark configurations
- can overlaps $\langle O_i | n \rangle$ give rigorous info on structure of hadrons

} currently qualitative

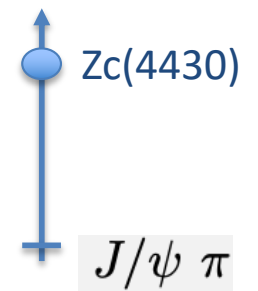
- vary masses of quarks u/d, s, c, b
explore position of the resonances with respect to thresholds
(one can not do it in experiment)

$$m_1 + m_2 \quad \text{---} \quad \text{---} \quad H_1 H_2$$


- QQ... or QQ... systems: determine Born Oppenheimer potentials
 - $V(r)$ on the lattice, $r > a$
 - $V(r)$ analytically at small $r < a$



- Lattice study of a high-lying resonance R: all energy region below it has to be extracted
 - find some approach that could address just a certain higher-lying energy region
 - this does not seem viable with currently used lattice methods



Lol: challenges to focus on (continued) & relation to other efforts

- determine relevant $\langle R_f | J | H_i \rangle$ and $\langle H_f | J | R_i \rangle$ J=EM or weak current

only $\langle \rho | J^{EM} | \pi \rangle$ determined [first by Had. spec. coll]

relation to other Snowmass efforts

$B \rightarrow K^* l^+ l^-$ (new physics search) , $B \rightarrow D^* l^+ l^-$ (lepton fl. universality) , $D \rightarrow \rho l \nu$ (semileptonic decays)

$\langle K^* | J | B \rangle$

$\langle D^* | J | B \rangle$

$\langle \rho | J | D \rangle$

- Improve analytical methods to extract poles of scattering amplitudes

relation to scattering amplitudes Snowmass efforts

- Relation to Theory Snowmass efforts
- Relation to Computational Snowmass efforts

Schedule

- march 2021: contact Lol authors and inquire who will actively contribute
- april 2021: discuss physics of white paper
- may, june 2021: write white-paper

What would I like to come out of the Snowmass process ?

- contribute to a (partial) solution of at least few of the given challenges
- it would be very welcome if scientists seriously think about some suggested simplifications

otherwise community will not be able to provide lattice QCD results for most interesting states soon

- identify some exotic hadrons that can be reliably studied in lattice and exp

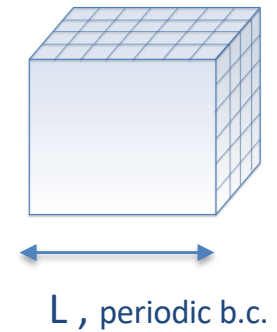
Conclusions

- lots of very interesting results on exotic and conventional hadrons from experiment
- many urgent challenges to resolve in ab-initio theoretical studies of these states

Backup

lattice QCD extracts energies E_n of eigenstates

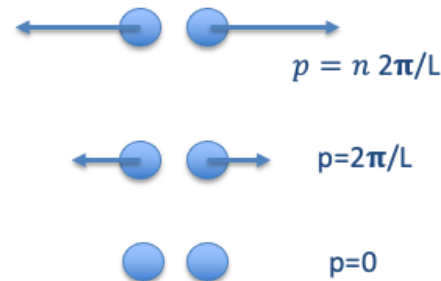
$$R \rightarrow H_1 H_2$$



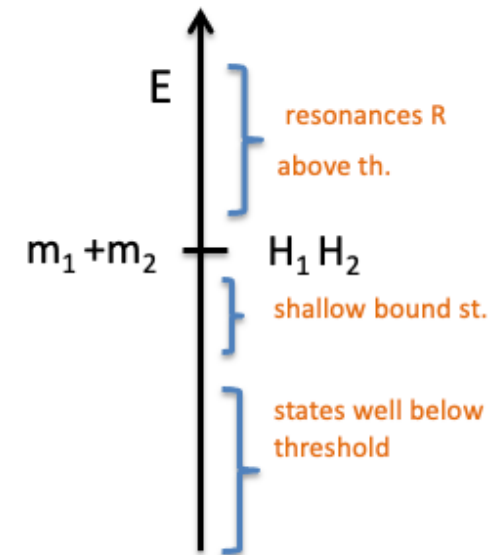
$E^{n.i.}$ (in non-interaction limit)

$$E^{n.i.} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$

$H_1 H_2$ (cm frame)



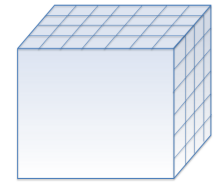
$$E = m_1 + m_2$$



one has to extract ALL eigen-energies below the energy of interest
(can not focus on just particular higher-lying energy window like in exp)

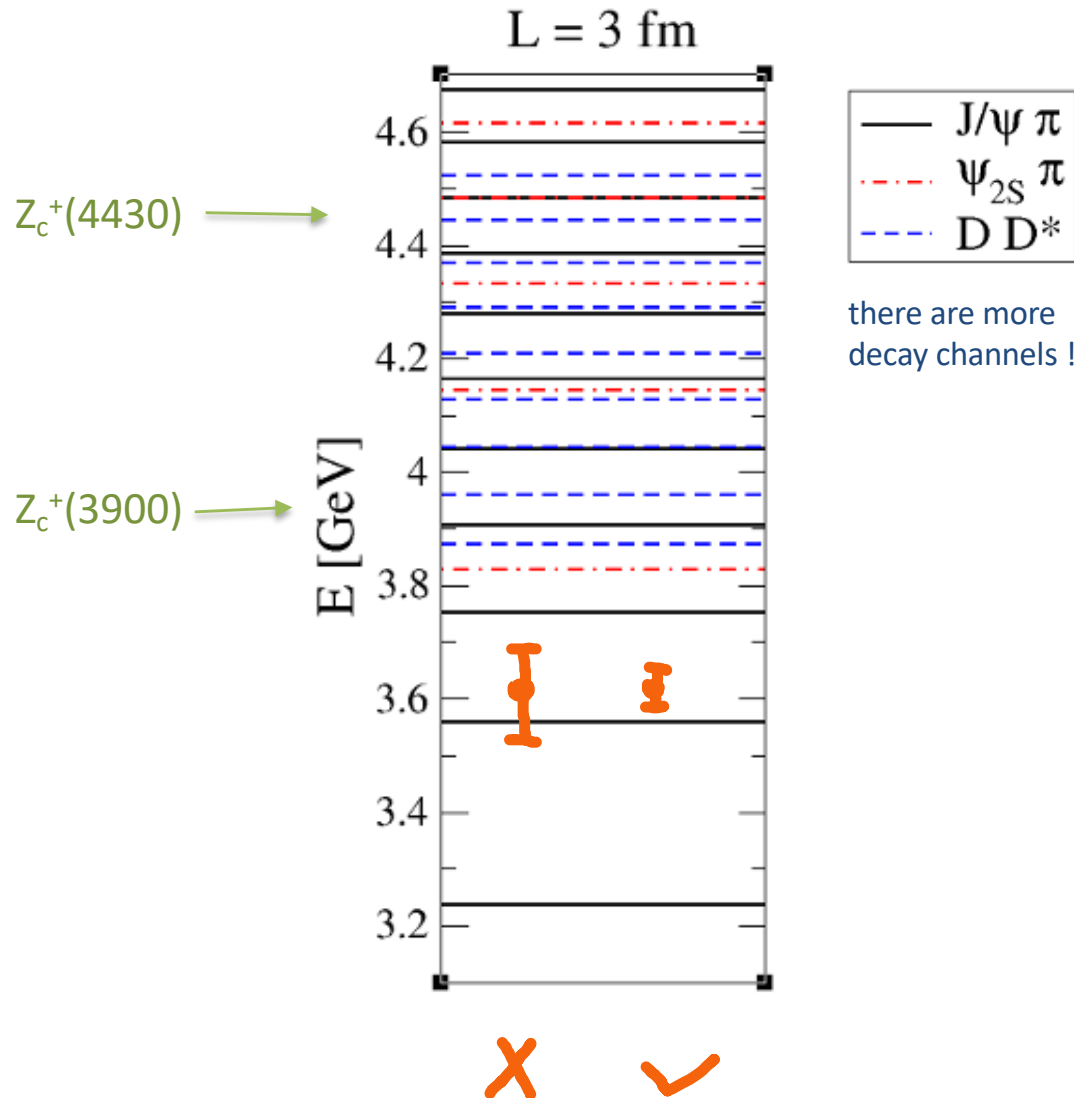
exp ($L=\infty$): continuous spectrum above th., lat ($L=\text{finite}$): discrete spectrum

Example: channel Z_c , $J^P=1^+$



plot of non-interacting energies

$$E^{n.i.} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} \quad , \quad p = n 2\pi/L$$



challenge:

very accurate determination of E is needed

higher E have larger statistical noise

One needs to resolve

$$\Delta E = E - E^{n.i.}$$

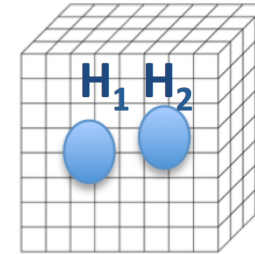
Scattering matrix T can be determined

only when ΔE are resolved

Resonance above one threshold

$$R \rightarrow H_1 H_2 \quad T(E) \quad \leftarrow E_n$$

Luscher's method



Lattice simulation of one-channel scattering via Luscher's method: doable

Resonance above two or more thresholds

most of exotic hadrons are above more than one threshold:
for example Zc(4430), X(6900), Zb

$$R \rightarrow H_1 H_2, H_1' H_2'$$

channel a : $H_1 H_2$
channel b : $H_1' H_2'$

$$T(E) = \begin{bmatrix} \overset{a \rightarrow a}{T_{aa}(E)} & \overset{a \rightarrow b}{T_{ab}(E)} \\ \overset{b \rightarrow a}{T_{ab}(E)} & \overset{b \rightarrow b}{T_{bb}(E)} \end{bmatrix} \quad \leftarrow E_n$$

Luscher's method

$\det[T(E) - f(E)] = 0$: at given E : one equation, three unknowns

↓
known f