Understanding Networks: From Real World to Models and Theory

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OUTLINE

- Introduction: Graphs & Real World Networks
- Numerical Modeling: Topology & Dynamics
- Theory—Two Faces: Evolving & Static Networks

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◇ Emergence of Topological & Functional Complexity
◇ Summary: Current Trends in Physics of Networks

I. Graphs & Real-World Networks (Physics View)

- What makes a Network? Patterns of Interactions

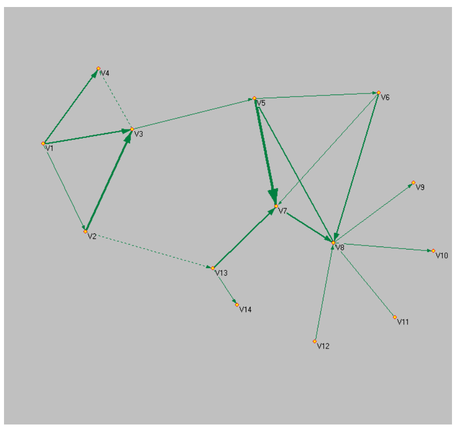
  -Social; -Economy; -Technological; -Biological;

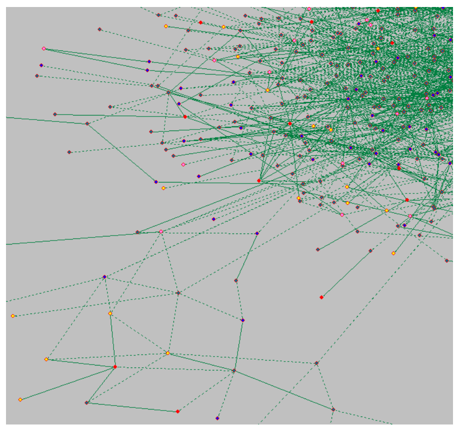
  -Virtual Networks (representing states & transitions in c.d.s. in physics, geology, ...);

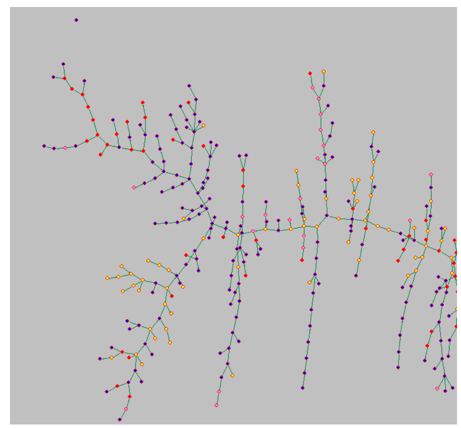
  -Random-Graphs instead of Compact Lattices: Spin Networks, etc.

- Graphs : (Nodes, Links)

  directed/undirected, random, structured...







- Another view: Evolving and Static Networks
  This property of network requires different approaches
  both in Numerical modeling and in Theoretical Methods

- Networks & Complexity
  why Networks are Complex systems?
  —Emergence of Properties: Structural & Functional !

- Two aspects of Physics Research on Networks:
  - (a) Search for the underlying **physics principles**
    Laws governing network's evolution and function;
    And how network topology serves network function;
  - (b) Understanding a particular network (i.e., from available real data, partial info.)
    Finding network structure and its function (*inverse problem !*);
    Make experiments/potential predictions;

Physics research so far built science basis; We show some details using numerical modeling approach; More/other aspects will be discussed during the WS

II. Numerical Modeling: Topology & Dynamics

◇ Modeling/more accessible comp. to theory...

◇ Two types of problems:

- 1. Topology, Construction & Reconstruction from some kind of data when only an "output" of the dynamics is known as a set of data or principle;
- 2. Function (Dynamics): How Networks work?
  - Transport (Information, Particles, Energy);
  - Support of a dynamic variable attached to a node (Examples: Genetic Networks, Spin Networks, ...)

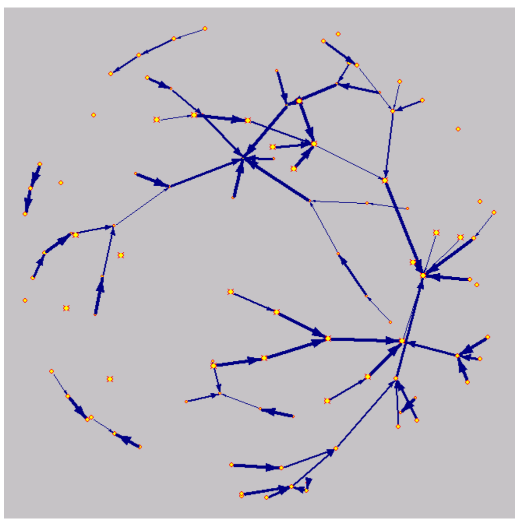
◇ Both approaches can be closely related and interconnected:
Simple models inspired by reality/Guessed rules/...

Physics of Networks: Short Introduction

◇ A. DEFINITIONS

Nodes + *Links* = *Graph* (Nodes with properties; Links with properties ⇒ Networks)

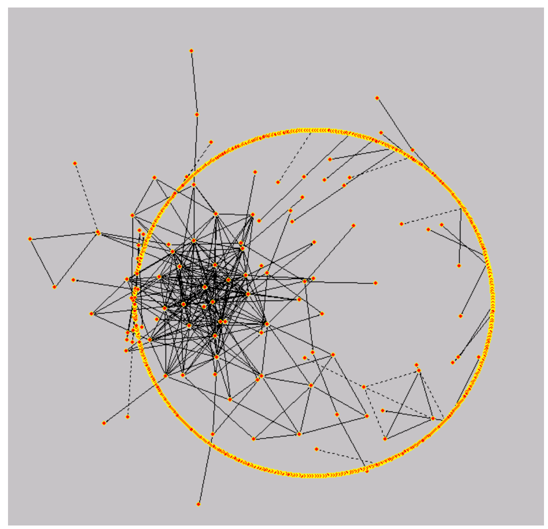
Basic characteristics:



Links

Directedness

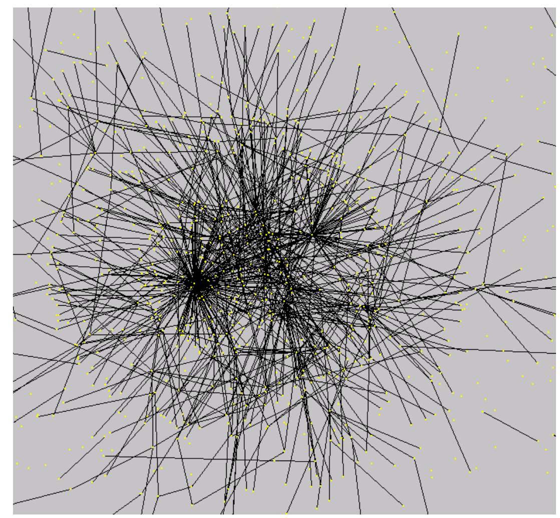
Weights



Nodes

Connectivity

Clustering



Graphs

Sparseness

Inhomogeneity

◇ B. LINKING:

How to link within a network? Network examples:

—Social Networks;

—Information (Technological);

—Biological Networks (virtual!):

— Physical systems; Chemical reactions; Spin Networks ...

Methods:

—Growth; —Linking; —Rewiring// Evolving Networks

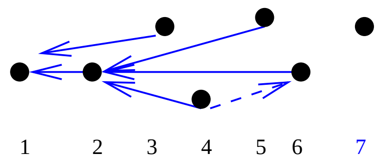
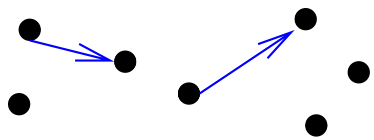
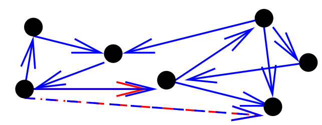
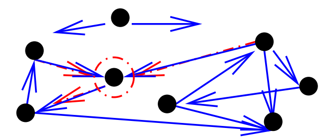
—Fixed N; —Linking; —Rewiring//Static Networks

◇ C. PHYSICS of NETWORKS: Objectives & Problems:

▷ Find Unifying Principles (Laws); Exploit it ⇒ to make predictions;

▷ Usually Network is NOT known; Information from Dynamics (or network function): Reconstruct the underlying network (*inverse problem*); then go to first task;

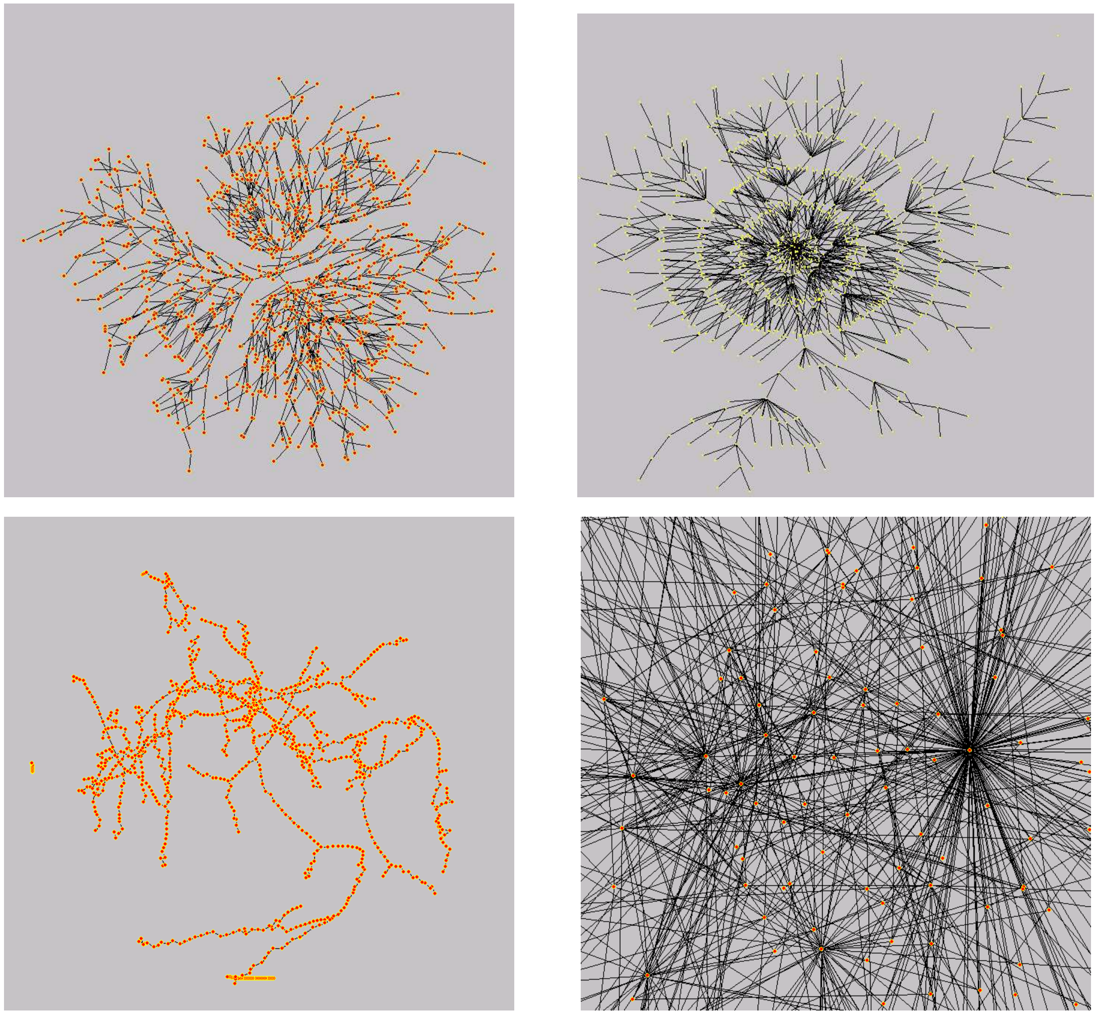
More about Methods: How Different Structures Emerge

add nodes & links $i = 1 \cdots N; \ell + +$	$i = N$; add links $\ell = 1 \cdots L$	fixed N, L replace link	N, L (fixed) repl. node & links
 			
growth (subspace) $+rewire!$	no multiple links $i = N$	sel. pair + 1 some rule	sel. unfixed config. some rule
WebGraph SF: in- & out-!	StatNet non-SF	SONs SF-in (?)	SON SF: $\tau \sim 2$

Linking Rules:

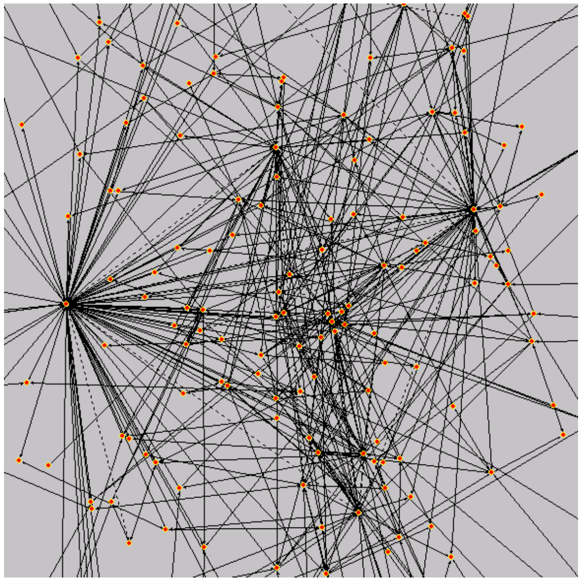
Preferential Linking Rules $p_{in}(k,i)$,$p_{out}(n,i)$: *in*- and *out*- probability rewiring & constraints	Different Self-Organized Schemes (incl. rewiring & constraints ...)

Some Network Structures: Randomly-Grown Tree; SW01;; SF Tree, WG

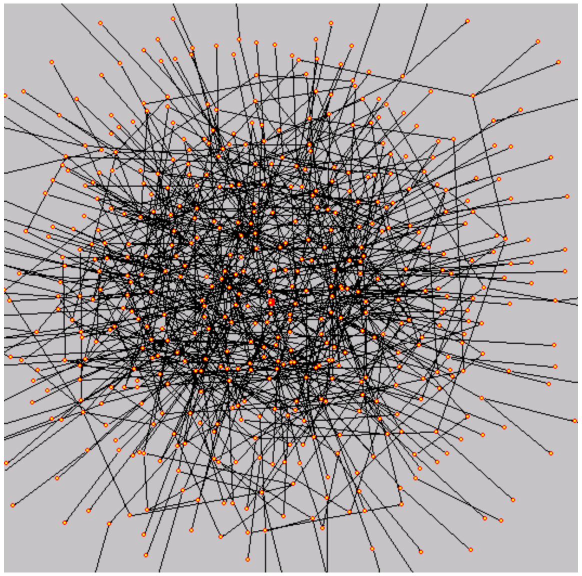


Rules & Emergent Structures:

Preferential *in*- and *out*- Linking	Preferential *in*- and *out*- Linking
$p_{in}(k,i) = \frac{\alpha+q_{in}(k,i)}{(1+\alpha)i}$	$p_{in}(k,i) = \frac{\alpha+q_{in}(k,i)}{(1+\alpha)i}$
$p_{out}(n,i) = \frac{\alpha+q_{out}(n,i)}{(1+\alpha)i}$	$p_{out}(n,i) = \frac{\alpha+q_{out}(n,i)}{(1+\alpha)i}$
Growth + Rewiring	Static: Link Fluctuations

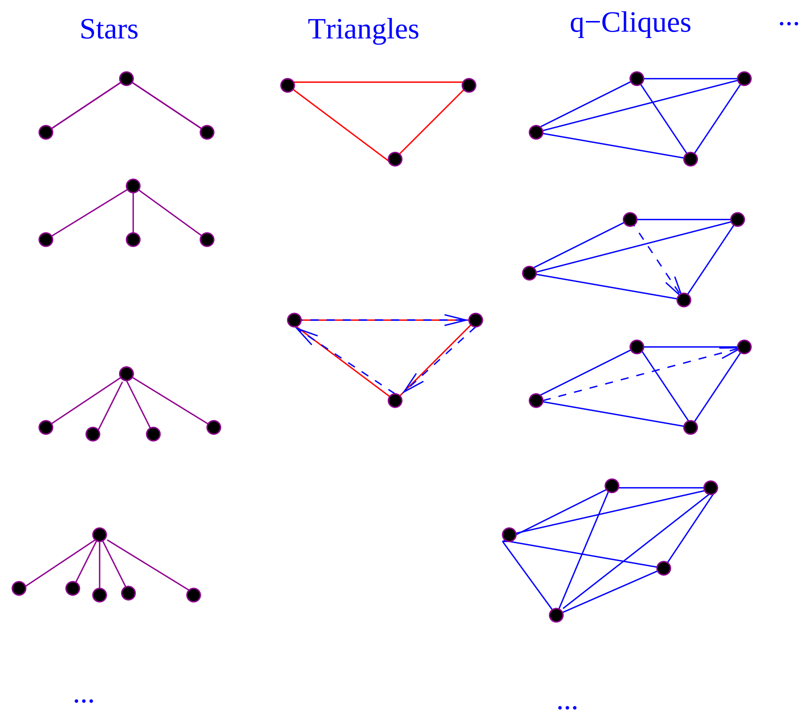


Web Graph



Preferential Static Net

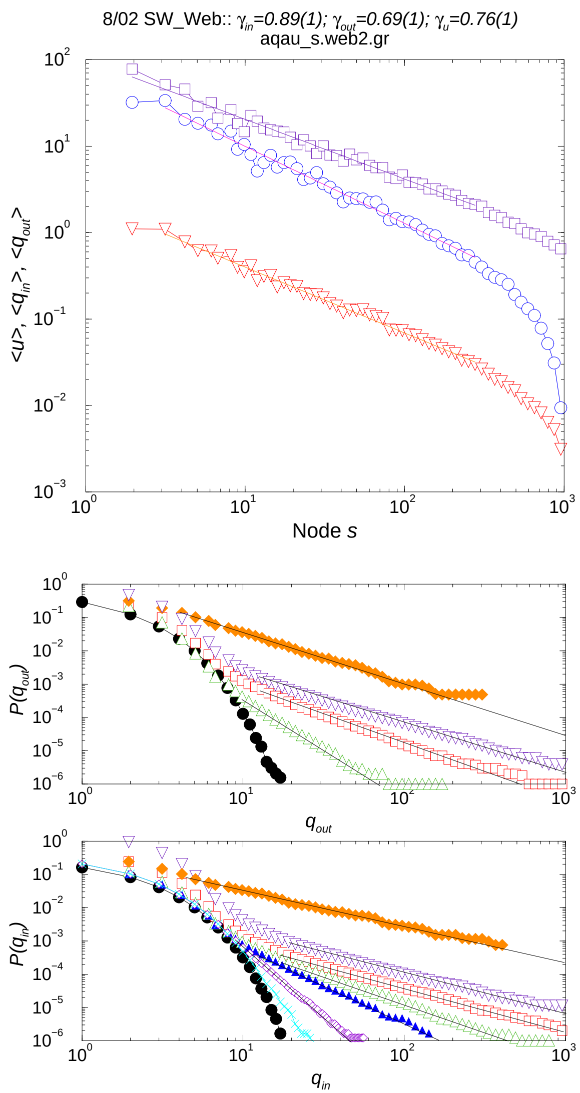
Basic Elements of Network Structure



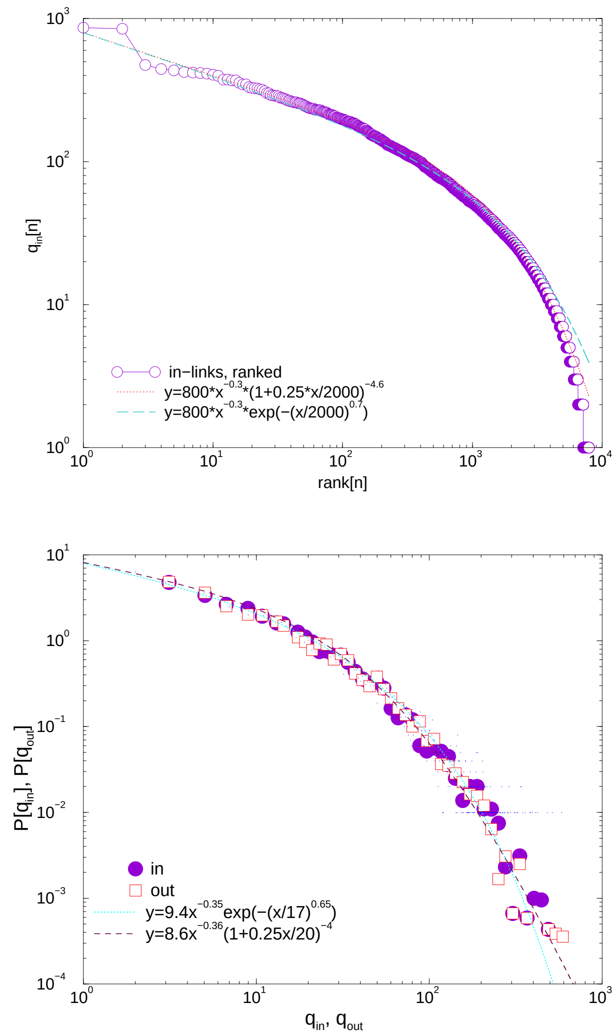
◇ Conditions required for these elements to appear on graph
Linking probabilities!

◇ Some elements can be induced by certain types of rules

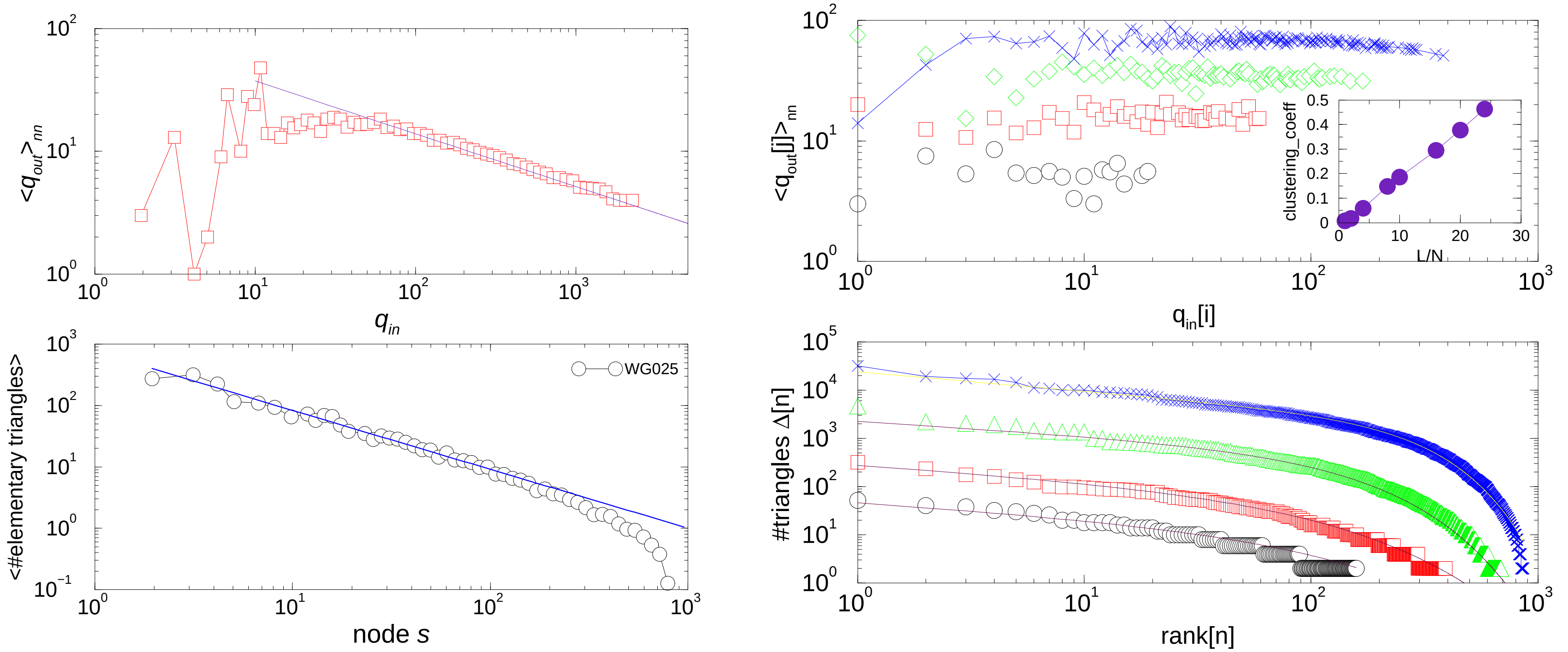
◇ v.v. Abundances of some structural elements $\Longrightarrow$ indicates ways in which network eveolved



Web Graph



Preferential Static Net



$$\phi_\Delta = 1.0$$

$$\kappa = 0.42$$

dis-assortative!

Web Graph

$$\xi(L/N) \sim (L/N);\ \sigma \in [0.7, 1.4]$$

$$C_k \sim (L/N)^1$$

un-correlated!

Preferential Static Net

Elementary Structural Characteristics: SUMMARY

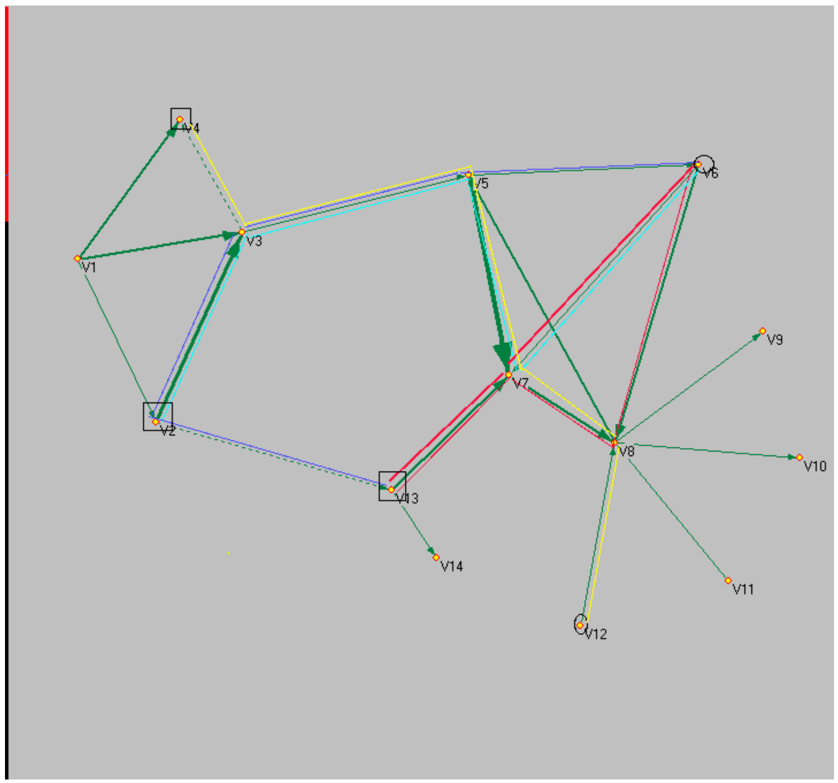
Elementary structures on local & global level; Hidden Structures

Property	Web Graph	Preferential Static Net
Connectivity Profile	$q_{in}(s) \sim s^{-\gamma_{in}}$ $q_{out}(s) \sim s^{-\gamma_{out}}$	$q_{in} = q_{out} =$ $Aq^{-\tau} \exp[-(q/\xi)^{\sigma}]$
Emergent Distributions	$P(q_{in}) \sim q_{in}^{-\tau_{in}}$; $P(q_{out}) \sim q_{out}^{-\tau_{out}}$	$P(q_{in}) \sim P(q_{out}) =$ $Bq^{-\tau} \exp[-(q/\xi)^{\sigma}]$
Clustering Profile	$n_{\Delta}(s) \sim s^{-\phi_{\Delta}}$	$Cq^{-\tau} \exp[-(q/\xi)^{\sigma}]$
in—out- Correlations	$< q_{out}(j) >_i \sim q_{in}(i)^{-\kappa}$	$\sim const.$
Scaling Relations:	$\tau_{in} = 1/\gamma_{in} + 1$; $\tau_{out} = 1/\gamma_{out} + 1$	$\tau = \sigma - 1$ -
Hidden	Superstructures !	Communities ?

NETWORK FUNCTION:: TRANSPORT NETWORKS

Random Walks on Networks: 1. Sequential Walkers

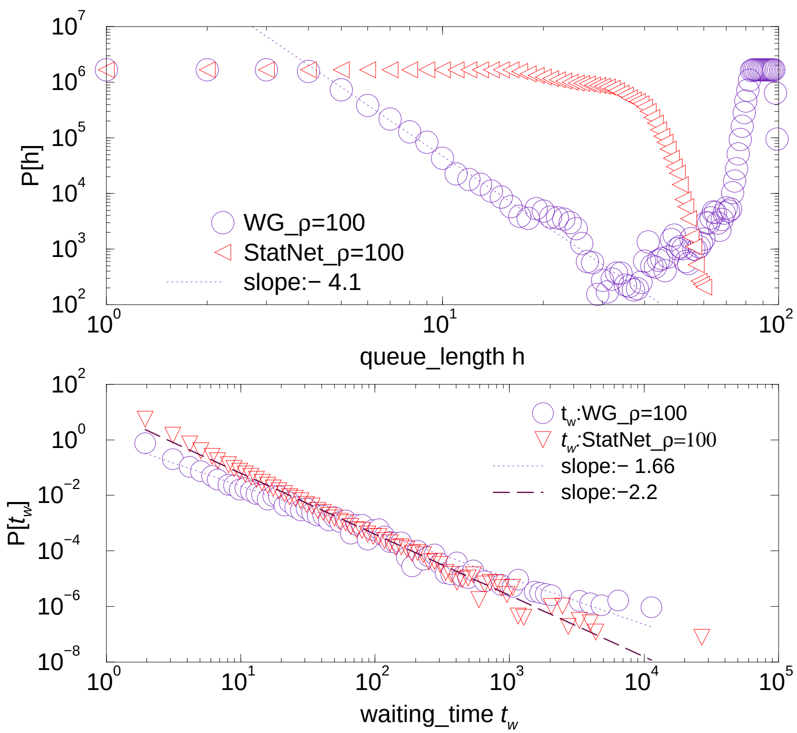
Origin → Destination : Random or Navigated Walk (Rules!)



Explore Graph	Survival Time T = Path Length

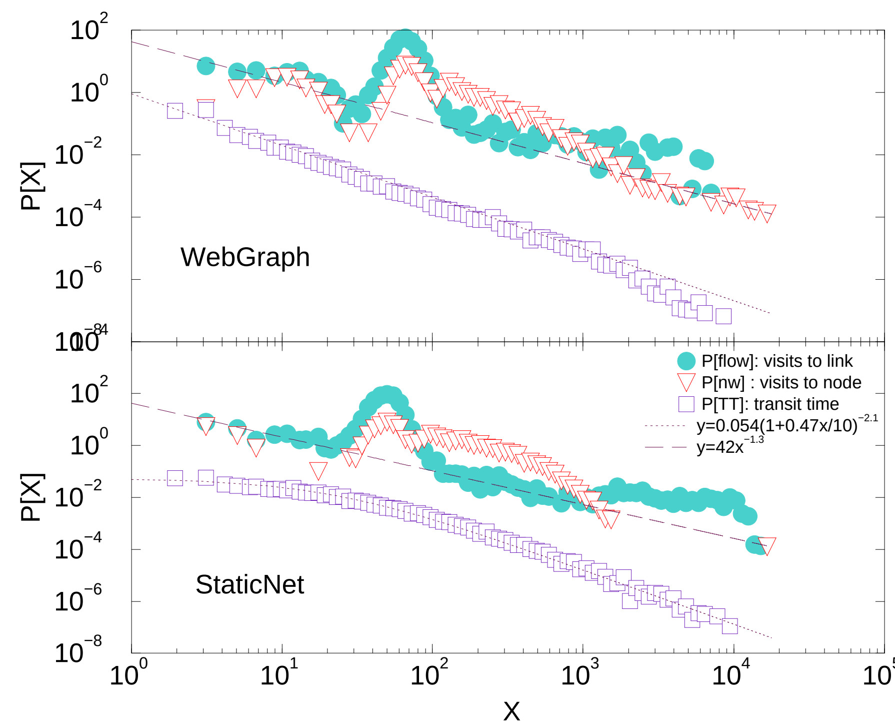
2. Parallel Walkers: Traffic at finite density; Queuing !

Additional Parameters: –Posting Rate R; –Max.Queue H; –Priority



Waiting Time $t_w(i)$ at node i

Queue Length $h(i)$ at node i



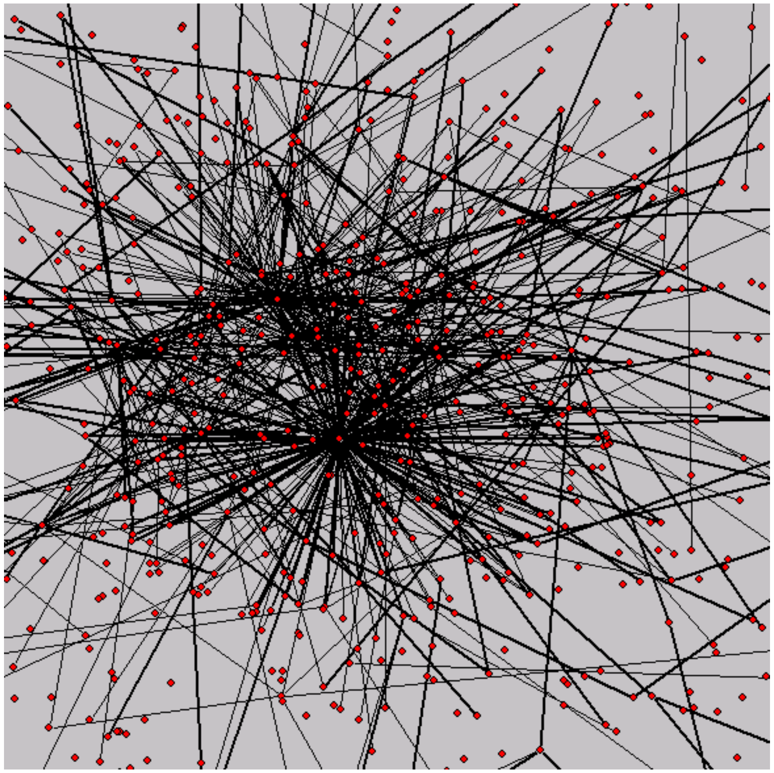
Transit Time $T_k = \sum_i t_w(i)$, Path k

Occupation Numbers, $\rho = const$

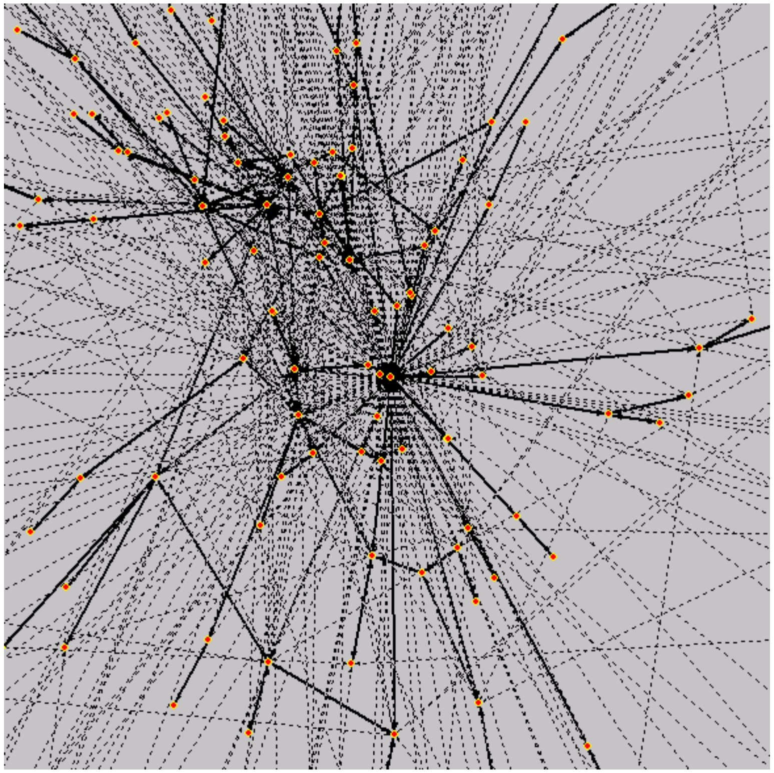
3. Network Flow: Defined as Number of Walkers on Link within T_{WIN}

—Dynamically generated Weights of Links!

—Depends on Structure & Navigation Rules



Flow on WebGraph: RD
uneven use of Links

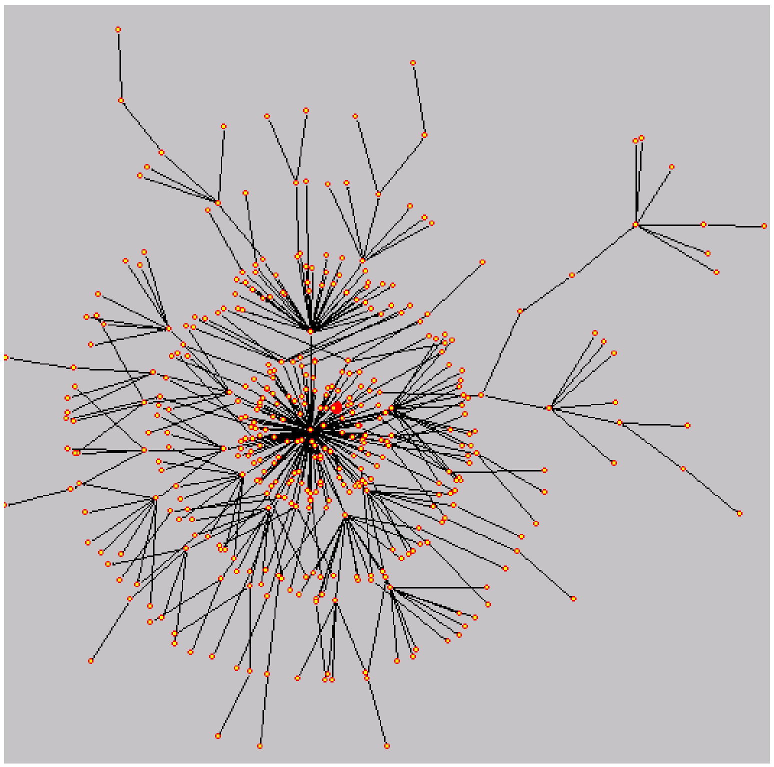


nnn-Search
... and Paths

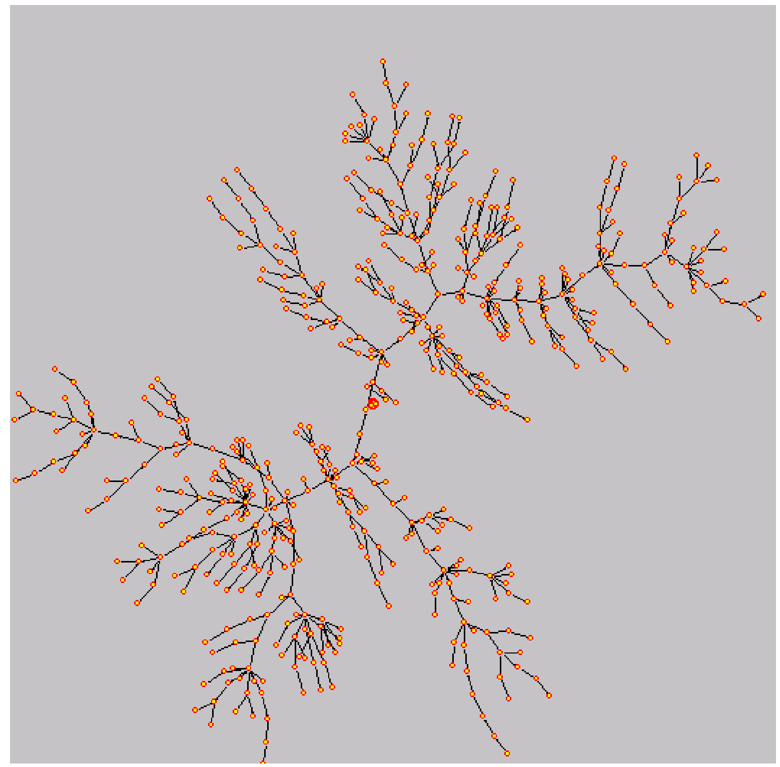
4. Maximum-Flow Spanning Trees:

–Way to characterize distribution of Network Flow

–Structure of the Spanning Tree vs. Structure of the underlying Graph!



$\rho = 1$ MSTree: Web Graph



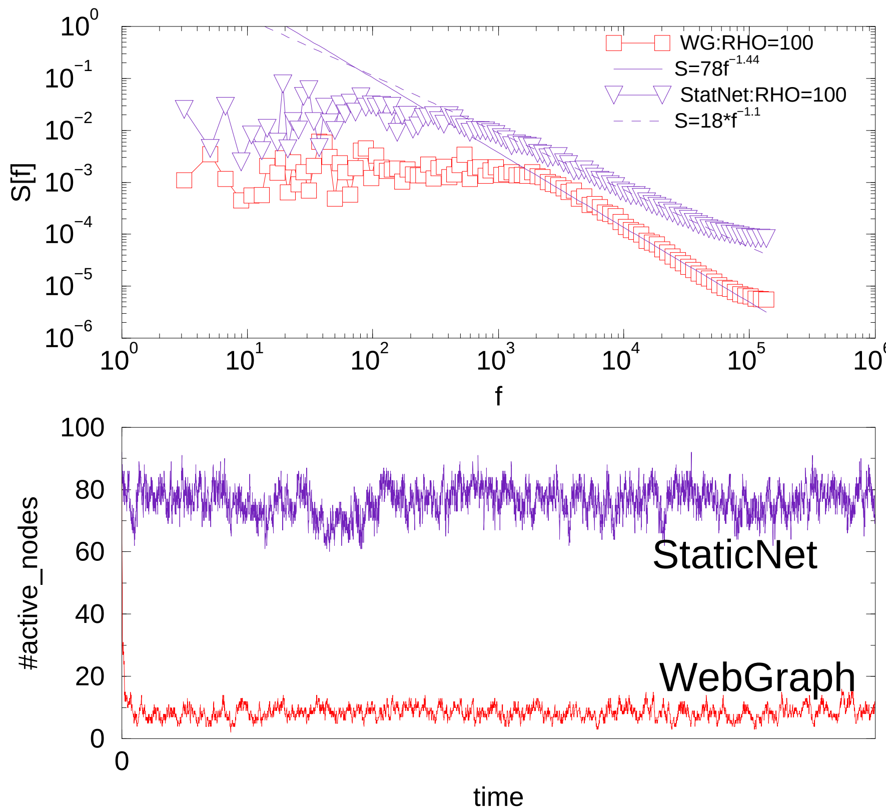
$\rho = 1$ MSTree: Preferential Static Net

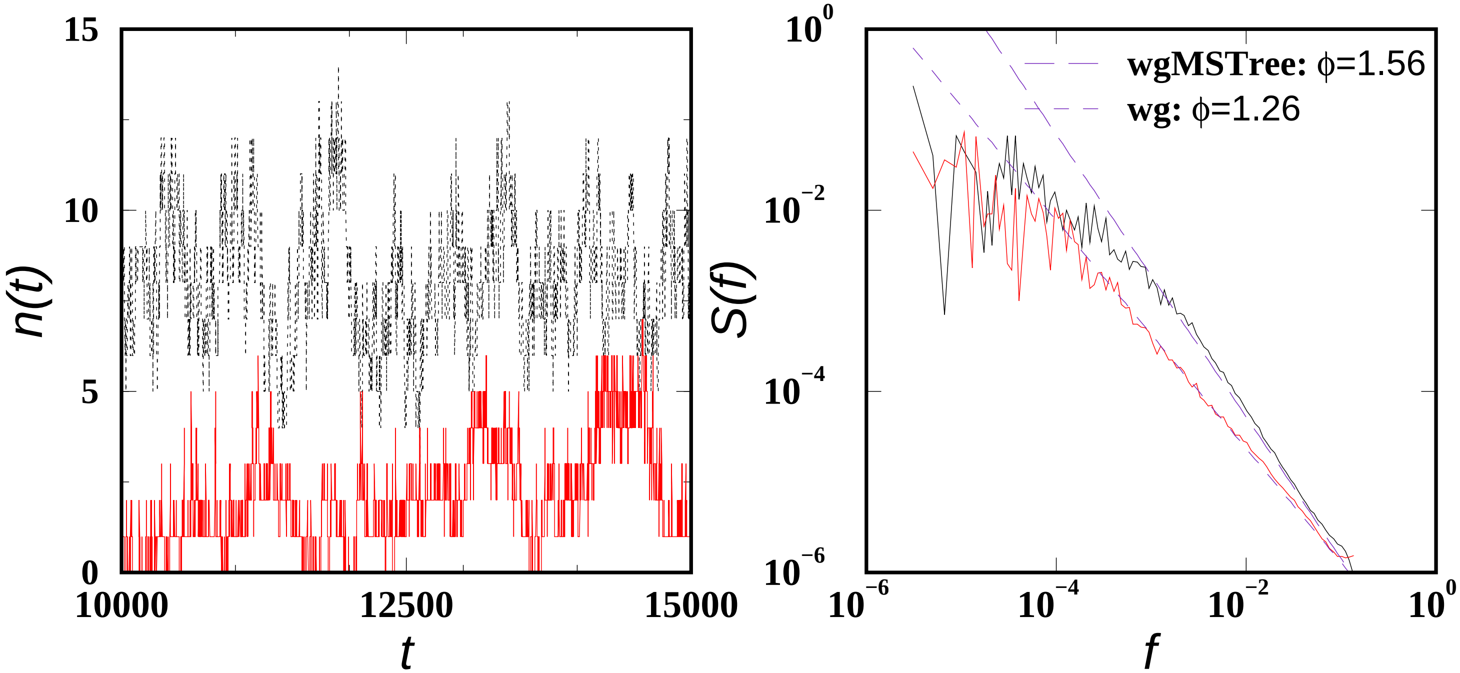
5. Traffic Noise on Networks:# of Walkers visiting a Node within T_{WIN}

—Dynamically generated "Loads" of Nodes;

—Depends on Network Structure & Navigation Rules;

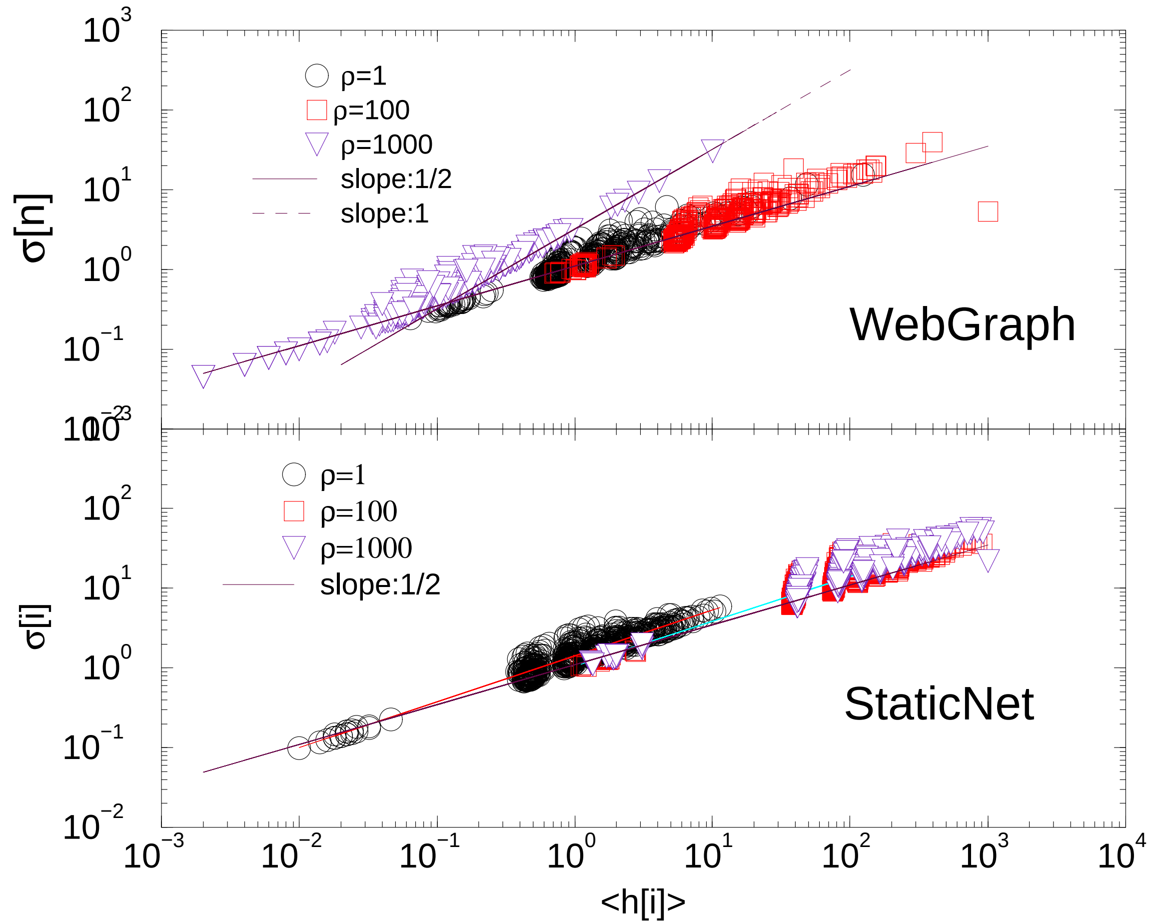
—Structured Noise: *Antipersistance*





Noise on WebGraph & its MST nnn-Search; & Power-Spectrum: *antipersistence*
(uneven use of Nodes due to Network Structure & Walker-Interactions)

6. More Regularity in Traffic Noise on Networks: Universal Dispersion-to-Average Ratio: $\sigma[i] \sim < h[i] >^{\mu}$; $\mu = 1/2$ or $\mu = 1$ (?)



III. Theory: Evolution & Fluctuatins—Two Faces of Networks

◇ Rate-Equation Methods: Evolving Networks;

◇ Field-Theory Methods: Link Fluctuations/Homogeneous Networks

- Special Subjects/Mixed Approaches:

—Dynamic Processes on Networks and Related Problems!
—Games on Networks

- Percolation, Phase Transitions on Networks

- . . .

• Master Equation for Network Evolution: Scale-Free Networks

When linking probabilities are known: $p(q,t) = \frac{\alpha+q(s,t)}{(1+\alpha)t}$

1.) Local connectivity evolution: $dq(s,t)/dt = p(q(s,t),t)$
Solution: $q(s,t) = B\left(\frac{t}{s}\right)^{\gamma}$, with $\gamma = 1/(1+\alpha)$

2.) Local connectivity distribution: $\rho(q,s,t)$

3.) Emergent connectivity distribution: $P(q,t) = \frac{1}{t}\sum_{s=1}^{t}\rho(q,s,t)$

$$\frac{\partial P(q,t)}{\partial t} = -r_1(t)P(q,t) - r(q)P(q,t) + r(q-1)P(q-1,t)$$

Rates r_1, $r(q)$ are related to the linking probabilities;

Stationary Limit: $\frac{dP(q,t)}{dt} = 0 \Longrightarrow$ solution: power-law

Ref.Book: Dorogovtsev and Mendes: *Network Evolution*, Oxford, 2003.

• Statistical Mechanics of Fluctuating Random Graphs

◇Ensemble of Networks: Probability distribution over many possible networks

◇ Suitable for fixed N, fluctuating Links (traditional RG approach; also for new structured (evolving!) graphs!)

◇ Incorporating constraints (growth, properties) in a systematic way;

◇ Select a set of graphs $\mathcal{G}$ to be studied (trees, k-regular, Hamiltonian, Eulerian, ...) and their properties (directed, cycles of appr. type, ..)
◇ Define probability $P(G)$ that graph $G \in \mathcal{G}$ by random sampling;
◇ Define entropy, for instance: $S = -\sum_{G\in\mathcal{G}} P(G) \ln P(G)$, or other !
◇ Maximize entropy under constraints:
$\sum_G P(G) X_i(G) = X_i^o$, where $X_i(G)$ is some graph's property!
and normalization: $\sum_G P(G) = 1$

– Depends on how $P(G)$ is defined;
– $Z_h = \sum_{lg} 1 \equiv \sum_g n_h(g) \rightarrow Z_h = \sum_{lg} w(q(1))w(q(2)) \cdots w(q(N))$
– Complex topologies/correlated graphs (u. Stat.Physics of SpinGlasses, Field Theory, ...)

References: Z. Burda *et al.*, Phys. Rev. E, series of papers (2001-2005), ...

Summary: Emergence of Topological & Functional Complexity in Networks

◇ Structural Complexity of Networks

–can be characterized quantitatively;

–by means: basic structural elements (Graph Theory) & Hidden (higher-order) Structures;

–related to: Cooperative behaviour of Nodes (esp. in structured networks);

–scale-free structures can result in some optimization processes (non-extensivity!)

◇ Structural Complexity $\Longrightarrow$ Functional Complexity!

–specific for given dynamic processe;

– Higher structural complexity can be made compatible with higher functional efficiency;

◇ Structure $\Longleftrightarrow$ Dynamics Synergy

–dynamic rules which "read" the structure adequately;

–some self-organized networks evolve so to optimize a specific function

Summary: Current Trends in Netork Research

Concepts to be discussed in the Workshop

◇ New Concepts to Explore Structural Complexity ($\neq$ RandomGraphs)
– relevant/related to Network Function; – Algorithms!

◇ Stat.Physics of Complex Systems applied to real-data analysis & network (re)construction (esp. bioinformatics, econophysics, ...)
– Specificity and Universality of these networks!

◇ New Network types discovered in specific areas and applications
– new (additional) characteristics of nodes, links, linking rules, and constraints;

◇ Theory backing up experimental and numerical results!

◇ Understanding structure–function interdependeces (new concepts!)

◇ Applying network science to new areas of self-assembled systems!

◇ Physics ON Networks;

VI. Basic References & Links

▷ S.N. Dorogovtsev, J.F.F. Mendes, *Evolution of Networks: From Biological Nets to the Internet and WWW*, Oxford (2003);
▷ R. Albert and A.-L. Barabasi: Rev. Mod. Phys. **74**, 47 (2002);

Bibliography Sites on the Web:

▷ http://www-f1.ijs.si/∼tadic/Networks/ : this lecture and other Refs. ;

▷http://sweet.ua.pt/∼f2064/bkm5.html :: Random Networks (by topics)

▷ http://tangra.si.umich.edu/clair/home/bib2html/clair.pdf

▷ http://complex.upf.es/∼ricard/complexnets.html ::

LINKS TO COMPLEX NETWORKS RESEARCH GROUPS from Ricard Sole's Home Page

Network dynamics at the Santa Fe Institute Self-organized networks at Notre Dame Small World and evolving networks, University of Porto Statistical physics of complex networks: Luis Amaral Lab Networks and graph theory: Mark Newman's homepage Complex Networks: Sidney Redner's Lab Ecological networks at Tiburon Center Networks and evolution at Leipzig: Stefan Bornholdt's Group Complex networks and cognitive systems,Ecole Normale Suprieure Gene and protein Networks: Wagner's Lab Graph theory, Neutral and Catalytic networks: Peter Stadler's Lab Networks and Chaos: Kaneko's Lab

Information traffic and the Web: Bernardo Huberman's Lab Selforganization and Complex Nets: Guido Caldarelli at Rome Signal transduction networks, Weizmann Institute: Uri Alon Lab Statistical Physics of Networks: Serguei Maslov's page Traffic in complex networks: Bosiljka Tadic homepage Selforganization in road networks: Frank Schweitzer Mapping Internet: ATlas of Cyberspace