Collective Dynamic Behavior in Transport Processes on Networks

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Outline

1. Transport on Networks
   - Model
   - Statistical Measures
2. Two Dynamical Classes of Behavior
   - Associated Network Structures
   - Simulated Traffic Features
3. Preferential Behaviour & Fluctuations
   - Nodes & Edges
4. Other Collective Dynamic Behaviors on Networks

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INFORMATION TRAFFIC ON NETWORKS

MODEL*:

Traffic of information packets on networks of $N = 1000$ nodes and given topology (cSF,seG); Model with:

- Creation and assignment; (Rate: R)
- Navigation; (Local; Depth $r = 2$)
- Queuing; (LIFO queue; Buffer: $H = 1000$)
- delivery (at destination; traffic stationarity)

PARAMETERS:

- Creation rate R (alt. constant density ρ);
- Buffer size H; Searched depth $r = 2$;

[Tadić & Rodgers, Adv. Compl. Syst. (2002), Tadić, LNCS (2003)]

Outline

STATISTICAL MEASURES OF TRAFFIC

Statistical properties of traffic are monitored at *local* (individual Nodes and Edges) and *global* level of the entire network:

(a) Traffic Noise Signals and Power Spectra:

- Number pf packets processed by a node $\{h_i(t)\}$ w. T_{WIN};
- Number pf packets processed by an edge $\{f_{ij}(t)\}$ w. T_{WIN};
- Network load; Network delivery rate; ...

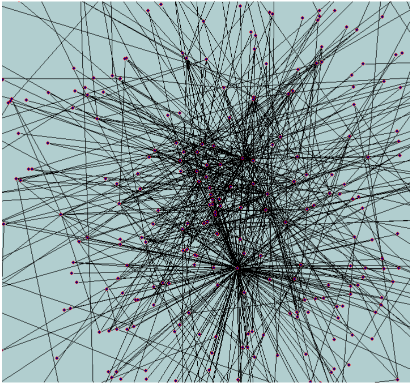
(b) various PDFs:

- Transit time of packets $P(T)$ (rtt);
- Waiting times in queues $P(t_w)$;
- Time intervals (return times) $P(\Delta t)$ to Nodes;Edges;
- Flow and Noise distributions; ...

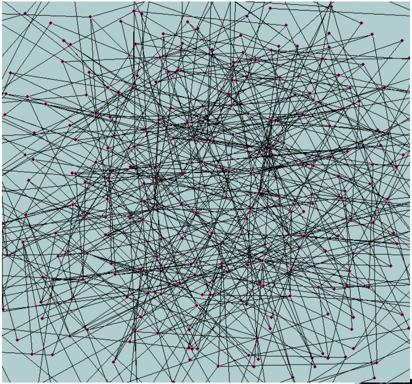
Outline

Two Network Structures

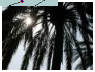
Low-traffic-density: cSF; *High-traffic-density: Homogeneous*



WebGraph



StatNet



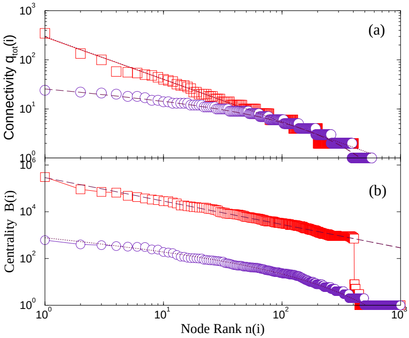
How these networks are grown

Preferential attachment AND rewiring rules: Prob. $\tilde{\alpha}$ attach new node i with a pref. target k; else Prob. $1-\tilde{\alpha}$ attach preferentially selected source node n to node k:

$$p_{in}(k,i) = \frac{\alpha + q_{in}(k,i)/M}{(1+\alpha)i}; \quad p_{out}(n,i) = \frac{\alpha + q_{out}(n,i)/M}{(1+\alpha)i}, \qquad (1)$$

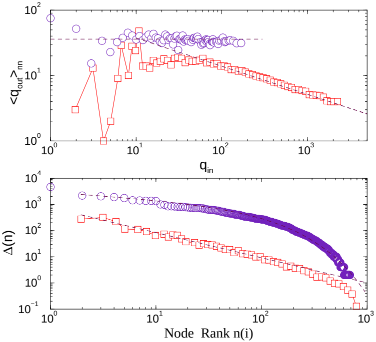
In results: if $M = 1$ some nodes are detached (fragmented graph) but nodes inside the giant cluster are highly connected: two hubs for in- and for out-links; large clustering coefficient; in–out link correlations: disassortativity;

Correlations in Network Structures



Connectivity

Centrality



Correlations

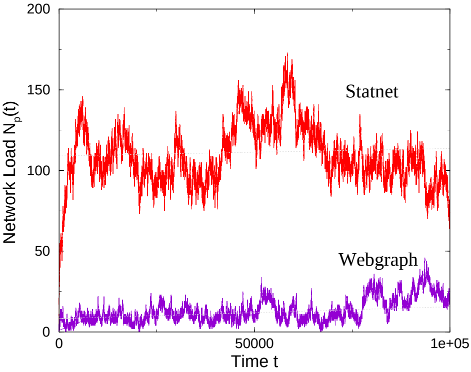
Clustering

Outline

How the simulations are done:

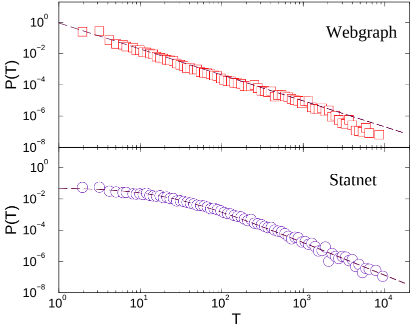
- Creation: packets are created at random nodes with rare R; each gets address to be delivered;
- Navigation: each node searches for the address of top packet and decides which direction to send it; nnn-search; ...; paths!
- Queuing: paths intersect; LIFO queue; Buffer finite $H = 1000$;
- Delivery: at its address packet is delivered and deleted; stationarity for $R \leq R_c$;

Effects: Rt packets created till time t; In the average, number of packets μ are being delivered par t.s.; $N_p(t)$ dynamically varying number of moving packets; $\mu == < \frac{dN_p(t)}{dt} >$ av. delivery rate

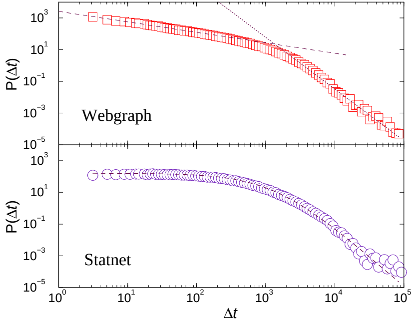


network load time series: $\rho = const$

PDFs: TIME STATISTICS



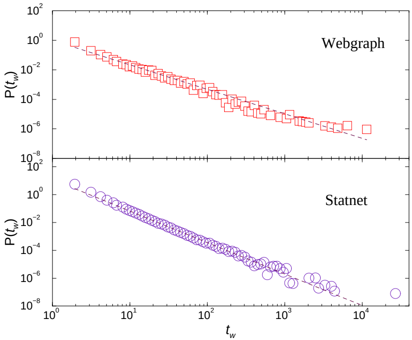
Travel Times ($\rho = 100$)



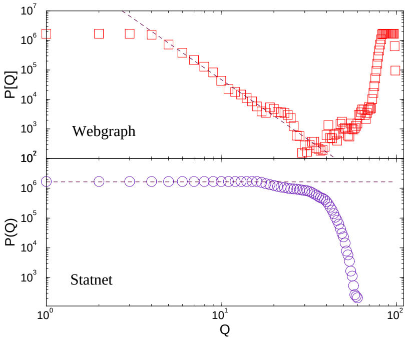
Return Times ($\rho = 1$)

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

PDFs: QUEUING STATISTICS



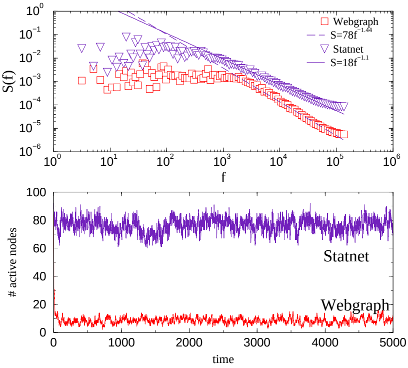
Waiting Times

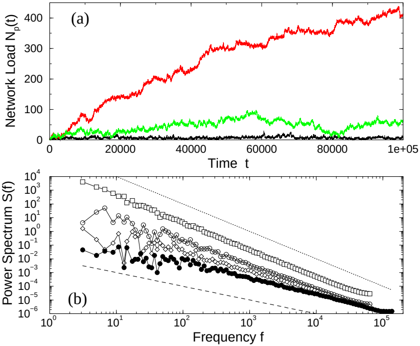


Queue Lengths

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

TIME SERIES ANALYSIS





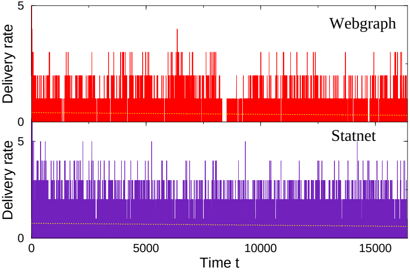
$\rho = const$: 2 Nets

R varied: WG

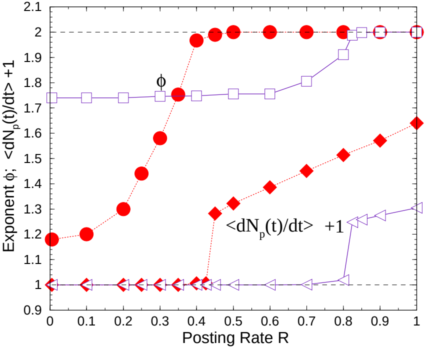
[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

Jamming on Networks

Def. jamming rate: $J \equiv < dN_p(t)/dt >= R - \lambda$



delivery rate λ



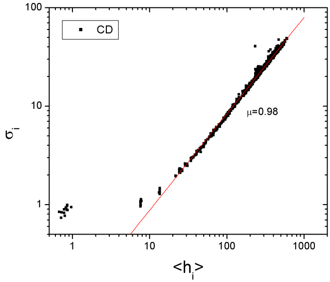
OP: 2 Nets

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

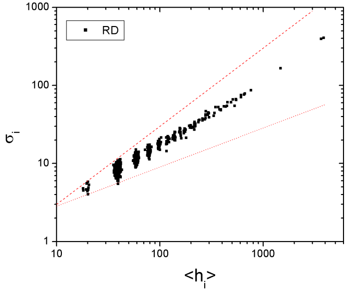
Outline

Scaling in Time-Series Fluctuations: NODES

Scaling relation: $\sigma \sim < X >^{\mu}$; Exponent μ increses with T_{WIN}



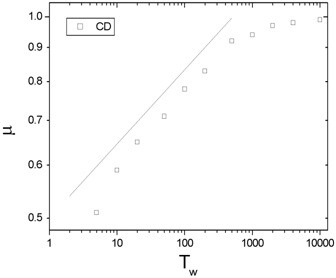
Preferred Navigation

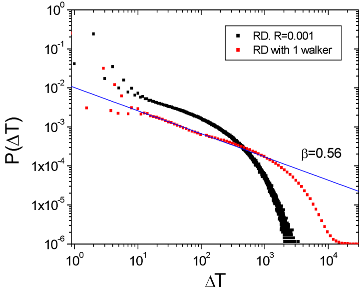


Random Diffusion

Time-Window Dependence & Return-Times

The scaling exponent μ is not universal: Dependence on the aquisition time-window is demonstrated in the simulations:

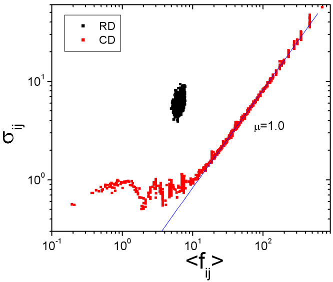




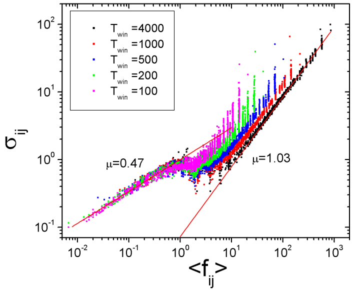
[Kujawski,Tadić & Rodgers, New J. Phys. **9**, 154 (2007)]

Scaling in Time-Series Fluctuations: EDGES

Dynamic **preferential behavior** *(navigation rule) or static* **topological preference** *(SF structure)* is necessary for the scaling to occur:



RD & Preferred Navigation



Preferred Navigation: T_{WIN}

[Kujawski,Tadić & Rodgers, New J. Phys. **9**, 154 (2007)]

Electron Transport; CCM

DIFFERENT type of Dynamics CCM: Collective Effects!

- Voltage driven electron transport;
  - multi-path dynamics · · ·
- 2D chaotic maps coupled through the networks links;
  - Localization (dissipation);
  - Synchronization; Dynamical patterns;
  - (quasi)Periodicity;
  - (non)Chaotic attractors & related Dynamic behavior;

[Blunt, Šuvakov, .. Tadić, NanoLett. 2007]
[Levnajić & Tadić: LNCS 2007; ++]

CONCLUSION

- Traffic models with search & Queuing
  - Diffusive motion but different from random walk!
- Emergent collective dynamics: two types $\Leftrightarrow$ two networks classes
  - power-law pdfs (travel times, waiting times, return times);
  - correlations in traffic time series: density dependence;
- Noise & Flow fluctuations: Scaling
  - some preferential behavior necessary; rel. to return-times;
- CCMaps:
  - variety of collective phenomena!

COLLABORATION & REFERENCES

- COLLABORATIONS:
  - Geoff Rodgers, Brunel University London
  - Stefan Thurner, Medical University Vienna
  - Bernard Kujawski, Brunel University London
  - Milovan Šuvakov, IJS Ljubljana
  - Zoran Levnajić, IJS Ljubljana

- GRANTS: P1-0044; COST-P10; MRTN-005728;

- REFERENCES: http://www-f1.ijs.si/∼tadic/Networks/