Complex Networks Hidden Geometry & Dynamics
Workshop within SigmaPhi 2023

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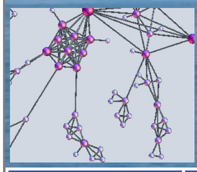

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Dominant Subjects detected in submitted abstracts

- Nonlinear dynamics of coupled units influenced by network's structure
- Dynamics on networks and simplicial complexes with(out) higher-order interactions
- Structural properties, Eigenvalue spectra & Phase transitions in networks
- Data–to-Networks with hidden geometry; Time-series graphs & more
- Networks applications & new discovery in materials science, biology and medicine

Key Words

Brain dynamics; clusters; cover time of Random Walks; communities; correlation networks; cross-diffusion terms; cyclical trends; data; diffusion innovation; dynamic criticality; Eigenvalue spectra; extreme-value theory; graphs; genomic data; Griffiths phases; hidden geometry; hierarchical communities; higher-order interactions; hyperbolic geometry; hypernetworks; information flow; K-means clustering; localization of eigenvectors; machine learning; max-flow spanning trees; message passing approach; microstructure networks; moment closure; multifractal analysis; networks; networked systems; oscillators; online social dynamics; power spectrum; percolation on networks; persistent homology; random walks; reaction-diffusion equations; social networks; self-organized criticality; simplicial complex; synchronization; sparse random regular graphs; time series; time-series networks;

Abstracts

- **Maria Letizia Bertotti, Giovanni Modanese**, Free University of Bozen-Bolzano, Bolzano, Italy: ***Innovation diffusion and Bass model on complex networks***

  A milestone in the study of diffusion of innovation, the Bass model has played, since its first appearance in the Sixties, a constantly present role in marketing. It has been extensively applied for the prediction of diffusion patterns and it has also been variously generalized. The original version is expressed by a single ordinary differential equation, a Riccati equation which is, albeit non-linear, analytically solvable. Its solution describes the evolution in time of the number of adopters of a new product within a population, under the assumption that there are two types of potential adopters, innovators and imitators. The perspective is an aggregate one since, for a fixed product, the two parameters appearing in the equation, the "innovation" and the "imitation" coefficient, are the same for the whole population. A thorough investigation however (as can be tackled with today's computational tools) should take heterogeneity of individuals into account. One way to do that is to consider the network of interpersonal connections. Especially in relation to the imitative aspect of the process, it can make a big difference whether individuals who have already adopted have few or many contacts.
  We here discuss how a network structure can be introduced into the model. We consider

networks whose nodes have at most a number n of links. In particular, we consider scale-free networks with degree distribution of the form P(k) = ck^(-γ) where 2<γ<3, because it is into this category and with power law exponent into this range that many real-world networks fall. Of further fundamental importance in relation to network topology are the degree correlations P(h|k), with P(h|k) expressing the conditional probability that an individual with k links is connected to one with h links. In this connection, we devised an algorithm to build correlation matrices and get assortative or disassortative networks. Following a heterogeneous mean- field approach, we reformulate the Bass model as a system of n ordinary differential equations with each of the n equations governing the evolution in time of the number of adopters among those who have j connections (with 1≤ j ≤n). We explore various aspects of the dynamics related to different classes of links for both correlated and uncorrelated networks. And we focus on the identification of two specific times, the takeoff time and the peak time, which play a significant role in the life cycle of an innovation/product.

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- **Ofer Biham, Ido Tishby, Eytan Katzav**, The Hebrew University, Jerusalem, Israel: ***The distribution of cover times of random walks on random regular graphs***

We present analytical results for the distribution of cover (C) times of random walks (RWs) on random regular graphs (RRGs) consisting of N nodes of degree c ≥3 [1]. Starting from a random initial node, at each time step an RW hops into a random neighbor of its previous node. In some of the time steps the RW visits new nodes that have not been visited before, while in other time steps it revisits nodes that have already been visited. As a result, the number of distinct nodes s visited up to time t is typically smaller than t. The cover time T is C the number of time steps required for the RW to visit every single node in the network at least once. In order to obtain the distribution of cover times, we first calculate the distribution $P(S = s)$ of the number of distinct nodes s visited by an RW up to time t. To this end we derive a master equation for the distribution $P(S = s)$. Using the generating function approach we obtain an analytical solution of the master equation. Inserting $s = N$ in the distribution $P(S = s)$ one obtains $P(S = N)$, which is the probability that the RW tt has visited all the nodes in the network up to time t. In fact, this coincides with the cumulative probability of the cover times, namely $P(T \le t) = P(S = N)$. The tail distribution of cover times is given by $P(T > t) = 1 - P(T \le t)$.Therefore, $P(T > t) = 1 - P(S=N)$. Using therelations above, we obtain the tail distribution of cover times.
In the long time limit the distribution of cover times converges towards a discrete Gumbel distribution, known from extreme value theory. The Gumbel distribution often emerges as the distribution of the maxima among sets of n independent random variables drawn from the same distribution. It is one of the three possible families of extreme value distributions specified by the extreme value theory.

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- **Stefano Boccaletti**, CNR- Institute for Complex Systems, Florence, Italy: ***The transition to synchronization of networked systems***

Synchronization of networked units is the collective behavior characterizing the normal functioning of most natural and man made systems. As an order parameter (typically the coupling strength in each link of the network) increases, a transition occurs between a fully disordered and gaseous-like phase (where the units evolve in a totally incoherent manner) to an ordered or solid-like phase (in which, instead, all units follow the same trajectory in time). The transition between such two phases can be discontinuous and irreversible, or

smooth and reversible. The first case is known as Explosive Synchronization, which refers to an abrupt onset of synchronization following an infinitesimally small change in the order parameter, as in a thermodynamic first-order phase transition. The second case is the most commonly observed one, and corresponds instead to a second-order phase transition, resulting in intermediate states emerging in between the two phases. Namely, the path to synchrony is here characterized by a sequence of events where different functional modules (or clusters) emerge, each one evolving in unison. The structural properties of the graph are responsible for the way nodes cluterize. In particular, it was argued that the clusters formed during the transition are to be connected to the symmetry orbits and/or to the equitable partitions of the graph.
In my talk, I will provide a full elucidation of the transition to synchronization in a network of identical systems. Namely, I will introduce a (simple, effective, and limited in computational demand) method which, with the only help of the eigenvalues and eigenvectors of the Laplacian matrix of a network, is able to: i) predict the entire sequence of events that are taking place during the transition; ii) identify exactly which graph's node is belonging to each of the emergent clusters, and iii) provide a rigorous calculation of the critical coupling strength value at which each of such clusters is observed to synchronize. I will also demonstrate that such a sequence is in fact universal, in that it is independent of the specific dynamical system operating in each network's node and depends, instead, only on the graph's structure. Moreover, I will clarify that the emerging clusters are those groups of nodes which are indistinguishable at the eyes of any other network's vertex. This means that all nodes in a cluster have the same connections (and the same weights) with nodes not belonging to the cluster, and therefore they receive the same dynamical input from the rest of the network. As such, synchronizable clusters in a network are subsets more general than those defined by the graph's symmetry orbits, and at the same time more specific than those described by equitable partitions. Finally, I will show extensive numerical simulations with both synthetic and real-world networks, which demonstrate how high is the accuracy of the predictions, and also report on synchronization features in heterogeneous networks showing that the predicted cluster sequence is maintained even for networks made of slightly non identical dynamical units.
References:

- **Taksu Cheon**, Kochi University of Technology, Kochi, Japan: ***Phase transition in urban agglomeration and segregation***

A model of the urban agglomeration and segregation is formulated, in which two types of agents move around on the square-lattice aligned cells. The model is shown to exhibit, when the density of agents are varied as the control parameter, various phase transitions representing appearance of urban aggregation, segregation and social disorder.
References:

- **Sergey N. Dorogovtsev**, Physics Depertment, University of Aveiro & I3N, Aveiro, Portugal: ***Different kinds of localization in complex networks***

We discuss localization effects observed in various matrix representations of complex networks and in processes taking place on networks [1]. The localization is indicated by a non-vanishing inverse participation ratio in large networks. The first basic example of localization is demonstrated by the quenched mean-field theory for the epidemic SIS model on a complex network with a hub or a cluster with high connectivity. This problem is reduced to the localization of the principal eigenvalue of the adjacency matrix of a network [2]. The explicit solution was found for a sparse random regular graphs and for Erdős-Rényi random graph with a hub. For a disease spreading, localization results in an island of infected vertices in a range of transmission rates below the endemic epidemic threshold.
Localization of the principal eigenvalues of the adjacency and Laplacian matrices of complex networks with a hub significantly hinders community detection. A non-backtracking matrix is widely used as a remedy against localization on isolated hubs. Nonetheless, the principal eigenvalues of this matrices still can be localized on clusters. We describe the explicit solution for localization of the principal eigenvector of the non-backtracking matrix on an arbitrary finite graph inserted into an arbitrary infinite tree-like network [3] derived in the framework of the non-backtracking expansion approach [4].
Importantly, the quenched mean-field approximation neglects the absorbing state in the SIS model and fluctuations, due to which a finite number of infected vertices all will finally

become susceptible due to fluctuations. Hence this solution with a finite number of infective vertices is meaningful only in the quasi- stationary, metastable state, and so localization on a hub in the SIS model is actually metastable [5]. This trouble disappears for localization on a large cluster still containing a vanishingly small fraction of an in infinite network. We outline the features and applications of the metastable localization.

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- **Aljaž Drašković-Bračun**, National Institute Of Chemistry, Ljubljana, & Faculty of Mathematics and Physics, Ljubljana, Slovenia: ***Acoustic metafluids based on random microstructure networks***

n metamaterials, the interaction between waves and matter leads to unusual wave-propagating properties of the medium, which can be used for various purposes [1]. In acoustic metamaterials, the parameters that determine sound propagation, i.e., the effective compressibility χ and density ρ of the wave-carrying medium, can become simultaneously negative in certain frequency windows due to resonance effects of sub- wavelength inclusions [1,2,3]. In our work [3] we focus on irregularly shaped inclusions - micro-oscillators – that are allowed to be randomly distributed throughout the host fluid and have irregular modal shapes. We show that the metafluid concept need not necessarily be based on position periodicity or correlation of the suspended micro-oscillators, and in this case not even on ideally designed micro-oscillators. We formulate the detailed operating principle of such a metafluid model, give explicit formulas for its effective dynamic moduli in terms of the modal structure of the micro-oscillators, and discuss basic practical issues of performance optimization in terms of their mass and size. In our model the micro-oscillators consist of point masses connected by harmonic potentials. Further we discuss how the amount and distribution of such connections affects the effectiveness of the micro-oscillators in modification of apparent acoustic parameters of the fluid. It turns out that the so called floppy-modes, which appear at very low frequencies due to under- constrained regions in the system [4], can have a substantial effect on acoustic parameters of the medium. This makes them acoustically accessible and additionally the absence of need for an intricately designed structure brings experimental realizations that much closer..

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- **Stanislaw Drozds & Marcin Watorek**, Cracow University of Technology and Institute of Nuclear Physics Polish Academy of Sciences, Krakow, Poland : ***Decomposition of cross-correlation networks by means of the concept of q-MST***

The dynamics of complex systems is commonly accessible through the multivariate time series. They are then used to determine the correlation matrices. By introducing an appropriate distance matrix, such matrices - for transparency - are usually converted into networks which are typically reduced to the minimum spanning tree (MST) representation. Of course, the traditional correlation coefficients, by construction, considerably compress

and reduce the amount of information contained in the original series. The related compression may result, at the first place, from the fact that such coefficients involve averaging correlations over the whole span of fluctuations and thus do not filter out some possible variability of the intensity of correlations at different values of their amplitudes. Within such an approach, in the case of strong correlations, the resulting MST may give rise to some false signals, such as promoting a peripheral node to play the role of a central hub.
Here the generalization of the concept of cross-correlation coefficient to the q-dependent detrended cross- correlation coefficient ρ(q,s) [1] is presented such that when varying the q-parameter, it acts selectively to cross-correlations between different fluctuation amplitudes at different time scales s of multivariate data. Following such a generalization, the family of q-dependent minimum spanning trees (q-MSTs [2]) is introduced, which allows to disentangle the composition and organization of correlations graphically and thus to study their varying network characteristics.
The utility of such a procedure in addressing the above indicated issues is illustrated on a recently vital subject of the world cryptocurrency market [3] and of the underlying cross-correlations [4]. During the periods of the relatively stable increases, accompanied by rather moderate cross-correlations, the overall structure of q-MSTs does not change significantly with varying q-values. This signals that the cross-correlations are rather uniformly distributed over the range of fluctuations. Also, the anticipated central hub - the Bitcoin - remains such at different values of q. On the other hand, during the periods of violent decreases and strong cross-correlations, the structure of q-MSTs sizeably varies, and for instance, in May 2021, it is the DASH which at q=2 is seen to constitute a node of comparable multiplicity to the Bitcoin. Even more, at q=4, the latter is seen as a peripheral node, and an overall structure of the corresponding MST is much more dispersed. This indicates that here the strength of cross-correlations is more diversified at the large amplitude of fluctuations and that q-MST offers a promising tool for a systematic study of such effects in many different areas.
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- **Deniz Eroglu**,Department of Bioinformatics and Genetics, Kadir Has University, 34083 Istanbul, Turkey: ***Reconstruction of Complex Networks Dynamics from Data: Emergent Higher-Order Interactions and Critical Phenomena***

Real-world complex systems such as ecological communities and neuron networks are essential to our everyday lives. These systems are composed of units that interact through intricate networks. The ability to predict sudden changes in network behavior (critical transitions) from data is essential to avert disastrous consequences of significant disruptions. Predicting such changes is a major challenge as it requires forecasting the behavior for parameter ranges for which no data on the system is available. We address this issue for networks with weak individual interactions and chaotic local dynamics by introducing a unified reconstruction scheme by blending dynamical systems theory and machine learning tools. Although our approach works perfectly under the given assumptions, the model reconstruction scheme can also surprisingly lead to recovering emergent hypernetworks with triplet and higher interactions among oscillators for slightly different settings. This appears paradoxical at first glance because, initially, such models are defined as oscillator networks with pairwise interactions. In this work, we uncover a general mechanism for emerging hypernetworks when recovering models of nonlinearly coupled oscillators from data. We present a full description of such emergent hypernetworks using normal form theory and the local tree structure of the original network. Our findings shed light on the apparent abundance of hypernetworks and provide a constructive way to predict their emergence. Using the approach, we can create a proxy of a complex system and thereby make predictions about the critical transitions in the system.
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- **Neelima Gupte & Rupali Sonone**, Department of Physics, IITM, Chennai, India: ***Climate network analysis of extreme events: Tropical Cyclones***

We construct climate networks based on surface air temperature data to identify distinct signatures of tropical cyclones in the region of the Indian Ocean, which have serious economic and ecological consequences. The climate network shows a discontinuous phase transition in the size of the normalised largest cluster and the susceptibility during cyclonic events. We analyze these quantities for a year (2016) which had three successive cyclones, viz Cyclone Kyant, cyclone Nada and cyclone Vardah, and compare these with years where a single cyclone, cyclone Ockhi (2017) and was seen. The microtransitions and jumps in susceptibility in these two cases show distinct patterns and scaling behaviour. The signatures of the cyclones can be seen in other quantities like the degree distributions and other network characterizers. The nodes of high degree show rough correlation with the cyclone paths, and also have potential as predictors. We discuss the implications of these results for further analysis.
References:

- **Jaroslav Hlinka(1,2), Anna Pidnebesna(1,2), Luigi Caputi1(1,3)** , 1Institute of Computer Science of the Czech Academy of Sciences, Prague, Czech Republic, 2 National Institute of Mental Health, Klecany, Czech Republic, 3University of Torino, Torino, Italy: ***Hidden geometry of brain dynamics revealed by persistent homology***

Characterizing the pattern of dependences between the dynamics of nodes of complex systems (such as interacting brain regions) by the tools of topological data analysis is an area of rapidly growing interest. It has been recently shown, that topological data analysis is sensitive to brain-disease-related alterations in both the structure of the functional connectivity (instantaneous statistical dependences) and effective connectivity (directed causal interactions), albeit the detectability of these alterations depends of their topological/topographical specificity [1]..
In this contribution we shall discuss the practical aspects of utilization of topological data analysis to characterize the hidden geometry of brain dynamics, as well as provide methodological approaches to gain qualitative insights concerning this hidden geometry. In particular, we present an investigation of the underlying curvature of data through the lens of topology [2]..
Building upon previous work [Giusti et al., 2015], we employ the tools of Persistent Homology, namely topological features derived from Betti curves. We first investigate the case of random and geometric matrices (distance matrices of points randomly uniformly distributed on manifolds of constant sectional curvature). We consider the three classical models given by the Euclidean space, the sphere, and the hyperbolic space. We show that Betti curves effectively distinguish these spaces. Thus we can use manifolds of constant curvature as comparison models to infer properties of the underlying curvature of manifold underlying real data.
We analyse brain dynamics data (while comparing with financial and climate data examples) and observe that their associated topological features appear to emerge from hyperbolic underlying geometry. This result is consistent with the general belief that their underlying data manifold is of non-positive curvature, however we also discuss alternative explanations related to the data sampling and processing steps, as well as more complex possible hidden geometries.
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- **Alkan Kabakcioglu**, Koc University, Istanbul, Turkey: ***Criticality of gene regulation networks and an exact bound on stability of Boolean systems***

We prove that nested canalizing functions are the minimum-sensitivity Boolean functions for any activity ratio and we determine the functional form of this boundary which has a nontrivial fractal structure. We further observe that the majority of the gene regulatory functions found in known biological networks (submitted to the Cell Collective database) lie on the line of minimum sensitivity which paradoxically remains largely in the unstable regime. Our results provide a quantitative basis for the argument that an evolutionary preference for nested canalizing functions in gene regulation (e.g., for higher robustness) and for plasticity of gene activity are sufficient for concentration of such systems near the "edge of chaos.".

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- **Reimer Kuehn**, King's College London, London, United Kingdom: ***Heternogeneous micro-structure of percolation in complex networks: theory and applications***

We review a message passing (or cavity) approach [1,2] that is able to uncover and analyze a considerable degree of microscopic heterogeneity in the percolation problem on complex networks. Indeed, the probability for individual nodes of a complex networks to remain part of the giant connected component (GCC) or percolating cluster, when bonds or nodes are randomly and independently removed with some given probability varies considerably across a network. Average percolation probabilities which measure the fraction of nodes that belong to a GCC are just the first moments of distributions of percolation probabilities. We evaluate these distributions, both on single large graph instances, and for configuration models in the thermodynamic limit. The underlying message passing approach can also be used to locate articulation points - vertices whose removal would break the cluster on which they sit into two or more smaller components. We discuss applications in the context of SIR/SEIR epidemics and in approaches to efficient network dismantling strategies [3,4].

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- **X. San Liang**, Fudan University, Shanghai, China: ***Measuring the importance of individual units to the structure integrity of a complex network***

A network may fail to function due to the depression or deterioration of certain individual nodes (e.g., the 2003 great blackout that darkened much of North America). How to identify the deteriorated/depressed unit(s) is thence of great importance. This study provides an easy and faithful way to fulfill the task, by generalizing information flow, a real physical notion which has just been rigorously formulated with causality naturally embedded (see Liang, 2016, and references therein), to cumulative cases. For a network, given the time series for the units, it is shown that a natural measure is the information flowing/transferring from the unit of concern to the rest units. This flow or transfer can be rigorously derived in the setting of a continuous- time dynamical system, either deterministic or stochastic. Under a linear assumption, a maximum likelihood estimator can be obtained, allowing for an estimation of it in an easy way. As expected, this "cumulative information flow" does not equal to the sum of the information flows to other individual units, reflecting the collective phenomenon that a group is not the addition of the individual members. For the purpose of demonstration and validation, we have examined a network made of Stuart-Landau oscillators. Depending on the topology, the computed information flow may differ. In some situations, the most crucial nodes for the network are not the hubs (i.e., those with highest degrees), in contrast to the traditional point of view; they may have low degrees, and, if depressed or attacked, will cause the failure of the entire network. This study provides an easy yet effective approach to measuring the importance of individual units in producing the collective behavior of a complex network. It can allow us to understand the potential damage to the structure integrity due to the failure of local nodes, and hence help diagnose neural network problems, control epidemic diseases, trace city traffic bottlenecks, identify the potential cause of power grid failure, build robust computer networks, and so forth.

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- **Marija Mitrovic Dankulov**, Scientific Computing Lab, Institute of Physics, Belgrade, Serbia: ***Complex networks analysis of time-series data: finding patterns in socio-economic systems***

The development of information-communication technologies enabled access to large-scale data about the structure and dynamics of different socio-economic systems. The data on system elements and interactions between them are not always explicitly available but are hidden in dynamical system outputs represented by time series. The time series contain information about the dynamics of each element and its coupling with the rest of the system. Mapping time series to graphs and topological analysis of these graphs are non-trivial problems. We demonstrate our approach by analyzing the time-series data from two social systems, SARS- CoV-2 epidemics infections and death rates [1] and Meta Data for good mobility data, and USA financial sector companies [2] to uncover how different forms of crisis in these systems change their structure. We combine approaches from complex network theory, computer science, and statistical physics to study the evolutions of these systems and uncover patterns and predominant drivers of the dynamics of each system. We use correlation-network mapping to map the data onto graphs and study the spectral properties of these graphs. Eigenvector localization reveals the mesoscopic organization of these graphs and the change in the structure due to systems evolution through the crisis. K-means clustering combined with multifractal time series analysis reveals a finer mesoscopic structure of these systems.
Our analysis of two phases of SARS-CoV-2 epidemics, the outbreak and immunization phase, show the existence of robust communities of different countries and regions that further break into clusters according to similar profiles of infection fluctuations. The structure of communities and clusters in the outbreak and immunization phase differ drastically, indicating a change in epidemic dynamics due to the start of immunization. Multifractal analysis of time series reveals that persistent fluctuations around the local trend occur in intervals smaller than 14 days. Analysis of the network of relations between USA companies operating in the financial sector further confirms that the system changes during crisis periods. We find that connectivity between communities is strongly influenced by the crisis and economic measures taken by the country. Furthermore, the occurrence of crisis is also seen in the patterns of eigenvector localization. Application of described methodology on mobility data in different countries during the SARS-CoV-2 epidemic shows that a combination of epidemic measures, culture, and the infectious curve influences both mobility patterns. .....**(with Roderick Melnik and Bosiljka Tadic)**

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- **Jan Moelter & Christian Kuehn**, Department of Mathematics, School of Computation, Information and Technology, Technical University of Munich, Munich, Germany: ***Preserving Bifurcations through Moment Closures***

Moment systems arise in a wide range of contexts and applications as high- or even infinite-dimensional systems of coupled equations. Hence, an indispensable step to obtain a low-dimensional representation that is amenable to further analysis is in many cases to apply a moment closure. The latter is a set of approximations that express certain higher-order moments in terms of lower-order ones, so that applying those breaks the hierarchical structure of the equations and leads to a closed system of equations for only the lower-

order moments. Closures are frequently found drawing on intuition and heuristics in trying to come up with quantitatively good approximations. Apart from that, from a dynamical systems' point of view, a key consideration when deriving closures has to be whether dynamical features such as bifurcations are preserved. Hence, we propose a change of perspective where we focus on closures giving rise to certain qualitative features such as bifurcations. Importantly, this provides us with the possibility to classify moment closures rigorously and makes the design and selection of the same more algorithmic, precise, and reliable. In this talk, we will revisit two paradigmatic network dynamical systems, the SIS epidemic and the adaptive voter model, and derive conditions that a moment closure has to satisfy so that the corresponding closed systems exhibit the transcritical bifurcation that one expects in these systems coming from the stochastic particle model. Finally, we examine existing moment closures for both systems in the light of these results and show that they indeed satisfy all the conditions.
References:

- **Alexandre V. Morozov**, Center for Quantitative Biology, Dept. of Physics & Astronomy, Rutgers University, Piscataway, USA: ***From networks to spin glasses: Machine learning and statistical inference in discrete systems through the lens of random walks***

Complex, large-scale networks represent a broad spectrum of systems in nature, science, technology, and human societies [1]. Computer networks such as the World Wide Web and the Internet, social networks such as Twitter and Facebook, and online knowledge-sharing platforms such as Wikipedia exert considerable influence on our everyday lives. Many of these networks are very large and may evolve with time, making predictions of their properties a challenging task. I will describe a novel methodology, based on random walks, for the inference of various properties of complex networks with weighted or unweighted symmetric edges [2]. I will show that this formalism yields reliable estimates of global network properties, such as the network size, after only a small fraction of nodes has been explored. I will also introduce a novel algorithm for partitioning network nodes into non-overlapping communities - a key step in revealing network modularity and hierarchical organization [3]. The problem of network community detection is similar to the well-known problem of clustering datapoints in machine learning. I will apply this algorithm to various benchmarks, including a state-of-the-art collection of synthetically generated networks with tunable community structure and a large-scale map of roads and intersections in the state of Colorado. Finally, I will demonstrate how these ideas can be used to estimate key thermodynamic quantities such as free energies in physical systems with discrete states, solely on the basis of small-size non-equilibrium samples. The main ingredient of the free energy reconstruction is so-called coincidence counting - the numbers of times the discrete states of the system are visited by random walks. In summary, random walks reveal modular organization and global structure of complex networks and at the same time provide a computationally efficient approach to inferring key statistical mechanics quantities in physical systems with discrete states.
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- **Geza Odor**,Centre For Energy Research, Complex Systems Department, Budapest, Hungary: ***Higher-order interactions generate mixed order phase transition and Griffiths phases on heterogeneous complex networks***

In d>2 dimensional, homogeneous threshold models discontinuous phase transition emerge, but the mean-field solution provides 1/t power-law activity decay and other power laws, and thus it is called mixed-order or hybrid type. This is in contrast with simple unary reaction spreading models, where continuous transition occur [1]. Quasi-static network heterogeneity can cause dynamical criticality below the transition point if the dimension is d<4 [1]. We provide numerical evidence that even in case of high graph dimensional hierarchical modular networks a Griffiths phase in the K=2 threshold model is present below the hybrid phase transition. This is due to the fragmentation of the activity propagation by modules, which are connected via single links. This delivers a widespread mechanism in

the case of the threshold type of heterogeneous systems, modeling the brain, socio or epidemic spreading for the occurrence of dynamical criticality in extended Griffiths phase parameter spaces [3]. References: [1] G. Odor, Universality classes in nonequilibrium lattice systems Rev. Mod. Phys. 76 (2004) 663. Phase transition classes in triplet and quadruplet reaction-diffusion models, Phys. Rev. E 67 (2003) 056114.
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- **Stefano Scialla, Marco Patriarca, Marius Yamakou, Els Heinsalu, Julyan H.E. Cartwright**, National Institute of Chemical Physics and Biophysics, Tallinn, Estonia: ***Are “hubs” in beta-cell clusters an emergent network property or do they exist independently?***

The cell network structure of the pancreatic islets of Langerhans has been the subject of numerous studies. A long-standing dilemma is whether the collective oscillations of beta-cells require the presence of specialized pacemaker cells, named “hubs”, or synchronization occurs through a “democratic” mechanism, where the collective network behavior is a nonlinear average of the properties of its elements. The topic has received so much attention to justify a review focused on the “hub” dilemma [1]. In a recent work [2] we mimicked the architecture of a beta-cell network by a cubic lattice of heterogeneous FitzHugh-Nagumo elements. This topology resembles the experimentally known features of a beta-cell islet. We introduced heterogeneity in the network through a diversified set of external currents Ji, drawn from a Gaussian distribution with standard deviation sigma, which we varied between sigma=0 and sigma=2. Our simulations showed a clear “Diversity-induced resonance”, with a maximum at sigma=0.5, corresponding to a 5% fraction of hubs (the units with Ji corresponding to an intrinsic oscillatory state), in good agreement with experiments [3]. While the above results support the existence of hubs, they do not allow us to decide whether these hubs are an emergent network property or they exist independently of the network. Trying to dig deeper into this, here we present the results of new simulations where we selectively disconnected either hubs or nonhubs from the network. We found rather surprising results, summarized in Fig. 1. We disconnected from the network 1/3 of the hubs, by setting their coupling constant C=0 in the coupled FHN equations. This means the corresponding FHN units had no interaction with other network units. As shown by the red bars in Fig. 1, this caused a dramatic drop of the collective oscillations vs. the reference network configuration, where no elements were disconnected, shown by the green bars. On the other hand, upon disconnecting the same number of nonhubs, we found virtually no change in oscillatory activity, as shown by the yellow bars. This suggests that hubs do play a crucial role as a distinct subset of network elements. However, if we build a truncated distribution of Ji values, where the central range corresponding to oscillatory FHN states is missing, therefore the network is formed by nonhubs only (without any disconnected units), then the global oscillatory activity is also maintained vs. the reference system (blue bars). Therefore, hubs seem to be crucial for global network oscillations if they are initially present and get disconnected, whereas their complete absence does not prevent the network from being in a resonant oscillatory state. In our contribution we will present additional data and hypotheses to explain this apparent contradiction. Our learnings help shed light on the “hub” dilemma but, at the same time, raise new questions that require more work to be fully understood.
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- **M. Angeles Serrano**, ICREA & University of Barcelona, Barcelona, Spain: ***Detecting the ultra low dimensionality of real networks***

Reducing dimension redundancy to find simplifying patterns in high-dimensional datasets and complex networks has become a major endeavor in many scientific fields. However, detecting the dimensionality of their latent space is challenging but necessary to generate efficient embeddings to be used in a multitude of downstream tasks. We have introduced models and methods to infer the dimensionality of real networks based on the ability of

hyperbolic geometry to capture their complex connectivity. Our analysis has revealed ultra low dimensionality as an ubiquitous feature, and unexpected regularities across different domains, including extremely low dimensionality in tissue-specific biomolecular networks, close-to-three-dimensional brain connectomes, and slightly higher dimensionality in social networks and the Internet.
References:

- **Alexander Shapoval**, Institute of Mathematics and Scientific Computing, University of Lodz, Poland: ***Bak-Tang-Wiesenfeld Sandpile as the Mechanism that Generates the 1/x Power-Law and the 1/f Spectrum***

Bak, Tang, and Wiesenfeld introduced the phenomenon of self-organized criticality with a sandpile model and positioned their model as the key to understanding the flicker noise. However, the BTW sandpile and its modifications are primarily known for the power-laws describing state variables, where a few power-law exponents characterize a great many isotropic model modifications defined on the square lattice. The BTW sandpile itself generates the power-law size-frequency relationship with the exponent equaled to 1.20, and the models with the exponent located closer to 1 are not known. The spectrum of basic quantities in sandpiles exhibits a constant at low frequencies, the 1/x^2 decay at high frequencies, and, for some models, the 1/x part in-between. This spectrum structure does not elucidate specific features underlying many processes including, for example, the superposition of pulses. Therefore, researchers created additional constructions to produce the “pure” 1/f spectrum with sandpiles. In this talk, I’ll show that the departure from the original BTW sandpile was premature, and the reciprocal function in both size-frequency distribution and spectrum is attained with the BTW mechanism. The 1/x size-frequency relationship is obtained with clustering of events in space and time in the BTW sandpile. The precise definition of the clustering, affecting the cluster volume, gives the tool to control the exponent of the size-frequency relationship. This clustering allows to fill in the range of exponents from 1.20 to 1 and, possibly, to values being smaller than 1. In the case of spectrum, I’ll focus on the dynamics of the average system stress in contrast to the sequence of event sizes, which is typically analyzed. The spectrum of the average system stress indeed exhibits a constant at low frequencies that turns to 1/x at moderate frequencies and finally to the 1/x^2 decay at high frequencies. However in the thermodynamic limit, the role of the parts precisely matches the expectations of researchers. The 1/f part extends over all time scales that represent the dynamics at the critical level of stress. The 1/f spectrum transits to a constant at the time scales that correspond to extremely rare drops of the system to the subcritical state. Finally, the 1/x^2 part is insignificant because it covers the time scales up to the system size, whereas only the scales related to the system area “survive” in the thermodynamic limit. I also note that this insignificant spectrum part is associated with the power-law segment of the size-frequency relationship, whereas the 1/x spectrum is associated with the tail of the event distribution. References: arxiv:2212.14726

- **Cinzia Soresina & Christian Kuehn**, Karl-Franzens-University of Gratz, Austria & TUM Munich, Germany: ***Cross-diffusion-induced instability on networks***

The concept of Turing instability, namely that diffusion can destabilize the uniform steady state, is well known either in the context of partial differential equations (PDEs) or in a network of dynamical systems. Recently, reaction-diffusion equations with cross-diffusion terms have been investigated, showing an analogous effect known as cross-diffusion-induced instability [1]. In [2], we extend this concept to a network of dynamical systems, showing that the spectrum of the graph Laplacian plays the role of the Laplace operator in reaction- diffusion equations and determines the instability appearance. In particular, we consider a network model for competing species, coming from the PDEs context. The influence of the topological structure on the cross- diffusion induced instability is highlighted, considering different topologies, both regular rings or lattices, but also small-world and Erdős-Rényi networks.
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- **Francesco Sorrentino**, University Of New Mexico, Albuquerque, United States: ***A master stability function for cluster synchronization in networks with adaptation***

Networks of neurons display many features that are sometimes hard to incorporate in dynamical models and numerical simulations. One of these is the presence of adaptation or synaptic plasticity, i.e., a mechanism by which the strength of a connection increases or decreases based on the activity of the neurons at its endpoints. Another one is the emergence of cluster synchronization, which is an intermediate state between complete synchronization (state of highest order) and lack of synchronization (state of lowest order.) We consider a general dynamical model that incorporates adaptation and allows for the emergence of cluster synchronization and develop a dimensionality reduction approach to study the stability of the emerging cluster synchronous states. We show how the method can be used as an effective replacement for time and memory intensive numerical simulations.
References:

- **Bosiljka Tadic**, Department of Theoretical Physics, Jozef Stefan Institute, Ljubljana, Slovenia: ***Emergence of modulated cycles in critical dynamics***

In complex systems, the existence of critical fluctuations, either near a nonequilibrium phase transition or a self-organized criticality attractor in dynamical systems driven out of equilibrium, indicate the states where the emergence of new features at a larger scale can occur. These stationary states are characterized by long- range spatiotemporal correlations and collective avalanching dynamics in response to endogenous or external forces. A closer inspection of state variables reveals a tendency towards cyclical trends in their temporal fluctuations, which can be related to the gradual accumulation and release of energy through a cascade of avalanches of different sizes, for example, in sandpile automata. Here, we investigate such cycles in two types of critical states, particularly near a jamming transition in traffic on complex networks [1] and in self-organized critical states observed in the empirical data of emotional messages in online social networks [2]. Even though their origins differ, our results show that the emergent cycles in both cases are irregular and modulated by the collective critical dynamics; they attain higher harmonics that can be appropriately described by multifractal analysis. The observed variations in the corresponding singularity spectra correlate with the nature of collective dynamics. .
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