Modeling Traffic on Networks as Complex Dynamical System

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Outline

CONTRIBUTORS

- Marija Mitrović, Spectral analysis, Blogs
- Stefan Thurner, Geoff Rodgers, Bernard Kujawski, Modeling Information Traffic on Networks
- Hong-Li Zeng, Applications to City Traffic

Outline

PHYSICS OF THE INTERNET

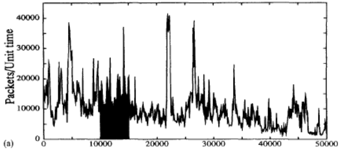
▷Internet is a *complex dynamical system*, endogeneously driven (by users); clustered-SF network structure; known protocols for information traffic;

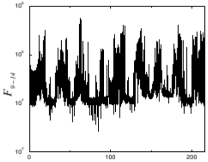
▷Research concerns:

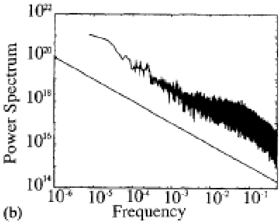
- Micro Rules ⇒ Macro Behavior (emergence! control?);
- Recognize relevant rules & quantities to describe the behavior;
- Complexity: Fractal time series; Long-range correlations; Power-law PDFs
- Scale-invariance (of the processes): Stable laws (SLP) & Reduced number of control parameters!

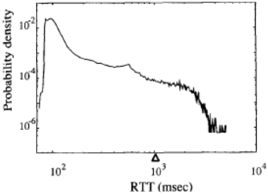
Empirical Evidence of complexity in traffic on networks

Internet traffic measurements: rtt, flow, jamming; time series









[Takayasu (1996), Moreno (2004), Barthelemy (2002), ...]

MODELING TRAFFIC & NETWORK STRUCTURE

- Minimal rules to take into account the traffic complexity (produce statistically the same type of behavior as in the empirical data)
- Understand the role of the network structure in the process (growing different network topologies/with given properties and simulating traffic on these different networks)

INFORMATION TRAFFIC ON NETWORKS: MINIMAL RULES

MODEL*:

Traffic of information packets on networks of $N = 1000$ nodes and given topology (cSF,seG); Model with:

- Creation and assignment; (Rate: R)
- Navigation; (Local; Depth $r = 2$)
- Queuing; (LIFO queue; Buffer: $H = 1000$)
- delivery (at destination; traffic stationarity)

PARAMETERS:

- Creation rate R (alt. constant density ρ);
- Buffer size H; Searched depth $r = 2$;

[Tadić & Rodgers, Adv. Compl. Syst. (2002), Tadić, LNCS (2003)]

STATISTICAL MEASURES OF TRAFFIC

Statistical properties of traffic are monitored at *local* (individual Nodes and Edges) and *global* level of the entire network:

(a) Traffic Noise Signals and Power Spectra:

- Number pf packets processed by a node $\{h_i(t)\}$ w. T_{WIN};
- Number pf packets processed by an edge $\{f_{ij}(t)\}$ w. T_{WIN};
- Network load; Network delivery rate; ...

(b) various PDFs:

- Transit time of packets $P(T)$ (rtt);
- Waiting times in queues $P(t_W)$;
- Time intervals (return times) $P(\Delta t)$ to Nodes;Edges;
- Flow and Noise distributions; ...

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NETWORK STRUCTURES & TRAFFIC DENSITY

▷ Packet interactions with queuing at nodes along the path:
▷ Work by BCN group [Guimerà et al (2002)]—small network reconstruction to *minimize transport time*: *two basic structures emerge*:

- Low traffic density: Highely Clustered Scale-Free Graph;
- High traffic density: Homogeneous (unclustered) Network;

▷ Here we simulate traffic on two prototype networks: Clustered SF (WebGraph) and Homogeneous (StatNet), with $N = 1000$ nodes, and *traffic rules with local navigation and queuing!*.

[Tadić, Rodgers & Thurner: Int. J. Bifurcation and Chaos (2007)]

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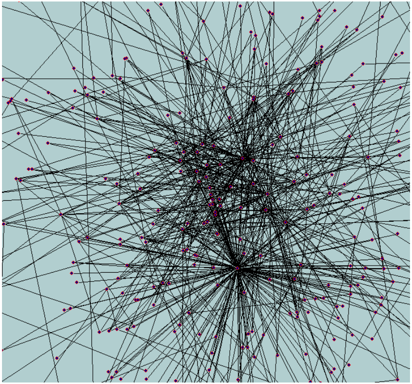
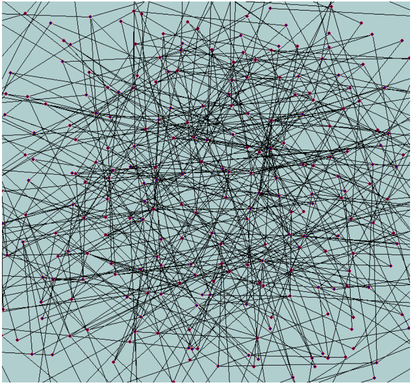
[Tadić, Rodgers & Thurner: Int. J. Bifurcation and Chaos (2007)]

Two Network Structures

Low-traffic-density: cSF; *High-traffic-density: Homogeneous*



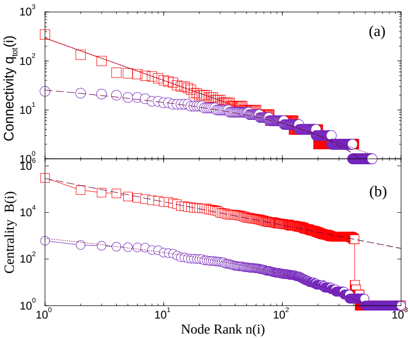
WebGraph



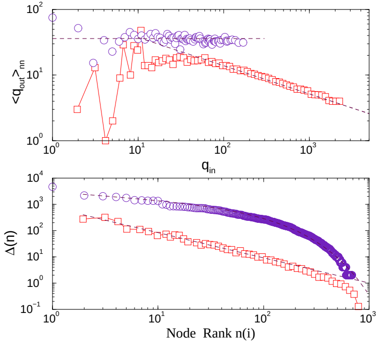
StatNet

Correlations in Network Structures

▷Networks' topol.properties, relevant to traffic:



Connectivity; Centrality



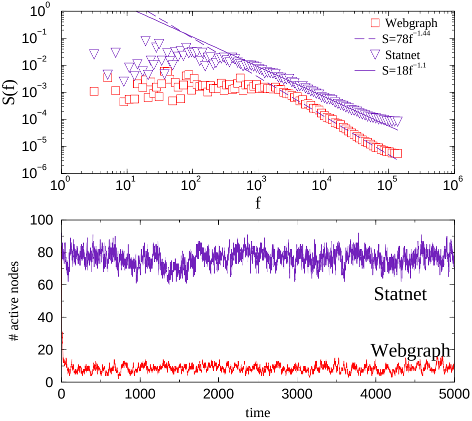
Correlations; Clustering

[Tadić, in "Systems self-assembly", elsevier (2008)]

Outline

1. Information Traffic on Fixed Network
   - Motivation, Model, Sampled Quantities
   - Collective Behavior in Traffic
   - Determining Statistical Indicators of Traffic
2. Network Spectra: Tools Between Structure & Dynamics
   - Outline of Spectral Methods, Example
   - Dynamical Communities in City Traffic
3. Networks from Dynamical Data
   - Blogs: Mapping Data to Networks
   - Structure of Communities

STATIONARITY of TIME SERIES



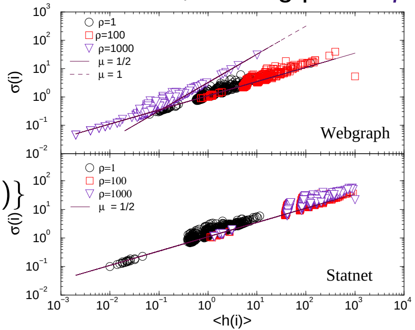
$\rho = const$: 2 Nets

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007) review]

more from TIME SERIES

Time-series of number of packets within a given time window processed by each node (or a link [Refs.]) in the network contain useful information about traffic; Scaling plot *dispersion*

vs. average of set $\{h_i(t_k)\}$

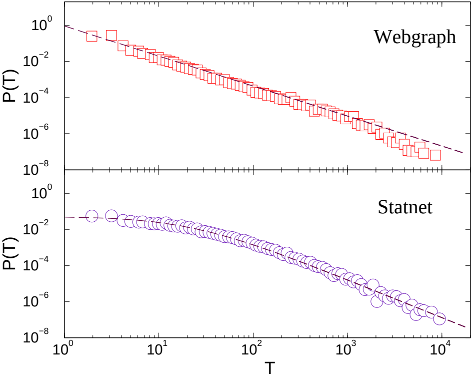


$\rho = const$: 2 Nets

[Kujawki et al. New J. Phys. 2007]

Outline

PDFs TIME STATISTICS: Travel times

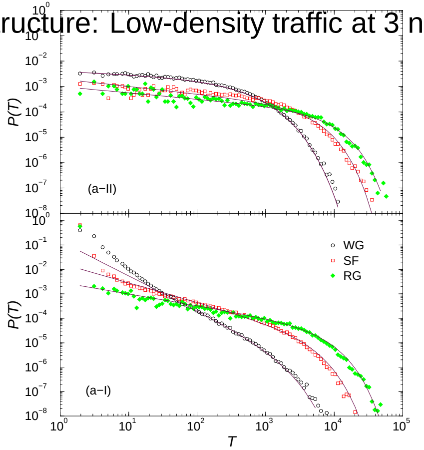


Levy; q-exponential

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

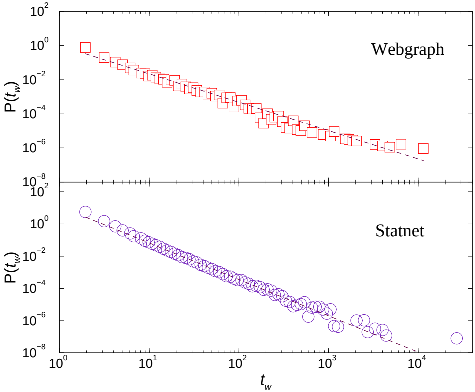
RTT on different TOPOLOGIES: Search vs. RW

Effects of network structure: Low-density traffic at 3 networks



[Tadić and Thurner, Physica A, Vol. 332, pp. 566 (2004)]

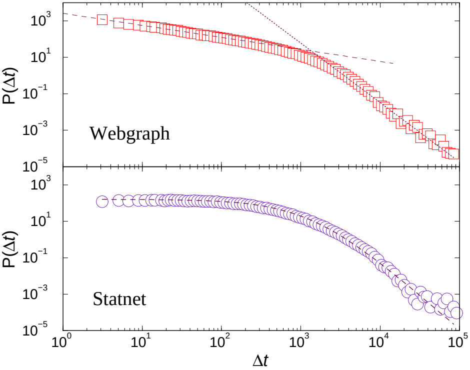
PDFs QUEUING STATISTICS: Waiting times



Power-laws: ρ-dependent!

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos, 2007]

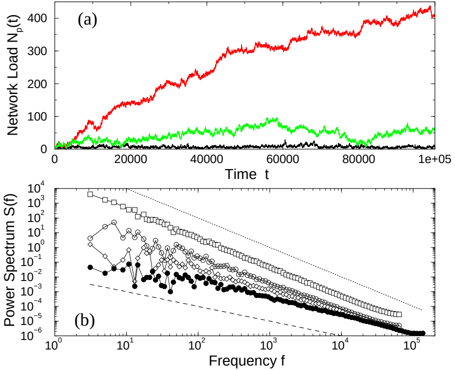
PDFs TIME STATISTICS: Return times



two slopes; q-exponential

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos, 2007]

TIME SERIES ANALYSIS: const.RATE & JAMMING

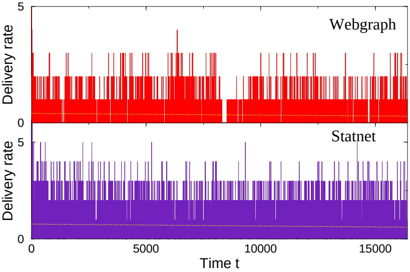


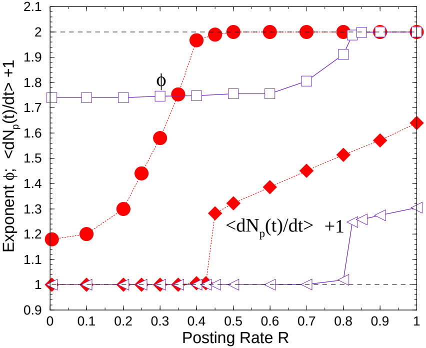
R varied: WG; Power-Spectrum de-correlations!

[Tadić, Thurner & Rodgers, Physical Review E, Vol. 69, 036102 (2004)]

Jamming on Networks: OrderParameter

Def. jamming rate: $J \equiv < dN_p(t)/dt >= R - \lambda$





delivery rate λ

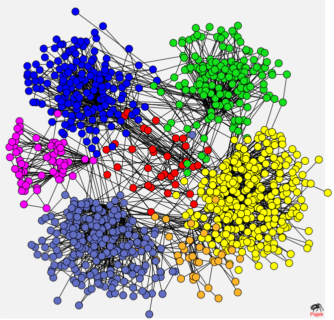
OP: 2 Nets

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

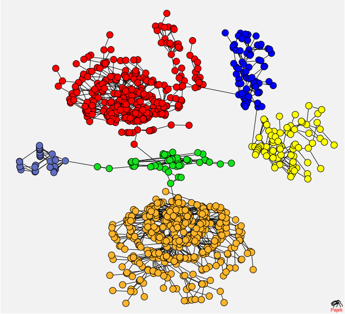
Outline

CASE OF MODULAR NETWORKS

▷Network modularity ⇔ Function; //Topological vs. Dynamical Modules



Clustered mdular network

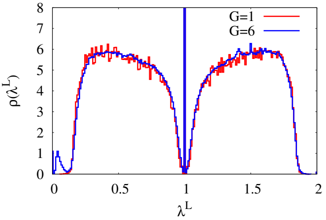


Tree-of-trees

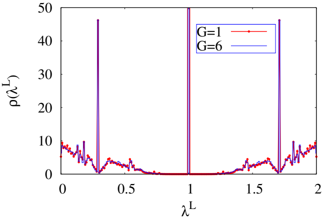
[Mitrović & Tadić: Physical Review E, vol. 80, 026123 (2009)]

SPECTRAL DENSITY vs. MODULARITY

▷Eigenvalue Spectrum & EV.Vectors have characteristic behavior on modular networks ↔ ways to detect modules



Clustered modular: M=2



Trees

[spectra of modular networks: Phys.Rev.E vol. 80, 026123 2009]

EV.LOCALIZATION

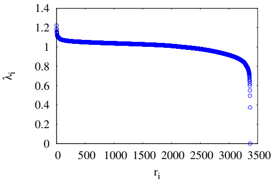
Adjasency matrix $\rightarrow$ Laplacian

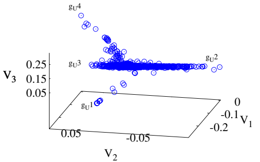
$$L_{ij} = \delta_{ij} - \frac{A_{ij}}{\sqrt{\ell_i \ell_j}}$$

$$\ell_i \equiv \sum_j A_{ij}$$

includes weights;
Eigenvectors $\lambda_i \sim 0$
Localize on network modules;

Example:





Outline

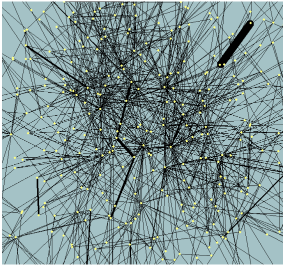
Application of the model:

Discrete-time dynamics on dual graph $\Leftrightarrow$ Queuing in city traffic

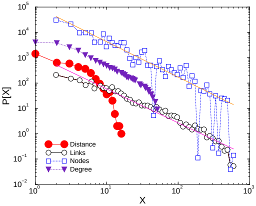
CITY MAP & DUAL GRAPH

Given geographical area of city with known streets & crossings
⇒ Dual graph [Refs.]; Topology of the graph fixed: measures;

map:



dual graph



topology

[H-L. Zeng et al., IEEE-Xplore 2009; Latora et al. PHYSA 2007]

(modified)TRAFFIC RULES

Traffic model with Local Navigation within *nnn* & Queuing:

- (c) Creation Rate R: pref. at large streets;
- (a) Assignement destination: random or preferred;
- (q) Queuing: FIFO; Max.queue-length $\sim$ street length;
- (n) Navigation: nnn-search *else* random (OR)

FEEDBACK RULE:

- (c),(a),(q) as above
- (n) Navigation depends on recent history of congestion: within nnn, if queue is above a threshold value, avoid that node, probabilistically! (FB)

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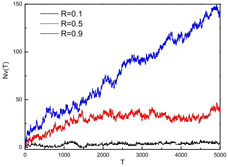
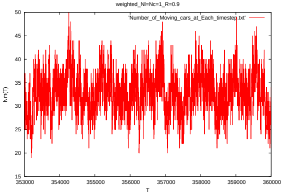
TRAFFIC TIME SERIES

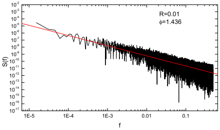
Traffic signals: temporal fluctuations of *number of cars on the network*, number of moving/waiting, *number processed by each node/edge in T_{WIN}*, ... are fractal t.s.; depending on density;



network Load



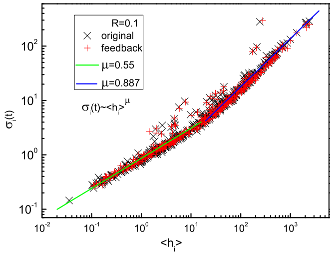
Moving Cars



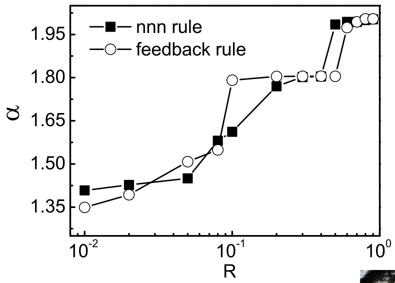
Correlations: PS

DIFFERENTIATING DYNAMIC REGIMES

Analysis of t.s. reveals two classes of nodes (by their participation in the dynamics) and three different dynamical regimes (by degree of correlations, R-dependent), different for OR and FB rule:



scaling: fixed R



varying PS exponent

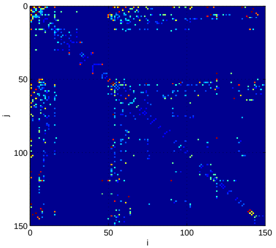
SPECTRAL ANALYSIS of weighted Flow Matrix

Use normalized Laplacian matrix (diffusion):

$$L_{ij} = \delta_{ij} - \frac{F_{ij}}{\sqrt{\ell_i \ell_j}}$$

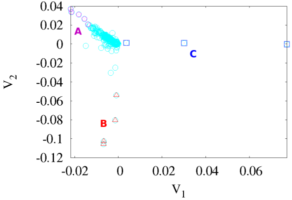
Spectrum $\lambda_i \in [0, 2]$;
Lowest nonzero λ's: E.Vectors localized on subgraphs!

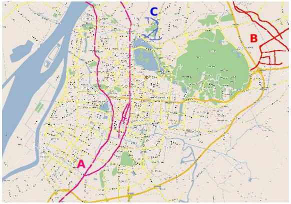
Flow Matrix F_{ij} : weighted



DYNAMIC COMMUNITIES revealed by Spectral Analysis

Eigenvector localization on weighted/topological subgraphs of the dual graph are seen in the Scatter-Plott as separate branches A, B, C: ⇒ here mapped back to the real space:





E.Vectrors scatter-plot Areas w. different dynamical behavior

[H-L. Zeng et al., IEEEXplore 2009, DOI: 10.1109/ICDSP.2009.5201238]

Networks from Data: Blogs

High-resolution Streaming Data $\Rightarrow$ Networks of Users & Posts

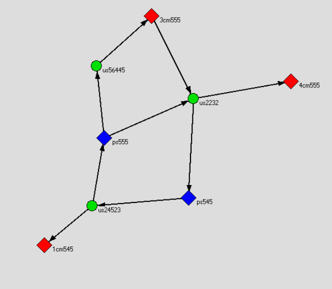
Outline

BIPARTITE NETWORKS REPRESENTATION

USERS + BLOGS
Users(==registered by IDs)
Blog(==Posts + Comments)
are two natural partitions

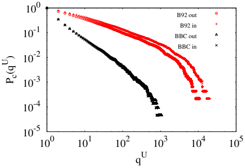
Linking $\Rightarrow$ network emerges:
U$\rightarrow$ P; P$\rightarrow$ U $\rightarrow$cmP, ..
U$\rightarrow$cmcmP, ...
directed links;

Graph: Linking ways...

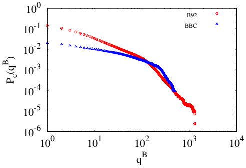


LINKS & COMMONS

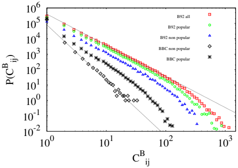
Complex topology of emergent networks; suggests how to select Normal- and very-Popular subjects (Posts& Comments):



U-degree



B-degree



Commons

Similar features found in movie-data, diggs, ... M. Mitrović & B. Tadić, Eur.Phys.J. B, 2009, DOI:10.1140/epjb/e2009-00431-9]

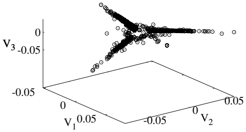
PROCEDURE TO IDENTIFY COMMUNITIES

DATA ⇒ bipartite.Network ⇒ projected.Network(weighted links == commons);
SpectralAnalysis (Laplacian) ⇒ E.Vectror-scatterplots ⇒ Community Branches! (⇒ IDs of nodes ⇒ IDs of Posts)

$$L^U_{ij} = \delta_{ij} - \frac{C^B_{ij}}{\sqrt{\ell_i \ell_j}}$$

$$C^B_{ij}\,,\ \ell_i \equiv \sum_j C^B_{ij}$$

Example:

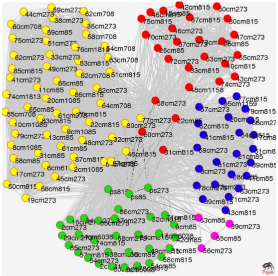


[M. Mitrović & B. Tadić, Eur.Phys.J. B, 2009, DOI:10.1140/epjb/e2009-00431-9]

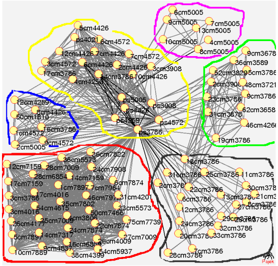
Outline

WHAT'S ON BLOGS?

Example: Posts and Comments made by users in 2 different groups on B92 Blogs: Further splitting od their posts into: Author-related or Subject-related groups!



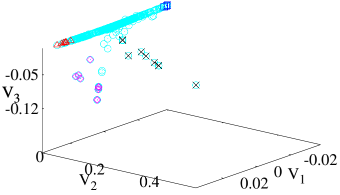
P+cmP of Ug4: urban architecture

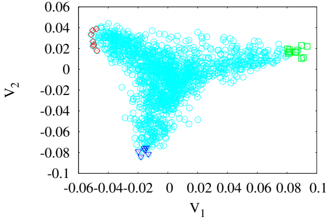


P+cmP of Ug1: sports

SPECTRAL ANALYSIS reveals USER COMMUNITY in Movie Database

▷ Weighted User/Movie Networks: Spectrum ⇒ Communities





user: unselective ... or fan   movie: three dominant genres

[Grujić et al., IEEE Xplore 2009, DOI:10.1109/ICDSP.2009.5201238]

CONCLUSIONS

- **Traffic on Networks: Rule & Structure:**
  Minimal rules that take into account effects of network structure; Collective traffic behavior on networks can be classified (Dynamic universality calsses);
- **Spectral Analysis of (weighted) Networks:**
  Methodology to determine emergent *dynamic subgraphs* on a given topology and known dynamic rules;
- **Users & Posts + Comments make Cyber society:**
  Data mapped onto Bipart.Network; Quantitative analysis using graph theory & spectral methods;

References & Support:

- LINK: http://www-f1.ijs.si/~tadic/Networks/
- GRANTS: P1-0044; FP6: MRTN-005728; FP7: CYBEREMOTIONS;
- ACTION: COST-MP0801 “Physics of Competition and Conflicts”

SOME RECENT REFERENCES

- M. Mitrovic and B. Tadic, *Bloggers behavior and emergent communities in Blog space* Europ. Phys. J. B: online version: DOI=10.1140/epjb/e2009-00431-9
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